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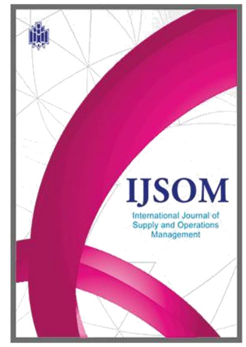

A Supplier-Stackleberg Game Model under Two Levels of Trade Credit Considering Default-Risk

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Abstract

In the context of a supply chain, individuals engage in cooperative efforts to optimize their financial gains. The present article represents a study of a supply chain model that incorporates a two-level trade credit policy. The model focuses on a scenario where a supplier offers a product with a market demand that is reliant on the credit duration. The study centers on the strategies employed by suppliers to motivate retailers to increase their orders through the use of trade credit, with the aim of maximizing their own profits. The analysis takes into account the influence of credit period-dependent default customers. A set of examples is provided and the sensitivity is studied to show the recommended technique. Finally, the equilibrium solution has been derived from the supplier Stackelberg game and examined using numerical examples in the scenario when the supply offers a higher credit than the retailer. The outcomes of the numerical illustrations indicate that a supplier can achieve greater profitability by electing an adequate range of credit periods. Furthermore, this study demonstrates a significant correlation between the duration of credit, market demand, and the profitability of both the retailer and supplier with the demand coefficient, the price involved in the model, and the coefficient of default risk.

Keywords: Two-Level Trade Credit; Delay in Payments; Default Risk; Supplier-Stackelberg.

1. Introduction

1.1. Motivation

In the last century, there have been many expansions to the traditional Economic Order Quantity (EOQ) and Joint Economic Lot Size (JELS) models. Glock (2012) undertook a thorough evaluation of the JELS literature. Incorporating credit policy into the EOQ and JELS models represents a highly notable advancement. Within a credit agreement, the purchaser has the option to defer payment in exchange for the immediate receipt of the goods. No interest will accrue during this period. As a result, the buyer may get interested in the sales money, while the vendor loses the interest collected during this period. If the buyer fails to pay after the grace period has expired, the vendor will impose interest on the outstanding sum. The vendor benefits from the allowed delay in payment in two ways: firstly, it attracts new buyers who observe it as a type of price discount, and secondly, it can be utilized as an alternative to reducing prices without encouraging other businesses to do the same and causing long-term price decreases.

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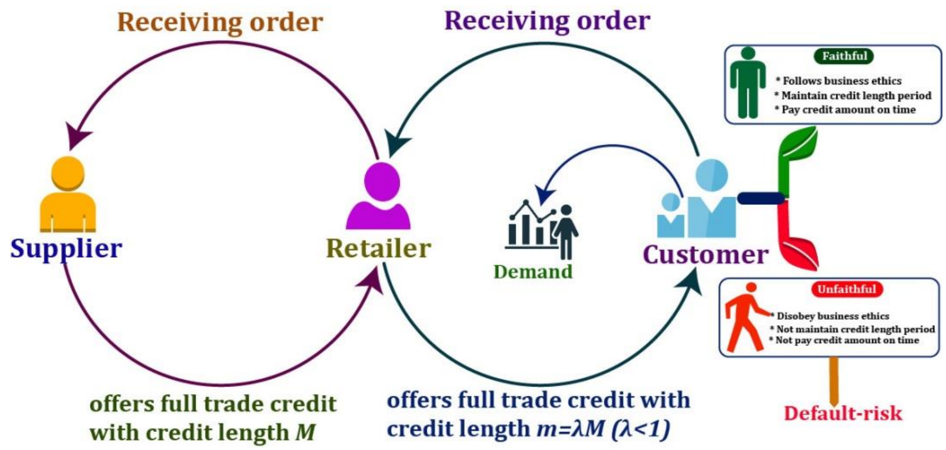

Figure 1. Flow-diagram of the proposed model.

Conversely, providing credit terms raises the vendor's chances of default while introducing an additional layer of expenditure. Early on, Shi and Zhang (2010) incorporated defaulters into the operational management decision model, where the fixed defaulters are the disadvantage. In practice, the supplier default risk increases with the length of the trade credit period to the retailer. In comparison to a five-year mortgage, the default risk associated with a fifteen-year mortgage is significantly larger. Some academicians in operation management have recently begun to pay attention to this form of comprehension. There are only a few (Teng and Lou, 2012; Chern et al., 2013; Lou and Wang, 2013) of the most renowned and up-to-date examples. Recently, Wu et al. (2017) developed under considering credit period demand and default risk under the Steckelberg game approach for the case where replenishment time is shorter than the credit length. Whereas they have not considered the point where the replenishment time is not greater than the credit length, this phenomenon is more realistic in business. Again, Shen and Liang (2009) suggest using a Stackelberg game model to determine the best ordering policy and delay-in-payments strategy, considering two levels of trade credit where the credit length influences the market demand linearly. However, they assume credit policy without taking default risk into account. So, this work removes the shortcomings mentioned earlier with proper justifications. This study will build upon prior research in the field to carry out new, relevant investigations.

1.2. Literature Review

Trade credit is commonly employed as a short-term business tactic in very competitive circumstances and substantially influences corporate operations. Several empirical studies have demonstrated the prevalence of credit policy over short-term bank lending in practical scenarios. Goyal (1985) conducted a comprehensive investigation on the impact of a particular allowable payment delay on a retailer's decision-making process regarding ordering. He showed that the retailer's economic lot-size and order cycle grew as a result of the permissible postponement in payments. Subsequently, other scholars have investigated the effects of this predetermined trade credit on the policy for replenishing inventory. Teng (2002) developed a straightforward analytical closed-form solution for Goyal's study by examining the disparity between wholesale and retail prices. Chang et al. (2003) expanded on Teng's research to create an economic order quantity (EOQ) model specifically for perishable products. This model incorporates credit policies that are based on lot sizes. Paul and Boden (2008) presented a concise overview of the existing research on the accessibility of trade credit and its main motives.

