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Inventory model for deteriorating items with quadratic time dependent demand under trade credits

R.P. Tripathi [a,1], D. Singh [b], Tushita Mishra [b]

[a] *Department of Mathematics, Graphic Era University, Dehradun (UK) India*
[b] *Department of Mathematics, S.G.R.R. PG College, Dehradun (UK) India*

Abstract
In this study, EOQ model is developed for a deteriorating item with quadratic time dependent demand rate under trade credit. Mathematical models are also derived under two different situations i.e. Case I; the credit period is less than the cycle time for settling the account and Case II; the credit period is greater than or equal to the cycle time for settling the account. The numerical examples are also given to validate the proposed model. Sensitivity analysis is given to study the effect of various parameters on ordering policy and optimal total profit. Mathematica 7.1 software is used to find optimal numerical solutions.

Keywords: Quadratic time–dependent demand; inventory; trade credits; deterioration; time dependent.

1. Introduction

In real life situation, supplier offers a delay period to the buyer to buy more items. Most of the inventory models are considered with infinite replenishment rate. The classical EOQ model was developed in 1995, the demand rate of an item was considered as constant. In reality, demand for physical goods or items may be time-dependent, stock-dependent and price-dependent.

In last few decades, inventory problems for deteriorating items have been widely studied. Maximum physical goods undergo decay or lose their originality over time. It is a major factor to control and maintain the inventories of deteriorating items for any business transaction. Many researchers have paid attention to time dependent demand rate. Silver and Meal (1969) first suggested a simple modification of the problem of inventory replenishment with linearly

[*] Corresponding author email addresses: tripathi_rp0231@rediffmail.com

time-dependent demand. Recently Khanna *et al.* (2011) developed an EOQ model for deteriorating item having time-dependent demand when delay in payment is permissible. Donaldon (1977) was the first who gave fully analytical solution to the problem of inventory replenishment with a linearly time-dependent demand. Many researchers like Mitra *et al.* (1984), Rilchie (1985) made valuable and significant contributors in this deviation. Teng *et al.* (1985) developed inventory model for linear non-decreasing demand under trade credits. In this In this direction the work of Dave and Patel (1981), Bahari and Kashani (1989) , Hariga (1995), Jalan *et al.* (1996), Giri and Chaudhuri (1996), Lin *et al.* (2000), Jalan and Chaudhari (1999), Chung and Teng (1993) are worth mentioning. Wee (1995) and Jalan and Chaudhary (1999presented their model taking exponentially time dependent demand rate.

Inventory model with trade credits was first developed by Goyal (1985), Chu *et al.* (1998) and Chung *et al.* (2001) also Goyal's (1985) model, for deteriorating items . Many researchers like Aggarwal & Jaggi (1995), Chung and Liao (2004), Mandal and Phanjdar (1989), Chung *et al.* (2001), Davis and Gaither (1985) developed inventory model taking trade credit. Chang and Huang (2003) developed an economic production quantity model (EPQ) for a retailer where the supplier offers a permissible delay in payment. Huang (2003) presented a model by extending Goyal's model to develop an EOQ model in which the supplier offers the retailer a permissible delay period M and the retailer in turn provides the trade credit period N (N≤ M) to its customers. Shah and Shah (1992) established the effect of uncertain demand under the condition of permissible delay in payment.

In this paper, an attempt is made to formulate the mathematical model for inventory system with quadratic time-dependent demand rate under trade credit. Demand rate is considered to be an increasing function of time. The objective function to be optimized is considered as the total relevant profit of an inventory system. The effect of parameters on the objective function is developed numerically.

The rest of paper organized as follows: Notations and assumptions are given in section 2 followed by mathematical model in section 3. Then optimal solutions are provided for both cases in section 4. Numerical examples are given in section 5. Sensitivity analysis is provided for variation of various parameters to illustrate the proposed model. In the last section, we discuss concluding remarks and suggestion for future studies.

