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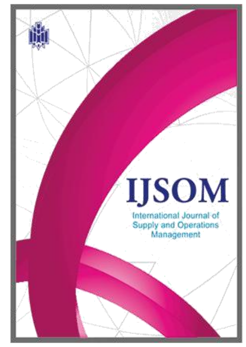

Inventory Optimization with Chance-Constrained Programming Under Demand Uncertainty

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Abstract

Uncertainty and variability in demand and supply processes make it difficult for companies to make inventory management decisions. In this study, a model is developed that will provide the maximum service level of a pharmaceutical warehouse under the budget constraint, taking into account stochastic demand. Due to stochastic demand, the chance constraint programming approach is used to achieve the desired service level at different levels. In this study, the problem of a pharmaceutical warehouse that supplies medicines to pharmacies and hospitals is considered as a real-world problem. The model is designed as a dynamic programming model based on periods. Since there are thousands of drugs in the pharmaceutical warehouse, as the number of products increases, it becomes difficult to find the appropriate solution in an acceptable time. The model is first solved as a mixed integer linear programming model in Lingo. A genetic algorithm (GA) approach is then proposed for large-scale problems. The simulation optimization method also applied to the problem and compared with the optimization method and GA. The GA approach yields better results in the shortest time as the number of periods increases. The developed integrated model demonstrated a numerical example in a pharmaceutical warehouse and was solved using three different approaches. This study is of great importance in terms of providing results that will enable managers to decide the amount of items they should keep in their warehouses by using their budgets in the most efficient way. Nine different scenarios have been derived with various chance constraint risk factors and budget values. Scenario analysis has revealed that the budget has a significant impact on the results at a 95% confidence level. If a pharmaceutical warehouse increases its budget by 10%, it can reduce its total annual inventory carrying costs by 70%.

Keywords: Inventory Management; Stochastic Demand; Chance-Constrained Approach; Genetic Algorithm.

Introduction

Inventory management is an important aspect discussed in the literature, especially in the field of supply chain management. The aim is to identify and implement activities that will increase the profit of the business by minimizing all inventory costs (Ahmadi et al., 2020). The primary goal is to increase customer service levels while minimizing associated inventory costs. Inventory models aim to answer fundamental questions about when and how much to order on an ongoing basis. This involves determining optimal stock levels to achieve desired service levels. Inventory management ensures that sufficient stock is maintained to meet service level targets. Effective inventory control in the face of conflicting objectives is important for the proper use of significant capital. Whether deterministic or probabilistic, demand structure plays a vital role and real-life scenarios often have a probabilistic demand structure. Cost and lead time are other critical factors in inventory models. Lead time, whether random or fixed, significantly affects the solution. Models may also need to include constraint conditions, such as budget limitations or storage space limitations.

Effective stock management can be achieved by thoroughly understanding the needs of the industry and choosing the appropriate stock strategy. Keeping excess stock allows us to eliminate uncertainties and, as a result, increase customer

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satisfaction. However, as a result, it requires more capital and may result in idle stocks due to constraints such as obsolescence or shelf life. On the other hand, in real life, delivery time and demand may vary. Due to this uncertainty, if the desired products cannot be provided at the desired time, customers may become dissatisfied (Qiu et al., 2022). To prevent such a situation and to ensure they can meet demand, businesses often turn to keeping excess inventory, also known as "safety stock."

At the beginning of the study, interviews were held with company managers and the necessary data for the problem was collected. The company under study is a large-scale wholesaler that purchases medical supplies from manufacturers, stocks them, and fulfills orders from pharmacies and hospitals. Figure 1 shows the flow of product from the suppliers to patients.

Figure 1. The supply chain in pharmaceutical items marketing

Inventory management establishes the required level of inventory to offset the cost of holding in the event that excessive inventory is kept, and the lost sales costs associated with inventory shortages. Multi-item storage, however, is a problem that frequently arises in real inventory systems. For example, department stores, large distribution chains, and wholesalers all oversee an inventory system that carries a variety of goods. Moreover, these products might have distinct features. They could differ in terms of cost, weight, size, lead time, shelf life, and/or storage capacity. However, over time, those items with different qualities will need to use the same resources, like money or storage space. Therefore, in these situations, inventory management becomes more difficult.

Whether a demand is deterministic or stochastic, whether it involves multiple products or just one product, can influence the approaches taken to solve inventory management problems. The majority of solutions to deterministic systems and single product problems have been found with mathematical techniques. Unfortunately, in multi-item inventory problems, the solution space's size leads to complexity, making it impossible to find optimal values. As a result, it is stated that multi-item inventory systems use evolutionary techniques such as heuristics and metaheuristics to reach near-optimal solutions within an acceptable time frame (Gómez-Rocha et al., 2021). Our problem is mathematically modeled and solved using a metaheuristic approach—the genetic algorithm (GA)—because it is a multi-item inventory problem. Most combinatorial optimization problems (COPs) are unsolvable. Because the solution run-time for these problems grows exponentially, they are referred to as NP-hard problems in the literature. An NP-hard problem is a type of computational problem that is known for its high level of difficulty. This means that any problem in NP can be translated or reduced to an NP-hard problem in polynomial time. In practical terms, an NP-hard problem is one for which there is no known algorithm that can solve it efficiently—that is, in polynomial time relative to the size of the input. Solving NP-hard problems often requires the use of approximation algorithms or heuristic methods, as finding an exact solution quickly is typically not feasible for large instances (Li et al., 2020). Genetic algorithms (GAs) are employed to address the computational complexity of large instances of the problem (Ahmadi et al., 2019). The approach for solving multi-item, multiple periods (s, S) inventory models is developed in this paper. It determines the best order-up-to inventory level and reorder point given budget constraints and an uncertain demand structure. There are two steps in the method utilized to solve this issue. First, the proposed inventory policy is solved with the Lingo optimization program considering a constant demand structure. The (s, S) values obtained with the mathematical model are defined as initial solution values in the GA approach where stochastic demand is considered. Also, OptQuest search engine is used to show the GA performance. OptQuest optimization tool is a heuristic based optimization tool integrates meta-heuristics from tabu search, neural networks, and scatter search into a single search heuristic (Ekren & Arslan, 2020; Kleijnen & Wan, 2007). In stochastic processes, we can get expected results, not exact results. Chance constrained programming (CCP) deals with the case where constraints have random variables. CCP is a type of programming that ensures that the constraints are satisfied with a given probability (Taha, 2007).