Moreover, prior studies mostly concentrated on examining the influence of credit alternatives on the retailer's ordering strategy in circumstances of consistent demand. The authors Teng et al. (2012) expanded the notion of fixed demand to include a demand that increases in a linear manner over time by integrating a credit policy. Previous studies neglected to account for the impact of price on customer demand. In pursuit of this objective, Shinn (1997) conducted an investigation on the matter of optimizing the selection of pricing and quantity for a merchant in order to maximize profits while also accommodating delayed payments. He illustrated the correlation between the buyer's order size and the credit length under the price sensitivity of demand. Chen and Kang (2010) worked to create integrated inventory models that took into account demand sensitivity to price and a credit policy with two levels. The researchers suggested a direct recursive solution approach by solving the integrated models and determining the optimal pricing and production/order strategy for the buyer. In a recent study, (Wu and Zhao, 2015) provided a theoretical foundation for an Economic Order Quantity (EOQ) model. This model considers a fixed degradation rate, inventory-dependent linearly increasing time-varying demand and trade credit. Several authors including (Moussawi-Haidar and Jaber, 2013; Chen et al., 2014; Yang et al., 2015) considered the degrading items in their research. Nevertheless, most of these models operate under the assumption that the duration of trade credit remains the same, and the retailer formulates its own strategy based on this assumption.

Conversely, the supply chain has thoroughly examined credit policy to increase profits by implementing strategies such as quantity discounts and price reductions. Specifically, Yang and Wee (2006) proposed a comprehensive model that considers the depreciation of goods and the demand that is influenced by the price. In addition, they implemented a policy on late payments and proposed a bargaining strategy to equitably distribute the extra profit between the seller and the buyer. The research done by Sarmah et al. (2008) examined the difficulty of efficiently coordinating a single supplier with several retailers, all operating on the same cycle length, via the use of trade credit. In a recent study, Wu and Zhao (2014a) proposed a cooperative model that incorporates trade credit and accounts for the demand, which is influenced by inventory levels and varies over time within a specified planning horizon. Multiple pertinent discussions have been conducted in the literature, such as those by (Teng et al., 2012; Giri and Maiti, 2013; Wu and Zhao, 2014b; Teng et al., 2014; Mondal et al., 2022, Pramanik and Maiti, 2019).

1.3. Research gap

However, there has been insufficient emphasis on determining the most advantageous credit duration from the supplier’s perspective. The policies of both the buyer and the seller were formulated by Abad and Jaggi (2003) in situations involving cooperation and non-cooperation. Granting a permissible delay did not affect the demand rate in their model. Jaggi et al. (2008) enhanced their model by demonstrating that an increase in the credit period leads to a higher demand, enabling the provider to determine the most effective credit and replenishment policy. However, they did not furnish information regarding the transactions between the supplier and the retailer; instead, their attention was solely directed towards the supplier. Su (2012) presented a replenishment method for an integrated inventory system that considers damaged items and allows for shortages while taking trade credit into account. The study conducted by Liu and Cruz (2012) introduces supply chain networks that incorporate corporate financial risks and trade credits within the framework of economic uncertainty. Purchasers often perceive the acceptance of trade credit or a brief extension of payment terms as a type of price reduction akin to receiving an interest-free loan from a supplier. The supplier will offer the trade credit to the retailers who are willing to boost their promotional activities. This trade credit aims to increase the size of retailers’ orders, stimulate potential demand, and attract a larger customer base. It is generally accepted that the supplier will earn a proportional increase in profit if the anticipated demand is met through tangible purchases. Nevertheless, no one in the field of operation management has provided rational arguments for it. Kim et al. (1995) presented a method to calculate the most advantageous price for the retailer and credit length for the supplier using a supplier Stackelberg technique. However, they did not explicitly include the credit period in their demand function. Teng and Lou (2012) conducted a follow-up research to investigate the increasing demand rate for loan length using the Stackelberg approach. Researchers are actively prioritizing the identification of the most effective trade credit approach. The aforementioned references establish the best credit duration from the credit provider’s point of view. Only a few articles from the viewpoint of games are now available in latest literature. Zhou and Zhou (2013) determined the supplier credit period through a supplier Stackelberg game in unconditional and conditional credit opportunity. In addition, the vendor’s optimal trade credit period was determined by Chern et al. (2013) using the model proposed by Lou and Wang (2013). The study also included compound interest rate and a relaxed lot-for-lot replenishment policy into a vendor-buyer Stackelberg model. Additional well-known recent works include those by (Dye et al., 2014; Shah and Cardenas-Barron, 2015; Wu et al., 2014), among others. However, these sources do not analyze the outcomes of a supplier-Stackelberg game in the presence of credit policy, also they do not consider the effects of decentralized decision-making in the absence of credit policy.

Table 1. Comparison of this study with other similar models

Article	Demand depends on	Demand Pattern	Credit Policy	Default risk	Default Risk Depends on	Game description
Teng et al. (2014)	Credit period	Exponential	One level	Yes	Credit period	No
Dye et al. (2014)	Credit period	Linear	One level	Yes	Credit period	No
Mondal et al. (2022)	Credit period	Exponential	Two-level	Yes	Credit Period	No
Pramanik and Maiti (2022)	Credit period	Exponential	Two-level	Yes	Fixed	No
Chern et al. (2013)	Credit period	Exponential	One level	Yes	Credit period	Stackelberg,Nash
Kim et al. (1995)	NA	Constant	One level	No	NA	Stackelberg
Wu et al. (2017)	Credit period	Exponential	One level	Yes	Credit period	Stackelberg
Shen and Liang (2009)	Credit period	Linear	Two level	No	NA	Stackelberg
This Paper	Credit period	Exponential	Two level	Yes	Credit period	Stackelberg

'NA' stands for Not Applicable

1.4. Contributions

Moreover, the advancements of this research work relative to the existing literatures is tabulated in Table-1. This study examines a two-level trade credit policy that considers the supplier-Stackelberg game when the credit period is associated with default and demand risk. In this scenario, we anticipate that the supplier possesses greater power or influence compared to the retailer. For example, a supplier possesses the capacity to establish its strategy and exert significant power over the adjacent grocery store. Thus, the dominating firm takes on the position of a market leader. Conversely, the tiny grocery store solely relies on the policies set by the big firm to formulate its market strategy. This model evaluates the disparities between the supplier-Stackelberg models incorporating trade credit. Next, examine the model to determine how profit and behavior vary depending on the trade credit period affects demand and default risk. In order to gain a deeper understanding of the interaction between the supplier and the customer in a non-cooperative Stackelberg business scenario, this model develops an EOQ (Economic Order Quantity) model for both parties that considers the following factors: (1) Sellers will allow retailers a grace period before they must make a payment. (2) Retailers will grant customers a grace period before they are required to make a payment. (3) A credit period affects market demand positively. (4) As the credit period lengthens, the likelihood of default rises. Within the framework of the non-cooperative Stackelberg equilibrium, we determine the specific criteria necessary to get the most favorable result for both the store and the buyer.

