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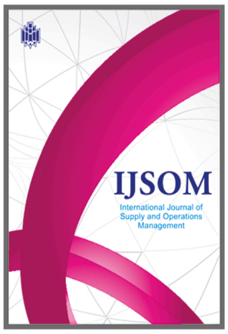

Joint Optimization of Star p-hub Median Problem and Seat Inventory Control Decisions Considering a Hybrid Routing Transportation System

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Abstract

This paper studies the problem of integrated capacitated hub location problem and seat inventory control considering concept and techniques of revenue management. We consider an airline company maximizes its revenue by utilizing the best network topology and providing proper booking limits for all itineraries and fare classes. The transportation system arises in the form of a star/star network and includes both hub-stop and non-stop flights. This problem is formulated as a two-stage stochastic integer program with mixed-integer recourse. We solve various instances carried out from the Turkish network data set. Due to the NP-hardness of the problem, we propose a hybrid optimization method, consisting of an evolutionary algorithm based on genetic algorithm and exact solution. The quality of the solutions found by the proposed meta-heuristic is compared with the original version of GA and the mathematical programming model. The results obtained by the proposed model imply that integrating hub location and seat inventory control problem would help to increase the total revenue of airline companies. Also, in the case of serving non-stop flights, the model can provide more profit by employing less number of hubs.

Keywords: perishable products; p-hub median; seat allocation; evolutionary algorithms; fare class segmentation; network revenue management.

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1. Introduction

After the deregulations in the airline industry, the revenue management techniques have become inevitable for airline seat inventory control. Revenue management is the process of selling the limited, perishable capacity to the right customers at the right prices so as to optimize the total revenue. Classic examples of RM can be found in the airline and hotel industry where there are finite number of seats and hotel rooms, respectively (Mou and Wang 2014). The main problem in airline revenue management is to determine booking control strategies. Airlines seldom charge the same fare for each seat on a flight, but instead price seats based on customer's willingness-to-pay. In fact, they segment the passengers into various subsets of customers with different behaviors and needs and then allocate their capacity to different fare classes in order to maximize the expected profit. The optimization methods and algorithms on seat allocation problems with multiple-fare classes can be divided into the single-leg optimization method and the network optimization method. The classical revenue management allocates the single flight leg capacity to different fare classes. However, most airlines have chosen network structures which causes the network version problem of revenue management to receive much interest as the flight legs are now shared by multiple origin-destination itineraries. Two network structures that have received particular attention in studies of the airline industry are hub-and-spoke networks and point-to-point networks. The hub-and-spoke network problem deals with locating hub facilities and allocating demand nodes to hubs to direct the flow between origin–destination pairs with less number of required links. In contrast, in a point to-point network, all cities are connected with each other through non-stop flights. In fact, route architecture is the foundation of an airline's product. Point-to-point (non-stop) and hub-and-spoke architectures lie at the poles of a continuum with most airlines operating some combination of the two. All passengers in a point-to-point system board at flight origin and deplane at the destination. In the hub and spoke system, by contrast, all passengers except those whose origin or destination is the hub, transfer at the hub for a second flight to their destination. Each architecture has some advantages. Most of the time, using hub and spoke networks helps to benefit the economies of scale and integration and results in lower cost carriers as compared to those of direct flight (Yaman and Elloumi 2012).

The counterpoint to the complexity of the hub-and-spoke transportation system is the simplicity of direct path between two non-hub nodes (non-stop flight). Non-stop flights reduce total travel time, primarily by eliminating the intermediate stops, but also by avoiding circuitous routings and increasing aircraft block speeds. Passengers value the reduction in travel time. Airlines can generate more revenue by charging a higher fare for direct flights. (Cook and Goodwin, 2008). Also, some airport hubs cannot consolidate traffic bound for many itineraries. Having this limitation and knowing the fact that some passengers prefer non-stop flights, consideration of both hub-stop and non-stop routing strategies can be more cost-efficient than a pure hub-and-spoke network (Jeng 1987). In other words, non-stop flights are always the most desirable in terms of convenience but on the other hand, less desirable in terms of price for price sensitive customers. Moreover, the stops at the hub airports increase the expenses for the airline companies due to the facility charges and landing fees. Therefore, the airlines can generate more revenue by considering these key parameters and applying best network routing strategies.

This paper investigates a hybrid routing system consisting of two types of routes: non-stop flight and hub and spoke flight. The capacity, ticket fare and demand of each route type are different. The booking limits and sales for each route are determined based on these factors to maximize the revenue of transportation networks.

Generally, in the airline market the demands of the nodes that can be referred to flows between origin/ destination pair nodes in the network have some uncertainties (Adibi and Razmi 2015). Since hub location problem and network design correspond to long-term strategic decisions, it seems more realistic to consider uncertainty into the modeling of the problem and as a result the obtained answer is more credible and practical (Alumur et al. 2012). For this reason this reason, in the current study, we assume that demand has a discrete distribution which is represented by a finite number of possible scenarios.

In this study, we present a two-stage stochastic integer programming model which maximizes expected revenue incurred by transfer activities. The first stage aims to determine hub locations and designs a transportation network to transport the passengers to their destination at the lowest operational cost while the second stage provides the decisions on sold tickets and optimal fare class booking limits for a fixed amount of capacity which are influenced by uncertain demands. We consider a hub location problem that formed in design of two-level star networks. The model selects *p* hubs and one central hub, then the hubs connect to the central hub with direct links and each non hub node is assigned to a hub. Beside the designed hub- and-spoke network, we consider non-stop link between each origin and destination except that origin or destination is the hub. Due to the complexity of the problem, we provide a hybrid optimization method based on an evolutionary genetic algorithm and exact solution improved by caching technique to solve the problem.

The rest of the paper is organized as follows. Section 2 discusses the relevant literature. In Section 3, we define the problem in more details and the mathematical formulation is presented. For solving the problem, we provide a meta-heuristic based on genetic algorithm by linking MATLAB software to GAMS in section 4. Computational results are presented in Section 5. And finally, the conclusions and some guidelines for further research are presented in Section 6.

2. Literature Review

The optimization methods on the seat inventory control problem firstly appeared in the research of Littlewood (1972). He extended a seat inventory control problem for two classes of customers on a single leg flight and presented a marginal seat revenue rule applied into proposed model. Belobaba (1987, 1989) considered this idea and presented the expected marginal seat revenue heuristic for multi fare class booking. His method was able to provide reasonable approximations of optimal booking limits and could easily be implemented in airline RM systems. Further, Curry (1990), Wollmer (1992), Brumelle and McGill (1993) and Robinson (1995) investigated the single-leg problem considering multiple customer classes. They developed various algorithms in order to determine the optimal solutions for booking control under various assumptions about the probability distribution of demand for each fare class. Lee and Hersh (1993) presented a discrete-time dynamic programming model to achieve an optimal booking control policy. Their proposed model does not require any assumptions about the arrival pattern for the different booking class. Feng and Xiao (2001) described a stochastic control model and developed optimal control rules. They considered the Poisson process for the demands of origins and fixed seat capacity for the hub-destinations.

With the appearance of hub-and-spoke networks in the airline industry in 1980s, researchers applied origin-destination control mechanisms to account for network effects. For the network optimization, Glover et al. (1982) initially presented a mathematical programming formulation in order to minimize the cost network flow with deterministic demand. They did not consider any stochastic element in their model. Wang (1983) provided a solution method for the sequential allocation of seats by consideration of stochastic demand. After that, Wollmer (1986) proposed a linear programming model that considered the uncertainty in demand. In the following, Boer et al. (2002) presented stochastic linear programming for network revenue management. Bertsimas and De Boer (2005) and van Ryzin and Vulcano (2008) utilized some methods based on simulation optimization that investigated seat inventory over the network. Yoon et al. (2012) focused on the seat allocation problem with stochastic demands. They presented a probabilistic non-linear programming model to determine booking limits by fare classes in airlines. For relevant research on seat allocation problems in airlines, we refer the interested reader to study McGill and van Ryzin (1999), Chiang et al. (2006) and Talluri and van Ryzin (2004).