The rest of this essay is structured as follows: A review of the literature is provided in Section 2. In Section 3, the problem formulation and statement are developed. Section 4 provides an illustrative experimental study. Finally, Section 5 offers conclusions and suggestions for additional study.

Literature review

This paper proposes a model that minimizes the costs for a wholesaler dealing with a large number of parts by treating demand as a random variable. (s, S) inventory policy that is considered in this study. The (s, S) inventory policy has been widely studied and utilized in literature. It was first analyzed by Sivazlian (1974), who assumed that the entire backlog of unmet demand could be managed by the (s, S) policy. The (s, S) policies have been applied to perishable items by several researchers. Gürler and Özkaya (2008), Saracoglu (2023) have contributed to this area, examining the unique challenges of managing perishable inventory. Veinott (1965) investigated multi-period inventory models primarily applied to production lot-sizing issues, focusing on dynamic nonstationary multi-item inventory models. Movahed and Zhang (2015) aimed to determine optimal inventory policy parameters for decision-makers facing variability in demand and lead time in supply chains. Noordhoek et al. (2018) presented a simulation-optimization (SO) model using scatter search to find (s, S) inventory policies for multi-echelon distribution networks, considering fill rate constraints. Ghalebsaz-Jeddi et al. (2004) studied a multi-item stochastic inventory system with shortages, backorders, and budgetary restrictions, focusing on payment timing.

Several methods exist for addressing uncertainties in optimization problems, with stochastic programming being one of the most widely utilized models (Shaw et al., 2016). Charnes and Cooper (1959) introduced chance-constrained programming, providing methodologies for decision-making under uncertainty. Kundu and Chakrabarti (2012) examined a multi-product continuous review inventory system under stochastic environments, incorporating budget constraints. Aggarwal (2018) proposed a supply chain configuration model for FMCG products, integrating chance constraints to address environmental concerns. Armagan Tarim and Kingsman (2004)), Rossi et al. (2008) and Xiang et al. (2023) investigated the stochastic dynamic production/inventory lot-sizing problem with service-level constraints. Xiang et al. (2018) introduced heuristics for computing non-stationary (s, S) policy parameters, addressing single item and single stocking location problems. Guerrero Campanur et al. (2018) introduce an inventory-location model for designing a four-echelon supply chain network. It considers warehouse and plant locations, transportation costs, inventory costs, and supplier selection.

Chen and Rossi (2021) emphasized the importance of cash constraints in inventory management for small retailers, proposing the (s, C(x), S) policy for near-optimal performance. Saracoglu (2023) focused on SO model for multi-product (s, S) inventory policies under stochastic demand conditions.

Žic et al. (2023) identified the relationships and equations needed to optimize transportation activities in supply chains operating under the (R, s, S) periodic review policy. Modibbo et al. (2022) presented a multi-objective optimization model for managing multi-product inventory and production planning under uncertainty, integrating fuzzy and stochastic elements. Xu et al. (2019) used a SO model for perishable items with stochastic demand, highlighting its advantages despite long iterative running times. Ali et al. (2021) constructed an approach on the problem as a multi-objective mixed-integer fuzzy nonlinear programming model to minimize the total cost of the organization. They found that the costs incurred at three different echelons, as demonstrated through multiple case scenarios in the analysis, were lower than the actual costs of the organization. Hooshangi-Tabrizi et al. (2022) addressed perishable inventory management under demand uncertainty with a two-stage robust optimization model. Perera and Sethi (2023) provided a comprehensive review of discrete-time stochastic inventory problems with fixed ordering costs, affirming the optimality of (s, S) policies.

In recent years, the most prominent topic of discussion is how to reduce environmental pollution. Environmental pollution causes significant damage to the earth, including climate change and global warming (Barman, Roy, et al., 2023). Nowadays, the urgent need for environmental sustainability has introduced an added layer of complexity to logistical decision-making (Das et al., 2024). A major global concern for industries is reducing their carbon emissions. We can give some researches about carbon emissions related with the supply chain and inventory problems (Barman et al. 2022; Paul et al. 2021; Pervin et al. 2023). Table 1 highlights the differences between the key contributions of the present work and those of several existing studies.

Table 1. Comparisons of previously proposed models

Authors	Demand pattern	Multi items	Multi period	Budget constraints	Carbon emission	Multi-echelon	Solution technique
(Xiang et al. 2018)	Stochastic		*				MILP, Heuristic
(Guerrero Campanur et al., 2018)	Stochastic					*	MINLP, MILP
(Chen & Rossi, 2021)	Stochastic		*				Heuristic
(Modibbo et al., 2022)	Stochastic	*	*	*		*	Multi-objective optimization, Chance-constrained programming
(Žic et al., 2023)	Stochastic		*		*	*	Numerical simulation, Symbolic regression
(Ekren & Arslan, 2020)	Stochastic		*			*	Simulation-based optimization
(Das et al., 2024)	Uncertain				*	*	Neutrosophic Multi-Objective Model
(Barman, Pervin, et al., 2023)	Price dependent				*	*	Game Theory Model
(Ali et al., 2021)	Uncertain	*	*			*	Fuzzy Non-Linear Programming
(Barman et al., 2022)	Price dependent		*		*	*	Fuzzy Goal Programming
(Barman, Roy, et al., 2023)	Price dependent		*		*	*	Neutrosophic Environment Model
(Pervin et al., 2023)	Composite		*		*		Mathematical Programming
(Paul et al., 2021)	Price dependent		*				Concave Fractional Programming
This Paper (2024)	Stochastic	*	*	*			MILP, Genetic Algorithm, Simulation optimization (OptQuest)

The aim of this paper is to introduce the MILP model and meta-heuristic method for computing near-optimal (s, S) policy parameters. Genetic Algorithm (GA) approach is offered and evaluated in this study because of the stochastic search engine. Among the various metaheuristics, this one is the most widely used algorithm to solve uncertainty and large-scale problems (Hiassat et al., 2017). GA was first proposed by (Holland, 1973) and models the evolution of biological systems according to Darwin's "survival of the fittest" theory (Goldberg & Samtani, 1986). In comparison to other heuristics, it is also one of the algorithms that is utilized in inventory problems the most. GA initially served to solve continuous non-linear optimization issues. It has since been successfully used to solve combinatorial optimization issues as well, including issues with inventory management, supply chain and transportation, job shop scheduling, traveling salesmen, and vehicle routing (Gen, M., & Cheng, 1997). A further explanation of evolution and the role of natural selection in the survival of the fittest can be found in the contemporary field of genetics. GAs selects parents to reproduce from a population, which is an initial set of random solutions. Every member of the population is referred to as a chromosome. In natural systems, a single chromosome or multiple chromosomes work together to form the entire genetic blueprint for an organism's development and functioning. A genotype is the entire set of chromosomes, and a phenotype is the resultant organism. The contribution of our work is to develop a solution methodology for a real-life problem of a pharmaceutical warehouse operating under uncertain demand conditions. A dynamic mathematical model is proposed, which can easily adapt to demand and price changes occurring within the planning horizon. The proposed model can adapt to seasonal demand fluctuations. We develop a GA that is capable of providing good solutions within a reasonable computational time.