1.5. Advantages and limitations of this study

The following is a list of the advantages of this work:

- The model enables providers to choose the best credit policy by considering the risk of default. This can aid in optimizing revenues while lowering the risk of default.

- Suppliers or retailers can gain a competitive advantage over rivals by strategically offering different levels of trade credit. By providing favorable lending terms, they can draw in new clients or keep hold of current ones.
- Vendors can more effectively control their exposure to credit risk by including default risk in the model. By assessing customers' creditworthiness, they can modify credit terms, which in turn decreases the risk of defaults.
- This approach provides suppliers with the ability to adjust their credit policies by evolving market conditions or alterations in default risk. Adaptability can improve their competitiveness and profitability in the long run.

However, some limitations of this work are the model assumes a static environment, failing to account for the dynamic nature of supply chain interactions. In reality, supply chain dynamics are influenced by various internal and external factors that evolve continuously, making it challenging to capture them accurately in a static model. Although the model takes into account default risk, it may not comprehensively encompass the intricacies of risk management measures utilized by vendors and purchasers. Practical risk management involves factors like hedging, insurance, and diversification, which may not be accurately shown in the model.
The rest of the paper is structured as follows: in Section 2, we provide the notation and the underlying assumptions. Section 3 presents a non-cooperative inventory model that incorporates trade credit and is based on the supplier-Stackelberg game. Significant management phenomenon findings are offered at the end of the paper. In the last section, we will briefly review the paper's findings.

2. Notations and Assumption

The following notations are used, and assumptions are made, throughout the work to create the suggested model, following industrial norms and our incentives to develop this model.

2.1. Notations

(i) $I(t)$, inventory level at any instant t.
(ii) Q, the order quantity, decision variable.
(iii) T, replenishment time.
(iv) A_i, per order ordering cost in \$, i = s, r.
(v) h_r, unit time inventory holding cost in \$.
(vi) c, production cost per unit, \$/unit.
(vii) p, unit wholesale price in \$.
(viii) s, unit retail price in \$.
(ix) I_e, rate of interest offered by the bank per \$ per unit time.
(x) I_p, rate of payable interest per \$ per unit time to the bank.
(xi) Z_i, per unit time profit \$, i = s, r, sc.
For convenience, Subscript i is used to denote several participants; specifically, $i = s$ stands for the supplier, $i = r$ for the retailer, and $i = sc$ for the entire supply chain.

2.2. Assumptions

(i) Here, to encourage the retailer to place a large purchase, the supplier offers a credit term (M) on the entire order amount. To take advantage of this chance, the retailer also provides a credit duration ($m = \lambda M$) to the customers. When the credit term ends, retailers are required to pay interest to suppliers on the remaining balance at a rate I_p. During the grace period, they can earn interest on sales revenue at a rate I_e.
(ii) Generally, customers are considered defaulters if they do not pay their credit amount during the grace period.
(iii) In a business policy, the default credit risk is influenced by the credit term. Specifically, a longer credit period increases the credit risk, while a shorter credit period decreases it. Due to this reason, here, the default risk, F(m); ($0 \leq F(m) < 1$) is assumed as a function of the credit period (m) and is of the form:

$F(m) = 1 - e^{-bm}$
i.e., $F(M) = 1 - e^{-b\lambda M}$ (1)

(iv) Allowable payment delays draw in new customers who view these terms as a form of price reduction. According to literature [26, 27, 33], among other authors, demand rate is assumed to be a polynomial or exponential function of the trade credit period. The demand rate D can be clearly expressed as

$D(m) = Ke^{am}$
i.e., $D(M) = Ke^{a\lambda M}$ (2)

where positive real values K, a, λ were selected based on the expert's assessment for best fit of demand.
(v) Here the scenario $m \leq M \leq T$ has been considered since the retailer is unable to earn interest from this circumstance, $M < m$.

3. Mathematical formulation of the supplier-Stackelberg game model incorporating trade credit

In a Stackelberg equilibrium with two players, the retailer (the follower) determines its optimal Q to maximize its profit for any given credit policies M offered by the supplier (the leader). The supplier chooses the optimal M to maximize its profit after understanding the retailer's optimal Q (a function of M). Therefore, we must approach the Stackelberg problem in reverse, as shown below:

Step 1. The retailer seeks to maximize its profit to find its optimal Q for any given supplier credit period M.
Step 2. To maximize its own profits, the supplier imports the retailer's optimal Q into its profit function before determining its own optimal policies M. Lastly, the supplier's optimal policies M is used to determine the retailer's optimal Q. Figure 3 provides a visual depiction of the method described in the text.

3.1. Supplier's optimal decisions. The supplier's profit per unit time consists of the following elements:

(i) Sales revenue, $SR = pD$ (3)
(ii) Ordering cost, $OC = \frac{A_s}{T}$ (4)
(iii) Purchase cost, $PC = cD$ (5)
(iv) Payable interest, $IC = I_p cDM$ (6)

3.2. Retailer's optimal decisions. For any credit period M offered by the supplier, the retailer's problem is to find the ordered quantity Q^* that maximizes his/her annual profit, which is made up of the following costs and revenues.

(i) Sales revenue, $SR = Ds[1 - F(m)] = Dse^{-bm}$ (7)
(ii) Ordering cost, $OC = \frac{A_r}{T}$ (8)
(iii) Purchase cost, $PC = pD$ (9)
(iv) Holding cost, $HC = \frac{h_r DT}{2}$ (10)
(v) Annual interest charge (payable/earning) for the retailer:

Since $M \leq T$, so the retailer's account should be settled at $t = T + m$, where all the revenues are collected from the customers. So for the following situations retailer has to pay interests:

(a) due to the unsold units after M,
(b) due to the customers' credit for the units sold in (M − m; M],
(c) due to the customers' credit for the units sold in (M; T],
(d) for the defaulters, who does not come back to pay the credit amount of the units sold in (0; T].

