



A robust programming approach to bi-objective optimization model in the disaster relief logistics response phase

Mohsen Saffarian^{a*}, Farnaz Barzinpour^b and Mohammad Ali Eghbali^a

^a Department of Industrial Engineering, Birjand University of Technology, Iran

^b Department of Industrial Engineering, Iran University of Science & Technology, Iran

Abstract

Accidents and natural disasters and crises coming out of them indicate the importance of an integrated planning to reduce their effects. Therefore, disaster relief logistics is one of the main activities in disaster management. In this paper, we study the response phase of the disaster management cycle. A bi-objective model has been developed for relief chain logistic in uncertainty condition including uncertainty in traveling time and also amount of demand in damaged areas. The proposed mathematical model has two objectives. The first one is to minimize the sum of arrival time to damaged area multiplying by amount of demand and the second objective is to maximize the minimum ratio of satisfied demands in total period to reach a fairness in distribution of goods. In the proposed model, the problem has been taken into account and to solve the mathematical model, Global Criterion method has been used and a case study has been done at South Khorasan.

Keywords: Relief logistic; Cumulative routing; Periodic approach; Inventory; Robust optimization

* Corresponding author email address: saffarian@birjandut.ac.ir

1. Introduction

Millions of people are affected by natural or man-made disasters and accidents in the recent decades; number of victims has been increased dramatically. The nature of the crisis is such that the responding time should be minimized (Thomas and Kopczak, 2005). Proper planning can be effective in reducing the side-effects of the crisis. Some studies estimate that the logistics and supply chain management include more than 80% of the total humanitarian relief operations (Van Wassenhove, 2006). As determined in the literature, Operation Research models in the operations support various types of humanitarian operations successfully (Van Wassenhove and Pedraza Martinez, 2012). Logistic could be coordinated in the delivery of regional warehouses and damaged areas. The purposes of relief logistic planning are to minimize the time of responding to the damaged areas and also maximize the satisfaction level and fairness in the distribution of commodities.

Research in the field of disaster management is very important and considerable research has been done in this field. It should be noted, there are few studies on disaster management. The brief review of the research in the relief chain response phase will be discussed. The first study on disaster phase response was conducted in 1988 by Knott. In this study, a linear programming model was used to determine the vehicle schedule for carrying food (Knott, 1988). (Oh and Haghani, 1996) provided a model to transport commodities like food, clothing, relief forces, and medicine through some kind of vehicle to relive operations. (Barbarosoglu and Ozdamar, 2002) studied development and provided a mathematical model for solving operational and tactical scheduling of helicopters activities. (Barbarosoglu and Arda, 2004) used a model in 2004 and discussed uncertainty and employed a two-phase random planning framework to plan shipment in disaster relief chain response phase. (Ozdamar et al., 2004) studied logistic planning in urgency to send article to distribution centers in the damaged areas. (Nolz et al., 2011) provided a multi-objective model to send commodities in the respond phase and their model consists of three target functions including reduce responding risk, reduce covering intervals of vehicle, and reduce total travel time. (Lin et al., 2011) have studied a multi-period, multi-commodity, and multi-vehicle logistic model in 2011 to provide logistic planning of important of high priority commodities in the disaster respond phase. Proposed model has two functions: one of them is to minimize an unanswered request and the other one is to minimize time travel. (Berkooone et al. 2012) provided a mathematical model to plan commodities shipment in the respond phase and to minimize travel time of vehicles. (Eshghi and Najafi, 2013) provided a two-objective model to reduce unfulfilled requests and the number of victims who have not transmitted to the hospital.

Periodic routing is an aspect of routing in this article. In this routing, servicing to customers should be done in the planning horizon and periodically. The purpose of periodic routing is to specify service direction to customers in each period, so that total routing price would be minimized in the planning horizon. Periodic routing was first proposed in 1974 (Beltrami and Bodin, 1974). First Mathematical model of periodic routing was proposed in 1984 (Christofides and Beasley, 1984).

(Rath and Gutjahr, 2011) provided a multi objective optimization model along with a medium-term economic department, a short-term economic department, and a disaster objective function and then designed an innovative model to solve it; moreover, they provided a meta-heuristic method based on genetic algorithm.

(Ulrich et al., 2010) provided an accumulated vehicle routing model to minimize vehicle arrival times to applicants. (Ke and Feng, 2013) provided a two-step innovative model to solve accumulated vehicle routing problem. In this model, vehicle leaves warehouse first and goes back to warehouse after servicing to some customers. There are two main limitations in above model. First, each customer delivers service just by one vehicle and once, as a result customer request should be lower than vehicle capacity. Another limitation is that each vehicle is used once. In this study, first periodic issue is considered and second servicing possibility which affects areas by several vehicles. This leads to solve problems in the field of request more than capacity of vehicle. The third issue is optimizing utilization of vehicles especially when there are not sufficient vehicles, for example one vehicle can be used several times during one period. Fourthly, one of the objective functions is a cumulated vehicle routing including minimization of total time to reach customers. In this study, fair distribution of commodity between affected areas is studied in addition to above purposes. (Bozorgi et al., 2013) provide a multi-objective robust contingency planning model. They studied demand, supply, cost and Vehicle cost as a non-deterministic. The purpose of their model is to minimize the expected variance of relief chain costs and maximize coverage level of affected areas. (Najafi et al., 2013) provided a multi-objective, multi-commodity, and multi-probable periodic model for commodity logistic management and victims in the earthquake respond phase. They used a robust approach for problem modeling. (Zhang et al., 2013) provide a bi-objective planning model to supply urgent services in the uncertain situation. They used robust approach to deal with uncertainty of the system parameters. (Rezaei-Malek and Tavakoli-Moghaddam, 2014) explained a bi-objective robust integer model for Humanitarian relief logistics and estimated optimal amount of relief commodities and optimal place of warehoused by taking the two variables of price and repose into account, simultaneously.