Problem statement and formulation

Proposed mathematical method

A mathematical model is developed to implement the (s, S) inventory policy, which is one of the continuous review control inventory policies, in order to establish an efficient inventory policy for a large pharmaceutical distribution warehouse. (s, S) inventory policy determines the order-up-to inventory level that should be in the warehouse. When the stock level reaches the specified reorder point, an order is placed to reach the maximum stock level. In this study, multiple products with variable demand and budget constraints are considered. Due to the large number of alternative warehouses where pharmacies can meet the demand for pharmaceuticals and the criticality of the items, the situation of backlogging for demand was not considered. The objective of this study is to minimize the total annual inventory cost with budget constraint. However, due to the high level of competition, it is desirable to meet the demand at a high service satisfaction level. Under these assumptions, MILP is constructed to decide the variables (s, S) by considering deterministic demand structure. The following notations are used in the model.

Index Set:

I	Number of items, $i = 1,2,\dots,N$.
T	Planning horizon, $t = 1,2,\dots,T$.

Parameters:

$d_{i,t}$	demand for item *i* in period *t*.
mud_i	expected average demand in lead time period for item *i*.
LT_i	lead time for item *i*.
upc_i	purchase price per unit for item *i*.
uoc_i	ordering cost per unit for item *i*.
usc_i	shortage cost per unit for item *i*.
uhc_i	inventory holding cost per unit for item *i*.
$y_{i,0}$	inventory level for item *i* in period 0.
ul_s_i	the upper bound of the reorder point for item *i*.
ul_S_i	the upper bound of the order-up-to level for item *i*.
Bgt	budget.
G	big number.

Decision variables:

$y_{i,t}$	inventory level for item *i* at the end of period *t*.
$y_{i,t}^{+}$	positive inventory level for item *i* in period *t*, $y_{i,t}^{+} = max\{0, y_{i,t}\}$
$y_{i,t}^{-}$	inventory shortage for item *i* in period *t*, $y_{i,t}^{-} = max\{0, -y_{i,t}\}$
$x_{i,t}$	order quantity for item *i* in period *t*.
S_i	Order-up-to level for item *i*.
s_i	reorder point for item *i*.
$V_{i,t}$	1, if inventory level of item *i* is less than s_i at the beginning of period *t*, 0 otherwise.
$Z_{i,t}$	1, if inventory level of item *i* at the ending of period *t* is positive, 0 otherwise.

The objective of this study is to minimize annual total inventory cost (TAIC). To achieve this, we consider the total cost of holding (THC), ordering (TOC), shortage (TSC), and purchasing (TPC) over the planning horizon. Objective functions are defined in the equations as follows:

a. Total holding cost for every period of the planning horizon THC:

$$= \sum_{i=1}^{N} \sum_{t=1}^{T} uhc_i . y_{i,t}^{+} \quad (1)$$

b. If replenishment order is opened in the planning horizon, ordering cost must be added in the cost function TOC:

$$= \sum_{i=1}^{N} \sum_{t=1}^{T} V_{i,t} . uoc_i \quad (2)$$

c. Total shortage cost for every period of the planning horizon TSC:

$$= \sum_{i=1}^{N}\sum_{t=1}^{T} usc_i . y_{i,t}^{-} \tag{3}$$

d. Total purchasing cost for the planning horizon TPC:

$$= \sum_{i=1}^{N}\sum_{t=1}^{T} upc_i . x_{it} \tag{4}$$

The equation below presents the objective function of the multi-item, multiple periods (s, S) policy model:

$$Min\ TAIC\ (S,s) = THC + TOC + TSC + TPC \tag{5}$$

Constraints:

The constraints of the model include inventory balance equations, order-up-to inventory level (S) and reorder point level (s), boundaries and linearization functions. These constraints are explained in detail as below equations:

<u>Inventory balance equations:</u>

<u>If</u>

$y_{i,t-1} \le s_i$	$\forall i \in N, \forall t \in T$	(6)
then		
$S_i - y_{i,t-1}^{+} - x_{i,t} = 0$	$\forall i \in N, \forall t \in T$	(7)
$x_{i,t}$ - $y_{i,t}^{+} + y_{i,t}^{-} + y_{i,t-1}^{+}$- $d_{i,t} = 0$	$\forall i \in N, \forall t \in T$	(8)
else		
$y_{i,t-1}^{+} - y_{i,t}^{+} + y_{i,t}^{-} - d_{i,t} = 0$	$\forall i \in N, \forall t \in T$	(9)
$y_{i,t-1}^{+}$– $s_i + 1 \le \mathrm{G}.(1 - V_{i,t})$	$\forall i \in N, \forall t \in T$	(10)
s_i– $y_{i,t-1}^{+} \le \mathrm{G}.V_{i,t}$	$\forall i \in N, \forall t \in T$	(11)
$x_{i,t}$ - $y_{i,t}^{+} + y_{i,t}^{-} + y_{i,t-1}^{+}$- $d_{i,t} \ge -\mathrm{G}.(1 - V_{i,t})$	$\forall i \in N, \forall t \in T$	(12)
$x_{i,t}$ - $y_{i,t}^{+} + y_{i,t}^{-} + y_{i,t-1}^{+}$- $d_{i,t} \le \mathrm{G}.(1 - V_{i,t})$	$\forall i \in N, \forall t \in T$	(13)
$y_{i,t-1}^{+} - y_{i,t}^{+} + y_{i,t}^{-} - d_{i,t} \ge -G.\left(1 - V_{i,t}\right)$	$\forall i \in N, \forall t \in T$	(14)
$y_{i,t-1}^{+} - y_{i,t}^{+} + y_{i,t}^{-} - d_{i,t} \le G.\left(1 - V_{i,t}\right)$	$\forall i \in N, \forall t \in T$	(15)
$x_{i,t} \le G.V_{i,t}$	$\forall i \in N, \forall t \in T$	(16)
$S_i - y_{i,t-1}^{+} - x_{i,t} \ge -\mathrm{G}.(1 - V_{i,t})$	$\forall i \in N, \forall t \in T$	(17)
$S_i - y_{i,t-1}^{+} - x_{i,t} \le \mathrm{G}.(1 - V_{i,t})$	$\forall i \in N, \forall t \in T$	(18)