2. Notations and Assumptions

The following notations and assumptions are used:

$R \equiv R(t) \equiv a + bt + ct^2$: $a > 0, 0 < b < 1, 0 \le c \le 1$; the annual demand

θ: deterioration rate
A: the ordering cost per order
C: the unit purchase cost
p: the unit selling price
M: the permissible credit period offered by the supplier to the retailer for settling the account

I_c : interest rate at which the interest is charged

I_e : interest rate at which the interest is earned

Q: the order quantity

I (t): inventory level at any instant of time

T: replenishment cycle time

K_i *(T)*: total profit per time unit; $i = 1, 2$.

Q_i*: optimal order quantity; $i = 1, 2$ for Case I and II respectively

T_i*: optimal replenishment cycle time; i = 1, 2.

$K_i (T_i*)$: optimal profit per time unit; $i = 1, 2$

Assumptions

(a) The inventory system under consideration deals with the single item.

(b) The planning horizon is infinite

(c) The demand of the product is a quadratic increasing function of time.

(d) Shortage is not allowed.

(e) Lead time is zero.

(f)The retailer can deposit generated sales revenue in an interest bearing account during the permissible credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day to day expenditure and paying the interest charges on the unsold items in the stock.

3. Mathematical Model

The inventory level I (t) depletes to meet the demand and deterioration. The differential equation governing the rate of change of inventory at any time t is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t) \qquad ; \quad 0 \le t \le T \tag{1}$$

With the initial condition I(0) = Q and boundary condition I(T) = 0 (2)

The solution of equation (1) is given by

$$I(t) = \frac{1}{\theta}\left(a - \frac{b}{\theta} + \frac{2c}{\theta^2}\right)\left(e^{\theta(T-t)} - 1\right) + \frac{1}{\theta}\left(b - \frac{2c}{\theta}\right)\left(Te^{\theta(T-t)} - t\right) + \frac{c}{\theta}\left(T^2 e^{\theta(T-t)} - t^2\right) \tag{3}$$

And the order quantity Q is given by

$$Q = \frac{1}{\theta}\left(a - \frac{b}{\theta} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \frac{1}{\theta}\left(b - \frac{2c}{\theta}\right)Te^{\theta T} + \frac{c}{\theta}T^2 e^{\theta T} \tag{4}$$

The total profit per time unit of inventory system consists of the following:

1. Ordering cost $OC = \frac{A}{T}$ (5)

2. Sales Revenue $SR = \frac{p}{T}\int_0^T R(t)dt = p\left(a + \frac{bT}{2} + \frac{cT^2}{3}\right)$ (6)

3. Deterioration Cost

$$DC=\frac{C}{T}\left[Q-\int_0^T R(t)dt\right]=\frac{C}{T}\left\{\frac{1}{\theta}\left(a-\frac{b}{\theta}+\frac{2c}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{1}{\theta}\left(b-\frac{2c}{\theta}\right)Te^{\theta T}+\frac{c}{\theta}T^2e^{\theta T}-aT-\frac{bT^2}{2}-\frac{cT^3}{3}\right\} \quad (7)$$

4. Holding Cost

$$\begin{aligned}HC&=\frac{h}{T}\int_0^T I(t)dt\\&=\frac{h}{\theta T}\left\{\left(a-\frac{b}{\theta}+\frac{2c}{\theta^2}\right)\left(\frac{e^{\theta T}-1}{\theta}-T\right)+\left(b-\frac{2c}{\theta}\right)T\left(\frac{e^{\theta T}-1}{\theta}-\frac{T}{2}\right)+cT^2\left(\frac{e^{\theta T}-1}{\theta}-\frac{T}{3}\right)\right\}\end{aligned} \quad (8)$$

Now two cases arise by considering interest charged and interest earned based on length of T and M.

