

Vendor-Buyer Collaboration in Supply Chain Management with Quality Inspection on the Buyer's Side

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Abstract

Effective vendor-buyer collaboration is a successful supply chain management (SCM) cornerstone. By working together, both parties can achieve significant benefits. This collaboration approach fosters open communication, which allows for better communication, improved efficiency, service levels, and reduced costs, i.e., a win-win situation. Ultimately, this partnership strengthens the entire supply chain, increasing efficiency and profitability for vendors and buyers. This study aims to investigate the dynamics of collaboration between supply chain partners. It focuses on a scenario where a single vendor supplies a single product to multiple buyers through a vendor-managed inventory (VMI) and consignment stock (CS) agreement. Managing inventory and product quality is a key challenge, as the components may be imperfect. Buyers are responsible for inspecting these items, but the process is susceptible to misclassification. To address these issues, the research aims to develop an optimization model that incorporates costs associated with collaboration, quality, inspection, and potential misclassification. In addition, developing a methodology to identify the optimal solution minimizes overall costs and maximizes benefits for both vendors and buyers. Furthermore, a numerical example is presented for tangible illustration, offering practical insights into the benefits of partner collaboration in SCM. The findings underscore the efficacy of implementing a collaborative approach between vendors and buyers to minimize supply chain costs. This collaborative policy necessitates a willingness from both parties to forgo their individual cost-minimization strategies in favor of optimizing the overall system cost through cooperation. Through this sacrifice, synergistic benefits emerge, resulting in enhanced efficiency, reduced expenses, and improved overall performance within the supply chain ecosystem.

Keywords: Inventory Management; Collaboration Policy; Consignment Agreement; Quality Inspection; Cost Reduction.

1. Introduction

Vendor-buyer collaboration in supply chain management (SCM) is paramount for achieving operational excellence and sustainable growth. Organizations can streamline processes, optimize inventory levels, and enhance supply chain efficiency by fostering close partnerships between vendors and buyers. Collaborative efforts enable real-time communication, allowing for proactive problem-solving and quicker responses to market changes. This synergy

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promotes innovation as both parties leverage their expertise to develop or improve new products. Moreover, by sharing information and aligning goals, vendors and buyers can mitigate risks, reduce costs, and improve the quality of products and services delivered to end customers. Ultimately, integrating vendor-buyer collaboration cultivates a culture of trust and transparency, driving long-term success in today's dynamic business environment. During disasters, e.g., the COVID-19 pandemic, robust SCM is crucial for mitigating risks and minimizing costs. Strategic measures like supplier diversification, inventory monitoring, and agile logistics networks enhance resilience (Abbasi et al., 2023, 2024). Proactive management identifies bottlenecks and adapts quickly, reducing disruption impact. Optimizing inventory levels and streamlining distribution channels minimizes excess costs and maximizes efficiency, enhancing competitiveness. Investing in robust SCM strengthens organizational resilience and contributes to long-term sustainability and success.

In SCN, incorporating vendor-managed inventory (VMI) and consignment stock (CS) is pivotal in optimizing operational efficiency and enhancing overall performance (Alfares & Attia, 2017; Ben-Daya et al., 2013; Blackstone & Cox, 2008). VMI allows vendors to assume responsibility for monitoring and replenishing inventory levels at the buyer's location based on agreed-upon criteria, such as sales data or demand forecasts, as shown in Figure 1. This arrangement minimizes stockouts, reduces excess inventory, and fosters smoother production schedules. Similarly, CS entails the transfer of inventory ownership to the buyer only upon consumption, thus mitigating financial risk for the buyer while ensuring product availability, as shown in Figure 1. Both VMI and CS promote closer collaboration between vendors and buyers, facilitating better demand forecasting, inventory management, and cost optimization. By aligning incentives and sharing risks, these strategies enable organizations to streamline their supply chains, enhance customer satisfaction, and gain a competitive edge in the market. Consequently, the main difference between these two strategies is who is responsible for ordering and paying for what: in CS, the buyer is responsible, but in VMI, the supplier is responsible (Anand et al., 2021; Gümüş et al., 2008).

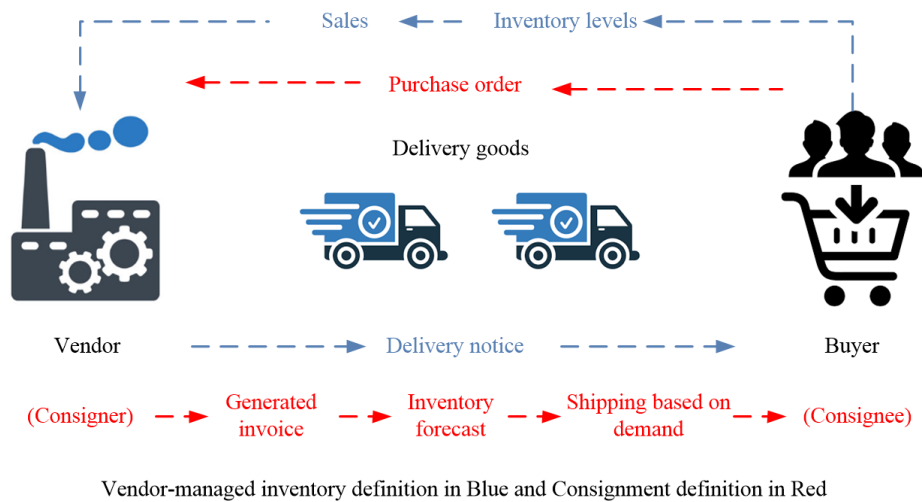


Figure 1. Definition of Vendor-managed inventory and Consignment stock policies

Venturing deeper into the intricacies of collaborative dynamics between vendor and buyers, (Ben-Daya et al., 2013) inferred that such symbiotic partnerships yield mutual advantages for both parties, particularly in reducing setup costs, a facet accentuated when the vendor showcases adaptable capacity. Elaborating further on this premise, (Khan et al., 2014) introduced a meticulous quality inspection regimen within a bifurcated supply chain model, wherein components may be subject to misclassification, such as erroneously designating a pristine item as defective. Remedying such lapses in inspection mandates a concerted effort toward bolstering collaboration management, refining both process and product design and fortifying personnel training within the confines of supply chain entities. This underscores the pressing need for a perpetual evolution of collaboration mechanisms to mitigate such discrepancies and fortify the overarching performance of the SC ecosystem.