<u>Inventory level constraints:</u>

$y_{i,t}^{-} \le \mathrm{G}.\mathrm{Z}(i,t)$	$\forall i \in N, \forall t \in T$	(19)
$y_{i,t}^{+} \le \mathrm{G}.(1 - \mathrm{Z}(i,t))$	$\forall i \in N, \forall t \in T$	(20)
$y_{i,t} = y_{i,t}^{+} - y_{i,t}^{-}$	$\forall i \in N, \forall t \in T$	(21)

<u>Budget constraints:</u>

$$\sum_{i=1}^{I}\sum_{t=1}^{T} x_{i,t}.upc_i \le Bgt \tag{22}$$

Bound constraints:

$$S_i \leq ul\_S_i \qquad \forall i \in N \qquad (23)$$

$$s_i \leq ul\_s_i \qquad \forall i \in N \qquad (24)$$

$$S_i - s_i \leq 0 \qquad \forall i \in N \qquad (25)$$

$$V_{i,t}, Z_{i,t} \ \in \{0,1\} \qquad \forall i \in N, \forall t \in T \qquad (26)$$

$$s_i, S_i, x_{i,t}, y_{i,t}, y_{i,t}^{+}, y_{i,t}^{-} \quad \geq 0, integer \qquad \forall i \in N, \forall t \in T \qquad (27)$$

According to constraints (6), if the stock level at the beginning of a period is lower than the “s”, an order quantity $x_{i,t}$ is placed. Constraint (7) asserts that the order quantity should be equal to the difference between the maximum inventory level and beginning inventory level. Inventory balance equation is enforced by Constraint (8). If constraint (6) is not applied, then constraint (9) should be evaluated for calculation of the ending inventory level. Conversely, if the stock level at the beginning of a period is greater than the reorder point, no order is placed, and the inventory balance equation is provided by Constraint (9). In order to model these relations linearly, Constraints (10) to (18) were added for the constructed ILP model. Constraint (19) defines the positive ending inventory level as $y_{i,t}^{+} = max\{y_{i,t}, 0\}$. Constraint (20) illustrates the negative ending inventory level as $y_{i,t}^{-} = max\{-y_{i,t}, 0\}$. According to these inventory levels, inventory holding cost and shortage cost should be calculated. Constraints (23) and (24) ensure that the decision variables are less than or equal to the upper bound. Constraint (25) makes S_i greater than the reorder point (s_i). Constraint (26) defines binary variables. Constraint (27) ensures that the decision variables are integers and non-negativity.

Methodology

In this article, five steps were used to solve the stochastic inventory problem for wholesaler company and shown as Fig. 2. Each step is described below:

1. Data Analysis: The demand data and unit purchasing costs were collected from the company. Initially, an ABC analysis was conducted based on annual usage, and items were selected for implementing the proposed model.
2. Mathematical Modeling: The proposed mathematical model was developed considering a deterministic demand structure. This model was solved using varying numbers of periods, and the model's performance was evaluated based on solution time. The mathematical model cannot produce optimal results in reasonable amounts of time when the number of products and periods rises.
3. Stochastic Inventory Model: The deterministic model was transformed into a stochastic inventory model, and a Genetic Algorithm was employed to solve the model. The demand for selected items was analyzed to determine the relevant distribution using Arena Input Analyzer software.
4. Genetic Algorithm: The proposed mathematical model was also solved using a Genetic Algorithm, and the solutions were compared with the optimization results. The Genetic Algorithm approach provided near-optimal solutions within a reasonable timeframe.

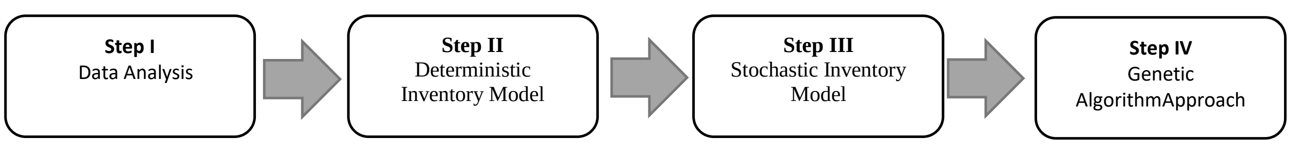


Figure 2. Steps of the solutions

When considering random demand, it is impossible to predict how much order should be open prior to the occurrence of this random event. In the case of random demand, the equations written in deterministic form are transformed into stochastic form as follows. In deterministic form, demand and end-of-period inventory level are known, whereas in stochastic form they can be expressed in terms of expected values.

$$\min E[TAIC] = \sum_{i=1}^{N}\sum_{t=1}^{T} uhc_i . E\{y_{i,t}^{+}\} + \sum_{i=1}^{N}\sum_{t=1}^{T} V_{i,t} . uoc_i + \sum_{i=1}^{N}\sum_{t=1}^{T} usc_i . E\{y_{i,t}^{-}\} + \sum_{i=1}^{N}\sum_{t=1}^{T} upc_i . x_{it} \tag{28}$$

$$E\{y_{it}\} = y_0 + \sum_{t=1}^{T} x_{it} - \sum_{t=1}^{T} E\{d_{it}\}\ for\ all\ i \qquad \forall i \in N, \forall t \in T \tag{29}$$

$$\text{Prb}\{E[y_{it}] \geq 0\} \geq \propto \qquad \forall i \in N, \forall t \in T \tag{30}$$

Constraint (28) shows the total expected annual inventory cost under the random demand event. Constraint (29) gives the expected value of the inventory balance equation. Constraint (30) shows the chance constraint that ensures that the expected end-of-period inventory level is not out of stock at the specified confidence level. The desired level of satisfaction is indicated with ∝. In the stochastic modelling stage, depending on the flexibility of the ordering process, order quantities can be adjusted at each time period t=, …,T based on known demand realizations up to that point. This scenario can be formulated as a multistage stochastic program, allowing for optimal decisions to be made at each time period using the available realizations of random data (Shapiro & Ruszczyn, 2003). In this study, different scenarios consistent with the distribution defined for the random variable demand have been generated, and decision variables that will minimize cost have been identified. The obtained results were tested using simulation optimization, and the expected cost values were calculated. The second key category of models in stochastic programming is Chance-Constrained Programming (CCP). Like general stochastic programs, CCP models incorporate random variables with known distributions. This model was solved with GA approach in Frontline Solver software. The Evolutionary Solver is based on GA. It relies heavily on controlled random search. Fig. 3 illustrates an example of chromosome representation. The first line in the chromosome structure shows the order-up-to level (S_i) and the second line shows the reorder point levels (s_i) for all items.