Case I : M < T

5. The interest charged per time unit for above case is

$$\begin{aligned}IC&=\frac{C\mathrm{I_c}}{T}\int_M^T I(t)dt\\&=\frac{CI_c}{\theta T}\left[\left(a-\frac{b}{\theta}+\frac{2c}{\theta^2}\right)\left(\frac{e^{\theta(T-M)}-1}{\theta}\right)+\left(b-\frac{2c}{\theta}\right)T\left(\frac{e^{\theta(T-M)}-1}{\theta}\right)+cT^2\left(\frac{e^{\theta(T-M)}-1}{\theta}\right)\right.\\&\qquad\left.-\left(a-\frac{b}{\theta}+\frac{2c}{\theta^2}\right)(T-M)-\left(b-\frac{2c}{\theta}\right)\left(\frac{T^2-M^2}{2}\right)-c\left(\frac{T^3-M^3}{3}\right)\right]\end{aligned} \quad (9)$$

6. During [0, M] retailer sells the product and deposits the revenue into an interest earning account at the rate I_e per unit per year. Therefore, the interest earned, IE_1 per time unit is given by

$$\mathrm{IE_1}=\frac{\mathrm{pI_e}}{T}\int_0^M R(t)dt=\frac{p\mathrm{Ie}}{T}\left\{\frac{a}{2}+\frac{bM}{3}+\frac{cM^2}{4}\right\}M^2 \quad (10)$$

Hence the total profit $K_1(T)$ of an inventory system per time unit is

$$K_1(T)=SR+IE_1-OC-DC-HC-IC \quad (11)$$

Putting the values of SR, IE, OC, DC, HC, IC from respective equations and solving by using Truncated Taylor's Series for approximating the exponential function i.e. $e^{\theta T}\approx 1+\theta T+\frac{\theta^2T^2}{2}$, etc , we get

$$K_1(T)=ap+\left(\frac{bp}{2}-\frac{aC\theta}{2}-\frac{cC}{\theta}-\frac{ah}{2}-\frac{aCI_c}{2}+\frac{2bCIc}{\theta}-\frac{2cCI_c}{\theta^2}\right)T+$$
$$\left(\frac{cp}{3}-\frac{bC\theta}{2}+\frac{cC}{2}-\frac{bh}{2}+\frac{ch}{3\theta}-\frac{bC\mathrm{I_c}}{2}+\frac{c\mathrm{I_c}\mathrm{C}}{3\theta}\right)T^2-\frac{c}{2}(h+C\theta)T^3$$
$$+\frac{aM^2}{2T}(\mathrm{pI_e}-CI_c)+\frac{1}{3}\left(bp\mathrm{I_e}-\frac{c\mathrm{I_c}\mathrm{C}}{\theta}\right)\frac{M^3}{T}+\frac{cp\mathrm{I_e}}{4}\frac{M^4}{T}-\frac{A}{T}+C\left(a\mathrm{I_c}-\frac{2c\mathrm{I_c}}{\theta^2}+2c\mathrm{I_c}\right)M \tag{12}$$
$$+CI_c\left(\frac{c\mathrm{I_c}}{\theta}-\frac{b\mathrm{I_c}}{2}\right)M^2+cI_cCMT^2+CI_c\left(b-\frac{c}{\theta}\right)MT-\frac{c\mathrm{I_c}\mathrm{C}}{2}M^2T$$

Case II: T ≤ M.

Hence the retailer sells R(T)T-units in all by the end of the cycle time and has CR(T)T to pay the supplier in full by the end of the credit period M, Hence interest charges
5) IC = 0
6) The interest earned per time unit is

$$\mathrm{IE}_2=\frac{\mathrm{pI_e}}{T}\left\{\int_0^T R(t)tdt+R(T)T(M-T)\right\}$$
$$=pI_e\left\{M\left(a+bT+cT^2\right)-T\left(\frac{a}{2}+\frac{2bT}{3}-\frac{3cT^2}{4}\right)\right\} \tag{13}$$