In supply chain management, product inspection serves as a cornerstone for ensuring the quality and reliability of goods, safeguarding against potential defects and discrepancies that could disrupt operations or compromise customer satisfaction (Dey et al., 2021; Dey & Seok, 2024). However, it is imperative to acknowledge the inherent risk of inspection errors, particularly in the context of product misclassification, as they can significantly impact inventory total cost functions. Misclassifying products can lead to erroneous inventory levels, resulting in excess or insufficient stock, increased carrying costs, stockouts, or obsolete inventory (Khan et al., 2014; Masanta et al., 2023). Supply chain managers must integrate accurate risk assessment models that account for potential inspection errors into their decision-making processes to achieve optimal coordination decisions. By considering the potential costs associated with misclassification, organizations can develop more robust inventory management strategies, such as implementing safety stock buffers or adjusting reorder points, to mitigate the adverse effects of inspection errors and achieve greater efficiency and resilience across the supply chain.

This research comprehensively examines a real SC scenario involving a single vendor, multiple buyers, and multiple products. It delves into the intricate dynamics of managing inventory through VMI and CS policies, highlighting the nuanced strategies required to optimize inventory levels and minimize total SC costs. By fostering close partnerships and aligning objectives, vendors and buyers can enhance communication, share information, and jointly develop strategies to improve supply chain efficiency and effectiveness. Moreover, the paper addresses the importance of integrating the costs of misclassification consequences into decision-making processes. By acknowledging and mitigating the impact of inspection errors on inventory management, the research offers novel insights into achieving a minimum SC total cost through vendor-buyer collaboration. It proposes innovative approaches to optimizing inventory levels, reducing costs associated with misclassification, and enhancing overall supply chain performance. In conclusion, the paper's comprehensive analysis and contributions provide valuable insights for practitioners and researchers alike, offering a practical mathematical model for achieving efficiency and cost-effectiveness in complex supply chain environments.

The paper is structured as follows: Section 2 discusses the current state-of-the-art literature, providing a comprehensive overview. Section 3 elucidates the assumptions underlying the model development process, along with the steps involved in constructing the model. Sections 4 and 5 outline the solution approach and demonstrate its applicability through a numerical example. Lastly, Section 6 draws conclusions based on the findings and recommendations for potential future research avenues.

2. Literature review

This section provides an overview of the latest advancements in SCM, focusing on vendor-buyer collaboration. The literature reviewed here is categorized into three main themes: (1) models involving a single vendor and multiple buyers; (2) collaborations between vendors and buyers, including profit-sharing arrangements; and (3) the integration of quality control practices with SCM. Within each category, recent studies from 2010 onward are examined in chronological order to highlight the evolving research interests in this field.

2.1. Single-vendor multiple-buyers

Under the VMI policy, which (Darwish and Odah, 2010) implemented, the vendor is responsible for charging the ordering costs as well as surplus inventory penalties. If the vendor's inventory levels exceed the limits specified in the contract and the SC has capacity restrictions, the vendor will be penalized. A more notable benefit for less efficient supply chains was shown by (Rouibi and Burlat, 2010), who modeled a three-echelon SC under (s, S) inventory policy and analyzed two VMI policies. In their model of a single-vendor multi-buyer SC, (Battini *et al.*, 2010) accounted for the space limitations of the buyer's warehouse and the risks of stockouts and material obsolescence.

In order to enhance SC performance, (Hoque, 2011) suggests varying the order quantity and size; in other words, using different sizes leads to a decrease in costs. The model imposes limitations on storage and transportation space, processing times, and minimum and maximum batch sizes. Moreover, the author presupposes that the buyer incurs the ordering cost a single time, independent of the quantity of orders. If the seller has an infinite supply and a lower unit holding cost than the buyers, then (Hariga *et al.*, 2013) minimized the joint inventory costs of a mixed integer

nonlinear program (MINLP) using an efficient heuristic. The effects of three distinct partnership policies were studied by (Ben-Daya et al., 2013): integration with a single decision maker, VMI-CS agreements, and no coordination agreement. Buyer cost savings under CS policy increase as ordering cost and demand variability increase, according to (Choudhary et al., 2014). The supplier would rather have a CS policy if the setup and shipment costs are very high. In a study conducted by (Zanoni and Jaber, 2015), the impact of buyer stock level on demand was investigated. Furthermore, a minimum stock level and an instantaneous replenishment policy are in place to prevent shortages, which are not permitted. It was presumed by (Hong et al., 2016) that demand is distributed uniformly and that any shortage results in a missed sale. A VMI policy was determined to be more cost-effective than a standard individual policy.

Time, delay without interest, and delay with interest were the three types of payment time that (Zahran et al., 2016a and 2016b) examined in their evaluation of the impact of CS policy. A delay with interest was determined to be the best-case scenario. According to (Lee et al., 2017), when the restocking level at the buyer's side is a decision variable, profit increases as the buyer embraces a minimum reorder level policy. A linear relationship was assumed between buyer-customer demand, stock level, and market price (Hemmati et al., 2017). Kumar & Uthayakumar (2019) explored a two-echelon inventory model in an integrated lead time controllable system and developed five models to find optimal solutions with minimum total cost. The model considered greenhouse gas (GHG) emission penalties and taxes. Comparisons with previous models showed that the proposed model provides the minimum cost. However, limitations include constant demand, 100% consumable production process, and single-vendor single-buyer limitation.

Gharaei et al. (2019) presented a mathematical model that considers the buyer and vendor total cost in an SC under penalty, green, and quality control policies and a VMI-CS agreement. The model differentiated between holding costs for financial and nonfinancial components. The objective was to determine the optimal batch-sizing policy with a minimum total cost in the integrated SC. An outer approximation with equality relaxation and augmented penalty algorithm is presented, minimizing the large-scale mixed-integer nonlinear programming problem.