These situations are respectively formulated mathematically in Eq. (12)-Eq. (15). Thus, in this scenario, the retailer has to pay per unit time interest, IP as

$$IP = \frac{1}{T}[IP_1 + IP_2 + IP_3 + IP_4] \quad (11)$$

where,

$$IP_1 = pI_p \int_M^T I(t)dt = pI_p D\frac{(T-M)^2}{2} \quad (12)$$

$$IP_2 = pI_p \int_{M-m}^{M} D(t+m-M)dt = pI_p D\frac{m^2}{2} \quad (13)$$

$$IP_3 = pI_p \int_M^T Dmdt = pI_p Dm(T-M) \quad (14)$$

$$IP_4 = pI_p \int_0^T DF(m)(T + m - M)dt = pI_p DF(m)(T + m - M)T \tag{15}$$

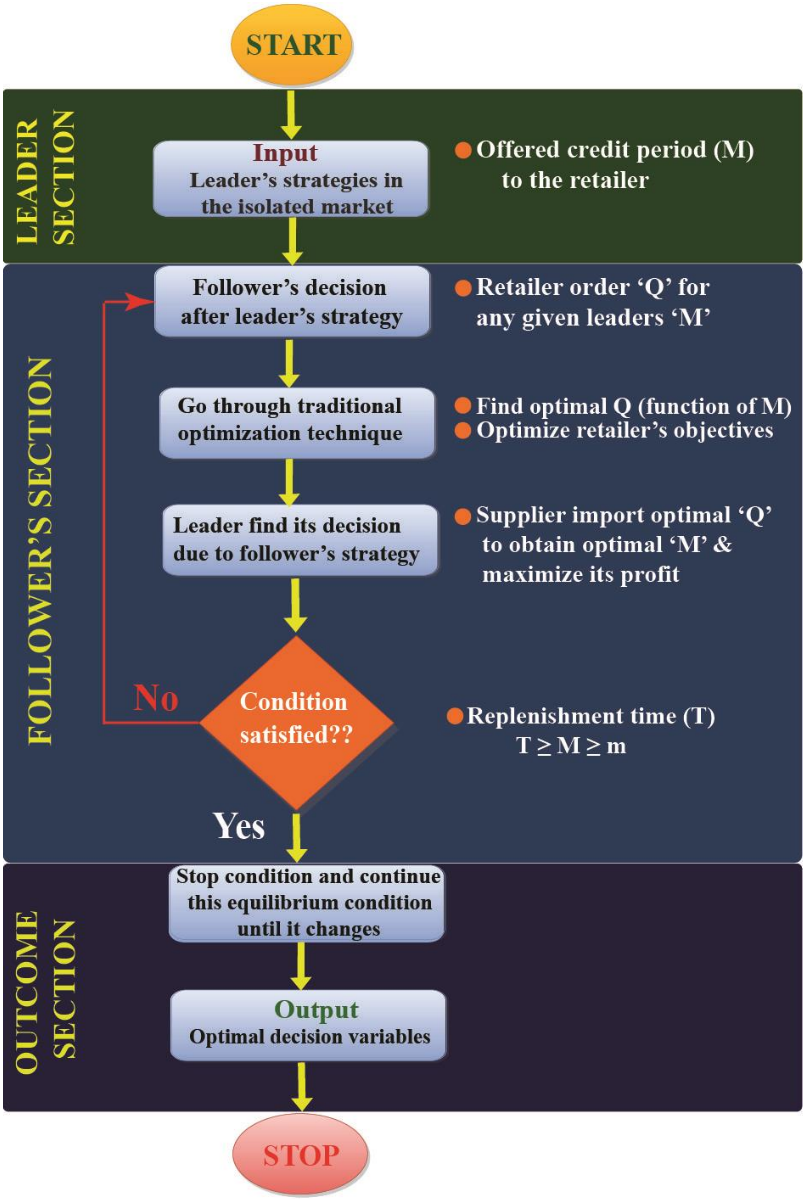

Figure 2. Flowchart of the Stackelberg game model

On the other hand, the retailer has an opportunity to earn interest on the credit payment of the customers for the units sold in (0, M-m]. This situation presented in Eq. (17). Thus, the per unit time interest, earned by the retailer is

$$IE = \frac{1}{T} IE_1 \tag{16}$$

where,

$$IE_1 = sI_e \int_0^{M-m} D[1 - F(m)](M - t - m)dt = sI_e e^{-bm} \frac{(M-m)^2}{2} \tag{17}$$

Therefore, the average profit of the retailer for Case-1 : m < M < T is given by

$$Z_r = SR - OC - PC - HC + IE - IP \tag{18}$$

Where SR, OC, PC, HC, IP and IE are in Eq.(7), Eq.(8), Eq.(9), Eq.(10), Eq.(11) and Eq.(16) respectively.

Therefore,

$$Z_r(T) = Dse^{-bm} - \frac{A_r}{T} - pD - \frac{h_r DT}{2} - pI_p D\left[\{1 + 2F(m)\}\frac{T}{2} + (m - M)\{1 + F(m)\} + \frac{(M-m)^2}{2T}\right] + sI_e e^{-bm}\frac{(M-m)^2}{2T}D \tag{19}$$

Theorem-3.1 For any given credit period M, the retailer's optimal credit period is given by

$$Q^*(M) = \sqrt{\frac{2A_r D + D^2M^2\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2}{h_r + pI_p\{1 + 2F(m)\}}}$$

Proof. For $m \le M \le T$, the annual profit of the retailer is:

$$Z_r(Q) = Dse^{-bm} - \frac{A_r D}{Q} - pD - \frac{h_r Q}{2} - pI_p D\left\{\frac{\{1+2F(m)\}Q}{2D} + (m - M)\{1 + F(m)\} + \frac{D(M-m)^2}{2Q}\right\} + sI_e e^{-bm}\frac{(M-m)^2}{2Q}D^2 \tag{20}$$

The first order derivative of Z_r with respect to Q is

$$\frac{dZ_r}{dQ} = \frac{D}{2Q^2}\left[2A_r + \left(pI_p - sI_e e^{-bm}\right)(M-m)^2 D\right] - \frac{1}{2}\left[h_r + pI_p\{1 + 2F(m)\}\right] \tag{21}$$

and

$$\frac{d^2Z_r}{dQ^2} = -\frac{2D}{Q^3}\left[2A_r + \left(pI_p - sI_e e^{-bm}\right)(M-m)^2 D\right] < 0, \tag{22}$$

if $2A_r + \left(pI_p - sI_e e^{-bm}\right)(M-m)^2 D > 0$

Therefore, in order to maximize the total profit $Z_r(Q)$ with respect to Q, the first order condition $\frac{dZ_r}{dQ} = 0$ must be satisfied. So, the optimal ordering lot size is given by

$$Q^*(M) = \sqrt{\frac{2A_r D + D^2M^2\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2}{h_r + pI_p\{1+2F(m)\}}} \tag{23}$$

which is a function of M.