Hence the total profit K(T) of an inventory system per time unit is

$$K_2(T)=SR+IE_2+-OC-DC-HC-IC$$

Putting the values of SR, IE, OC, DC, HC, and IC from respective equations and solving by using Truncated Taylor's Series for approximating the exponential function, we get

$$K_2(T)=ap+\left(\frac{bp}{2}-\frac{aC\theta}{2}-\frac{cC}{\theta}-\frac{ah}{2}-\frac{ap\mathrm{I_e}}{2}\right)T+\left(\frac{cp}{3}-\frac{bC\theta}{2}+\frac{cC}{3}-\frac{bh}{2}+\frac{ch}{3\theta}-\frac{2bp\mathrm{I_e}}{3}\right)T^2$$
$$+\left(\frac{cp}{3}-\frac{bC\theta}{2}+\frac{cC}{3}-\frac{bh}{2}+\frac{ch}{3\theta}-\frac{2bp\mathrm{I_e}}{3}\right)T^2-c\left(\frac{3p\mathrm{I_e}}{4}+\frac{C\theta}{2}+\frac{h}{2}\right)T^3-\frac{A}{T} \tag{14}$$
$$+pI_eM\left(a+bT+cT^2\right)$$

4. Determination of optimal solution

To find the optimal solution for the problem, we maximize K(T) for Case I and Case II respectively and then compare them to obtain maximum value. Our aim is to find maximum average profit per time unit for both cases i.e. Case I and II respectively with respect to T. The necessary and sufficient condition to maximise K(T) ; i=1,2 for given values of T are respectively

$$\frac{dK_1}{dT}=0,\frac{dK_2}{dT}=0,\quad \text{and}\quad \frac{d^2K_1}{dT^2}<0,\frac{d^2K_2}{dT^2}<0.$$

Differentiating (12) and (14) two times with respect to T, we get

$$
\begin{aligned}
&9c\theta^2(C\theta+h)T^4-\theta\left(4cp\theta-6bC\theta^2+4cC\theta-6bh\theta+4ch-6bC\mathrm{I_c}\theta+4cC\mathrm{I_c}\right)T^3\\
&-12cC\mathrm{Ic}MT^3-(3bp\theta^2-3aC\theta^3-6cC\theta-3ah\theta^2-3aC\mathrm{I_c}\theta^2+12cC\mathrm{Ic})T^2\\
&-b\theta\left(6C\mathrm{I_c}\theta+cC\mathrm{I_c}\right)MT^2+3cC\mathrm{Ic}\theta^2M^2T^2+3a\theta^2(p\mathrm{I_e}-C\mathrm{I_c})M^2\\
&+2\theta(bp\mathrm{Ie}\theta-cC\mathrm{Ic})M^3+1.5cp\mathrm{Ie}\theta^2M^4-6A\theta^2=0\ ,
\end{aligned}\tag{15}
$$

$$
\begin{aligned}
&9c\theta(1.5p\mathrm{Ie}+C\theta+h)T^4-(4cp\theta-8bp\mathrm{Ie}\theta-6bC\theta^2+4cC\theta-6bh\theta+4ch)T^3\\
&-(3bp\theta-3ap\mathrm{I_e}\theta-3aC\theta^2-6cC-3ah\theta)T^2-12cp\theta MT^3-6bp\mathrm{Ie}\theta MT^2-6A\theta=0\ .
\end{aligned}\tag{16}
$$

and

$$
\begin{aligned}
\frac{d^2K_1}{dT^2}&=\frac{2cp}{3}-bC\theta+cC-bh+\frac{2ch}{3\theta}-bC\mathrm{I_c}+\frac{2c\mathrm{I_c}C}{3\theta}-3c(C\theta+h)T\\
&-\frac{aM^2}{2T^2}\left(pI_e-CI_c\right)-\frac{M^3}{3T^2}\left(bpI_e-\frac{cCI_c}{\theta}\right)-\frac{cp\mathrm{I_e}}{4}\frac{M^4}{T^2}+\frac{A}{T^2}+2cC\mathrm{I_c}M<0
\end{aligned}\tag{17}
$$

$$
\begin{aligned}
\frac{d^2K_2}{dT^2}&=-\frac{1}{3}\left\{3bC\theta+3bh+4bpI_e-2c\left(\frac{h}{\theta}+p\right)\right\}-3c\left(1.5p\mathrm{Ie}+C\theta+h\right)T\\
&+\frac{A}{T^2}+2cp\mathrm{Ie}M<0
\end{aligned}\tag{18}
$$