The main purpose of this study is to provide a model to make decision about relief logistic in which delivery time which affects areas should be minimized and level of servicing to victims should maximized to meet their satisfaction. In this article, periodic issue is studied. To this end relief chain management is divided in several periods, and there are different requests for relief commodities in each period. Other note which is studied in this article is a possibility of doing several travel in one period, it means one vehicle could move from central warehouse in one period and goes back to the central warehouse after visiting several affected areas and then start another travel in the same period. Also, in this model, the unmet request in one period is transferred to next period and level of inventory is considered. In the designed model in this study, there are several central warehouses, several vehicle and several commodities. Regarding all the above mentioned issues contribute to the novelty of the present study. In addition, South Khorasan province where seems to be under examined was selected as a case study in this model. Also, given the uncertain environment of the problem, fuzzy sets are used to deal with uncertainty. The purposes of fuzzy approach in this model are: first, in most of actual problems, there are not definite historical data for parameters adequately, so rarely; appropriate distribution is obtained for uncertain parameters. Secondly, in most of previous work in the field of relief logistic designing in uncertainty situation, scenario-based random planning is used to model uncertainty of parameters and in this state high number of scenarios to explain uncertainty leads to computational problems and challenges (Pishvae and Torabi, 2010).

The organization of the paper is as follows: in the second section, the proposed mathematical model and its solution are provided. In the third section, information of a case study is presented and in the fourth section conclusion and suggestions for further studies are presented.

2. Proposed Mathematical Model

The model is described as: (1) several central warehouses as supplier of relief commodities, (2) several regional warehouses in affected areas as request points which the quantity of the requests in different periods are triangular fuzzy numbers. The model is periodic and the periods are daily. In each period, first the vehicle leaves central warehouse and goes back to the same central warehouses after visiting regional warehouses. It should be noted that a vehicle has several travels in each period, provided that duration of its travel is not exceeded time limitation.

2.1 Assumptions of the Model

In this model, several central warehouses are studied in which there are different relief inventories in different periods and several vehicles with various capacity and speed. Each vehicle can travel more than one time in each period. The vehicle leaves corresponding central warehouse in each travel and supply services to the affected areas and then goes back to the same central warehouse and if needed next travel would be done. Each affected area could deliver service by several vehicles. In the designed model, sending several commodities to the affected area is possible. Model is planned periodically and the amount of request and supply as well as available vehicles could be changed.

2.2 Model Indexes

Model indexes include:

i, j : Index of the affected areas

t : Index for the period

m : Index of vehicle

c : Index of commodity

v : Index of the number of travel

w : Index to the central warehouse

s : Index of the scenarios

2.3 Model Parameters

p_s : Probability of scenario s

d_{icts} : The amount of affected area i for commodity c in period t of scenario s

wh_c : Weight per unit of commodity c

v_c : Volume per unit of commodity c

w_{wmijts} : Interval between two regions i and j by vehicle m to central warehouse w , in period t of scenario s

$wcap_{wmts}$: Vehicle weight capacity m of warehouse w , in period t of scenario s

$vcap_{wmts}$: The volumetric capacity of the vehicle m of warehouse w , in period t of scenario s

inv_{wcts} : The Inventory c at the central warehouse w , in period t of scenario s

α_c : Duration of download of a single commodity c

LT_{wmts} : Time of vehicle loading m of warehouse w , in period t of scenario s

h_{wmts} : Allowable time to vehicle m of central warehouse w , in period t of scenario s

$Mbig$: Very large number

$I_s = \{k + 1, k + 2, \dots, k + n_s\}$: Total damaged areas in scenario s

$M_{wts} = \{1, 2, \dots, m_{wts}\}$: Total Vehicles of central warehouse w , in period t of scenario s

m_{wts} : The number of available vehicles in the central warehouse w , in period t of scenario s

$C = \{1, 2, \dots, c\}$: Types of commodities

$T = \{1, 2, \dots, t\}$: The number of periods of the planning horizon

$I'_s = \{1, \dots, k, k + 1, \dots, k + n_s + k\}$: Total of damaged area and central warehouse, It should be noted that 1, $k + n_s + 1$ index corresponding to first central warehouse in scenario s and 2, $k + n_s + 2$ index corresponding to second central warehouse in scenario s and etc.

$TR = \{1, 2, \dots, v\}$: Total travels

$W = \{1, 2, \dots, w\}$: Total of central warehouses

2.4 Decision Variables

x_{wjmvt} : If the vehicle m corresponds to the central warehouse w is traveling v from period t in scenario s from region i to region j is 1, otherwise 0.