Poshtahani & Pasandideh (2020) proposed a bi-objective model for a two-echelon single vendor-single buyer green SCn under VMI policy. The model aimed to minimize total cost and greenhouse gas emissions, allowing back-ordering shortages. The model used stochastic programming and GAMS software to solve non-linear programming. Results showed that multi-choice goal programming with utility function (MCGP-U) has better efficiency than Goal attainment and LP-metrics, and Goal attainment has better performance than LP-metrics in terms of objective functions, second objective functions, and CPU time criteria. However, significant differences exist between LP-metrics, Goal attainment, and MCGP-U methods regarding CPU time. Woo et al. (2021) introduced a coordination policy for controlling a production-inventory system in a vertically integrated SC. It relaxed the assumption of fixed production rates and aimed to determine optimal order sizes, replenishment cycles, and production rates without inventory shortages. The model addressed limitations in dependent replenishment cycles and ensured manufacturers could replenish raw materials independently. The model can be extended to address location-allocations in SCs, aiming to provide more cost-effective decisions by considering various facility types and yields.

SCM often employs both CS and VMI policies; however, VMI is associated with increased ordering frequency, decreased shortage quantity, and decreased inventory cost (Owusu Kwateng et al., 2022). Research by (Hariga et al., 2022, 2023) investigated how carbon cap and tax policies, as well as other CO2 regulation measures, influenced SCM choices. They discovered that a considerably smaller carbon footprint can be accomplished with little changes to operations, although there will be a slight increase in operational costs. The costs of reworking, disposal, holding, and inspection were attributed to the vendor in the study by (Gharaei et al., 2023), which assumed that the vendor was responsible for error-free product inspection. The model optimized the number and quantity of shipments using mixed integer nonlinear programming (MINLP) and generalized bender decomposition (GBD). However the model failed to take into account how beneficial it would be for various SC parties to work together.

Utama et al. (2024) highlighted the importance of optimizing production capacity and SC planning in the single-vendor multi-buyer problem. The study suggested increased demand leads to shorter production cycles, requiring vendors to adapt their strategies. It also highlighted the need for efficient shipping strategies, as higher costs lead to

decreased JTP and longer production cycles. Additionally, the study suggested prioritizing product quality to ensure customer satisfaction and minimize financial losses.

2.2. Vendor-buyer collaboration and profit sharing

The authors of the relevant studies (Ben-Daya et al., 2013; Hu et al., 2014) dealt with the problem of increased costs for vendors or buyers due to collaboration and offered solutions to that problem. In the event that the vendor's financial situation worsens, a portion of the buyers' savings can be reinvested in the vendor through an increase in the unit price. The collaboration cannot go forward if at least one potential customer is scared off by the proposed price hike. The vendor is not worse off after the cooperation ends, even with a slight price increase. Just as the vendor can lower prices to entice some buyers to accept the partnership, they can do the same if some or all of the buyers are worse off.

(Wettasinghe & Luong, 2020) devised two mathematical models to calculate the most advantageous base stock level, delivery quantity to the retailer, and cycle length of a VMI system for an SC consisting of a single vendor and retailer. They conducted numerical experiments and sensitivity analysis to evaluate the optimal solution under different cost parameters. The suggested heuristic, derived based on the rate of demand, can be utilized for expansions where the precise method is infeasible. The sensitivity analysis demonstrated that the demand rate impacts all three decision variables, explicitly leading to changes in the system's base stock level and the quantity of deliveries made to the retailer. To enhance the model, one should consider service centers SCs that have Poisson/compound Poisson customer demands and inventory transshipment. The possibility of a developer and distribution platform splitting the profits was explored by (Avinadav et al., 2022). Platforms like Google Firebase and Apple App Analytics recommend sharing data with developers, and when developers are aware of this, the platform benefits.

Juman et al. (2023) presented an integrated multi-vendor multi-buyer production-inventory model for supplying a single product, synchronizing production flow, and transferring batches of different sizes. The model optimized joint inventory costs, improving performance and potentially establishing mutually beneficial relationships. The approach used combinatorial optimization to minimize total costs for both vendors and buyers, including ordering, setup, inventory holding, and transportation. The results showed that the integrated model outperforms single-vendor multi-buyer systems under uniform demand. The model can be extended to synchronized production flow, limited capacities on transportation vehicles, storage space, and variable production/demand rates. Khakbaz et al. (2024) developed a cross-docking EOQ-based model for a retail company, considering a two-stage inventory procurement process. The model minimized total inventory costs by analyzing holding costs at the central warehouse, retail store holding, fixed ordering, and central warehouse holding costs. The model also allows managers to analyze key factors affecting system costs, such as annual retailer demand, central warehouse ordering costs, retail store ordering costs, and holding costs at each store. It can be extended to consider the returned products from the inspection process.

Ru (2024) addressed the conflict regarding the impact of VMI on retailer performance in a two-product SC. The research incorporated consumer response to retail stockouts and explored how product substitution and consumers' store loyalty affect VMI value. Key findings included that retailers benefit from VMI if the wholesale price is greater than a critical value, manufacturers benefit from VMI if the wholesale price is greater than another critical value, and retailers are more likely to benefit from VMI for higher consumer loyalty.

2.3. Quality control procedures and inventory management in SCM

Inspection can be conducted by either the vendor or the buyer. Chiu *et al.* (2013a and 2013b) examined the existence of faulty items that can be repaired on the vendor's end. Furthermore, a failure may happen during rework and be considered a standard cycle time for multiple items. Jaber et al. (2014a) posited that sending back faulty items for repair was impractical due to the distance between the supplier and the buyer. Jaber et al. (2014b) later posited that the supplier is responsible for managing production, repair, and waste disposal. Two inspection scenarios are being examined from the perspective of the vendor and the buyer. The model operates under the assumption that both new and repaired products are of equal quality. Defective items generated during production and remanufacturing are consequently disregarded. In 2014, Bazan and colleagues addressed imperfect production by incorporating reworking and interrupting the production run. They applied modest setups to bring the process back to its in-control state.

Sana (2011) implemented an unequal shipment policy among different SC echelons. Defective items are inspected and then sent back to the previous echelon in a single shipment. (Khan et al., 2014, 2016) analyzed inspection errors from the buyer's perspective and the learning rate from the vendor's perspective. They implemented two strategies for handling defective items: either scrapping them or reworking them, which incurs costs for remanufacturing and disposal. Akbarzadeh *et al.* (2016) later examined a scenario where the rework of faulty items is imperfect, potentially resulting in the production of more imperfect items. Hariga *et al.* (2017) analyzed faulty products requiring remanufacturing and developed a model to optimize the order quantity and shipment quantities for newly manufactured and remanufactured components. In 2017, Giri and colleagues examined the impact of varying shipment sizes, buyer-led inspections, and handling defective items through scrapping or repair.