The retailer's optimal replenishment time under the supplier's credit period M is determined by the theorem 3.1 and it is given as

$$T^* = \sqrt{\frac{2A_r + DM^2\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2}{D\left[h_r + pI_p\{1+2F(m)\}\right]}}, \text{ if } 2A_r + \left(pI_p - sI_e e^{-bm}\right)(M-m)^2 D > 0 \tag{24}$$

3.3. Theoretical Discussion for Existence of Supplier's Optimal Response Solution.

In the role of the leader, the supplier has the ability to observe the retailer's most advantageous reaction to a specific value of M, as determined by Equation (23). As a result, the supplier replaces the retailer's best strategy for restocking with their own total profit equation. Then, the retailers choose the optimal value for M^* in order to maximize their overall yearly profit.

The total profit of the supplier is given by

$$Z_s(Q^*) = D(p - c) - \frac{DA_w}{Q^*} - I_p cDM \tag{25}$$

Consequently, the Z_s can be changed to a new function of M by replacing Q^*.

3.3.1. Discussion for an Optimal Solution to the $m \le M \le T$.

$$Z_s(M) = ke^{a\lambda M}(p - c) - A_w\sqrt{\frac{ke^{a\lambda M}\left[h_r + pI_p\{3 - 2e^{-b\lambda M}\}\right]}{2A_r + e^{a\lambda M}\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2M^2}} - I_p cke^{a\lambda M}M \tag{26}$$

In order to maximize $Z_s(M)$ in equation (26), we obtain

$$\frac{dZ_s}{dM} = -\frac{A_w}{2}\frac{a\lambda D\{h_r + pI_p[3-2e^{-b\lambda M}]\} + 2DpI_p b\lambda e^{-b\lambda M}}{\sqrt{D\{h_r + pI_p[3-2e^{-b\lambda M}]\}}\sqrt{\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2 DM^2}}$$
$$- \frac{\sqrt{D[h_r + pI_p\{3-2e^{-b\lambda M}\}]}[MsI_e b\lambda e^{-bm} + (2+a\lambda M)(pI_p - sI_e e^{-bm})(1-\lambda)^2 DM]}{[2A_r + e^{a\lambda M}\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2 M^2]^{\frac{3}{2}}}$$
$$-I_p cke^{a\lambda M}(1+a\lambda M) + (p-c)ak\lambda e^{a\lambda M} \quad (27)$$

which includes a single decision variable, M and

$$\frac{d^2Z_s}{dM} = \frac{d}{dM}\left(\frac{dZ_s}{dM}\right) = \frac{d}{dM}\left[(p-c)ak\lambda e^{a\lambda M} - \frac{A_w}{2}\left\{\frac{z}{\sqrt{xy}} - \frac{u\sqrt{x}}{y^{\frac{3}{2}}}\right\} - I_p cke^{a\lambda M}(1+a\lambda M)\right] \quad (28)$$

where,

$$x = ke^{a\lambda M}[h_r + pI_p\{3-2e^{-b\lambda M}\}]$$
$$y = 2A_r + e^{a\lambda M}\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2 M^2$$
$$z = a\lambda x + 2pI_p b\lambda e^{(a-b)\lambda M}$$
$$u = [MsI_e b\lambda e^{-b\lambda M} + (2+a\lambda M)(pI_p - sI_e e^{-b\lambda M})](1-\lambda)^2 ke^{a\lambda M}M$$

Therefore,

$$\frac{d^2Z_s}{dM^2} = (p-c)a^2k\lambda^2 e^{a\lambda M} - \frac{A_w}{2}\left\{\frac{\sqrt{xy}\frac{dz}{dM} - z\frac{x\frac{dy}{dM} + y\frac{dx}{dM}}{2\sqrt{xy}}}{xy} - \frac{y^{\frac{3}{2}}\left(u\frac{\frac{dx}{dM}}{2\sqrt{x}} + x\frac{du}{dM}\right)}{y^3}\right\} \quad (29)$$

where,

$$\frac{dx}{dM} = ke^{a\lambda M}[a\lambda(h_r + 3pI_p) + 2pI_p\lambda(b-a)e^{-b\lambda M}]$$
$$\frac{dy}{dM} = k(1-\lambda)^2[sI_e b\lambda M^2 e^{(a-b)\lambda M} + (pI_p - sI_e e^{-b\lambda M})(a\lambda M + 2)Me^{a\lambda M}]$$
$$\frac{dz}{dM} = a\lambda ke^{a\lambda M}[a\lambda(h_r + 3pI_p) + 2pI_p\lambda(b-a)e^{-b\lambda M}] + 2pI_p(a-b)b\lambda e^{(a-b)\lambda M}$$
$$\frac{du}{dM} = k(1-\lambda)^2[sI_e b\lambda(1-b\lambda M)e^{-b\lambda M} + a\lambda(pI_p - sI_e e^{-b\lambda M}) + b\lambda sI_e e^{-b\lambda M}(a\lambda M + 2) + \{MsI_e b\lambda e^{-b\lambda M} + (a\lambda M + 2)(pI_p - sI_e e^{-b\lambda M})\}(1 + a\lambda M)e^{a\lambda M}]$$

Establishing sufficiency conditions analytically for optimality is difficult due to the complex mathematical representation of the second-order derivative. Therefore, the software MATLAB is utilized to determine the concavity.

Now, using equation (27), the optimal credit length offered by the supplier is given by

$$M^* = \frac{1}{I_p a\lambda cke^{a\lambda M}}\left\{-\frac{A_w}{2}\left[\frac{a\lambda D\{h_r + pI_p[3-2e^{-b\lambda M}]\} + 2DpI_p b\lambda e^{-b\lambda M}}{\sqrt{D\{h_r + pI_p[3-2e^{-b\lambda M}]\}}\sqrt{2A_r + \{pI_p - sI_e e^{-bm}\}(1-\lambda)^2 DM^2}} - \frac{\sqrt{D\{h_r + pI_p[3-2e^{-b\lambda M}]\}}[MsI_e b\lambda e^{-b\lambda M} + (2+a\lambda M)\{pI_p - sI_e e^{-bm}\}](1-\lambda)^2 DM^2}{[2A_r + (\{pI_p - sI_e e^{-bm}\}(1-\lambda)^2 DM^2)]^{\frac{3}{2}}}\right] + (p-c)ak\lambda e^{a\lambda M} - I_p cke^{a\lambda M}(1+a\lambda M)\right\} \quad (30)$$

Therefore, the optimal profit of the retailer and the supplier are given by

$$Z_r(Q^*) = Dse^{-bm^*} - pD - \frac{A_rD}{Q^*} - \frac{h_rQ^*}{2} - pI_pD\left[\frac{\{1+2F(m^*)\}Q^*}{2D} + (m^* - M^*)\{1 + F(m^*)\} + \frac{D(m^*-M^*)^2}{2Q^*}\right] + sI_ee^{-bm}\frac{(M-m)^2}{2Q^*}D^2 \quad (31)$$

and

$$Z_s(Q^*) = D(p-c) - \frac{DA_w}{Q^*} - I_pcDM^* \quad (32)$$

, respectively.