The performance of a transportation system, substantially depends on designing the optimal networks for routing the traffic. Utilization of hubs in the networks can reduce the transportation costs by using fewer arcs and helps to benefit the economies of scale. After the seminal papers by O'kelly (1987), hub location problem has been known as a one of the most appealing fields in facility location which attracted many researchers in recent years. After that, many of other researchers (for example, Campbell, 1994, 1996; Ernst Krishnamoorthy 1996, and Mayer and Wagner 2002) worked on different models of hub location. The p-hub median problem, and the capacitated and uncapacitated hub location problems are the most frequently studied have been addressed in this area, See (Labbé and Yaman ,2004, Labbé et al.,2005, Contreras et al.,2009, Yaman ,2011, and Lin et al. ,2012).

In the field of network design problems, Yaman (2008) studied transportation system with hub and spokes that arises in the design of a star–star network. Labbé and Yaman (2008) studied the problem of designing a star/star network with minimum routing cost. Yaman and Elloumi (2012) introduced two related star p-hub location problems, include the star p-hub center problem and the star p-hub median problem considering bounded path lengths. It is worthwhile mentioning that in the aforementioned works, researchers considered that a fixed central hub is given and *p* additional hubs should be chosen among the user nodes, but in the current study we provide the star p-hub location with no fixed central hub.

Considering uncertain data in many decisions making problems plays an important rule, especially, at the process its implementation may take considerable time. Transportation network design problems are no exception. Nonetheless, only a few papers exist in the literature addressing various sources of uncertainty in the hub location problems. Marianov and Serra (2003) studied a hub location problem in the airline transportation. They focused on uncertainty at the hub nodes by representing hub airports as M/D/c queues. Sim et al. (2009) introduced the stochastic p-hub center problem considering chance constraints. The only source of uncertainty in the model is travel time. Contreras et al. (2011) presented stochastic uncapacitated hub location problems in which uncertainty in demand and transportation costs involved in the model. Alumur et al. (2012) considered two sources of uncertainty include the set-up costs for the hubs and the demands to be transported between the nodes. Yang (2009) provided a stochastic formulation to address the air freight hub location and flight routes planning with seasonal demand variations. In this model the demand is transported through the non-stop and hub-stop path. Adibi and Razmi (2015) developed a two stage stochastic programming for hub location problems under uncertainty in the set-up costs and in the demands.

Previous studies in hub location problem only focused on this problem in terms of minimizing the total transportation cost under various sources of uncertainty. In the current study, we investigate the problem from the viewpoint of increasing the revenue. To the best of the authors' knowledge, there is no other paper considering the joint hub location and revenue management problem. In this integrated model, different classes of customers with stochastic demands are considered, in which a part of the demands can be satisfied with determining the booking limit based on maximizing the network's revenue. The second contribution of the presented model is to provide a star/star network which does not require any assumption about the location of the central hub. The location of the central hob is determined by the model optimization. And finally, the last novelty is to consider both non-stop and hub-stop flights in a star/star transportation network under uncertainty. In the subsequent section, the structure of the problem is explained in more details.

3. Model formulation

3.1. Problem definition

In this section, we describe the problem of integrated hub location which arises in the form of a star/star network and seat inventory control based on principal of revenue management. Two important key issues in airline operations and related planning activities are designing an efficient network structure and managing the limited capacity of the transportation network. In practice, the airline network design problem aims to:

- Determine the location of hubs and decisions about the configuration of the service network.
- Design network structure which can be a combination of non-stop and hub-and-spoke connections.
- Specify the routes that transfer the traffic between all origin–destination pairs.
- Allocate the flows on each route by considering the capacity of the links.

In the air transportation market usually we face various customers with different behaviors and needs. Consequently, airlines often segment their passengers to different fare classes and then distribute the available capacity (airplane seats) to their customers. The airplane seats can be recognized as a perishable asset, because when the aircraft departs, all empty seats remaining become worthless. Furthermore, demands in air industry have obvious seasonal variations during the planning process and it is impossible to have precise forecasting of future demand. Therefore, deterministic models are not capable of responding to these variations (Yang 2010). In order to consider demand uncertainties, we applied a scenario-based approach and provided a model with a two-stage stochastic formulation. The aim of the model is to find the optimal locations for central hub and non-central hubs, the assignments of non-hub nodes to the p located hubs, control of seat inventory within the context of network revenue management in both non-stop and hub-stop flights. Our hub and spoke network configuration is an extended case of star p-hub median problem with modular arc capacities (SM) which proposed by Yaman (2008). This problem has various applications in telecommunications and transportation. Unlike mentioned work, the model presented in this study does not require any assumption about location of a central hub and it is defined by model optimization.

In Figure 1, a star–star network is depicted. In this structure, there is a designated central hub and all the traffic flow that is consolidated at hub nodes goes through this special hub. Also, all hub nodes are connected to the central hub by direct links.

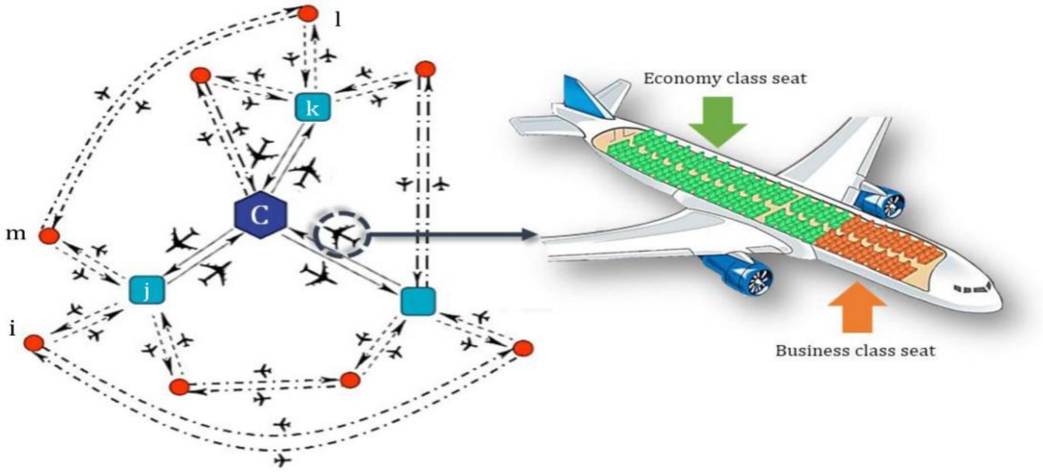


Figure1. Example of seat allocation and star p-hub median problem

Consider that we have to transfer passengers between two nodes i and m, if both nodes are assigned to the same hub like j , the passenger from node i to node m first goes to hub j and then from hub j goes to node m. In a situation that node i and node l are connected to different hubs the passengers traffic from node i to node l first goes from node i to its hub like j, and then from hub j to the central hub and then to hub k and lastly the traffic goes from hub k to node l. We also consider the availability of non-stop transportation between nodes which are not connected directly into the designed hub- and-spoke network. In fact, the pairwise traffic demand can be transferred by non-stop flights. For example, in Figure 1 it is possible for the passengers to travel from node m to node l directly. After that the limited capacity on arcs is shared between different itineraries and customer classes. In addition, due to the uncertain demands between nodes the sales decision is made after the arrival of customer orders as well as when more definitive information becomes available.

3.2. An integrated star p-hub median and seat allocation problem under uncertainty

Consider a flight network consisting the set of origin-destination itineraries. All nodes are candidates to be a hub or central hub and the number of hubs is predefined. Each ordinary node is assigned to only one hub and direct transportation between them is allowed. We consider three types of flight leg in the network. The first link type transfers the traffic flow between non hub node and a hub node. The second types are those which transfer the traffic between the hubs and central hub and the third type is used to transfer the traffic between nodes with non-stop flight. We applied the discount factor α for the transportation costs between a non-central hub and central hub by considering $C2 = \alpha\, C1$ so as to approximate the economies of scale for the transportation costs between hubs. The decisions about network design, location of hubs and protection levels are made at the beginning of horizon and customer classes arrive with a stochastic demand. To deal with the stochastic nature of demand in the model, we assume demand has a discrete distribution with finite number of possible realizations (scenarios) with probabilities pr_{imk}^{s}, $s = 1 \dots S$, of the booking demand.