5. Numerical Example

Example 1. Case I. a =1000, b = 0.2, c = 0.2, M = 30/365, θ = 0.1, h =1, p = 40, C = 20, A= 250, I_c = 0.12, I_e = 0.09 , in appropriate units. Optimal cycle time $T = T_1^*$ = 0.305009 year, optimal total relevant profit $K_1(T) = K_1^*(T_1^*)$ = $ 86354.9, and optimal order quantity $Q = Q_1^*$= 309.67 units .
Example 2. Case II. a = 600, b = 0.1, c = 0.1, M = 60/365, θ = 0.2, h =1, p = 35, C = 30, A = 50, I_e = 0.09, in appropriate units. Optimal cycle time $T = T_2^*$ = 0.127874 year, optimal total relevant profit $K_2(T) = K_2^*(T_2^*)$ = $20528.6 , and optimal order quantity $Q = Q_2^*$ = 77.7063 units.

6. Sensitivity Analysis

To find sensitivity analysis, the effects of parameters a, b, c, h, p, θ, M, I_e, I_c, A and C on the optimal solution. Let us take the set of values a = 500, 700, 1200, 1500, 1700, 2000, b = 0.05, 0.10, 0.15, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, c = 0.05, 0.10, 0.15, 0.25, 0.30, 0.35, 0.40, h = 0.5, 5, 10, 15, p = 10, 20, 30, 50, 60, 70, θ = 0.5, 0.15, 0.20, 0.25, 0.30, 0.35, M = 20/365, 40/365, 50/365, 60/365, I_e = 0.01, 0.05, 1.0, 1.5, I_c = 0.05, 0.10, 0.15, 0.20, 0.25. 0.30, 0.35, A = 100, 150, 200, 300, 350, 400, C = 5, 10, 15, 25, 30, 35, 40, 45 in appropriate units for case I.

And, let us take , the set of values a = 400,500, 800, 1000, 1500, 2000, b = 0.05, 0.15, 0.25, 0.30, 0.35, 0.40, c = 0.05, 0.15, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, h = 0.5, 5, 10, 15, 20,25, p = 10, 15, 20, 25, 45, 55, θ = 0.5,0.10, 0.15, 0.25, 0.30, 0.35, 0.40, M = 50/365, 70/365, 80/365, 100/365,150/365, I_e = 0.01, 0.05, 0.1, 0.5, 1.0. A = 10, 20, 30, 40, 60, 100, C = 20, 40, 50, 60, 80, 100, in appropriate units (For Case II).
The result of sensitivity analysis is given in Tables 1.

Case I:

Table 1(A). Variation of 'a' keeping all parameters same as in given example 1.

a	$T = T_1{}^*$	$Q = Q_1{}^*$	$K_1(T_1{}^*)$
500	0.438227	223.933	27259.4
700	0.367250	261.809	46215.8
1200	0.277460	337.579	122111
1500	0.247102	375.239	191067
1700	0.231530	398.163	248228
2000	0.212725	429.980	352316

Table 1(B). Variation of 'b' keeping all parameters same as in given example 1.

b	$T = T_1{}^*$	$Q = Q_1{}^*$	$K_1(T_1{}^*)$
0.05	0.304848	309.497	86379.3
0.10	0.304901	309.554	86371.3
0.15	0.304955	309.612	86363.1
0.25	0.305063	309.728	86346.8
0.30	0.305116	309.785	86338.6
0.35	0.305170	309.843	86330.6
0.40	0.305224	309.901	86322.5
0.45	0.305278	309.959	86314.3
0.50	0.305332	310.017	86306.2

Table 1(C). Variation of 'c' keeping all parameters same as in given example 1.

c	$T = T_1{}^*$	$Q = Q_1{}^*$	$K_1(T_1{}^*)$
0.05	0.302759	307.351	86704.8
0.10	0.303503	308.118	86588.6
0.15	0.304253	308.891	86471.9
0.25	0.305771	310.455	86237.6
0.30	0.306539	311.247	86119.9
0.35	0.307313	312.247	86002.0
0.40	0.308094	312.849	86123.5