$time_{wmivts}$: Arrival time of vehicle m corresponds to the central warehouse w is traveling v from period t to damaged area i in scenario s

d'_{wicmvt} : The Amount of met demand in the damaged area i of commodity c in travel v from period t by means of vehicle m corresponds to the central warehouse w , in scenario s

inv'_{wcts} : Surplus of commodity c in central warehouse w , in scenario s which is transferred from period t to period $t + 1$

de_{ict} : The Amount of unmet demand of commodity c in the damaged area of i , in the scenario s which is transferred from period t to period $t + 1$

2.5 Proposed Mathematical Model

$$\min f_1 = \sum_{s \in S} p_s \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I'_s} \sum_{m \in M_{wts}} time_{wmivts} \right] \quad (1)$$

$$\max f_2 = \sum_{s \in S} p_s \left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I'_s} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts}} \sum_{v \in TR} d'_{wicmvt}}{d_{ict}} \right] \right] \quad (2)$$

$$\sum_{w \in W} \sum_{v \in TR} \sum_{j \in I'_s \setminus \{i, n_s + k + 1, \dots, n_s + 2k\}} \sum_{m \in M_{wts}} x_{wjmvt} \geq 1 \quad \forall s \in S, i \in I'_s, t \in T \quad (3)$$

$$\sum_{w \in W} \sum_{v \in TR} \sum_{j \in I'_s \setminus \{i, 1, \dots, k\}} \sum_{m \in M_{wts}} x_{wjmvt} \geq 1 \quad \forall s \in S, i \in I'_s, t \in T \quad (4)$$

$$\sum_{i \in I'_s} x_{wwimvt} - \sum_{i \in I'_s} x_{wi(k+n+w)mvts} = 0 \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (5)$$

$$\sum_{j \in I'_s \setminus \{i\}} x_{wjmvt} - \sum_{j \in I'_s \setminus \{i\}} x_{wijmvts} = 0 \quad \forall s \in S, i \in I'_s, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (6)$$

$$\sum_{i \in I_s} \sum_{j \in I_s \setminus \{i\}} x_{wijmvt} \leq \left(\sum_{i \in I_s} x_{wwimvt} \right) Mbig \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (7)$$

$$\sum_{i \in I'_s \setminus \{n_s+k+1, \dots, n_s+2k\}} \sum_{j \in I_s \setminus \{i\}} \sum_{c \in C} wh_c x_{wijmvt} d'_{wjcmvt} \leq wcap_{wmts} \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (8)$$

$$\sum_{i \in I'_s \setminus \{n_s+k+1, \dots, n_s+2k\}} \sum_{j \in I_s \setminus \{i\}} \sum_{c \in C} v_c x_{wijmvt} d'_{wjcmvt} \leq vcap_{wmts} \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (9)$$

$$time_{wm(n+k+w)(v-1)ts} + LT_{wmts} + w_{wmw}jts + \sum_{c \in C} \alpha_c d'_{wjcmvt} - (1 - x_{wwjmvt})Mbig \leq time_{wmjvt} \quad \forall s \in S, j \in I_s, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (10)$$

$$time_{wmivts} + w_{wmi}jts + \sum_{c \in C} \alpha_c d'_{wjcmvt} - (1 - x_{wijmvt})Mbig \leq time_{wmjvt} \quad \forall s \in S, i \in I'_s \setminus \{n_s+k+1, \dots, n_s+2k\}, j \in I'_s \setminus \{1, \dots, k\}, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (11)$$

$$time_{(j-n-k)mjvt} \leq h_{(j-n-k)mts} \quad \forall s \in S, j \in \{n_s+k+1, \dots, n_s+2k\}, m \in M_{wts}, t \in T, v \in TR \quad (12)$$

$$inv'_{wct} = inv_{wcts} + inv'_{wc(t-1)s} - \sum_{i \in I_s} \sum_{m \in M_{wts}} \sum_{v \in TR} d'_{wicmvt} \quad \forall s \in S, c \in C, t \in T, w \in W \quad (13)$$

$$de_{icts} = d_{icts} + de_{ic(t-1)s} - \sum_{w \in W} \sum_{m \in M_{wts}} \sum_{v \in TR} d'_{wicmvt} \quad \forall s \in S, i \in I_s, c \in C, t \in T \quad (14)$$

$$d'_{wjcmvt} \leq d_{jcts} \sum_{i \in I'_s \setminus \{n_s+k+1, \dots, n_s+2k\}} x_{wijmvt} \quad \forall s \in S, j \in I_s, c \in C, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (15)$$

$$\sum_{j \in S} x_{wwjmvt} \leq 1 \quad \forall s \in S, m \in M_{wts}, v \in TR, t \in T, w \in W \quad (16)$$

$$time_{wmwvts} \geq time_{wm(n+K+W)(v-1)ts} \quad \forall s \in S, m \in M_{wts}, v \in TR, t \in T, w \in W \quad (17)$$

$$time_{wmwvts} \leq time_{wm(n+K+w)vts} \quad \forall s \in S, m \in M_{wts}, v \in TR, t \in T, w \in W \quad (18)$$

$$\sum_{t \in T} \sum_{v \in TR} \sum_{w \in W} \sum_{m \in M_{wts}} \sum_{i \in W, i \neq w} \sum_{j \in I_s} x_{wijmvt_s} = 0 \quad \forall s \in S \quad (19)$$

$$\sum_{t \in T} \sum_{v \in TR} \sum_{w \in W} \sum_{m \in M_{wts}} \sum_{i \in I_s} \sum_{j \in \{n_s+k+1, \dots, n_s+2k\}, j \neq n_s+k+w} x_{wijmvt_s} = 0 \quad \forall s \in S \quad (20)$$

$$x_{wijmvt_s} \in \{0,1\} \quad \forall s \in S, i \in I'_s, j \in I'_s, m \in M_{wts}, t \in T, w \in W, v \in TR \quad (21)$$

$$inv'_{wct_s} \geq 0 \quad \& \text{integer} \quad \forall s \in S, w \in \{1, \dots, k\}, c \in C, t \in T \quad (22)$$

$$de_{ict_s} \geq 0 \quad \& \text{integer} \quad \forall s \in S, i \in I_s, c \in C, t \in T \quad (23)$$

$$d'_{wicmvt_s} \geq 0 \quad \& \text{integer} \quad \forall s \in S, i \in I_s, c \in C, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (24)$$

$$time_{wmivt_s} \geq 0 \quad \forall s \in S, i \in I_s, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (25)$$