The notation and variables for mathematical formulation are as follows:

Sets and Parameters

N	The number of nodes (spokes) in the hub-spoke network
P	The number of hubs
S	The number of scenarios
K	The number of customer classes
i, m	Indices for nodes $i, m = 1 \dots N$

j	Indices for non-central hubs
j''	Index of central hub
s	Indices for scenarios
k	Indices for customer classes
f_{im}	Distance from node i to node m
C_{1k}	Transfer cost (per unit flight and unit distance) between origin and hub which is defined as leg type 1 for customer class k
C_{2k}	Transfer cost (per unit flight and unit distance) between a hub node and central hub node which is defined as leg type 2 for customer class k
$C_k^{non.stop}$	Transfer cost (per unit flight and unit distance) between nodes in leg type 3 for customer class k which is defined as non-stop flight
$d_{imks}^{non.stop}$	Traffic demand for non-stop flight between origin i and destination m for customer class k under scenario s
$d_{imks}^{hub.stop}$	Traffic demand for hub-stop flight between origin i and destination m for customer class k under scenario s
$pr_{imks}^{non.stop}$	The probability of traffic demand for non-stop flight between origin i and destination m for customer class k under scenario s
$pr_{imks}^{hub.stop}$	The probability of traffic demand for hub-stop flight between origin i and destination m for customer class k under scenario s
No_{ij}	Number of hub-stop flights available for itinerary between node i and hub j
$No'_{jj''}$	Number of hub-stop flights available for itinerary between hub j and the central hub j''
$No_{im}^{non.stop}$	Number of non-stop flights available for itinerary between origin i and destination m
Q_1	Number of seats available in a service cabin at leg type 1
Q_2	Number of seats available in a service cabin at leg type 2
$Q_{non.stop}$	Number of seats available in a service cabin of non-stop flight
F_j	Fixed cost for establishing non-central hub j
$F'_{j''}$	Fixed cost for establishing central hub j"
$price_{imk}^{non.stop}$	Ticket price for non-stop flight of itinerary between origin i and destination m for customer class k
$price_{imk}^{hub.stop}$	Ticket price for hub-stop flight of itinerary between origin i and destination m for customer class k

Decision variables

$y_{imks}^{non.stop}$	Number of tickets sold for non-stop itinerary between origin i and destination m for customer class k under scenario s
$y_{imks}^{hub.stop}$	Number of tickets sold for hub-stop itinerary between origin i and destination m for customer class k under scenario s
$z_{imk}^{non.stop}$	Protection level for non-stop itinerary between origin i and destination m for customer class k
$z_{imk}^{hub.stop}$	Protection level for hub-stop itinerary between origin i and destination m for customer class k
$n_{j''}$	A binary decision variable, which is 1 if node j'' is selected as central hub and 0 otherwise

w_{ij}	A binary decision variable, which is 1 if node i is connected to hub j and 0 otherwise
$x_{ijj''j'm}$	A binary decision variable, which is 1 if traffic flow between node i and node m must goes through hub j, central hub j'' and hub j' ,respectively otherwise it takes 0

3.3. Non-linear formulation of the problem

The probabilistic non-linear model of capacitated star p-hub median and seat allocation problem with different customer classes is provided in the following (The parameter M is sufficiently large):

$$Max \sum_i \sum_m \sum_k \sum_s pr^{hub.stop}_{imks}\, y^{hub.stop}_{imks}\, price^{hub.stop}_{imk} + \sum_i \sum_m \sum_k \sum_s pr^{non.stop}_{imks}\, y^{non.stop}_{imks}\, price^{non.stop}_{imk} - \sum_i \sum_{m \neq i} \sum_k \sum_s C^{non.stop}_k \frac{y^{non.stop}_{imks}}{Q_{non.stop}} f_{im} - \sum_i \sum_j \sum_k C_{1k}\Big[f_{ij}\Big(\sum_{m\in I/\{i\}} \sum_s pr^s_{imk} \frac{y^{hub.stop}_{imks}}{Q_1}\Big) + f_{ji}\Big(\sum_{m\in I/\{i\}} \sum_s pr^s_{mik} \frac{y^{hub.stop}_{miks}}{Q_1}\Big)\Big]w_{ij} - \sum_k \sum_j \sum_{j'\in I/\{j\}} \sum_i \sum_{m\in I/\{i\}} C_{2k}\left[\left(f_{jj''}\left(pr^{hub.stop}_{imks}\frac{y^{hub.stop}_{imks}}{Q_2}\right) + f_{j''j}\left(pr^{hub.stop}_{miks}\frac{y^{hub.stop}_{miks}}{Q_2}\right)\right)x_{ijj''j'm}\right] - \sum_{j\in I/\{j''\}} F_j w_{jj} - \sum_{j''} F'_{j''} n_{j''} \quad (1)$$

$$\sum_j \sum_{j'} \sum_{j''} x_{ijj''j'm} = 1 \qquad \forall i,m \quad (2)$$

$$x_{ijj''j'm} \leq w_{ij} \qquad \forall i,j,j',j'',m \quad (3)$$

$$x_{ijj''j'm} \leq w_{mj'} \qquad \forall i,j,j',j'',m \quad (4)$$

$$w_{ij} \leq w_{jj} \qquad \forall i,j \quad (5)$$

$$\sum_j w_{ij} = 1 \qquad \forall i \quad (6)$$

$$\sum_j w_{jj} = P \quad (7)$$

$$\sum_{j''} n_{j''} = 1 \quad (8)$$

$$n_j \leq w_{jj} \qquad \forall j \quad (9)$$

$$\sum_i \sum_{j'} \sum_m x_{ijj''j'm} \leq M n_{j''} \qquad \forall j,j'' \quad (10)$$

$$\sum_i \sum_j \sum_m x_{ijj''j'm} \leq M n_{j''} \qquad \forall j',j'' \quad (11)$$

$$y^{non.stop}_{imks} \leq d^{non.stop}_{imks}\,(1 - w_{im} - w_{mi} - x_{iiimm} - x_{iimmm}) \qquad \forall i,m,k,s, i \neq m \quad (12)$$

$$y^{hub.stop}_{imks} \leq d^{hub.stop}_{imks} + d^{non.stop}_{imks}\,(w_{im} + w_{mi} + x_{iiimm} + x_{iimmm}) \qquad \forall i,m,k,s, i \neq m \quad (13)$$

$$y^{hub.stop}_{imks} \leq Z^{hub.stop}_{imk} \qquad \forall i,m,k,s \quad (14)$$

$$y^{non.stop}_{imks} \leq Z^{non.stop}_{imk} \qquad \forall i,m,k,s \quad (15)$$

$$\sum_{m\in I/\{i\}} \sum_k \frac{z^{hub.stop}_{imk}}{Q_1} + \sum_{m\in I/\{i\}} \sum_k \frac{z^{hub.stop}_{imk}}{Q_1} \leq \sum_j NO_{ij}\, w_{ij} + M\, w_{ii} \qquad \forall i \quad (16)$$

$$\sum_i \sum_{m\in I/\{i\}} \sum_k \frac{z^{hub.stop}_{imk}}{Q_2} x_{ijj''j'm} + \frac{z^{hub.stop}_{imk}}{Q_2} x_{ijj''j'm} \leq NO'_{jj''} \qquad \forall j,j'',j'' \neq j \quad (17)$$

$$\sum_{m\in I/\{i\}} \sum_k \frac{z^{non.stop}_{imk}}{Q_{non.stop}} + \sum_{m\in I/\{i\}} \sum_k \frac{z^{non.stop}_{mik}}{Q_{non.stop}} \leq \sum_{m\neq i} NO^{non.stop}_{im}\,(1 - w_{im} - w_{mi}) \quad \forall i \quad (18)$$

$$\sum_{m\in I/\{i\}} \sum_k \frac{z^{non.stop}_{imk}}{Q_{non.stop}} + \sum_{m\in I/\{i\}} \sum_k \frac{z^{non.stop}_{mik}}{Q_{non.stop}} \leq \sum_{m\neq i} NO^{non.stop}_{im}\,(1 - (x_{iiimm} + x_{iimmm})) \quad \forall i \quad (19)$$

$$w_{ij} \in \{0,1\} \qquad \forall i,j \quad (20)$$

$$n_{j''} \in \{0,1\} \qquad \forall j'' \quad (21)$$

$$x_{ijj''j'm} \in \{0,1\} \qquad \forall i,j,j',m \quad (22)$$

$$y^{non.stop}_{imks}, Z^{non.stop}_{imk}, y^{hub.stop}_{imks}, Z^{hub.stop}_{imk} \in Z_+ \ \forall i,m,k,s \quad (23)$$