S_i	300	400	500
s	50	100	70

Figure 3. Chromosome structure of (s,S) inventory model

The objective function values of the solutions can be determined by their fitness values. It is crucial to calculate these values accurately as the selection method relies on them. A penalty function is utilized in this study to account for constraints. To minimize cost, the fitness value is calculated by adding the penalty function to the objective function. The penalty method is the most used method to deal with infeasible solutions in GA for constrained optimization problems. GA generates a sequence of parameters to be tested objective function and constraints. The model was solved, and then objective function was evaluated, and checked the constraints whether violated or not. If the constraints are violated, the solution is infeasible, and no fitness is obtained. This approach is adequate, but many practical issues come with significant constraints. Identifying a feasible point can be nearly as challenging as locating the optimal one. Consequently, we typically aim to gather information from infeasible solutions, potentially by adjusting their fitness ranking based on the extent of constraint violation. This is the approach taken in a penalty method. In a penalty method, a constrained optimization problem is converted into an unconstrained one by imposing a cost or penalty for any violations of constraints. For example, the original constrained problem in minimization form can be written as follows:

$minimize\ f(x)$
Subject to $h_i(x) \geq 0, i = 1,2, \dots, n$
where x is an m vector
The model transforms to the unconstrained form:

$$minimize\ f(x) + r . \sum_{i=1}^{n} \theta[h_i(x)]$$

where $\theta = penalty\ function, r = penalty\ coefficient$
Goldberg (1989) usually used the square the violation of the constraint, $\theta[h_i(x)] = h_i^2(x)$, for all violated constraints i.

Experimental Study

In this study, a wholesaler warehouse in Turkey that sells regional pharmaceutical items is considered. From the 2022 sales information, 4 products in class A, whose sales are in accordance with the normal distribution, are selected. Pharmacies may request one or more pharmaceutical items from a distributor. If the distributor is out of stock for any of these items, the pharmacy sources its supply from another distributor with whom it cooperates. Due to the intense competition in the industry, it is not feasible for the pharmacy to wait for orders to be fulfilled. The pharmacy incurs a loss if an order cannot be met five times. (Nahmias, 2008) demonstrated how Type I and Type II service levels are calculated. The Type I service level is the probability, denoted as α, of not experiencing a stockout during the specified lead time. The Type II service level measures the rate, β, at which the proportion of demands is met from stock. In this study, we estimated the stockout probability using the Type I service level. We considered a one-year planning horizon, divided into 52 periods (weeks). The average lead time for drugs from the manufacturing company is one week. If a pharmacy loses a customer after five unmet requests during the planning period, it is expected to meet demand in 47 out of 52 instances. Therefore, α is calculated as $\alpha = 47/52 = 0.90$. The objective of the warehouse is to find the order quantity to meet the demand for one year under budget constraint and the reorder point to meet 90% customer satisfaction. Firstly, the problem is considered as small size and the performance of the mathematical model compared with GA and OptQuest optimization tool. The mathematical model is solved in Lingo optimization software. For more products and more periods, GA approach was used in Frontline Solver software. ARENA 16.0 version was used to solve the OptQuest search heuristic algorithm. Solutions are obtained by using Intel® Core TM i7-1051U CPU running at 1.80GHz and 8.00 GB of RAM computer. The data of the products used in this research are given in Table 2.

Table 2. First data for products used in this study.

Items	1	2	3	4
EOQ_i (unit)	1829	1173	1832	4876
upc_i ($)	12.83	15.30	13.17	3.04
uhc_i ($)	0.09	0.11	0.09	0.02
poc_i ($)	30	30	30	30
usc_i ($)	14.14	16.83	14.49	3.34
$y_{i,0}$ (unit)	0	0	0	0
d_{it}	N~ (107,46)	N~ (49,19)	N~ (96,45)	N~ (161,40)

The resulting optimum was compared with optima estimated by two other meta-heuristic algorithms. The percentage difference was calculated using with $\text{GAP}\ (\%) = \left((\text{GA Solution} - \text{MILP Solution})\right) / (\text{MILP Solution})\ x\ 100$ to compare the results obtained from the MILP model and GA approach. The comparison results of the optimization model and the other two methods are given in Table 3. The simulation model of the inventory problem has been set up in Arena simulation software to obtain the OptQuest optimization results. The decision variable values obtained with the optimum result are defined as initial data. A total of 150 simulation runs were conducted for the OptQuest optimization study. In order to further improve the results, if the simulation study is considered for 1000 iterations over 24 periods, the simulation study time increases from 370 seconds to 2139 seconds. However, the total cost decreases from $35,867.7 to $32,986. The results indicate that GA provides optimum or near-optimum results in a shorter time.

Table 3. Comparative results of the MILP and GA solution

Period no	Variables	Integers	Constraints	Total Inventory Cost ($)				Computational time CPU (sec)			
				MILP	Proposed GA	OptQuest	GAP (%)	MILP	Proposed GA	OptQuest	GAP (%)
4	46	22	60	5277.99	5277.99	6414.05	0	0.19	16.59	93	8631.57
8	78	42	113	10223.3	10223.3	15675.61	0	0.54	22.91	291	4142.59
12	110	62	165	14993.73	14993.73	18865.74	0	143	29.41	352	-79.43
24	206	122	321	31337.72	31089.37	35867.70	-0.79	3600*	126	370	-96.5

(*): After 1 hour the solution is stopped.