Table 1(D): Variation of 'h' keeping all parameters same as in given example 1.

h	$T = T_1{}^*$	$Q = Q_1{}^*$	$K_1(T_1{}^*)$
0.5	0.325490	330.798	59646.9
5	0.230127	232.780	124127
10	0.185543	187.268	173395
15	0.159695	160.973	234746

Table 1(E). Variation of 'p' keeping all parameters same as in given example 1.

p	$T = T_1*$	$Q = Q_1*$	$K_1(T_1*)$
10	0.310369	315.195	20302.8
20	0.308595	313.366	42180.1
30	0.306808	311.524	64196.2
50	0.303197	307.803	108660
60	0.301371	305.921	131117
70	0.299532	304.027	153729

Table 1(F). Variation of 'θ' keeping all parameters same as in given example 1.

θ	$T = T_1*$	$Q = Q_1*$	$K_1(T_1*)$
0.05	0.360355	363.614	79236.8
0.15	0.277983	283.786	90848.3
0.20	0.258028	264.692	94767.6
0.25	0.242039	249.368	98373.5
0.30	0.228756	236.611	101752
0.35	0.217465	225.746	104948

Table 1(G). Variation of 'M' keeping all parameters same as in given example 1.

M (in days)	$T = T_1*$	$Q = Q_1*$	$K_1(T_1*)$
20	0.306355	311.057	59646.9
40	0.303093	307.695	124127
50	0.300596	305.123	173395
60	0.297503	301.937	234746

Table 1(H). Variation of 'I_e' keeping all parameters same as in given example 1.

I_e	$T = T_1*$	$Q = Q_1*$	$K_1(T_1*)$
0.01	0.311646	316.512	43759.2
0.05	0.308345	313.108	64826.4
1.0	0.215625	217.954	789622
1.5	0.144729	145.778	1716760

Table 1(I). Variation of 'I_c' keeping all parameters same as in given example 1.

I_c	$T = T_1*$	$Q = Q_1*$	$K_1(T_1*)$
0.05	0.347678	353.734	56134.8
0.10	0.315420	320.404	77101.3
0.15	0.291335	295.587	101071
0.20	0.272506	276.226	127630
0.25	0.257289	260.605	156467
0.30	0.244679	247.678	187338

0.35	0.234021	247.678	220045

Table 1(J). Variation of 'A' keeping all parameters same as in given example 1.

A	$T = T_1{}^*$	$Q = Q_1{}^*$	$K_1(T_1{}^*)$
100	0.190428	192.245	115710
150	0.234899	237.663	100986
200	0.272214	275.926	92284.5
300	0.334614	340.223	81773.5
350	0.361813	368.371	78553.5

Table 1(K). Variation of 'C' keeping all parameters same as in given example 1.

C	$T = T_1{}^*$	$Q = Q_1{}^*$	$K_1(T_1{}^*)$
5	0.482620	494.289	46581.9
10	0.392796	400.526	57396.8
15	0.340345	346.148	70812.6
25	0.279172	283.077	103722
30	0.259248	262.625	122698

Case II:

Table 2(A). Variation of 'a' keeping all parameters same as in given example 2.

a	$\mathrm{T} = \mathrm{T}_2{}^*$	$Q = Q_2{}^*$	$K_2(T_2{}^*)$
400	0.156454	63.5619	13567.9
500	0.140022	70.9923	17044.7
800	0.110799	89.6219	27511.7
1000	0.099133	100.116	34509.0
1500	0.080975	122.447	52041.8
2000	0.070142	141.267	69609.9

Table 2(B). Variation of 'b' keeping all parameters same as in given example 2.