The objective function (1) maximizes the total profit of the transportation network considering the revenue of selling tickets to different classes and itineraries for both non-stop and hub-stop flights, total transportation cost between all non-hub nodes and the hub nodes, total transportation cost from all the hubs to the central hub and total fixed cost of installing hubs at nodes. Equation (2) forces the traffic to go through hubs. Equations (3) and (4) force the traffic through hubs j' and j and central hub to be zero unless there are nodes such i and m that are allocated to j' and j respectively. Equation (5) ensures that node i cannot be assigned to j unless there is a hub at node j. Equation (6) ensures that every node is assigned to exactly one hub. Equation (7) states that there must be exactly P hubs contain central hub. Equation (8) ensures that one hub is selected as central hub. Equation (9) ensures that central hub should be chosen among the set of hubs. Equations (10) and (11) force the traffic from hubs j and j' through central hub to be zero unless hub j'' is assigned to be central hub. Equation (12) insures that the number of sold tickets for each non-stop flight should be less than the direct demand if the regarded origin and destination are not connected directly by hub-and-spoke network design. Otherwise the hub-stop flight and non-stop flight demands added together, which is modeled by equation (13). Equations (14) and (15) guarantee that the number of sold tickets for each origin destination itineraries should be less than the protection level for both non-stop and hub-stop flights. Equation (16) and (17) establish the capacity constraints of link type one and two respectively. Equation (18) and (19) establish the capacity constraints of link type three. Equations (20-23) define the types of decision variables.

Since proposed model is non-linear we add some variables and a set of linear constraints to obtain the linear integer programming. To linearize the proposed non-linear model, following equations are added to the model:

$$V^s_{ijmk} \geq (1 - w_{ij})(-M) + y^{hub.stop}_{imks} \quad \forall i, m, j, k, s \tag{24}$$

$$V^s_{mjik} \geq (1 - w_{ij})(-M) + y^{hub.stop}_{miks} \quad \forall i, m, j, k, s \tag{25}$$

$$O^s_{ijj''j'mk} \geq (1 - x_{ijj''j'm})(-M) + y^{hub.stop}_{imks} \quad \forall i, m, j, j'', j', k, s \tag{26}$$

$$O^s_{mjj''j'ik} \geq (1 - x_{mjj''j'i})(-M) + y^{hub.stop}_{miks} \quad \forall i, m, j, j'', j', k, s \tag{27}$$

$$G_{ijj''j'mk} \geq (1 - x_{ijj''j'm})(-M) + Z^{hub.stop}_{imk} \quad \forall i, m, j, j'', j', k \tag{28}$$

$$G_{mjj''j'ik} \geq (1 - x_{ijj''j'm})(-M) + Z^{hub.stop}_{mik} \quad \forall i, m, j, j'', j', k \tag{29}$$

$$O^s_{ijj''j'mk}, V^s_{ijmk}, G_{ijj''j'mk} \geq 0 \quad \forall i, m, j, j'', j', k, s \tag{30}$$

Using constraints (24-25), the variable $y^{hub.stop}_{imks}$ becomes the lower bound for V^s_{ijmk}, if $w_{ij} = 1$, and using constraints (26-29), the variable y^s_{imk} and Z_{imk} becomes the lower bound for $O^s_{ijj'mk}$ and $G_{ijj''j'mk}$ respectively, if $x_{ijj'm} = 1$. As we minimize the value of $O^s_{ijj''j'mk}$ and V^s_{ijmk} in the objective function and according to the utility of the lower value of $G_{ijj''j'mk}$ in the capacity constraint, they will attain the lower bound. In line with additional constraints (24-30), the objective function and constraint (17) are also changed to equations (31) and (32), respectively.

$$
\begin{aligned}
&Max\ \sum_i \sum_m \sum_k \sum_s pr_{imks}^{hub.stop}\, y_{imks}^{hub.stop}\, price_{imk}^{hub.stop} + \sum_i \sum_m \sum_k \sum_s pr_{imks}^{non.stop}\, y_{imks}^{non.stop}\, price_{imk}^{non.stop} - \\
&\sum_i \sum_{m \neq i} \sum_k \sum_s C_k^{non.stop} \frac{y_{imks}^{non.stop}}{Q_{non.stop}} f_{im} - \\
&\sum_i \sum_j \sum_k C_{1k} [\, f_{ij} (\sum_{m \in I/\{i\}} \sum_s pr_{imks}^{hub.stop} \frac{V_{ijmk}^s}{Q_1}) + f_{ji} (\sum_{m \in I/\{i\}} \sum_s pr_{miks}^{hub.stop} \frac{V_{mjik}^s}{Q_1})] - \\
&\sum_k \sum_j \sum_{j' \in I/\{j\}} \sum_i \sum_{m \in I/\{i\}} C_{2k} \left[(f_{j j'} \left(pr_{imks}^{hub.stop} \frac{O_{mjj^{"}j'ik}^s}{Q_2} \right) + f_{j j'} \left(pr_{miks}^{hub.stop} \frac{O_{mjj^{"}j'ik}^s}{Q_2} \right)) \right] - \sum_{j \in I/\{j^{"}\}} F_j w_{jj} - \\
&\sum_{j^{"}} F'_{j^{"}} n_{j^{"}}
\end{aligned}
\tag{31}
$$

$$
\sum_i \sum_{m \in I/\{i\}} \sum_k \sum_{j'} \frac{G_{ijj^{"}j'mk}}{Q_2} + \frac{G_{mjj^{"}j'ik}}{Q_2} \leq NO_{jj^{"}} \qquad \forall j, j^{"}, j^{"} \neq j
\tag{32}
$$

Since the p-median hub location problem is an NP-hard (Kara 2000) so proposed model which is an extended version of the p-hub location problem is also NP-hard. Thus in the next section a hybrid meta-heuristic algorithm combined the genetic algorithm and the exact solution approach is provided to find near optimal solutions with reasonable computational time.

4. Genetic Algorithm

A genetic algorithm (GA) is a metaheuristic search algorithm which has emerged as an effective and robust optimization for finding the near-optimal solutions in large spaces. GA rooted in the mechanisms of evolution and natural genetics. The initial idea was proposed by Holland (1975). It starts with a population of individuals that are generated randomly. Each population is represented by chromosomes, which are encoded solutions of the problem. Evaluating the fitness of all individuals in a population is calculated according to the objective function. Then individuals of a given population go through evolution process consist of selection, cross over and mutation. Cross over and mutation are used to generate population. The cycle of reproduction and population replacement is repeated until a well-defined stopping criterion is met.

Caching GA is a modified version of the genetic algorithm with the purpose of avoiding unnecessary calculation of objective values for repetitive individuals during GA operation. In the caching process, the objective function value of each individual is stored in a cache table which is called hash-row table. The main advantage of this method is that if we meet the same genetic code during the operation of the genetic algorithm, we can use the stored information in cache-table instead of repeating the calculations (Kratica et al. 2007 and Kratica et al. 2011).