The Proposed mathematical model has two objective functions. The First objective function is to minimize total of time to reach the damaged areas. The Second objective function is to take account of fairness in the commodity distribution and maximum total of delivered commodities to actual request in the damaged areas. Limitation (3) shows that for each damaged area in each period, minimum one warehouse or another damaged area has been founded. Limitation (4) shows that each damaged area in each period goes to central warehouse or other damaged area. Limitation (5) is related the beginning and end of each travel route and which indicates that started from the central warehouse and is finished by the same central warehouse. Limitation (6) shows that input and output of each region in each period is equal and guarantees route unity. Limitation (7) indicates that a region when a vehicle leaves the corresponding warehouse. Limitation (8) guarantees weight capacity of vehicle. Limitation (9) suggests volume limit of vehicle. Limitations (10) and (11) show the relationship between arrival times to damaged area and prevent to create sub tour (Ulrich et al., 2010). Limitation (12) shows limitation in the vehicle performance time duration vehicle m related to central warehouse w , in period t and shows that arrival time to central warehouse by vehicle should be less than its performance time duration. Limitation (13) determines relationship between inventories in different periods. Limitation (14) specifies transferred request from one period to another period for the damaged areas. Limitation (15) shows when damaged area j by vehicle m in travel v delivers commodity from period t which vehicle m in travel v enters from period t to point j . Limitation (16) shows a vehicle related to the central warehouse which probably may not be used. This is likely to occur when request volume in one period is low. Limitation (17) shows travel time unity; this means that When a travel reaches the end the next travel starts. Limitation (18) sequences number of travels and if there is no sequence, the prior travel has been done. Limitation (19) shows that the vehicle leaves warehouse which is not belonged to it. Limitation (20) shows when a vehicle ends up to warehouse $n+k+w$ and starts corresponded warehouse w . limitations (21) to (25) define permissible limit of decision variables.

2.6 Robust Optimization Model

In this study, Mulvey robust optimization model is used (Mulvey et al., 1995). Based on this model, the designed model is converted to the following format.

$$\begin{aligned}
 \min f_1 = & \sum_{s \in S} p_s \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_s} \sum_{m \in M_{wts}} time_{wmivts} \right] \\
 & + \lambda_1 \sum_{s \in S} p_s \left[\left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_s} \sum_{m \in M_{wts}} time_{wmivts} \right. \right. \\
 & \left. \left. - \sum_{s' \in S} p_{s'} \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_{s'}} \sum_{m \in M_{wts'}} time_{wmivts'} \right] \right] + 2\theta_{1s} \right]
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 \max f_2 = & \sum_{s \in S} p_s \left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I_s} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts}} \sum_{v \in TR} d'_{wicmvts}}{d_{icts}} \right] \right] \\
 & + \lambda_2 \sum_{s \in S} p_s \left[\left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I_s} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts}} \sum_{v \in TR} d'_{wicmvts}}{d_{icts}} \right] \right. \right. \\
 & \left. \left. - \sum_{s' \in S} p_{s'} \left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I_{s'}} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts'}} \sum_{v \in TR} d'_{wicmvts'}}{d_{icts'}} \right] \right] \right] + 2\theta_{2s} \right]
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 & \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_s} \sum_{m \in M_{wts}} time_{wmivts} \right. \\
 & \left. - \sum_{s' \in S} p_{s'} \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_{s'}} \sum_{m \in M_{wts'}} time_{wmivts'} \right] \right] \\
 & + \theta_{1s} \geq 0 \qquad \forall s \in S
 \end{aligned} \tag{28}$$

$$\left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I_s} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts}} \sum_{v \in TR} d'_{wicmvts}}{d_{icts}} \right] \right] \quad \forall s \in S$$

$$- \sum_{s' \in S} p_{s'} \left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I_{s'}} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts'}} \sum_{v \in TR} d'_{wicmvts'}}{d_{icts'}} \right] \right] + \theta_{2s}$$

$$\geq 0$$

$$\theta_{1s} \geq 0, \theta_{2s} \geq 0 \quad \forall s \in S \quad (30)$$

(3)-(25)

2.7 Linearization of the proposed Model

Objective function (27) is non-linear, so auxiliary variable y_{tcs} is defined to be approved in limitation (32). Now, objective function (27) is defined as equation (31) and two limitations (32) and (33); therefore, second objective function is linear (Lai and Hwang, 1992).

$$\max f_2 = \sum_{s \in S} p_s \left[\sum_{t \in T} \sum_{c \in C} y_{tcs} \right]$$

$$+ \lambda_2 \sum_{s \in S} p_s \left[\left[\sum_{t \in T} \sum_{c \in C} y_{tcs} - \sum_{s' \in S} p_{s'} \left[\sum_{t \in T} \sum_{c \in C} y_{tcs'} \right] \right] + 2\theta_{2s} \right] \quad (31)$$

$$y_{tcs} \leq \frac{\sum_{w \in W} \sum_{m \in M_{wt}} \sum_{v \in TR} d'_{wicmvts}}{d_{icts}} \quad \forall s \in S, i \in I, c \in C, t \in T \quad (32)$$

$$y_{tcs} \geq 0 \quad \forall s \in S, i \in I, c \in C, t \in T \quad (33)$$

$x_{wijcmvts} d'_{wjcvmvts}$ Sentence is non-linear in limitations (8) and (9) and the following processes are used to convert them to a linear model.