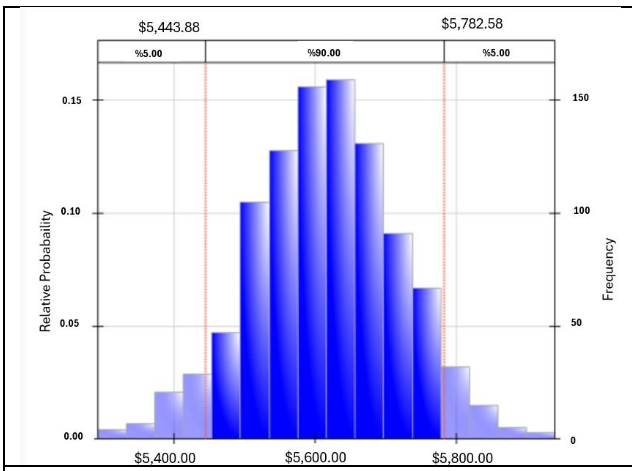
First, the case of 4 products and a budget of 210,000$ with a 90% probability of not being out of stock for all periods. Budget and stock level constraints are defined as chance constraints. Demand is entered as a normal distribution. The expected results obtained under these conditions using 100 populations and a mutation rate of 0.075 are summarized in Table 4.

Table 4. The results of Scenario 1

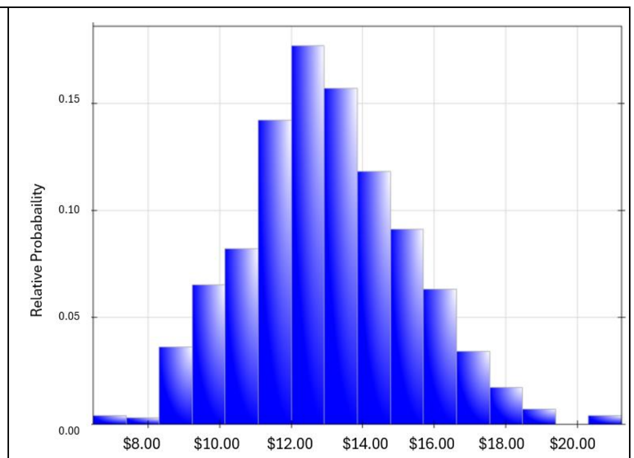
Items	1	2	3	4	Total
S_i	706	258	318	4,631	
s_i	229	17	0	254	
THC ($)	$1,807.74	$620.71	$480.32	$2,720.32	$5,629.09
TOC ($)	$300.00	$270.00	$390.00	$60.00	$1,020.00
TSC ($)	$0.00	$2,036.43	$13,994.44	$0.00	$16,030.87
TPC ($)	$72,705	$35,282	$54,445	$27,770	$190,202
TAIC ($)	$74,813	$38,209	$69,310	$30,551	$212,882
Service Level	1.00	0.95	0.81	1.00	0.94

Over a 52-week period, using the chosen values of S_i and s_i, a simulation with 5 replications of 1000 trials was conducted to evaluate whether the budget remains under $210,000 for 90% of the time in Scenario 1. The variability of budget utilization as a result of the simulation is illustrated in Fig. 5. Scenarios with different budget levels and probabilities were developed and tested with budgets of $210,000, $220,000, and $230,000. The results of these scenarios are presented in Table 5. Fig. 4 provides a summary of the expected cost information from Scenario 1. The total holding cost is projected to range between $5,443.88 and $5,782.58 at a 90% confidence level (Fig. 4(a)). If a budget of $220,000 can be maintained 90% of the time, the cost of stockouts is expected to range from $9,376.41 to $16,971.06 (Fig. 4(b)). The total purchasing cost could range from $181,770 to $221,877 at a 90% confidence level (Fig. 4(c)). Fig. 4(d) shows that the total cost expectation could range between $203,152.16 and $242,152.28.

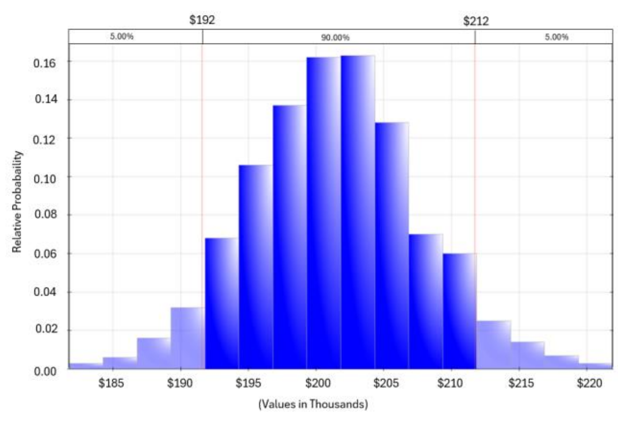
In the 2nd scenario, the budget of 210 thousand dollars is met 95% of the time. In the 3rd scenario, the case of meeting the budget with 95% probability was analyzed. In the 4th scenario, the change in costs in case of having a budget of 220 thousand dollars was analyzed. In the 7th scenario, the change in results was observed with a higher budget. In order to decide on the most appropriate inventory policy, 9 different scenario studies were conducted by considering 3 different levels for budget and chance constraint realization probability. For the budget constraint, three levels of 210, 220 and 230 thousand dollars were considered. As another factor, three levels of 90, 95 and 99% were considered as the probability of realization of these budget constraints. The results of all scenarios are given in Table 5. TIC shows the overall of the THC, TOC and TSC. As the budget level increases, the number of items purchased increases and as a result, the probability of not meeting the demand decreases. If a budget of 210 thousand dollars is required to be met at 90% level, it can be seen that under 1000 different demand scenarios, a maximum expected stockout cost of 21,242.56$ can be encountered. By increasing the budget to 220 thousand dollars, this amount would decrease to 14,389.99$. If the budget is 230 thousand dollars, the maximum expected stockout cost will be 3,624.12$.