Proposed algorithm for solving large-scale problem is composed of two basic parts: the first part is based on the evolutionary Genetic algorithm. The indices of established hubs are obtained from the individual's genetic code and after decoding process the feasible solution for hub network is input into the second part which produce an exact solution by CPLEX solver for protection levels and sold tickets. Since the hubs are established, the values of $x_{ijj^{"}j'm}$ and w_{ij} in the model are fixed and known. Therefore, by fixing these values, we have reduced the initial large mixed linear integer programming problem (MILP) to a smaller sub problem This sub problem is easily solved by CPLEX. This algorithm is coded by linking MATLAB software to GAMS.

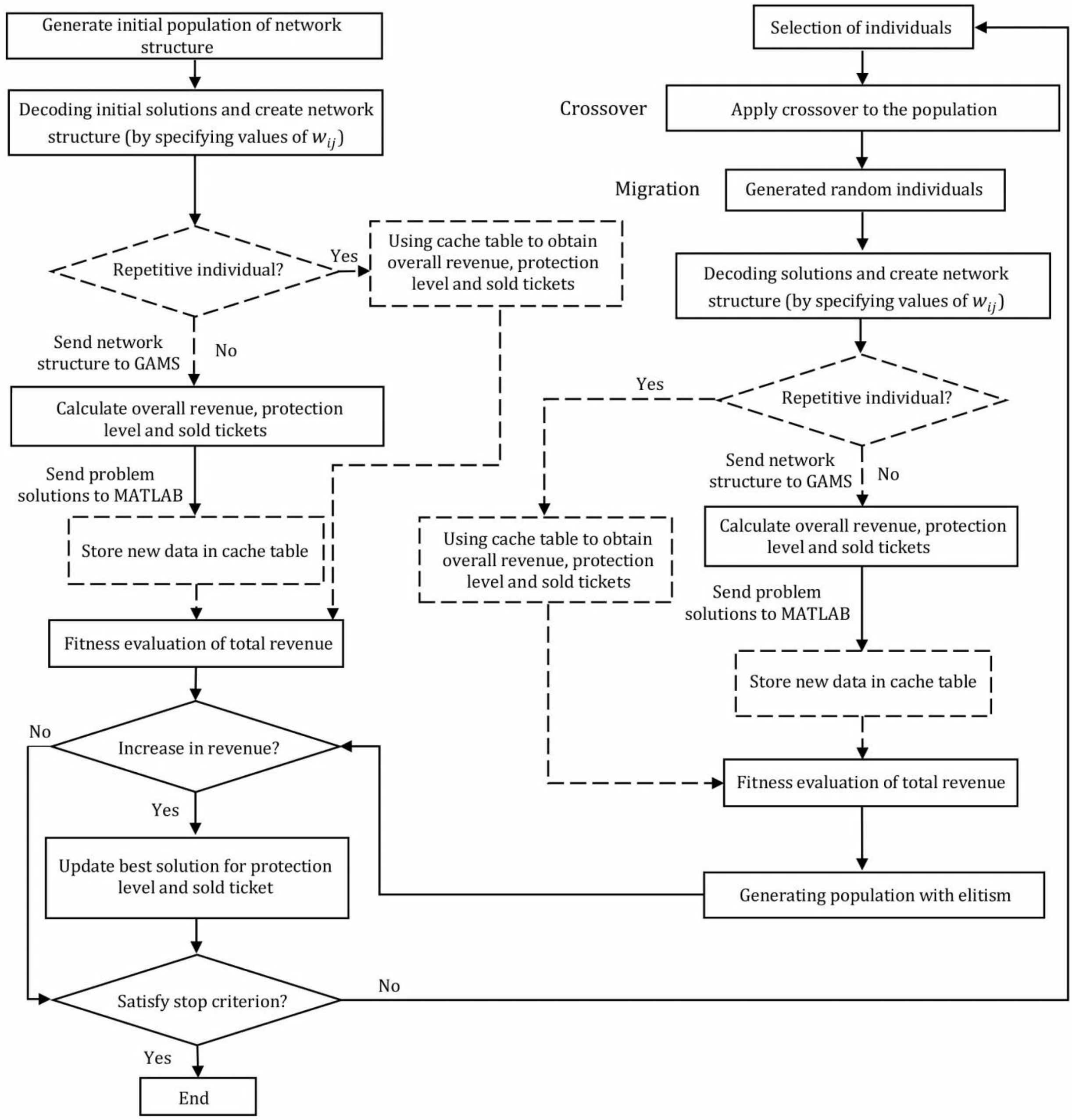


Figure 2. Flowchart of process of proposed hybrid evolutionary algorithm based on genetic algorithm

It is noteworthy that after obtaining the network configuration from the individual's genetic code, the initial mixed non-linear integer programming problem (NMILP) is changed to a linear mixed integer programming (MILP) sub problem that could be solved by CPLEX without any complexity of the linearization. The flowchart process of the algorithm is provided in Figure 2. We consider roulette wheel selection as a fitness proportional selection. The selection probability is represented in section 5.1 using Taguchi design method.

4.1. Chromosome representation

A chromosome in a hub location problem presents the network configuration by determining the location of hubs and allocation of nodes to hubs. In the present context, the direct use of allocation matrix as chromosomes is too complicated to implement because it is difficult to develop corresponding crossover and mutation operations. So we designed a decoding process to obtain the network structure of the given chromosome. Figure 3 illustrates the proposed decoding process.

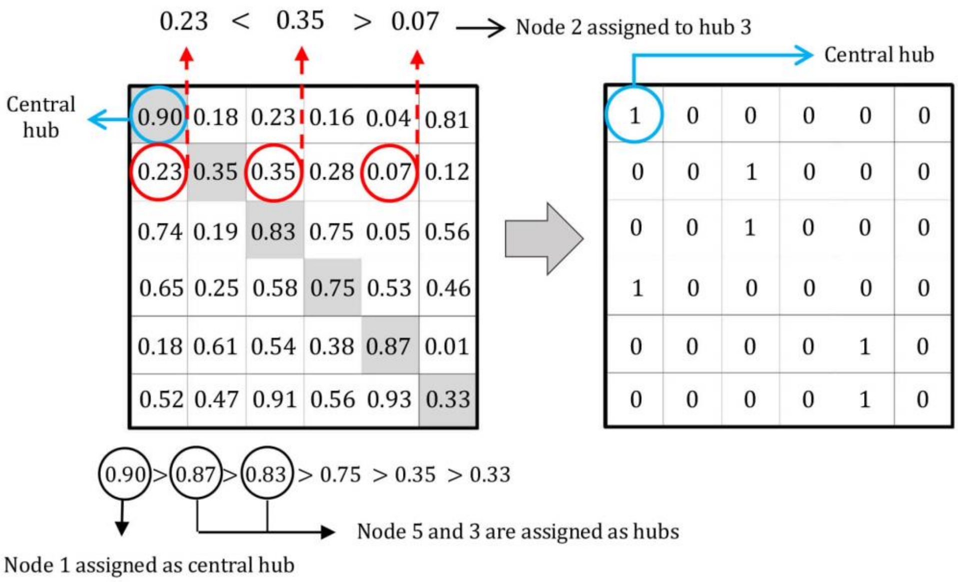


Figure 3. Decoding process

In this example, there are six nodes that two of them should be determined as hubs and one of them as central hub. First the numbers in diagonal of matrix are sorted, the largest number denotes the central hub and the next large numbers are assigned as a hub until the number of hubs are completed. For example, node 1 is determined to be central hub and node 3 and 5 are chosen as hubs. In the following, the non-hubs nodes are assigned to hubs by comparing the values at the intersection of non-hub node's column and the rows which are assigned as hubs, and the highest number determines the hub number which the mentioned node should be assigned to. Since the diagonal of the matrix is sorted in descending order and the nodes are assigned to the hubs respectively, until the number of hubs is complete, the proposed approach avoids infeasible solutions and guarantees that exactly p distinct hub indices including one central hub are obtained from the genetic chromosome.

The values which are equal to one of the main diagonal will be considered as a hub and on the other elements with value 1 show the allocated demand nodes. A sample solution related to Figure 3 is depicted in Figure 4.