$$XX_{wijcmvts} \leq d'_{wjcvmvts} \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts} \\ t \in T, c \in C, v \in TR, w \in W \quad (34)$$

$$XX_{wijcmvts} \leq Mbig \ x_{wijcmvts} \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts} \\ t \in T, c \in C, v \in TR, w \in W \quad (35)$$

$$XX_{wijcmvts} \geq d'_{wjcvmvts} - Mbig(1 - x_{wijcmvts}) \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts} \\ t \in T, c \in C, v \in TR, w \in W \quad (36)$$

$$XX_{wijcmvts} \in Integer \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts}, t \in T, c \in C, v \in TR, w \in W \quad (37)$$

Therefore, a multi-objective linear planning model is obtained.

$$\begin{aligned} \min f_1 = & \sum_{s \in S} p_s \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_s} \sum_{m \in M_{wts}} time_{wmivts} \right] \\ & + \lambda_1 \sum_{s \in S} p_s \left[\left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_s} \sum_{m \in M_{wts}} time_{wmivts} \right. \right. \\ & \left. \left. - \sum_{s' \in S} p_{s'} \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_{s'}} \sum_{m \in M_{wts'}} time_{wmivts'} \right] + 2\theta_{1s} \right] \right] \end{aligned} \quad (38)$$

$$\max f_2 = \sum_{s \in S} p_s \left[\sum_{t \in T} \sum_{c \in C} y_{tcs} \right] + \lambda_2 \sum_{s \in S} p_s \left[\left[\sum_{t \in T} \sum_{c \in C} y_{tcs} - \sum_{s' \in S} p_{s'} \left[\sum_{t \in T} \sum_{c \in C} y_{tcs'} \right] \right] + 2\theta_{2s} \right] \quad (39)$$

$$\sum_{w \in W} \sum_{v \in TR} \sum_{j \in I_s \setminus \{i, n_s+k+1, \dots, n_s+2k\}} \sum_{m \in M_{wts}} x_{wjimvts} \geq 1 \quad \forall s \in S, i \in I_s, t \in T \quad (40)$$

$$\sum_{w \in W} \sum_{v \in TR} \sum_{j \in I_s \setminus \{i, 1, \dots, k\}} \sum_{m \in M_{wts}} x_{wijmvt} \geq 1 \quad \forall s \in S, i \in I_s, t \in T \quad (41)$$

$$\sum_{i \in I_s} x_{wwimvts} - \sum_{i \in I_s} x_{wi(k+n+w)mvt} = 0 \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (42)$$

$$\sum_{j \in I_s \setminus \{i\}} x_{wjimvts} - \sum_{j \in I_s \setminus \{i\}} x_{wijmvt} = 0 \quad \forall s \in S, i \in I_s, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (43)$$

$$\sum_{i \in I_s} \sum_{j \in I_s \setminus \{i\}} x_{wijmvt} \leq \left(\sum_{i \in I_s} x_{wwimvts} \right) Mbig \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (44)$$

$$\sum_{i \in I_s \setminus \{n_s+k+1, \dots, n_s+2k\}} \sum_{j \in I_s \setminus \{i\}} \sum_{c \in C} wh_c XX_{wijcmvts} \leq wcap_{wmvts} \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (45)$$

$$\sum_{i \in I_s \setminus \{n_s+k+1, \dots, n_s+2k\}} \sum_{j \in I_s \setminus \{i\}} \sum_{c \in C} v_c XX_{wijcmvts} \leq vcap_{wmvts} \quad \forall s \in S, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (46)$$

$$\left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_s} \sum_{m \in M_{wts}} time_{wmivts} \right. \quad \forall s \in S \quad (58)$$

$$\left. - \sum_{s' \in S} p_{s'} \left[\sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I_{s'}} \sum_{m \in M_{wts'}} time_{wmivts'} \right] \right] + \theta_{1s} \geq 0$$

$$\left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I_s} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts}} \sum_{v \in TR} d'_{wicmvts}}{d_{icts}} \right] \right. \quad \forall s \in S \quad (59)$$

$$\left. - \sum_{s' \in S} p_{s'} \left[\sum_{t \in T} \sum_{c \in C} \min_{i \in I_{s'}} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wts'}} \sum_{v \in TR} d'_{wicmvts'}}{d_{icts'}} \right] \right] \right] + \theta_{2s} \geq 0$$

$$y_{tcs} \leq \frac{\sum_{w \in W} \sum_{m \in M_{wt}} \sum_{v \in TR} d'_{wicmvts}}{d_{icts}} \quad \forall s \in S, i \in I_s, c \in C, t \in T \quad (60)$$

$$XX_{wijcmvts} \leq d'_{wjcmvts} \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts}, t \in T, c \in C, v \in TR, w \in W \quad (61)$$

$$XX_{wijcmvts} \leq Mbig \ x_{wijmvts} \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts}, t \in T, c \in C, v \in TR, w \in W \quad (62)$$

$$XX_{wijcmvts} \geq d'_{wjcmvts} - Mbig(1 - x_{wijmvts}) \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts}, t \in T, c \in C, v \in TR, w \in W \quad (63)$$

$$x_{wijmvts} \in \{0,1\} \quad \forall s \in S, i \in I'_s, j \in I'_s, m \in M_{wts}, t \in T, w \in W, v \in TR \quad (64)$$

$$inv'_{wcts} \geq 0 \quad \& \text{integer} \quad \forall s \in S, w \in \{1, \dots, k\}, c \in C, t \in T \quad (65)$$

$$de_{icts} \geq 0 \quad \& \text{integer} \quad \forall s \in S, i \in I_s, c \in C, t \in T \quad (66)$$

$$d'_{wicmvts} \geq 0 \quad \& \text{integer} \quad \forall s \in S, i \in I_s, c \in C, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (67)$$

$$time_{wmivts} \geq 0 \quad \forall s \in S, i \in I_s, m \in M_{wts}, t \in T, v \in TR, w \in W \quad (68)$$