Alfares & Afzal (2021) introduced an inventory model that addresses the management of imperfect quality items, allowable shortages, and holding costs. The model hypothesized that a portion of fully matured items are of substandard quality and must be eliminated following inspection. Shortages are authorized and placed on backorder to minimize warehousing and inventory maintenance expenses. The model's objective was to minimize the overall expenses associated with the inventory system, encompassing procurement, setup, replenishment, quality control, stockouts, and storage costs. Extensions incorporate practical elements such as quantity discounts and lost sales to modify existing assumptions.

Salas-Navarro et al. (2023) hypothesized a probabilistic demand rate for the customer and a consistent deterioration rate for the vendor. The model takes into account various costs, including deterioration, screening, disposal, and labor. A vendor-buyer partnership is established by exchanging information regarding sales forecasts, operating costs, storage policies, and quality attributes. Masanta et al. (2023) studied a closed-loop SC model involving a single producer, retailer, and collector, considering retail pricing, green innovation, and marketing efforts. They considered an S-shaped learning curve to mitigate inspection errors and benefit both consignors and consignees. They indicated that the learning effect has a strong influence and should be included in all repetitive processes, especially the production, inspection, and handling of objects.

2.4. Summary of the literature and contribution of the paper

Table 1. Summary of the relevant reviewed literature provides an overview of the pertinent literature on vendor-buyer collaboration within SCM, encompassing studies that incorporate quality considerations and those that do not. The table outlines key elements such as the objective function, decision variables, and solution approach. It shows the different policies and approaches researchers have considered to deal with the difficulties of vendor-buyer collaboration in SCM.

In summary, the proposed research aims to address a research gap by comprehensively analyzing the intricate dynamics within a single-vendor, multiple-buyers SC environment. The study focuses on integrating VMI and CS policies for complete collaboration, highlighting their implications for inventory management and total SC cost optimization. Furthermore, it delves into the role of vendor-buyer collaboration in improving supply chain efficiency and effectiveness, particularly in achieving a minimum total cost. One of the main aspects of the research is its investigation into the impact of product inspection and the consequences of misclassification errors on inventory management within this SC setting. By developing a mathematical model and decision-making framework, the paper aims to provide practitioners and researchers with valuable insights into optimizing inventory levels, reducing costs associated with misclassification, and fostering collaboration between vendors and buyers to achieve greater efficiency and cost-effectiveness. Overall, this research seeks to fill a gap in the literature and contribute to advancing knowledge and practices in supply chain management.

Table 1. Summary of the relevant reviewed literature

Paper	Vendor	Buyer	Items	Collaboration policy	Inspection	Inspection error	Objective function	Decision variables	Solution approach
(Hariga et al., 2013)	Single	Multiple	Single	VMI	Not considered	Not considered	Minimize total cost	n, Q, T	Heuristic
(Choudhary et al., 2014)	Single	Single	Single	VMI and RMI	Not considered	Not considered	Minimize total cost	o, z	Analytical
(Zanoni & Jaber, 2015)	Single	Single	Single	VMI and CS	Not considered	Not considered	Maximize total profit	n, Q	Analytical
(Hong et al., 2016)	Multiple	Multiple	Single	VMI	Not considered	Not considered	Minimize total cost	Q, T, S, r	Heuristic
(Zahran et al., 2016a)	Single	Single	Single	CS	Not considered	Not considered	Maximize total profit	n, Q	Heuristic
(Alfares & Attia, 2017)	Single	Multiple	Single	VMI and CS	Considered	Considered	Minimize total cost	n, T	Exact
(Giri et al., 2017)	Single	Single	Single	CS	Considered	Not considered	Maximize total profit	n, Q	Exact
(Gharaei et al., 2019)	Single	Multiple	Multiple	VMI and CS	Not considered	Not considered	Minimize total cost	n, Q	OA&AP
(Kumar & Uthayakumar, 2019)	Single	Single	Single	VMI	Not considered	Not considered	Minimize total cost	n, Q, L	GA
(Poshtahani & Pasandideh, 2020)	Single	Single	Multiple	VMI	Not considered	Not considered	Minimize total cost Minimize GHG	Q, s	Exact
(Wettasinghe & Luong, 2020)	Single	Single	Single	VMI	Not considered	Not considered	Minimize total cost	Q, T, S	GA
(Alfares & Afzal, 2021)	Single	Single	Single	VMI	Considered	Not considered	Minimize total cost	T, s	Exact
(Woo et al., 2021)	Multiple	Multiple	Multiple	JC	Not considered	Not considered	Minimize total cost	Q, T, P	Heuristic
(Hariga et al., 2022)	Single	Multiple	Single	VMI	Not considered	Not considered	Minimize total cost	n, Q, z	Exact
(Gharaei et al., 2023)	Single	Multiple	Multiple	VMI	Considered	Not considered	Minimize total cost	n, Q	Exact
(Salas-Navarro et al., 2023)	Multiple	Multiple	Multiple	VMI	Considered	Not considered	Maximize total profit	T, P	Analytical
(Hariga et al., 2023)	Single	Multiple	Single	VMI	Not considered	Not considered	Minimize total cost	n, Q, z	Exact
(Juman et al., 2023)	Multiple	Multiple	Single	JC	Not considered	Not considered	Minimize total cost	n, Q	Exact
(Masanta et al., 2023)	Single	Single	Single	Not considered	Considered	Considered	Maximize total profit	n, Q, <i>p</i>	Exact
(Khakbaz et al., 2024)	Multiple	Multiple	Multiple	Not considered	Not considered	Not considered	Minimize total cost	T	Exact
(Ru, 2024)	Multiple	Single	Multiple	VMI and RMI	Not considered	Not considered	Maximize total profit	<i>p</i>	Exact
(Utama et al., 2024)	Single	Multiple	Multiple	Not considered	Not considered	Not considered	Maximize total profit	n, m, T	WOA
Current work	Single	Multiple	Single	VMI, CS, and JC	Considered	Considered	Minimize total cost	n, T	Exact

Abbreviations: JC: Joint Collaboration, VMI: Vendor-Managed Inventory, RMI: Retailer-Managed Inventory, CS: Consignment Stock, Q: order quantity, T: replenishment cycle length, n: number of orders, L: lead time, S: safety stock, r: reorder point, P: production rate, m: number of raw material orders, *p*: price, z: overstock quantity, s: out-of-stock (shortage) quantity, o: order or not (binary), GHG: greenhouse gasses, GA: Genetic Algorithm, WOA: Whale Optimization Algorithm, OA&AP: outer approximation and augmented penalty algorithm.