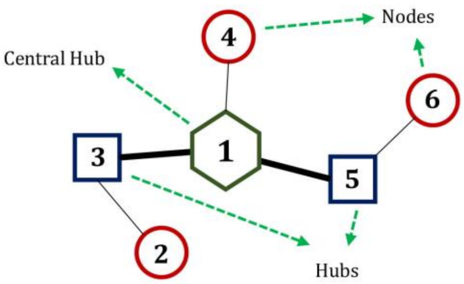


Figure 4. Sample network solution

4.2. Crossover operator

In crossover operator, two individuals are randomly chosen to act as parents so as to create one or more offspring. There are different methods to combine variable values of given parents. In the current study we applied parameterized uniform crossover (Mendes et al. 2009). We choose the first parent amongst the best individuals in the population while the other one is chosen from the whole of population, randomly. Then, a real random number in the interval [0,1] for each row is produced. If the random number is larger than a predetermined threshold value, called crossover probability (CProb), then the allele of the first parent is applied. Otherwise, the allele of the second parent is applied to generate the offspring. An example process of crossover is provided in Figure 5.

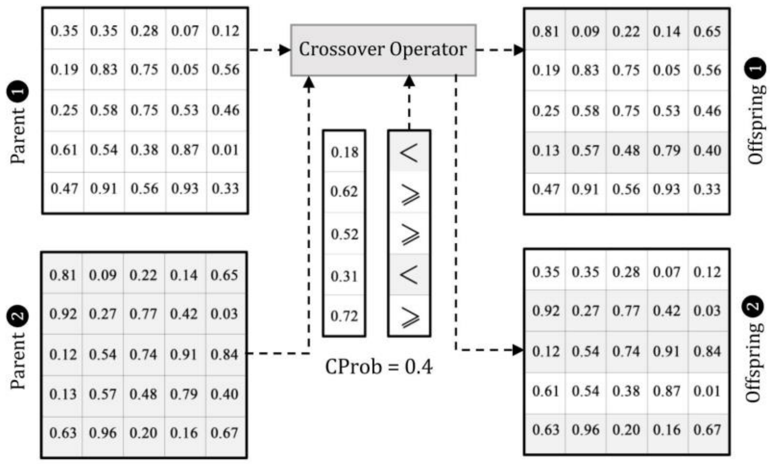


Figure 5. Example of parameterized uniform crossover with crossover probability equal to 0.4.

In this example the offspring 1 inherits the gene of parent 1 with probability 0.6 and inherits the gene of parent 2 with probability 0.4.

4.3. Migration operator

In this study, instead of using mutation operator we applied immigration operator. Same as mutation operator, immigration operator helps to prevent premature convergence of the population. This idea inspired form many real-world societies in which there is a set of individuals named immigrants which enter to the existing population permanently. These new immigrants are randomly created from the same distribution as the original population and thereupon, no genetic material of the current population is brought in (Zade et al. 2014).

Altogether, in the proposed algorithm, number of offsprings and immigrants added to the main population, and then after sorting, better individuals enter to the next generation. Figure 6 demonstrates the transitional process between two consecutive generations.

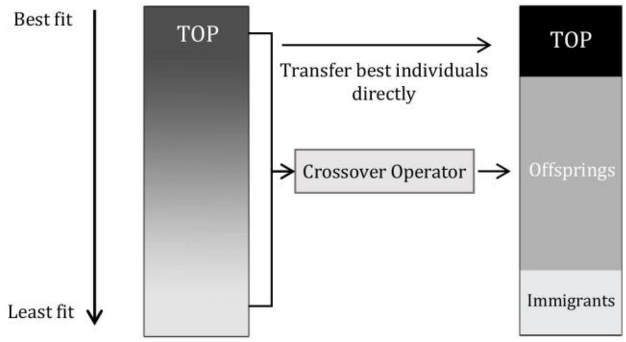


Figure 6. Transitional process between consecutive generations.

5. Computational Results

In this section, we reported the results of computational experiments by implementing the model on five subsets of Turkish network data set presented by Tan and Kara (2007). These subsets are selected from the first 5, 6, 10, 15 and 20 elements of the aforementioned dataset and other required parameters are generated according to the Table 1. We also evaluated the performance of our modified GA in comparison with the exact solution from GAMS SOLVER 24.1.3 and pure GA. The meta-heuristic algorithms are coded and implemented in the MATLAB program and linked to the GAMS with ILOG CPLEX 12.5 64-Bit optimization routines. All programs are run on a PC with Intel core i5-3337U (1.8 GHz) with 6 GB of RAM.

Table1. Generated input parameters

Number of flights between hubs and central hub	$DU(3,6)$
Number of flights between nodes and hubs	$DU(1,3)$
Number of available non-stop flights between nodes	$DU(1,3)$
The setup costs for a hub	$U(1200000,2400000)$
The setup cost for a central hub	$20\times$ fixed cost for corresponding central hub
The unit transportation costs (C)	$U\ [7,14]$
The unit transportation costs of non-stop flights	$U\ [9,16]$
Demand scenarios (high/nominal /low)	125%/100%/75%
Demand classes (economic / business)	80%/20%
Scenario Probabilities	0.19/0.59/0.22
Ticket price for each itinerary for economic class	$U(600,3600)$
Ticket price for each itinerary for business class	$1.5\times$ ticket price for corresponding economic class
Ticket price for each itinerary for economic class and non-stop flight	$1.4\times$ ticket price for corresponding economic class
Ticket price for each itinerary for business class and non-stop flight	ticket price for corresponding business class $1.7\times$
Capacity of aircrafts in leg type 1	150
Capacity of aircrafts in leg type 2	200
Capacity of aircrafts for non-stop flights (leg type 3)	100

In Table 1, U[a,b] and DU(a,b) denote a continuous uniform distribution and discrete uniform distribution function between a and b, respectively. Parameter C represents the transfer cost per unit flight and unit distance between origin and hub for business class seats ($C11 = C$). And for economic seats $C12 = \theta C$ where $\theta \sim U\ [0.5,1]$. Since the demand in the airline market usually varies periodically, we considered that demand variation has a discrete distribution with three

numbers of possible scenarios. We assume deviations from the nominal scenario as 125%, and 75%.
To archieve better and more robust solutions, in the next section, the Taguchi method is applied in order to set the parameters of meta-heuristic algorithms.

5.1. Parameter tuning

In this paper, we utilized the Taguchi experimental design method so as to calibrate the parameters of meta-heuristic algorithms. This method has been introduced by Taguchi in the early 1960s, and it is applicable in the designing of processes. The orthogonal arrays of the method are employed for the evaluation of a large number of factors with a few experiments.

In the current problem, the L9 design of the Taguchi method is performed for the algorithms by using the Minitab 16.2 software. The Taguchi method strives to minimize variances of quality characteristics obtained from S/N ratio (Taguchi et al. 2000). Quality characteristic of this paper is considered as a relative percentage deviation (RPD), which is employed to change objective function values to non-scale. Accordingly, we prefer "the smaller-the better" type. RPD is determined as follows:

$$RPD = \frac{|Obj_i - Obj_{best}|}{|Obj_{best}|} \times 100 \qquad (33)$$

Where Obj_{best} and Obj_i are the best obtained objective value for a particular instance and the objective value obtained for the ith trial, respectively.