A robust programming approach to bi-objective optimization model in the disaster relief logistics response phase

$$\theta_{1s} \geq 0 \quad \forall s \in S \quad (69)$$

$$\theta_{2s} \geq 0 \quad \forall s \in S \quad (70)$$

$$y_{tcs} \geq 0 \quad \forall s \in S, c \in C, t \in T \quad (71)$$

$$XX_{wijcmvts} \geq 0 \text{ \&integer} \quad \forall s \in S, i \in I_s, j \in I_s, m \in M_{wts}, t \in T, c \in C, v \in TR, w \in W \quad (72)$$

2.8 Global Criterion to Solve Multi-Objective Problems

Suppose below multi-objective model

$$\max \setminus \min [f_1(x), \dots, f_k(x)] \quad (73)$$

$$g_i(x) \geq 0$$

Each of the objective functions of model is independent and is optimized on limitations. Suppose, independent optimized answers resulted by each problem which are shown through $f_i^*(x)$

Below, the model is formed.

$$\min F(x) = \left[\sum_{i=1}^k w_i \left| \frac{f_i^*(x) - f_i(x)}{f_i^*(x)} \right| \right] \quad (74)$$

$$g_i(x) \geq 0$$

In this model, w_i represents the importance of the corresponding objective function which is specified by a decision maker. Therefore, solving single-objective model leads to solve primary multi-objective model (Rao, 1996 and Zeleny, 1982).

3. Case Study

To increase the accuracy, the model is applied by small-sized problem and expert opinions in research and operation field. For this objective, the researchers supposed that the problem has one scenario and there is one commodity in kind 2 according to table (8) to distribute between four damaged areas and shown requests in the table (2). The time interval between different areas and inventories in the central warehouse is shown in Tables (1) and (3), respectively. Supposed, numbers of vehicles in different periods are respectively two, three, and three.

Table 1. Time of movement of commodities between central warehouse and damaged areas (min)

		Central depot	Area 1	Area 2	Area 3	Area 4
Period 1	Central depot	0	102	135	205	80
	Area 1	102	0	70	220	125
	Area 2	135	70	0	284	186
	Area 3	205	220	284	0	100
	Area 4	80	125	186	100	0
Period 2	Central depot	0	80	130	175	60
	Area 1	80	0	65	195	110
	Area 2	130	65	0	255	168
	Area 3	175	195	255	0	85
	Area 4	60	110	168	85	0
Period 3	Central depot	0	65	108	125	55
	Area 1	65	0	60	180	95
	Area 2	108	60	0	225	150
	Area 3	125	180	225	0	78
	Area 4	55	95	150	78	0

Table 2. The amount of demand for commodities in the damaged areas in different periods (* 10)

	Area 1	Area 2	Area 3	Area 4
Period 1	300	100	140	61
Period 2	200	250	100	200
Period 3	300	300	100	150

Table (4) shows for first period, vehicle 1 in first travel moves toward with 1500 commodity unit and first moves to region 4 and then goes to region 3 and goes back to central warehouse, based on table (1), The best route was selected. According to the demand of region 1, the vehicle No 2 can only cover that region and return to warehouse. According to the travel durations of both vehicles, vehicle 2 goes back sooner to the central warehouse, so for the second travel it choose regions 1 and 2 for covering the demands. Based on table 1, this is the best rout also.

Table 3. The amount of inventories of central warehouse at various periods

Period 1	Period 2	Period 3
450	1100	310

Table 4. Results of the proposed model to a problem with small size

period	vehicle	tour	path	Vehicle inventory(*10)
Period 1	1	1	0-4-3-0	150
	2	1	0-1-0	156
	2	2	0-1-2-0	144
Period 2	1	1	0-1-0	158
	2	1	0-2-0	160
	3	1	0-4-0	157
	1	2	0-1-2-0	160
	2	2	0-2-0	72
	3	2	0-4-3-0	160
	3	3	0-3-0	33
Period 3	1	1	0-1-0	156
	2	1	0-2-0	158
	3	1	0-4-3-0	150
	1	2	0-1-2-0	46

Based on the second objective function and fair distribution, the best state is to maximize request. Table 5 shows all damaged regions which have received 75.01% of their requests. Analyzing other periods shows that the model has not good performance. After solving model, first objective function is 4194 minutes and second objective function is 2.71.

Table 5. Cover percent of the damaged area at various periods

Period 1	Period 2	Period 3
75.01%	100%	59.95%

3.1 Explaining Case Study

In this section, the results of solving the model are explained as a case study. In this case study, the information related to South Khorasan is considered and geographical situation is shown in figure (1).