b	$\mathrm{T} = \mathrm{T}_2{}^*$	$Q = Q_2{}^*$	$K_2(T_2{}^*)$
0.05	0.127857	77.6955	20528.5
0.15	0.127892	77.7178	20528.7
0.20	0.127909	77.7287	20528.9
0.25	0.127926	77.7395	20529.0
0.30	0.127944	77.7510	20529.1
0.35	0.127961	77.7691	20529.2
0.40	0.127978	77.7728	20529.3

Table 2(C). Variation of ‘c’ keeping all parameters same as in given example 2.

c	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
0.05	0.128025	77.7992	20529.6
0.15	0.127724	77.6140	20527.7
0.20	0.127574	77.5217	20526.8
0.25	0.127424	77.4294	20525.8
0.30	0.127275	77.3377	20524.9
0.35	0.127127	77.2467	20523.9
0.40	0.126979	77.1556	20523.0
0.45	0.126832	77.0652	20522.1
0.50	0.126685	76.9747	20521.1

Table 2(D). Variation of ‘h’ keeping all parameters same as in given example 2.

h	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
0.50	0.1311310	50.1511	20548.1
5.00	0.1083670	47.7316	20387.9
10.0	0.0931882	36.0378	20237.6
15.0	0.0830017	29.0566	20105.9
20.0	0.0755600	25.0060	19987.2
25.0	0.0698172	22.2813	19878.3

Table 2(E). Variation of ‘p’ keeping all parameters same in example 2.

p	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
10	0.144815	88.1483	5398.21
15	0.140890	85.7260	8423.35
20	0.137267	83.4971	11449.0
25	0.133910	81.4228	14475.1
45	0.122587	74.4546	26583.7
55	0.117906	71.5784	32640.1

Table 2(F). Variation of ‘θ’ keeping all parameters same in given example 2.

θ	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
0.10	0.151704	91.7140	20651.5
0.15	0.138338	83.8649	20587.8
0.25	0.119441	72.7353	20437.4
0.30	0.112468	68.6198	20421.5
0.35	0.106583	65.1432	20371.4
0.40	0.101530	62.1555	20325.7

Table 2(G). Variation of 'M' keeping all parameters same as in given example 2.

M (in days)	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
50	0.127874	77.7063	20476.9
70	0.127875	77.7069	20580.4
80	0.127875	77.7069	20632.2
90	0.127875	77.7069	20684.0
100	0.127875	77.7069	20735.8
150	0.127876	77.7076	20994.7

Table 2(H). Variation of 'I_e' keeping all parameters same as in given example 2.

I_e	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
0.01	0.1501540	91.4463	20368.5
0.05	0.1376800	83.7463	20446.2
0.1	0.1257340	76.3897	20549.8
0.5	0.0824068	49.8519	21512.5
1.0	0.0629624	38.0155	22863.8

Table 2(I). Variation of '*A*' keeping all parameters same in example 2.

A	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
10	0.0571849	34.5073	20960.9
20	0.0808728	48.9194	20816.1
30	0.0990495	60.0188	20704.9
40	0..114373	69.4093	20611.2
60	0.14008	85.2263	20454.0
70	0.151308	92.1577	20385.4

Table 2(J). Variation of 'C' keeping all parameters same in example 2.

C	$T = T_2*$	$Q = Q_2*$	$K_2(T_2*)$
20	0.142776	86.8897	20610.3
40	0.116837	70.9219	20454.8
50	0.108240	65.6475	20386.8
60	0.101298	61.3950	20323.5
80	0.0906645	54.8923	20207.7
100	0.0828023	50.0931	20103.0

All the above observations from Table 1 to 19 sum up as follows:

Case I

- From Table 1(A): It is observed that, increase of '*a*' results decrease in optimal cycle time $T = T_1*$, increase in optimal order quantity $Q = Q_1*$ and optimal total relevant profit $K_1(T_1*)$.