Also the signal-to-noise for “the smaller-the better” characteristic is calculated as follows:

$$\text{S/N} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) \qquad (34)$$

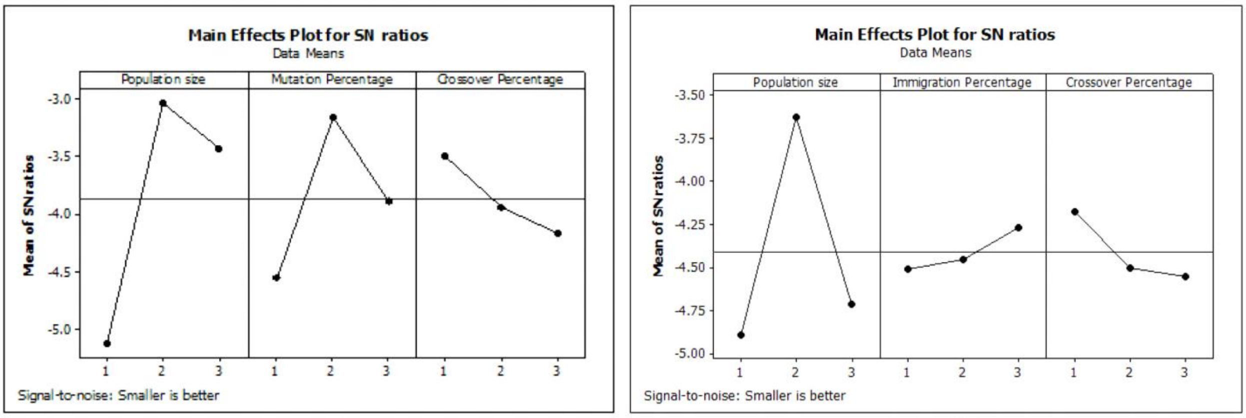
Where y_i represents the response value in the ith replication and n denotes the number of replications in experiments replications.

We considered three factors that can have salient effects on the proposed evolutionary algorithms. The considered levels of the parameters are presented in Table 2.

Table 2. Considered Levels of parameters of genetic algorithms

Modified GA	Parameter	Level 1	Level 2	Level 3
	Crossover percentage	0.60	0.70	0.80
	Immigration percentage	0.10	0.15	0.20
	Population size	50	70	90
Pure GA	Crossover percentage	0.60	0.70	0.80
	Mutation percentage	0.05	0.10	0.15
	Population size	50	70	90

The mean S/N ratio is calculated for each level of control factors. Figure 7 depicts the level of control factors versus control factors. A larger value of the S/N ratio in the graphs is more desirable.



(a) (b)

Figure 7. Mean S/N ratios for the pure GA (a), and proposed GA (b)

Desired parameters for each of the algorithms are shown in Table 3.

Table 3. Tuned parameters for the proposed GA and pure GA

Parameter (GA)	Modified GA	Pure GA
Crossover percentage	0.60	0.60
Mutation percentage	-	0.10
Immigration percentage	0.20	-
Population size	70	70

5.2. Experimental results

In order to evaluate the efficiency of the proposed GA, we compared the quality of its optimal solutions obtained by the developed mathematical programming for small size instances, and with the solutions obtained by a pure GA algorithm for small and large size instances. The results of the comparison of various instances of the problem with discount factor $\alpha = 0.2, 0.4$ are given in Table 4. For each class of the problem, an instance is generated according to Table 1. We run proposed GA and pure GA ten times to solve it.

The average CPU-time and best founded solution of objective function value from all GA runs and the variance of ten obtained solutions by the algorithms are reported in Table 4. Moreover, the quality of obtained solutions is evaluated by Gap and standard deviation of average Gap which are calculated as below:

$$Gap_i = \frac{|Obj_{Meta-heuristic} - Obj_{solution.best}|}{|Obj_{solution.best}|} \times 100 \tag{36}$$

$$Std = \sqrt{\frac{1}{10}\sum_{i=1}^{10}(Gap_i - A_{Gap})^2} \tag{37}$$

In equation (36), $Obj_{solution.best}$ denotes the optimum solution of mathematical programming of the problem if it is existed and otherwise it is the best solution obtained from all GA runs. And $Obj_{Meta-heuristic}$ shows the best obtained solution in the i th execution. In equation (37), $A_{Gap} = \frac{1}{10}\sum_{i=1}^{10} Gap_i$ .We set the total running time to 1 hour as a criterion and both algorithms terminate if the best achieved solution is not changed for 10 iterations

.

Table 4. Computational comparison of the mathematical programming model and the meta-heuristic algorithms.

α=0.2													
n	p	GAMS		Pure GA				Modified GA					
		Obj value	CPU (sec)	Obj value	CPU (sec)	A_{Gap} %	Std %	Obj value	CPU (sec)	Gap %	A_{Gap} %	NC	NE
5	2	7863051.74	16.83	opt	174.73	0	0	opt	43.46	0	0	0.80	130
6	2	14518624.14	56.37	opt	295.73	0.08	0.11	opt	77.25	0	0	0.69	220
10	3	-	-	52087259.16	865.33	2.02	2.15	52087259.16	616.97	1.07	0.85	0.35	874
	5	-	-	51814872.63	982.70	2.70	2.44	51814872.63	587.22	0.96	1.14	0.45	917
	7	-	-	51377735.10	1067.7	2.31	2.74	51377735.10	644.22	1.22	1.03	0.47	920
15	3	-	-	101157554.12	3290.6	3.58	2.97	103314322.77	2863.3	2.31	2.17	0.22	1346
	5	-	-	102446898.40	3226.4	3.61	3.59	102446898.40	2923.8	2.14	2.21	0.23	1343
	7	-	-	101836709.52	3359.3	3.91	3.83	102197315.02	2937.1	2.91	2.33	0.17	1347
20	3	-	-	171847391.64	3405.1	4.48	5.08	172287289.85	3137.6	3.64	3.11	0.03	451
	5	-	-	171571516.40	3396.9	4.86	4.94	171571516.40	3034.9	3.42	3.54	0.03	422
	7	-	-	170187510.04	3417.3	5.09	5.20	170965499.81	3251.7	3.74	3.51	0.04	413
α=0.4													
5	2	7586792.23	15.71	opt	168.11	0	0	opt	39.22	0	0	0.82	121
6	2	14676701.36	56.37	opt	300.18	0.10	0.09	opt	80.06	0	0	0.66	234
10	3	-	-	52041994.17	859.40	2.25	2.06	52041994.17	627.16	1.13	0.76	0.33	886
	5	-	-	51224172.32	1000.6	2.64	2.31	51224172.32	579.91	1.50	1.52	0.45	920
	7	-	-	51316166.89	1013.6	2.55	2.14	51316166.89	632.17	1.66	2.01	0.50	909
15	3	-	-	102923318.96	3290.6	3.18	3.11	102923318.96	2890.2	2.72	2.29	0.20	1351
	5	-	-	102093416.41	3246.0	3.40	3.88	102112844.02	2897.3	2.69	2.18	0.24	1329
	7	-	-	102016827.16	3299.3	4.01	3.70	102016827.16	2901.7	3.11	2.83	0.19	1338
20	3	-	-	172261674.05	3461.5	4.81	4.89	172261674.05	3098.4	3.55	3.70	0.03	447
	5	-	-	171349507.39	3387.4	4.69	4.44	171527086.95	3062.7	3.19	3.43	0.04	416
	7	-	-	170904676.72	3452.1	5.15	5.29	170904676.72	3260.2	3.86	4.01	0.04	417
Total Average					2134.5	2.97	2.95		1826.6	2.03	1.93	0.31	761
Maximum					3461.5	5.15	5.29		3260.2	3.86	4.01	0.82	1351

The average number of fitness function evaluations is shown with NE while the average percentage of using cache table to obtain fitness function, is shown with NC in Table 4. In practice, this parameter could be considered as a measure of run-time saving during the executions.

The results of the comparison of various problem instances indicate the superiority of the proposed algorithm to pure GA in all instances according to the quality of the solutions and computational time requirements.

5. 3. Airline Revenue Improvement by Using Proposed Model

In order to evaluate the impact of using an integrated model for airline hub location and network revenue management, assume that the company sells tickets to the single class by sum of demands for two classes and mean prices and determine the network configuration. We solve the revenue management problem with this predefined network structure. The result solution obtained from this method in which we used predefined network configuration (Method 1 in Table 5) is compared with a profit achieved by the proposed model (Method 2 in Table 5). The obtained profit from these two methods for some different instances are as follows:

Table 5. Comparing the revenue of predefined network model and proposed integrated model ($\alpha = 0.2$).