4. Real-life applications

Numerous sectors, including supply chain management and finance, have implemented the Supplier-Stackelberg game model under two levels of trade credit. Here are some possible practical uses:

- Manufacturing Industry: This model is utilized in manufacturing supply chains to examine the relationship dynamics between a supplier and a manufacturer. The two levels of trade credit can indicate the payment conditions between them. Accounting for default risk introduces complexity by showing the financial stability of both parties. This model can be utilized to enhance order volumes and credit terms to boost the efficiency of the entire supply chain.
- Retail Industry: Retailers and suppliers often engage in negotiations regarding credit terms and order volumes. The model can improve decision-making processes in these negotiations. Taking default risk into account assists in achieving a balance that benefits all sides.
- Risk Management: The model can help businesses create plans to reduce the risk of default in trade credit agreements. This involves establishing suitable credit limits, overseeing creditworthiness, and introducing risk-sharing systems to guarantee financial stability within the supply chain.
- E-commerce Platforms: The Supplier-Stackelberg game model can optimize credit terms and order volumes in the e-commerce sector, where suppliers provide commodities to online retailers. It is essential for sustaining a good and sustainable relationship between suppliers and retailers.

The Supplier-Stackelberg game model with two levels of trade credit and default risk is relevant in several businesses where supply chain dynamics, credit terms, and default risks are significant factors in decision-making.

5. Numerical Experiments and Sensitivity analysis

In this section, a numerical experiment is performed to illustrate the proposed model. From the perspective view in the mathematical formulation section, the prime goal is to determine the retailer's optimal order quantity (Q) and supplier's optimal credit period (M) to maximize the average profit of the retailer and supplier. Here appropriate parametric values are considered to discuss the proposed model. Different assumed parametric values for the example are given below.

Example-1: A = 200; b = 0.2; a1 = 0.15; s = 39; p = 25; c = 13; A_r = 10; hr = 0:05; I_p = 0:080; I_e = 0.060; Aw = 30; λ = 0.5.

Table 2. Results of the supplier Stackelberg game model.

Example	$m \leq M \leq T$								
	T	M^*	m^*	F(m)	D	Q^*	Z_r	Z_s	Z_{sc}
1	0.2164	0.1186	0.059	0.0089	202.3858	43.8054	2696.40	2265.10	4961.50

From the Table 2, the optimal credit period offered by supplier (M^*) = 0.1186, Consequently, the optimal order quantity of the retailer Q^* = 43.81, the optimal retailer's profit Z_r = 2696.40, the optimal supplier's profit Z_s = 2265.10 and the total supply chain profit Z_{sc} = 4961.50. However, the optimal default risk is F(m) = 0.0089.

5.1. Sensitivity Analysis

This study uses sensitivity analysis to provide managerial insights to examine the effects of changing parameter values on optimal values. The fundamental parameter values are identical to those in the example. The sensitivity analysis discloses the subsequent findings:

Table 3. Sensitivity analysis on parameters for the parameters c, p and s.

Parameters	Parameter's values	$m \leq M \leq T$						
		D	Q^*	M^*	T	Z_r	Z_s	Z_{sc}
c	12.70	204.2344	43.8721	0.2095	0.2148	2673.30	2329.00	5002.30
	12.80	203.6135	44.0047	0.1791	0.2161	2680.20	2307.90	4988.10
	12.90	203.0014	44.1077	0.1490	0.2173	2686.50	2287.00	4973.50
	13.00	202.3998	44.1809	0.1193	0.2183	2692.20	2266.30	4958.50
	13.10	201.8103	44.2245	0.0901	0.2191	2697.20	2245.60	4942.80
p	25.20	203.2000	43.9290	0.1587	0.2162	2643.80	2306.70	4950.50
	25.10	202.7872	44.0598	0.1384	0.2173	2668.20	2286.50	4954.70
	25.00	202.3998	44.1809	0.1193	0.2183	2692.20	2266.30	4958.50
	24.90	202.0345	44.2931	0.1010	0.2192	2715.60	2246.10	4961.70
	24.80	201.6891	44.3972	0.0841	0.2201	2738.70	2226.00	4964.70
s	41	201.7911	44.1722	0.0892	0.2189	3098.10	2265.70	5363.90
	40	202.0486	44.1754	0.1019	0.2186	2895.70	2266.00	5161.60
	39	202.3998	44.1809	0.1193	0.2183	2692.20	2266.30	4958.50
	38	202.9144	44.1906	0.1447	0.2178	2486.80	2266.70	4753.50
	37	203.7733	44.2106	0.1869	0.2170	2277.20	2267.40	4544.60

5.1.1. Effect of production cost, wholesale price and retail price on the proposed model

To gain some managerial insights for the players involve in the supply chain, parametric study with respect to production cost (c), wholesale price (p) and retail price (s) is performed here. The findings are derived using the specific parameter values of the
Example and displayed in Table 3. Based on the results of Table 3, some managerial insights are found, which are listed below:

- It is observed that, when production cost (c) of an item increases, the credit length (M) provided by the supplier decreases. Also, the market demand (D) decreases as the credit length directly proportional to the market demand. Again, for this situation the supplier's optimal profit (Zs) decreases and the total supply chain profit (Zsc) decreases. But the retailer's optimal profit increases since the retailer's order quantity increase.
- Also, it is noticed that for increasing the wholesale price(p), the credit length(M) provided by the supplier as well as the market demand decreases. Here, the supplier's optimal profit (Zs) But the retailer's optimal profit increase since the retailer's order quantity increases.
- If the retail price (s) decreases, replenishment time (T) and the retailer's optimal profit decreases.