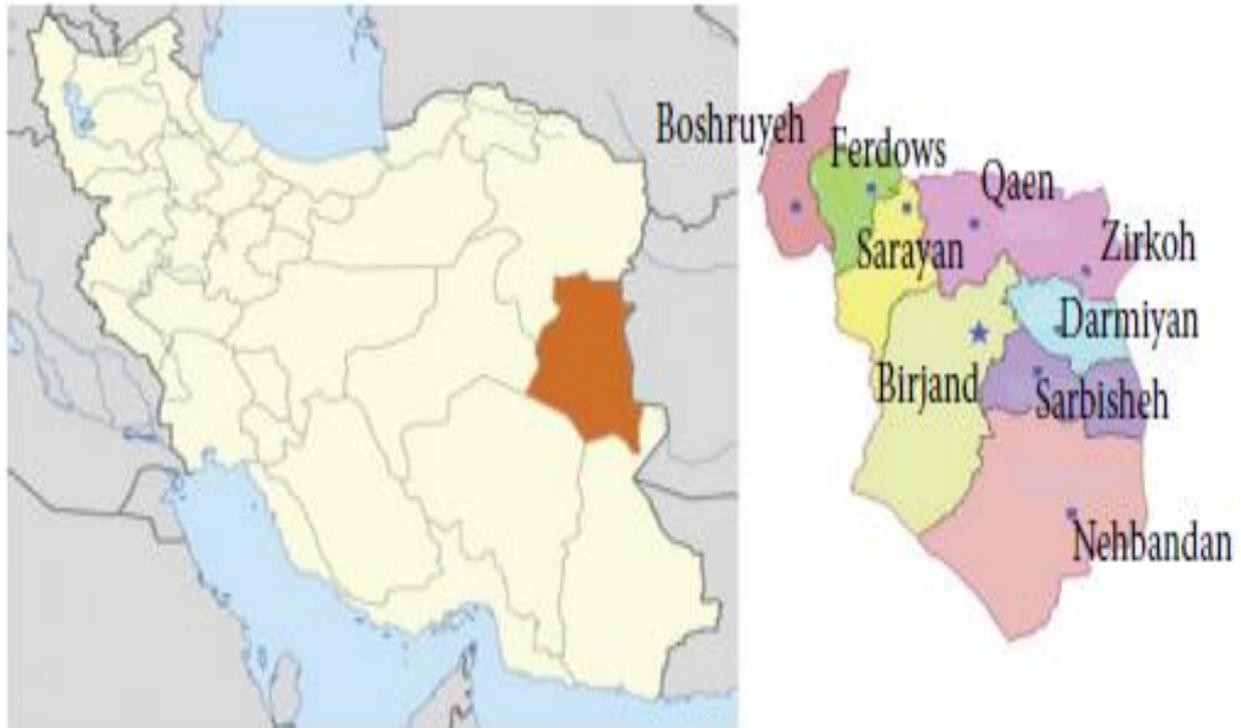


Figure 1. Geographical map of South Khorasan province

In this model, there are two central warehouses in Birjand and Qaen and 7 cities including: Sarbisheh, Nehbandan, Darmiyan, Zirkoh, Ferdows, Sarayan and Boshruyeh which were considered as damaged areas of the problem. Also, there are 4 different scenarios possibility 0.4, 0.2, 0.1 and 0.3 which its data are provided.

Information like displacement time between regions in different periods by different vehicles are considered based on information of experts and request related to damaged regions in different scenarios. In this study there are two central warehouses which send relief commodities for damaged regions. The model is suitable for number of planning period equal to 3 periods and 72 hours. It is supposed, the time of vehicle availability in each period is 8 hour.

In this study, $\lambda_2 = 1$ and $\lambda_1 = 1$ are considered. This amount shows weight parameter to difference between average of function and amount of function in each scenario. Table (6) shows volume and weight of kind of commodities. In table (7), the number and capacity of different vehicles in the central warehouses are shown. Table (8) shows inventory in different periods in central warehouses. In table (9), request of the damaged regions or regional warehouses in different scenarios are shown.

Table 6. Weight, volume and time of unloading

Commodity type	weight (kg)	volume (m ³)	Unloading time (min)
Commodity 1	0.5	0.002	0.02
Commodity 2	1.5	0.005	0.02

Table 7. Number and capacity of available vehicles in central warehouses at different periods

period	warehouse	vehicle	Capacity of weight (kg)	Capacity of volume (m ³)
1	1	1	15000	60
	2	1	15000	60
2	1	1	15000	60
	2	2	18000	75
3	1	1	10000	50
		2	15000	60
		3	18000	75
	2	1	15000	60
		2	2	15000

Table 8. The amount of inventories of central warehouse at different periods (* 100)

	commodity	Warehouse 1 (Birjand)	Warehouse 2 (Qaen)
Period 1	1	177	211
	2	101	129
Period 2	1	283	162
	2	169	113
Period 3	1	241	263
	2	183	167

Table 9. The demand for commodities in the damaged areas in different periods (* 100)

scenario	period	commodity	Area 1	Area 2	Area 3	Area 4	Area 5	Area 6	Area 7
			Zirkoh	Nehbandan	sarbisheh	sarayan	darmian	boshruyeh	ferdows
1	1	1	273	30	20	31	229	30	41
		2	167	35	24	33	157	34	44
	2	1	231	28	19	31	226	26	40
		2	162	31	21	30	148	28	42
	3	1	225	25	18	28	219	22	35
		2	153	27	21	28	147	26	38
2	1	1	120	40	30	0	300	0	0
		2	60	20	15	0	150	0	0
	2	1	140	50	40	0	300	0	0
		2	70	25	20	0	150	0	0
	3	1	100	40	30	0	280	0	0
		2	50	20	15	0	140	0	0
3	1	1	200	0	0	40	0	40	50
		2	120	0	0	25	0	25	25
	2	1	180	0	0	35	0	35	50
		2	100	0	0	22	0	22	25
	3	1	160	0	0	32	0	35	45
		2	100	0	0	20	0	22	23
4	1	1	100	25	20	30	100	40	50
		2	50	12	10	15	50	20	25
	2	1	120	28	24	34	120	46	58
		2	60	14	12	17	60	23	29
	3	1	100	20	22	32	100	42	48
		2	50	10	11	16	50	21	24