3. Model development

This section describes the steps to formulate the mathematical model and the suggested optimization procedure. Table 1 summarizes the key notations used throughout this paper. It includes definitions for sets and indices, primary and dependent decision variables, and various buyer, vendor, and inspection parameters.

Table 1. Model notations

Sets/Indices	
N	set of buyers, denoted by index i
Decision variables	
Primary variables	
n	number of orders from the vendor to buyer i , assumed to be equal
T	replenishment cycle length
Dependent variables	
t	time to produce one order = Q/P
q_i	order quantity to buyer i
Q	total orders quantity to all buyers = $\sum_{i=1}^N q_i$
TC_v	total cost for vendor per unit of time
TC_i	total cost for buyer i per unit of time
TC	total cost for the supply chain per unit of time
Parameters	
Buyer parameters	
K_{bpi}	cost of placing an order at buyer i (\$/order)
K_{bri}	cost of receiving an order at buyer i (\$/order)
h_{boi}	cost of tied-up inventory at buyer i charged by the vendor (\$/unit/unit time)
h_{bsi}	cost of physical storage at buyer i (\$/unit/unit time)
d_i	demand at buyer i (units/unit time)
D	total demand of buyers = $\sum_{i=1}^N d_i$ (units/unit time)
Vendor parameters	
K_{vs}	cost of setup at the vendor (\$/order)
K_{vri}	cost of order release at the vendor to buyer i (\$/order)
h_v	cost of holding one unit at the vendor (\$/unit/unit time)
P	production rate of the vendor (units/unit time)
Inspection parameters	
e_1	probability of type I error, i.e., a non-defective item being classified as defective
e_2	probability of type II error, i.e., a defective item being classified as non-defective
c_{fa}	cost of falsely accepting a defective item
c_{fr}	cost of falsely rejecting a non-defective item

Table 2. Model notations (*Continued*)

c_{cr}	cost of correctly rejecting a defective item
p	percentage of defective supplied by the vendor
p_e	percentage of rejected items, either correctly or falsely rejected
I_r	buyer's inspection rate (units /unit time)
t_{si}	inspection time of buyer i (time/order) = q_i/I_r

3.1. Model assumptions

The primary assumptions are presented to enhance convenience. Many of these assumptions are commonly found in the literature. Additional assumptions have been incorporated to formulate a more accurate and realistic model that reflects real-life scenarios.

1. The vendor charges the costs of setup and release of orders and holding (Ben-Daya et al., 2013).
2. Under the consignment policy, the vendor is responsible for placing an order and tying up inventory costs (Ben-Daya et al., 2013).
3. The buyers charge the costs of receiving an order and physical storage (Ben-Daya et al., 2013).
4. The vendor produces each cycle nQ units continuously throughout the first nt time units.
5. The vendor's production rate exceeds the combined demand from all buyers; $P > D$.
6. Buyers receive the same number of orders, n , during each cycle.
7. A buyer receives equal-size orders. However, different buyers receive orders of different sizes based on their demand rates, where the q_i/d_i ratio is the same for all buyers.
8. A buyer receives an order, and then this cycle is repeated until all orders are fulfilled, i.e., a cyclic shipping plan.
9. Buyers have an equal inspection rate, which is higher than the demand rate of any buyer; $I > d_i, \forall i$.
10. Buyers inspect all received items and consume only the accepted ones. While falsely accepted items are used to produce low-value items.
11. At the end of the inspection process, the items buyers reject are disposed of collectively (Khan et al., 2014; Salameh & Jaber, 2000).
12. The inspection process is subjected to classification errors. Type I error, in which the non-defective item can be classified as defective, and Type II error, in which the defective item can be classified as non-defective, are shown in Table 2.
13. The percentage of defective items and the probabilities of type I and type II errors are fixed constants that represent the expected values of random variables with probability density functions $f(p)$, $f(e_1)$, and $f(e_2)$, respectively, (Khan et al., 2011).

Table 2. Item classification after the inspection activity

		Item classification after inspection	
		Non-defective	Defective
Item true classification	Non-defective (1-p)	$(1-p)(1-e_1)$ Correctly accepted	$(1-p)(e_1)$ Falsely rejected Type I error
	Defective (p)	$(p)(e_2)$ Falsely accepted Type II error	$(p)(1-e_2)$ Correctly rejected

3.2. Inventory modeling

The stocking procedure commences with the initial order, q_1 , dispatched by the vendor to the first buyer. Subsequently, the vendor sends the first order, q_2 , to the second buyer, as illustrated in Figure 2. This sequential process continues until all buyers receive their initial orders from the vendor. Subsequently, after a time period has elapsed, t , buyers receive the second order before depleting the first one. This results in the buyers' inventory buildup, consequently increasing holding costs. However, it is noteworthy that the vendor bears the burden of the buyers' inventory costs under the consignment policy partnership. Initially, upon receiving an order, the buyer's inventory stands at l_i . After that, the processes of consumption and inspection commence at rates d_i and l_r , respectively, resulting in a decline in inventory levels, as depicted in Figure 2. The inspection process eliminates defective items, while the consumption process satisfies the demand for non-defective ones. The inventory level at the end of the cycle reaches l_4 prior to the receipt of the subsequent order.