5.1.2. Effect of holding cost, retailer's ordering cost and supplier's ordering cost on the proposed model

Table 4. Sensitivity analysis on the parameters for the parameter h_r, A_r and A_s

Parameters	Parameter's values	$m \le M \le T$						
		D	Q^*	M^*	T	Z_r	Z_s	Z_{sc}
h_r	0.09	202.3840	43.7576	0.1185	0.2162	2691.40	2264.90	4956.30
	0.08	202.3880	43.8622	0.1187	0.2167	2691.60	2265.20	4956.80
	0.07	202.3919	43.9677	0.1189	0.2172	2691.80	2265.60	4957.40
	0.06	202.3958	44.0739	0.1191	0.2178	2692.00	4957.90	4957.90
	0.05	202.3998	44.1809	0.1193	0.2183	2692.20	2266.30	4958.50
A_r	11	202.8714	46.3143	0.1426	0.2283	2683.60	2273.00	4956.60
	12	203.3635	48.3449	0.1668	0.2377	2674.80	2278.90	4953.70
	13	203.8729	50.2846	0.1918	0.2466	2665.90	2284.20	4950.00
	14	204.3967	52.1437	0.2175	0.2551	2656.70	2288.90	4945.60
	15	204.9324	53.9308	0.2436	0.2632	2647.30	2293.30	4940.50
A_s	27	202.9337	44.1173	0.1456	0.2174	2687.20	2280.30	4967.50
	26	203.1570	44.0875	0.1556	0.2170	2685.20	2285.00	4970.20
	25	203.3516	44.0522	0.1662	0.2166	2683.00	2289.70	4972.70
	24	203.5829	44.0106	0.1776	0.2162	2680.60	2294.40	4975.00
	23	203.8310	43.9614	0.1897	0.2157	2677.90	2299.10	4977.00

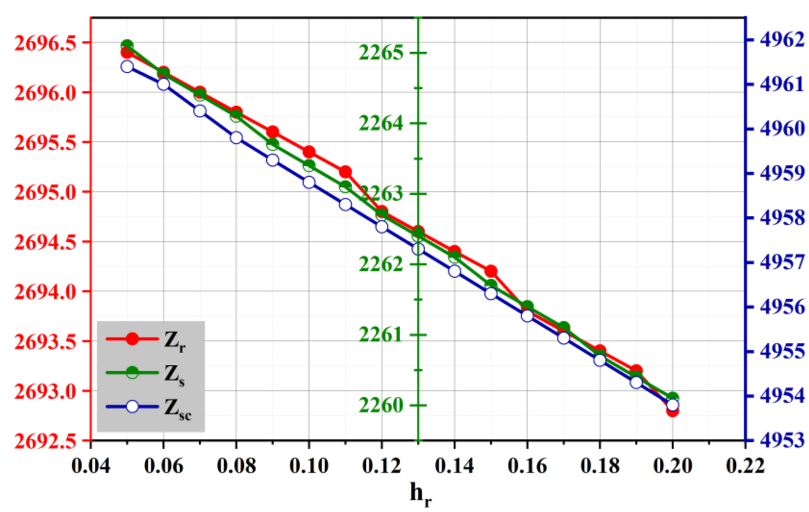

Figure 3. Impact of variations in parameter h_r on the supply chains profits.

Here are some management lessons that can be gained from the results of Table 4:

- When the holding cost (h_r) of the retailer decreases, the retailer orders more quantity as well as his profit will increase. Figure 3 depicts this phenomenon graphically. Also, for this example the replenishment time will increase.

- Also, it is observed that when the retailer's ordering cost (A_r) increase, his average profit will decrease. The graphical representation of these phenomena is illustrated in Figure 4.
- Again, if the supplier's ordering cost (A_s) decrease, the credit period provided by the supplier as well as the supplier average profit increases. This is visually shown in Figure 5.

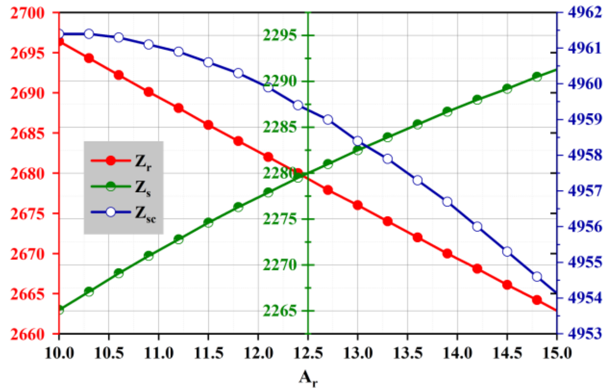

Figure 4. Impact of variations in parameter A_r on the supply chains profits.

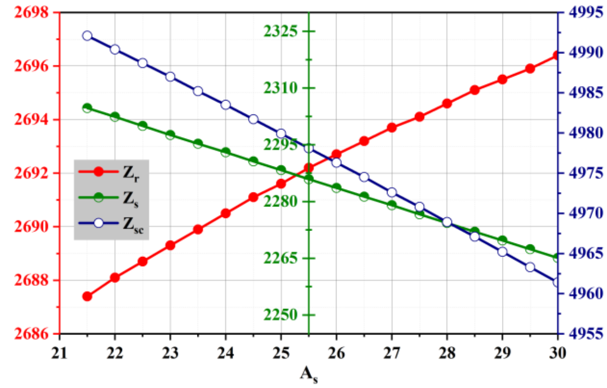

Figure 5. Impact of variations in parameter A_s on the supply chains profits.

5.1.3. Effect of rate of payable interest and earning interest on the proposed model

The following managerial insights are derived from the data presented in Table 5:

- It is observed that, if the payable interest (I_p) increases, the order quantity of the retailer and the credit length provided by the supplier decreases. Also, the average profit of the retailer, supplier and total supply chain will decrease.
- If the earning interest (I_e) decreases, the average profit of the retailer and total supply chain will decrease.

5.1.4. Parametric study of s and A_r on M and T

- It is noted that if the retail price increases the credit length provided by both the supplier and retailer increase, whereas the replenishment time decreases. The graphical representation of this phenomenon is illustrated in Figure 6.
- Again, when the ordering cost of the retailer increases the credit length provided by both the supplier and retailer increase, whereas the replenishment time also increases. This phenomenon is illustrated graphically in Figure 7.