- From Table 1(B): It is observed that, increase of 'b' results slight increase in optimal cycle time $T = T_1*$, optimal order quantity, optimal order quantity $Q = Q_1*$ and optimal total relevant profit $K_1 (T_1*)$.
- From Table 1(C): We see that, increase of '*c*' results slight increase in optimal order quantity $T = T_1*$, $Q = Q_1*$, and decrease in optimal total relevant profit $K_1(T_1*)$.
- From Table 1(D): We see that, increase of holding cost 'h' results in decrease in optimal cycle time $T = T_1*$, optimal order quantity $Q = Q_1*$ and increase in optimal relevant profit $K_1(T_1*)$..
- From Table 1(E), we see that, increase of unit selling price '*p*' results slight decrease in optimal cycle time $T = T_1*$, and optimal order quantity $Q = Q_1*$ and increase in optimal total relevant profit $K_1(T_1*)$.
- From Table 1(F), we wee that, increase of deterioration rate 'θ' results decrease in optimal cycle time $T = T_1*$, and optimal order quantity $Q = Q_1*$ and increase in optimal total relevant profit $K_1(T_1*)$.
- From Table 1(G), we see that , increase of credit period 'M' results decrease in optimal cycle time $T = T_1*$, and optimal order quantity $Q = Q_1*$ and increase in optimal total relevant profit $K_1(T_1*)$.
- From Table 1(H), we see that, increase of interest earned 'I_e' results decrease in optimal cycle time $T = T_1*$, and optimal order quantity $Q = Q_1*$ and increase in optimal total relevant profit $K_1(T_1*)$.
- From Table 1(I), we see that, increase of interest charged 'I_c' results decrease in optimal cycle time $T = T_1*$, and optimal order quantity $Q = Q_1*$ and increase in optimal total relevant profit $K_1(T_1*)$.
- From Table 1(J), we see that increase of ordering cost 'A' results increase in optimal cycle time $T = T_1*$, and optimal order quantity $Q = Q_1*$ and decrease in optimal total relevant profit $K_1(T_1*)$.
- From Table1(*K*), we see that, increase of purchase cost '*C*' results decrease in optimal cycle time $T = T_1*$, and optimal order quantity $Q = Q_1*$ and increase in optimal total relevant profit $K_1(T_1*)$.

Case II

- From Table 2(A), we see that increase of parameter '*a*' results decrease in optimal cycle time $T = T_2*$, increase in optimal order quantity $Q = Q_2*$ and optimal total relevant profit $K_2(T_2*)$.
- From Table 2(B), we see that, increase of parameter 'b' results slight increase in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and optimal total relevant profit $K_2(T_2*)$.
- From Table 2(C), we see that, increase of parameter '*c*' results slight decrease in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and optimal total relevant profit $K_2(T_2*)$.
- From Table *2(D)*, we see that, increase of holding cost '*h*' results decrease in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and optimal total relevant profit $K_2(T_2*)$.
- From Table *2 (E)*, we see that, increase of unit selling price '*p*' results decrease in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and increase in optimal total relevant profit $K_2(T_2*)$.
- From Table 2(F), we see that, increase of deterioration rate '*θ*' results decrease in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and optimal total relevant profit $K_2(T_2*)$.

- From Table 2(G), we see that, increase of credit period '*M*' results slight increase in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and increase in optimal total relevant profit $K_2(T_2*)$.
- From Table 2(H), we see that, increase of interest earned 'I_e' results decrease in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and increase in optimal total relevant profit $K_2(T_2*)$.
- From Table 2(I), we see that ,increase ordering cost '*A*' results increase in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and optimal total relevant profit $K_2(T_2*)$.
- From Table 2(J), we see that, increase of '*C*' results decrease in optimal cycle time $T = T_2*$, optimal order quantity $Q = Q_2*$ and optimal total relevant profit $K_2(T_2*)$.

7. Conclusion and Suggestion for Future Studies

In this paper, we develop an EOQ model for deteriorating items with quadratic demand rate. We provide numerical solution to find the optimal cycle time, optimal order quantity and total relevant profit. From the sensitivity analysis we conclude that the results are quite sensitive with respect to variation of different parameters. Truncated Taylor's series expansion is used for finding closed form optimal solutions.This paper can be extended in different ways. We would like to consider the deteriorating items as time varying deterioration rate and demand rate as stock level dependent demand rate.

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