Due to potential inspection inaccuracies by buyers, where both accepted and rejected items might be either defective or non-defective, the effective proportion of rejected items, denoted as p_e , encompasses both correctly rejected items, calculated as $p(1 - e_2)$, and falsely rejected items, represented by $(1 - p)e_1$, as shown in Eq. (1). Consequently, the fulfillment of an order, q_i , only consumes $q_i(1 - p_e)$ accepted units. Moreover, over a cycle T , the number of utilized accepted items, expressed as $nq_i(1 - p_e)$, must meet the demand for buyer i , Td_i , as illustrated in Eq. (2). Eq. (3) mathematically signifies the required time to produce an order for all buyers.

$$p_e = p(1 - e_2) + (1 - p)e_1 \tag{1}$$

$$T = \frac{nq_i(1 - p_e)}{d_i} = \frac{nQ(1 - p_e)}{D} \tag{2}$$

$$t = \frac{DT}{nP(1 - p_e)} \tag{3}$$

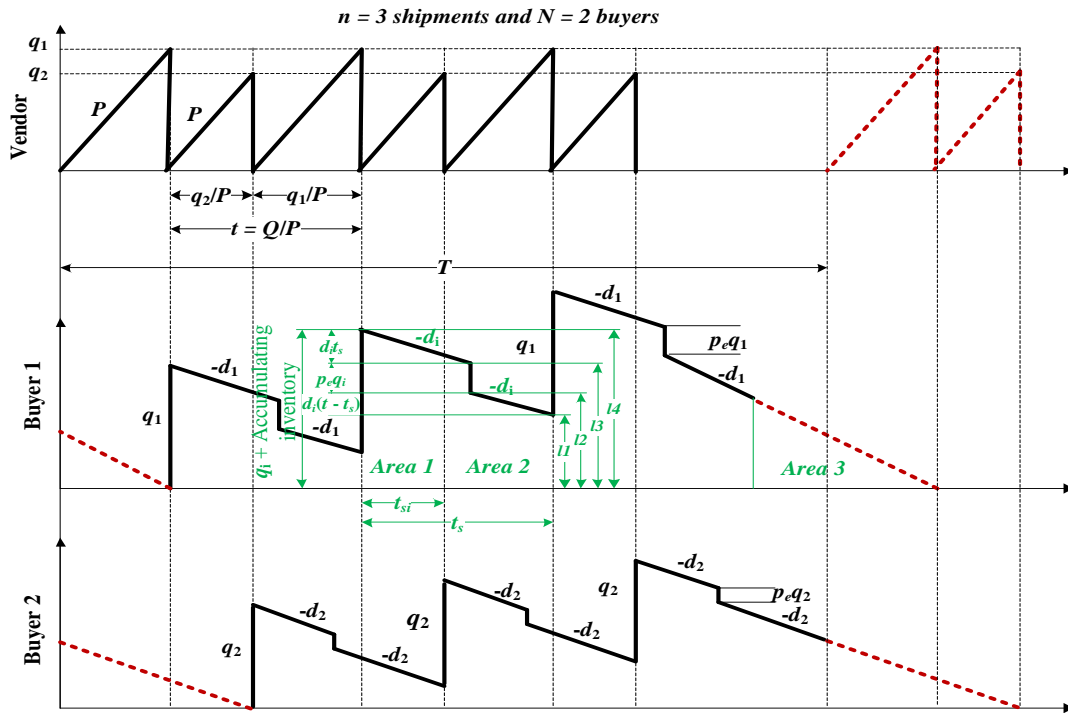


Figure 2. Inventory profile for 1 vendor-2 buyers collaboration

Referring to Figure 2, the average inventory of the vendor, \bar{I}_v , is calculated using the total area of the triangles, where the height corresponds to q_i and the base to q_i/P . Therefore, the total n triangular areas for each of the N buyers are divided by the cycle time T , so it can be formulated mathematically as follows:

$$\bar{I}_v = \frac{T}{2Pn(1-p_e)^2} \sum_{i=1}^N d_i^2 \quad (4)$$

Contrarily, referring to Figure 2, the average inventory of a buyer equals the sum of the areas under the inventory profile divided by the cycle time, Eq. (5). The area is n pairs of trapezoids, *Area 1* and *Area 2*, and at the end of the cycle, one triangle, *TRI*. After formulating the area and performing simplification, the mathematical expression in (6) can be used for the average inventory of a buyer.

$$\bar{I}_{b,i} = \frac{1}{T} \left[\left(\sum_{j=1}^n Area\ 1_{i,j} + Area\ 2_{i,j} \right) + Area\ 3_i \right] \quad (5)$$

$$\bar{I}_{b,i} = \frac{d_i T}{2Pn(1-p_e)^2} \left[\alpha + \frac{2Pp_e d_i}{I_r} \right] \quad (6)$$

$$\alpha = (1-p_e)[D + n(P - Pp_e - D)] \quad (7)$$

where i is the buyer number, and j is the order number in the cycle

3.3. Mathematical model

The vendor is in charge of determining both the timing and the quantity for each order, which results in the seven components of the vendor's total cost, as shown in Eq. (8). The first three components primarily encompass the vendor's setup cost, K_{vs} , shipment release cost, K_{vri} , and holding cost, $h_v \bar{I}_v$. Conversely, within the framework of the consignment policy, the subsequent fourth and fifth components incorporate the buyers' ordering costs, nK_{bpi} , and holding costs, $h_{b oi} \bar{I}_{b,i}$. Ultimately, the final two components encapsulate the expenses attributed to rejected items, encompassing false and correct rejections.

$$TC_v(n, T) = \frac{K_{vs}}{T} + \frac{n \sum_{i=1}^N K_{vri}}{T} + h_v \frac{T}{2Pn(1-p_e)^2} \sum_{i=1}^N d_i^2 + \frac{n \sum_{i=1}^N K_{bpi}}{T} + \frac{\sum_{i=1}^N h_{b oi} d_i \left[\alpha + \frac{2Pp_e d_i}{I_r} \right]}{2Pn(1-p_e)^2} \quad (8)$$

$$+ \frac{c_{fr} D(1-p)e_1}{1-p_e} + \frac{c_{cr} Dp(1-e_2)}{1-p_e}$$

For each buyer, i , there are three cost components, Eq. (9): (1) cost of receiving orders, $n K_{bri}$, (2) holding cost: the cost of physical storage per unit, h_{bsi} , times the average inventory level, $\bar{I}_{b,i}$, and (3) cost of false acceptance of defective items: the cost of false acceptance per unit, c_{fa} , times the expected number of falsely accepted units. Adding up the costs of the vendor and all buyers, Eqs.

(8) and (9), the total cost for the supply chain system is expressed mathematically in Eq.