5.1.5. Managerial implications

A supplier-Stackelberg game model under two levels of trade credit taking default risk has several managerial ramifications that can have a big impact on supply chain management strategic decision-making. Here are a few significant implications:

- The model facilitates managers in determining the most advantageous trade credit policies for the supplier and the retailer within the supply chain. Managers can achieve a balance between stimulating demand by giving favorable lending conditions and minimizing the risk of default by considering the possibility of default.
- A decision maker can utilize the knowledge gained from the model to formulate risk management strategies to reduce the adverse effects of default risk on the supply chain. This may entail establishing suitable credit limits, supervising customers' creditworthiness, and implementing credit insurance or risk-sharing

arrangements.

- The model offers valuable insights into the financial consequences of various trade credit rules and situations, including default risk. Managers can utilize this information to aid in financial planning, namely in areas such as managing cash flow, optimizing working capital, and provisioning for credit risk.
- By implementing trade credit rules that consider default risk, businesses can gain a competitive edge by increasing customer satisfaction, optimizing cash flow, and minimizing the likelihood of financial difficulties within the supply chain.
- A decision maker can enhance the long-term sustainability of the supply chain by considering the possibility of default risk when making trade credit choices. This entails the delicate equilibrium between profits and the imperative of minimizing risk and ensuring long-term financial stability.

Table 5. Sensitivity analysis on parameters for the parameters I_p and I_e

Parameters	Parameter's values	$m \le M \le T$						
		D	Q^*	M^*	T	Z_r	Z_s	Z_{sc}
	0.060	204.7065	48.4666	0.2326	0.2368	2692.20	2292.60	4984.80
I_p	0.065	204.4542	47.2908	0.2203	0.2313	2682.70	2285.70	4968.40
	0.070	204.0782	46.2251	0.2019	0.2265	2676.40	2279.00	4955.40
	0.075	203.4707	45.2189	0.1720	0.2222	2672.20	2272.50	4977.70
	0.080	202.3998	44.1809	0.1195	0.2183	2669.30	2266.30	4935.60
	0.080	200.7656	44.1669	0.0382	0.2200	2704.70	2264.80	4969.50
	0.075	200.9198	44.1668	0.0459	0.2198	2703.70	2265.00	4968.60
I_e	0.070	201.1531	44.1674	0.0575	0.2196	2702.00	2265.20	4967.20
	0.065	201.5505	44.1698	0.0772	0.2192	2699.10	2265.50	4964.60
	0.060	202.3998	44.1809	0.1193	0.2183	2692.20	2266.30	4958.40

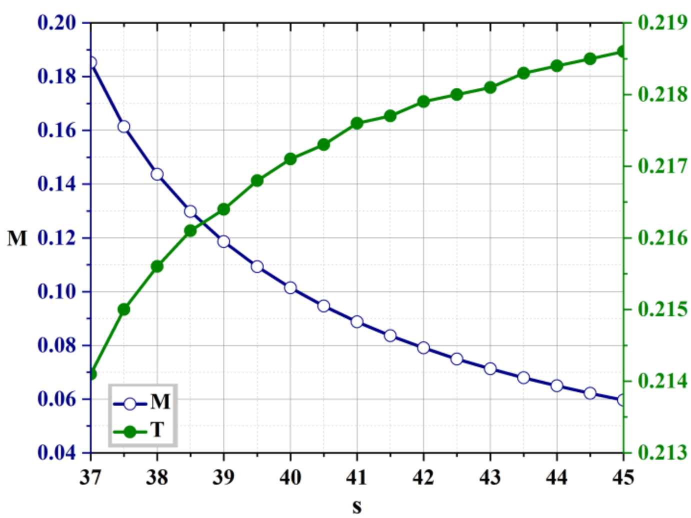

Figure 6. Effect of changes of parameter s on M and T.

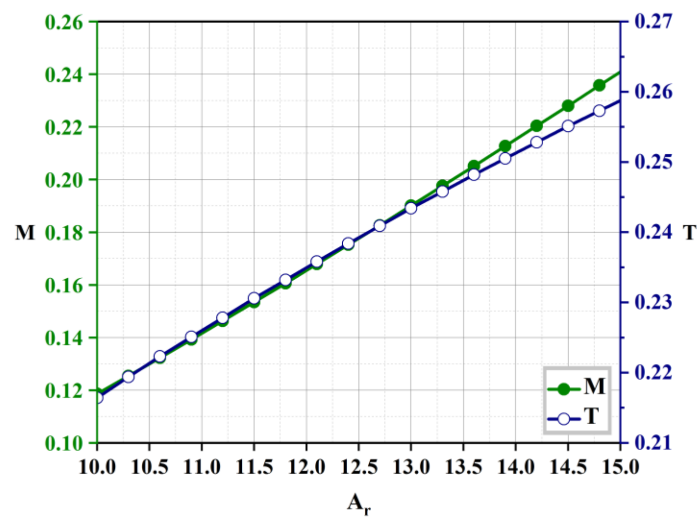

Figure 7. Effect of changes of parameter A_r on M and T.

6. Conclusion

This study examines how credit-linked demand affects the supplier's optimum strategy for delaying payments and the retailer's best policy for replenishing inventory. The suggested model has many favorable characteristics, as listed below:

- Since, trade credit is a crucial funding source for merchants, particularly small, micro, and state-up business that are cash-strapped, market demand influences the credit period exponentially. Additionally, default risk is an exponential function of the delay period.
- Two level credit policy has been considered where supplier offers credit opportunity to the retailer as well as the retailer provides delay period to the customers to improve the market demand.
- A Stackelberg game model is proposed to determine the ideal values for both the credit period supplied by the supplier and the credit period offered by the retailer in a two-level credit policy with default risk.
- The credit period influences the decision.

In a supplier-Stackelberg situation, this work examines how the supplier formulates a trade credit policy to maximize their profit. Also, this study analyzes the impact of trade credit on order decisions, the profit of both parties involved, and the overall profit of the channel. Also, from this work, the retailer must exercise caution and meticulously choose reputable customers to mitigate the default risk associated with trade credit from shops. Furthermore, it is crucial for the retailer to establish a strong credit history in the market, as well as cultivate a long-standing partnership with the supplier. Also, a numerical example is provided to demonstrate the theoretical findings. The findings indicate that extending the credit time to consumers has a beneficial effect on the market demand. A thorough analysis has been given for the outcomes of the supplier-Stackelberg model with trade credit. Finally, a fruitful comparative performance of the Stackelberg solutions has been carried out by providing few numerical attributes.
Numerous potential extensions might be implemented for future research on the suggested model. The model can be expanded by incorporating decaying products, capacity restrictions, and other factors. Various demand functions can be examined, including the quadratic function related to the credit period. Multi-retailers instead of single retailers could be chosen for future projects.

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