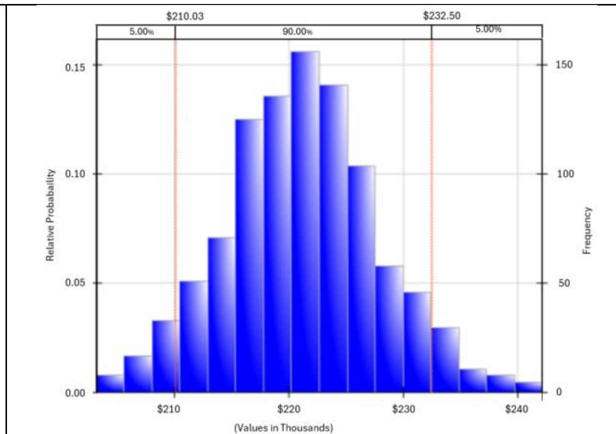
a) Expected THC for Scenario 1



b) Expected TSC for Scenario 1



c) Expected TPC for Scenario 1



d) Expected TAIC for Scenario 1

Figure 4. Expected cost functions for Scenario 1

Table 5. The results of the GA for different scenarios

Scenario	Budget	%chance	Cost criteria	Average ($)	Std.deviation ($)	Minimum ($)	Maximum ($)
S1	210000	90	TPC	201,459.00	6,235.00	181,770.00	221,877.00
			TSC	13,004.43	2,281.13	6,455.00	21,242.56
			TAIC	221,175.86	6,655.44	203,152.16	242,152.28
			TIC	19717.3	2270.81	13235.03	27942.34
S2	210000	95	TPC	193,556.00	4,767.00	176,201.00	212,574.00
			TSC	14,363.24	2,413.55	7,425.41	23,332.80
			TAIC	218,365.80	6,049.32	199,840.26	237,023.05
			TIC	24810.03	2407.41	18137.98	33711.06
S3	210000	99	TPC	198,733.00	4,364.00	183,312.00	216,443.00
			TSC	18,440.71	3,165.83	9,219.84	30,053.66
			TAIC	225,102.84	5,914.72	204,507.49	246,190.61
			TIC	26337.68	3160.85	16981.02	38018.77
S4	220000	90	TPC	204,169.00	5,253.00	186,419.00	220,336.00
			TSC	8,560.54	1,689.60	4,338.80	14,389.99
			TAIC	221,524.34	5,946.70	203,610.98	239,373.30
			TIC	17355.17	1681.53	13133.46	23044.65
S5	220000	95	TPC	204,061.00	5,331.00	186,477.00	220,843.00
			TSC	5,895.53	1,462.78	2,288.88	11,848.60
			TAIC	217,414.59	5,934.31	199,589.78	235,278.31
			TIC	13353.46	1449.8	9819.73	19190.61
S6	220000	99	TPC	204,032.00	5,413.00	186,837.00	221,832.00
			TSC	5,336.70	1,663.11	988.19	11,655.55
			TAIC	216,181.86	5,986.47	198,003.06	233,888.14
			TIC	12,150.20	1,647.75	7,904.04	18,255.37
S7	230000	90	TPC	207,767.00	6,007.00	189,310.00	227,303.00
			TSC	613.21	475.97	0.00	3,624.12
			TAIC	214,598.97	6,090.22	196,033.10	233,870.89
			TIC	6,831.98	468.88	6,069.31	9,703.95
S8	230000	95	TPC	211,509.00	6,158.00	191,971.00	228,858.00
			TSC	614.06	476.02	0.00	3,624.12
			TAIC	218,229.73	6,206.14	198,991.29	235,738.95
			TIC	8,823.46	249.08	8,378.46	10,094.65
S9	230000	99	TPC	208,090.00	5,948.00	188,530.00	226,981.00
			TSC	338.31	198.91	0.00	660.86
			TAIC	214,755.76	6,003.85	195,195.90	233,719.66
			TIC	6,033.55	199.49	5,630.57	6,272.60

TPC: Total Purchasing Cost; TSC: Total Shortage Cost; TAIC: Total Annual Inventory Cost. TIC: Total Inventory Cost

The effects of the models run at different levels on the results were analyzed by ANOVA analysis. Table 6 shows that both factors create a significant difference in the TAIC. The significant level value of the budget factor at 95% confidence level is 0.028 and the budget makes a statistically significant difference on total cost. The significance level of 0.855 for the three levels of the chance constraint shows that the percentage values have not an equal impact on total cost. Statistical analyses were conducted using Minitab® 21.4.2, and results were considered significant at $p < 0.05$.

Table 6. Analysis of variance for the effects on TAIC function

Factor	Type	Levels	Values
Budget	Fixed	3	210000, 220000, 230000
SL%	Fixed	3	90.00%, 95.00%, 99.00%

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Budget	2	189007660	94503830	10.01	0.028
Chance%	2	3073934	1536967	0.16	0.855
Error	4	37765247	9441312		
Total	8	229846840			

The change in total cost when all factors are considered is shown as the main effects plot in Fig. 5. The best inventory level can be achieved if the budget of 230 thousand dollars is not to be exceeded with 90% probability according to Fig.5.