Node	P	Method 1	Method 2	Improvement
10	3	50029086.97	52087259.16	2058172.19
10	5	49640762.54	51814872.63	2174110.09
15	3	93912347.94	103314322.77	9401974.82
15	5	92658949.19	102446898.40	9787949.21

As can be seen in Table 5, in four studied instances, proposed model helps to gain more profit by using the optimal transportation network. The results clearly show the benefits of using an integrated model to gain more profit.

5. 4. Impact of serving non-stop flights on the airline revenue

In order to investigate the impact of serving non-stop flights on the total revenue of the transportation system, we consider that no non-stop services are permitted to bypass the hubs. Indeed the airline loses a portion of the demands related to non-stop flights. In Figure 8 (a), we present the revenue of the hybrid routing transportation system considering both non-stop and hub-stop flights and the total revenue for the case which the airline does not consider any capacity to meet the demand of non-stop flights. As we can see from Figure 8 (a), serving non-stop flights helps to significantly increase the total revenue. It is noteworthy that in the case of serving non-stop flights, the model can provide more profit by employing less number of hubs. But in practice, if airlines do not offer any non-stop flights, the customers would tend to use hub-stop flights instead, and also a portion of the demands would be shifted towards other airlines.

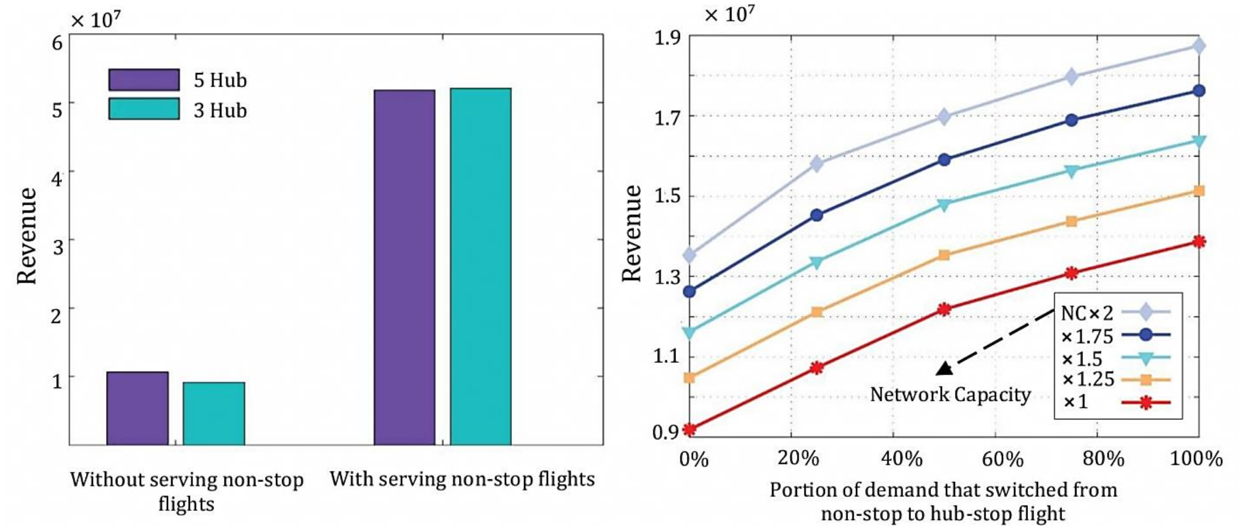


Figure 8. The objective function with and without serving non-stop flights for a case with 10 nodes (a), The objective function with respect to the different portion of demands that are switched and different network capacity for the case with 10 nodes and 3 hubs (b)

Depending on the portion of the demand that the airline loses, the total revenue will be different. Figure 8 (b) presents the total revenue achieved according to different portions of the demands that may be switched to hub-stop flights when the airline does not offer any non-stop flights. We also compare how profitability of the airline is affected by increasing the network capacity. It can be seen from Figure 8 (b) that increasing the network capacity in different routes can increase the profit.

5. 5. Value of Stochastic Programming

So as to measure the impact of uncertainty parameters on the planning decisions, we applied two stochastic well-known concepts: the expected value of perfect information (EVPI) and the value of stochastic solution (VSS) (Birge and Louveaus 2011).

Expected Value of Perfect Information which estimates the maximum amount a decision maker would be ready to pay in return for complete information in the future. Consider Q^* is the objective value of the stochastic programming and ξ is the random variable which realizations correspond to the different scenarios, and $\bar{Q}(\xi)$ is the optimal value of deterministic problem correspond to each scenario. The wait-and-see value (WS), corresponds to the expected value of the objective value for each scenario$\mathrm{WS} = E_{\xi}(\bar{Q}(\xi))$. WS model would help to always make the best decision regardless of the uncertain parameters which is not beneficial in practice. Accordingly, the expected value of perfect information (EVPI) is calculated as blew:

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$$EVPI = WS - Q^* \tag{38}$$

The other concept is VSS implies that whether putting extra effort into modeling and solving stochastic programming is valuable. Let $\bar{Z}(\bar{\xi})$ be the optimal decision of the first stage in deterministic problem where all random variables are replaced by their expected values. The VSS is obtained as follows:

$$VSS = Q^* - EEV \qquad \text{(h39)}$$

Where expected result of using EV is calculated by$EEV = E_{\xi}(\bar{Q}(\bar{\xi}), \xi)$. In conclusion, a higher value for VSS implies the advantages of using stochastic programming approach and denotes that the solutions of stochastic programming model are more valuable in comparison with deterministic model.

The objective values of two-stage stochastic programming model, which optimized with three scenarios for four problem instances and the corresponding results for WS, EVPI and VSS are reported in Table 6.

Table 6. Computational results for wait-and-see and EVPI of two-stage stochastic programming.

n	p	Q_0^*	WS	EVPI	EEV	VSS
10	3	52087259.16	53959620.52	1872361.36	50960314.38	1126944.77
10	5	51814872.63	53666108.78	1851236.15	50705194.12	1109678.51
15	3	103314322.77	105458342.03	2144019.26	101957450.28	1356872.49
15	5	102446898.40	104607032.49	2160134.09	101098959.33	1347939.07

According to the results of Table 6, the large values for VSS signify that it is worthwhile to use more complicated modeling techniques. Moreover, high EVPI in studied instances show that have perfect information about future would be helpful to substantially improve the objective function.

6. Conclusion

In this article, we propose a novel model to jointly optimize air transportation network, which arises in the form of a star/star network and booking limits for a two-product seat allocation problem within the context of network revenue management. We presented a hybrid route transportation system considering both non-stop and hub-stop flights. The model strives to maximize airline profit by designing optimal network transportation, classifying customers and determining protection levels in a stochastic environment. In order to capture the demand uncertainty in the model, a two-stage stochastic programming approach was applied to formulate the problem. Since the decisions about the location of hubs is a long-term investment and will not change according to demand variations, hub and spoke network design will not be influenced by the randomness of stochastic environment and belongs to the first stage decision of the model. Conversely, the decisions of the flight routes to transport, flows from origins to destinations and booking limits determination based upon the hub location and realized uncertain scenario are

related to the second stage. Computational experiments with various number of hubs and nodes was carried out based on the Turkish network data set. The results intelligibly imply that the integrated model for hub location and seat inventory decision problem would help to increase the total revenue of airline companies. Hence the problem is NP-hard, it is not possible to solve large scale problem in a reasonable time. Therefore, we proposed the evolutionary algorithm includes standard genetic operators and exact method by linking MATLAB software to GAMS. In the proposed modified GA an immigration operator is utilized in purpose of increasing the exploration and better search in the solution space, also caching technique is implemented to improve the computational performance of the algorithm. The efficiency of the algorithm is compared with the pure GA and mathematical programming model. The results showed the capability of the proposed algorithm regarding to the computational time requirements and high-quality solutions.
An Interesting avenue of research for future work consists in addressing this problem with other meta-heuristics and larger scale data. Also analyzing the problem with hub capacity constraints as the assumption, finding more robust optimization techniques or improving the process of solving the proposed problem by incorporating decomposition methods could be valuable future research subjects.

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