(10).

$$TC_{bi}(n, T) = \frac{nK_{bri}}{T} + \frac{h_{bsi}d_i}{2Pn(1-p_e)^2} \left[\alpha + \frac{2Pp_e d_i}{I_r} \right] T + \frac{c_{fa}p_e d_i}{1-p_e} \quad (9)$$

$$TC(n, T) = \frac{K_{vs} + n \sum_{i=1}^N (K_{vri} + K_{bpi} + K_{bri})}{T} + \frac{D[c_{fr}(1-p)e_1 + c_{cr}p(1-e_2) + c_{fa}p_e d_i]}{1-p_e} \quad (10)$$

$$+ \frac{h_v \sum_{i=1}^N d_i^2 + \sum_{i=1}^N (h_{boi} + h_{bsi})d_i \left[\alpha + \frac{2Pp_e d_i}{I_r} \right]}{2Pn(1-p_e)^2} T$$

4. Solution approach

For a given n, the total cost expressed in Eq.

(10) is convex regarding the cycle time, T , as its second partial derivative with respect to T is positive for all $T > 0$. Consequently, the optimum cycle time, T , in Eq. (11) is obtained by taking the partial derivative of Eq.

(10) with respect to the cycle time, T , and equating it to zero. Then, Eq.

(12) is derived by substituting the value of T into Eq.

(10).

$$T = \sqrt{\frac{2Pn(1-p_e)^2 [K_{vs} + n \sum_{i=1}^N (K_{vri} + K_{bpi} + K_{bri})]}{h_v \sum_{i=1}^N d_i^2 + \sum_{i=1}^N (h_{boi} + h_{bsi})d_i \left[\alpha + \frac{2Pp_e d_i}{I_r} \right]}} \quad (11)$$

$$TC(n, T) = \frac{D[c_{fr}(1-p)e_1 + c_{cr}p(1-e_2) + c_{fa}p_e d_i]}{1-p_e} \quad (12)$$

$$+ \sqrt{\frac{2[K_{vs} + n \sum_{i=1}^N (K_{vri} + K_{bpi} + K_{bri})] \left(h_v \sum_{i=1}^N d_i^2 + \sum_{i=1}^N (h_{boi} + h_{bsi})d_i \left[\alpha + \frac{2Pp_e d_i}{I_r} \right] \right)}{Pn(1-p_e)^2}}$$

In Eq.

(12), the term under the square root should be minimized to minimize the total cost. This can be achieved by applying the first difference approach, by setting Term (n) < Term($n+1$) and Term (n) < Term($n-1$), deleting redundant terms, and combining similar terms. Then, by getting the positive roots of the quadratic function, Eq. (13) expresses the optimum number of cycles.

$$n = \sqrt{\frac{1 + 4K_{vs} \left[h_v \sum_{i=1}^N d_i^2 + \sum_{i=1}^N (h_{boi} + h_{bsi})d_i \left(D - Dp_e + \frac{2Pp_e d_i}{I_r} \right) \right]}{4(1-p_e)(P - Pp_e - D) \left[\sum_{i=1}^N (h_{boi} + h_{bsi})d_i \right] \left[\sum_{i=1}^N (K_{vri} + K_{bpi} + K_{bri}) \right]}} \quad (13)$$

In summary, the following steps are used to optimize the decision variables and to achieve the minimum total cost:

Step 1: Determine the number of cycles, n , using Eq. (13)

If n is an integer, go to step 2

else, n is not integer, round it to the nearest lower integer $\lfloor n \rfloor$ and higher integer $\lceil n \rceil$.

Step 2: Compute the total cost, $TC(n, T)$, for the values of n using Eq.

(12).

Step 3: Choose the n value that yields the minimum total cost.

Step 4: Calculate the optimum cycle length, T , by substituting the optimum number of cycles, n , into Eq. (11).

5. A numerical example and sensitivity analysis

A numerical example is provided herein to elucidate the managerial implications of the model under consideration. The dataset pertains to a supply chain configuration featuring a single vendor and two buyers, drawn from prior studies and detailed in Table 3 (Ben-Daya et al., 2013; Khan et al., 2016). This empirical demonstration underscores the proposed framework's practical applicability and managerial insights.

Initially, the expected values of the inspection errors and percentage of defective items based on the given uniform distributions are ($p = 0.02$, $e_1 = 0.02$, $e_2 = 0.02$). Then, computing the number of shipments using Eq. (13), $n = 2.361$, and substituting both $\lfloor n \rfloor = 2$ and $\lceil n \rceil = 3$ into Eq.

(12), the minimum total cost $TC = 7,511.59$ is associated with $n = 2$ and $T = 0.429$, as shown in Table 4.

Table 3. Numerical example data

$P = 3200$ units/year	$K_{bp1} = \$15$ /shipment	$S_r = 175,200$ units/year
$D = 1500$ units/year	$K_{bp2} = \$50$ /shipment	$f(e_1) = U(0, 0.04)$
$d_1 = 500$ units/year	$K_{br1} = \$10$ /shipment	$f(e_2) = U(0, 0.04)$
$d_2 = 1000$ units/year	$K_{br2} = \$25$ /shipment	$f(p) = U(0, 0.04)$
$h_{bo1} = \$2.5$ /unit/year	$h_v = \$4$ /unit/year	$c_{cr} = \$100$ /unit
$h_{bo2} = \$2$ /unit/year	$K_{vr1} = \$0$ /shipment	$c_{fr} = \$50$ /unit
$h_{bs1} = \$2.5$ /unit/year	$K_{vr2} = \$0$ /shipment	$c_{fa} = \$200$ /unit
$h_{bs2} = \$3$ /unit/year	$K_{vs} = \$400$ /setup	

Table 4. Solutions for vendor-buyer collaboration

n	t	T	q_1	q_2	TC_v	TC_{b1}	TC_{b2}	TC
1	0.161	0.33	172	343	6,815.17	278.1	654	7,747.25
2	0.105	0.429	112	223	6,534.01	291	686.6	7,511.59
3	0.081	0.499	87	173	6,470.47	312.2	738.6	7,521.27
4	0.068	0.556	72	145	6,467.79	333.9	791.9	7,593.60

This section conducts a sensitivity analysis to garner managerial insights and implications concerning the comparative efficacy of vendor-buyer collaboration versus individualistic approaches. The cost functions govern strategic interactions between parties in the supply chain (SC), Eqs.

(8) and (9), are minimized using methods similar to those used to find the minimum total costs when parties collaborate, which can be seen in Table 5, Table 6, and Table 7. The findings underscore that a collaborative strategy between vendors and buyers yields the most advantageous outcome, culminating in a minimized SC cost of \$7,511.59. However, embracing such a collaborative stance necessitates a degree of sacrifice from the involved parties, requiring

them to forgo the pursuit of individual cost minimization in favor of optimizing the system's overall cost efficiency through collaboration.