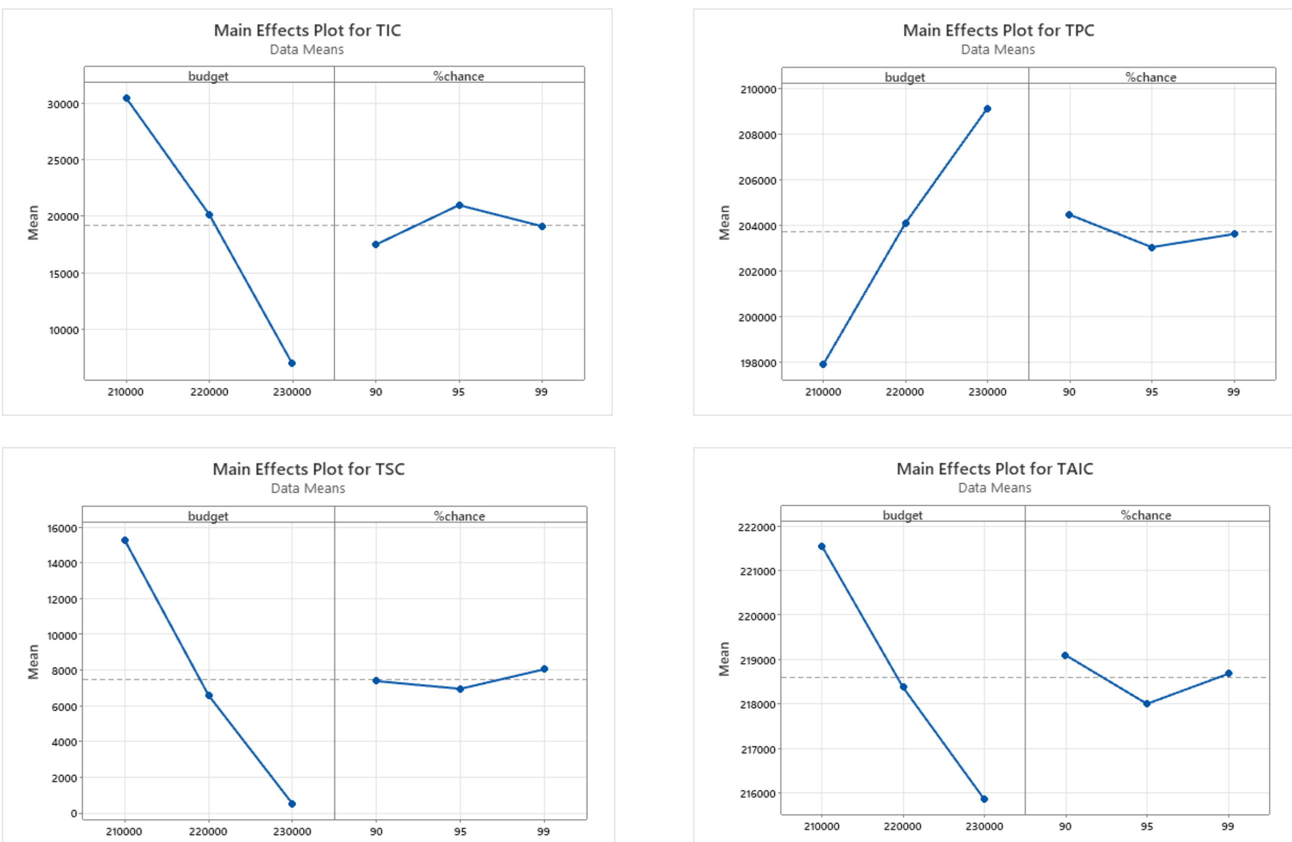


Figure 5. Main effects plot for TPC, TSC, TIC and TAIC by Budget - Chance constraint %

The company's current inventory management policy involves a weekly review of stock levels, with orders placed based on available budget, recent sales data from the past three months, and current promotions. Industry consultations have confirmed that calculating reorder points is highly effective. This study aims to offer a useful framework for companies, enabling efficient control over numerous products. Through ongoing collaboration with the company, we have identified substantial cost savings when our recommended purchasing strategies are implemented. For example, for item 1, the company's current approach resulted in a cost of $97,077, whereas our algorithm would have reduced this to $74,813. This underscores the need for a systematic approach, as the company currently encounters issues such as product spoilage due to limited shelf life and customer loss from budget constraints and suboptimal purchasing practices. The pharmaceutical sector's dynamic nature requires that any changes in input variables necessitate rerunning the algorithm to maintain optimal inventory management. However, initiating a study involving approximately 5000 products demands a considerable investment of time to generate and analyze the initial data. Implementing our proposed model can help companies achieve more efficient inventory management, optimize budget utilization, and enhance service levels, ultimately leading to reduced costs and improved customer satisfaction.

Managerial insights

The study presents a robust framework for optimizing inventory management in a pharmaceutical warehouse, addressing the complexities of stochastic demand. Key insights for managers include:

- The proposed model focuses on maximizing service levels within a budget constraint, crucial for maintaining customer satisfaction and operational efficiency. Managers should prioritize strategies that balance inventory holding costs against service level objectives.
- The use of mixed integer linear programming (MILP) and genetic algorithms (GA) demonstrates the importance of leveraging advanced optimization techniques for complex, large-scale inventory problems. Implementing such methods can provide near-optimal solutions within reasonable timeframes, essential for dynamic and high-demand environments.
- Managing a diverse inventory with various products having different costs, sizes, and demand patterns requires sophisticated models. The study's approach to multi-item inventory systems underscores the necessity of iterative and metaheuristic techniques in achieving feasible solutions.
- Recognizing the inherent uncertainty in demand, the model integrates chance-constrained programming to account for probabilistic constraints. This approach ensures that inventory decisions are robust under varying demand scenarios, helping managers to mitigate risks associated with stockouts and overstocking.
- The experimental study's scenario analysis provides valuable insights into the impact of different budget levels and service probabilities on total inventory costs. Managers can use similar analyses to evaluate various operational scenarios, enhancing decision-making under budgetary and service constraints.

By integrating these insights into their inventory management practices, managers can enhance their ability to maintain high service levels, optimize resource allocation, and respond effectively to demand uncertainties in the pharmaceutical industry.

Conclusion

In this study, an analysis is made to decide the optimal inventory policy for a large pharmaceutical distribution warehouse considering the budget constraint and variable demand structure by using chance constraint. The (s, S) model of continuous review control policies were constructed mathematically. As the number of products and the number of periods increase, we have found that it is not possible to find an optimal solution in a reasonable amount of time using MILP. Therefore, the GA approach was used since an appropriate solution could not be found at the desired time. The simulation optimization technique is also used to evaluate the performance of the GA, and GA shows better performance than the simulation optimization. This study will help to decide how much budget the company needs to reach the desired customer service level and the stock levels of the pharmaceutical items it sells.

There are two main limitations in this proposed study. First, this paper addresses the inventory problem of pharmaceutical items but does not include the concept of perishability restrictions, which should be addressed in future studies. The planning horizon has been set to one year, and since the lifetime of the items are longer than the planning horizon, they have not been included in this model. Additionally, it should be considered that there might be issues with selling pharmaceutical items that are nearing their expiration dates. Secondly, in the multi-period model, the potential price changes due to the effects of inflation have been ignored.

Pharmaceutical items consumption has been increasing over the years. Global healthcare expenditures accounted for approximately 9.8% of the global GDP in 2019, reaching $8.5 trillion (WHO 2021). Proper planning of these items is crucial for sustainability to reduce medical waste and prevent medication shortages (Ahmadi et al., 2022). This study addresses the stock policy of a pharmaceutical warehouse, which is a part of the supply chain. The proposed model is designed to be user-friendly and flexible, allowing the addition of new constraints. It will be beneficial for determining inventory levels in multi-item wholesalers.

This model can be further generalized in subsequent studies to take into account various constraints like capacity, lifetime, and service level, as well as the variable lead time. Furthermore, the model will be expanded to address supply chain management issues, incorporating multi-echelon components. Machine Learning techniques can be used to forecast demand of pharmaceutical items. For this type of problem, a multiple objective optimization method can be evaluated.

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