3.2 Solving Model Steps

The Steps of the proposed models are as follows:

- ✓ The First step: modeling two-objective planning model
- ✓ The Second step: determining inaccurate data of objective function and limitation coefficients with triangular possibility distribution
- ✓ The Third step: determining inaccurate limitations to new certain limitations based on equations (28) to (33)
- ✓ The Fourth step: solving proposed model using global standard method

Proposed model was solved by Lingo11 software on computers Intel(r) core(tm) i3 CPU M330@2.13Ghzand4G RAM and results are shown in tables (10) and (11). In order to implement fourth step, each of objective function should be solved independently and optimized amount should be determined. Then the global standard method was used with 0.3 coefficient for first objective function and 0.7 coefficient for second objective function as well as $r=1$, model is changed to single-objective problem. It was found that reach time to the damaged region is more important and are determined elites. In table (10), for each vehicle, the first and last number of list shows the number of central warehouse that vehicle start its travel from and go back to that.

As it is shown in Table (10), it could move from the central warehouse on a travel in one period and spend several damaged regions and it could go back to the same warehouse after covering regional warehouse requests and do next travel if needed. After solving first objective function for 6849 minute and for second objective function 4.58 have been estimated. The results of fist

scenario are shown in tables 10 and 11. In table (10), efficiency of a vehicle in a travel from one period is shown. It is probable that inventory of central warehouse is lower than requests, so all request are not met and based on fair distribution, the damaged regions should deliver commodities equally.

Table (11) shows percent of its requests which have been delivered in related period. For example, in the first period, damaged region 1 delivers 61.6% of corresponding request to commodity kind 1 and 45.5% of request related to commodity kind 2. Based on tables (10) and (11), the damaged regions should deliver fairness in different travels. Indeed, in order to maximize level of coverage and fairness, coverage percent for all damaged regions should be equal.

Table 10. The results of the proposed model (scenario 1)

period	Central warehouse	vehicle	tour	path	Amount of commodity 1 in vehicle(*100)	Amount of commodity 2 in vehicle(*100)
1	1	1	1	1-5-3-2-1	177	40
			2	1-5-1	0	61
	2	1	1	2-4-7-6-1-2	141	53
			2	2-1-2	70	76
2	1	1	1	1-5-1-1	35	88
			2	1-1-1	46	29
	2	1	1	1-5-3-2-1	202	52
			2	2-4-7-6-2	72	61
	2	1	1	2-1-2	90	52
			2	2-1-2	90	52
3	1	1	1	1-3-2-1	36	39
			2	1-5-1	182	38
	1	2	1	1-5-1-1	23	106
			2	2-4-7-6-2	72	74
	2	1	1	2-1-2	164	45
			2	2-1-2	0	48

Table 11. Coverage percent of regional warehouses demand in different periods (scenario 1)

period	commodity	Area 1 Zirkoh	Area 2 Nehbandan	Area 3 sarbisheh	Area 4 sarayan	Area 5 darmiyan	Area 6 boshruyeh	Area 7 ferdows
1	1	61.6	63.3	65	64.5	63.3	63.3	63.4
	2	45.5	48.5	45.8	48.4	46.4	47	47.7
2	1	74	75	73.6	74.1	73.8	73	75
	2	61.1	61.2	61.9	60	60.8	60.7	61.9
3	1	83.1	84	83.3	85.7	83.1	86.3	82.8
	2	79	81.4	80.9	82.1	78.9	80.7	78.9

For example in first period, corresponding vehicle to first central warehouse moves toward regions 5, 3 and 2, in the second travel the vehicle moves toward region 5. During this travel, 63.3 % of requests in regions 5 and 2, 65% if request in region 3 for commodity kind 1 and 46.4 % of request of region 5, 45.8% of request of region 3 and 48.5 % of request of region 2 for commodity kind 2 are met.

Also, in the same period, corresponding vehicle moves toward central warehouse 2 and supplies regions 4, 7, 6 and 1 in the first travel and in second travel moves toward region 1, so 64.5% of

request of region 4. 63.4% of request in region 7, 63.3% of request in region 6 and 61.6% of request of region 1 are covered of one kind commodity. For regions 4, 7, 6 and 1 request of two kind commodities, they are 48.4%, 47.7%, 47% and 45.5% respectively.

4. Conclusion and further Studies

Logistic activities are most important actions in relief chain management. Respond phase and integrated planning can increase efficiency when there is a disaster. In this study, a two-objective planning model has been developed for vehicles routing in disaster relief logistic. This could leads to an effective planning during natural disasters like earthquake. The purposes of the model are to maximize the occurrence of met requests and minimize reach time to the damaged areas and finally lead to fairness commodity distribution. In this study, a brief review was done in this field and then problem solving, assumptions, and its purposes are presented. A number of issues pertaining to the proposed model such as uncertainty in amount of request, displacement time and multi-commodity, several central warehouse, several vehicles and multi-periodic have been studied.

Then, the proposed model has been solved by a global standard method. The most efficient routes have been found to supply services to the damaged areas, in addition, it can be concluded that by solving the model the level of coverage and victims satisfactions could be enhanced. It can be suggested that the use of a heuristic and meta-heuristic model can be used in future studies. Finally, a dynamic planning can be employed in future research.

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