Furthermore, this collaborative paradigm incurs adjustments in the minimum costs incurred by individual entities within the SC. Specifically, the vendor's minimum cost experiences a marginal increase of 2.1%, ascending from \$6,400.79 to \$6,534.01. Similarly, the cost for Buyer 1 witnesses a substantial escalation of 46%, rising from \$199.77 to \$291.03, while Buyer 2 encounters a parallel increase of 46%, surging from \$470.66 to \$686.55. These insights illuminate the dynamic interplay between collaborative strategies and individual cost optimization within the SC, underscoring the nuanced trade-offs inherent in pursuing collaborative efficiencies.

Table 5. Solutions if the vendor has chosen to work individually

n	t	T	q_1	q_2	TC_v	TC_{b1}	TC_{b2}	TC
1	0.212	0.434	226	451	6,734.17	335.8	791.8	7,861.80
2	0.138	0.566	147	295	6,461.39	344.6	814.1	7,620.12
3	0.107	0.657	114	228	6,401.42	364.4	862.5	7,628.26
4	0.089	0.729	95	190	6,400.79	385.4	913.9	7,700.11

Table 6. Solutions if buyer 1 has chosen to work individually

n	t	T	q_1	q_2	TC_v	TC_{b1}	TC_{b2}	TC
1	0.062	0.126	66	132	8,579.16	199.8	470.7	8,976.59
2	0.05	0.206	54	107	7,506.18	236.1	559.7	7,976.99
3	0.043	0.267	46	93	7,188.62	266.6	634.5	7,696.05
4	0.039	0.318	41	83	7,062.08	293.5	700.3	7,628.89

Table 7. Solutions if buyer 2 has chosen to work individually

n	t	T	q_1	q_2	TC_v	TC_{b1}	TC_{b2}	TC
1	0.063	0.129	67	134	8,511.72	199.8	470.7	9,059.86
2	0.051	0.21	55	109	7,461.31	236.1	559.6	8,032.91
3	0.044	0.272	47	94	7,151.23	266.7	634.4	7,816.72
4	0.04	0.324	42	84	7,028.32	293.5	700.1	7,768.73

The managerial implications of the contrast between vendor-buyer collaboration and individualistic approaches in the context of SCM are significant, such as the following key considerations:

1. **Collaborative strategy benefits:** The findings suggest that a collaborative strategy between vendors and buyers leads to the most advantageous outcomes, notably minimizing SC costs. Managers need to recognize the potential benefits of collaboration, such as reduced costs, improved efficiency, and enhanced responsiveness to market changes.
2. **System optimization vs. Individual optimization:** Embracing collaboration necessitates a shift in mindset from individual cost minimization to optimizing the system's overall efficiency. Managers should understand that while individual entities may incur higher costs initially, the collective benefits outweigh these individual sacrifices. This entails a strategic realignment of organizational goals towards broader SC objectives.
3. **Sacrifice and commitment:** Implementing a collaborative approach demands sacrifice and commitment from both vendors and buyers. Managers must foster a culture of trust, transparency, and mutual benefits to encourage cooperation among SC partners. This may involve renegotiating contracts, sharing information, and aligning incentives to incentivize collaborative behaviors.
4. **Cost adjustments:** Managers must anticipate and manage the minimum cost adjustments incurred by individual entities within the supply chain. It is crucial to consider the overall cost savings achieved through

collaboration, even though some parties may experience marginal cost increases. Strategies such as process optimization and inventory management can help mitigate any adverse effects on individual costs.

5. Performance measurement: Establishing metrics and performance indicators is crucial for evaluating the effectiveness of collaborative efforts. Managers should track the key performance indicators (KPIs) related to cost savings, lead times, inventory levels, and customer satisfaction to gauge the success of collaboration initiatives. Regular performance reviews and feedback mechanisms can help identify the areas for improvement and drive continuous optimization.

In summary, the managerial implications of vendor-buyer collaboration versus individualistic approaches in supply chain management underscore the importance of fostering collaborative relationships, embracing system-wide optimization, and proactively managing costs and risks. Organizations can enhance their competitive advantage and adaptability in an increasingly complex and dynamic business environment by prioritizing collaboration and adopting a holistic approach to SCM.

6. Conclusion

This study investigated supply chain management (SCM) within the context of collaborative endeavors between buyers and vendors and the implementation of item inspection protocols on the buyer's end. When viewed in this light, the SC system incorporates a single vendor and multiple buyers operating within inventory partnership policies. This research paper has investigated various aspects of SCM, specifically emphasizing vendor-buyer collaboration, vendor-managed inventory, consignment stock policy, and inspection practices. The analysis considered two significant errors that occur during the inspection process: the acceptance of a defective unit and the rejection of a unit that was not defective. In order to investigate the complexities of managing these interrelated components, a mathematical model and optimization procedure were developed. This procedure aimed to find the optimal number of shipments and cycle length by reducing the total costs. In addition, a numerical example is provided to shed light on the procedural complexities of the solution method and highlight the primary benefits resulting from the collaboration between the buyer and the vendors. It has been observed that inspection errors significantly influence the total costs incurred by the SC parties, with the costs of the vendor being especially susceptible to fluctuations of this kind. It is important to note that the findings highlight the viability of a collaborative policy in order to achieve the lowest possible SC cost. Nevertheless, implementing such a policy calls for the involved parties to abandon their efforts to minimize their individual costs in favor of maximizing the efficiency of the system as a whole through the utilization of collaborative efforts. Future research endeavors must address the primary limitations inherent in the assumptions used in the current study. Therefore, these may include, but are not limited to, considerations pertaining to the size, quantity, and sequencing of shipments, thereby paving the way for more robust and comprehensive investigations in this domain.

References

- Abbasi, S., Daneshmand-Mehr, M., & Ghane Kanafi, A. (2023). Green Closed-Loop Supply Chain Network Design During the Coronavirus (COVID-19) Pandemic: A Case Study in the Iranian Automotive Industry. *Environmental Modeling & Assessment*, 28(1), 69–103. <https://doi.org/10.1007/s10666-022-09863-0>
- Abbasi, S., Vlachos, I., Samadzadeh, A., Etemadifar, S., Afshar, M., & Amra, M. (2024). Modelling a Logistics and Financial Supply Chain Network during the COVID-19 Era. *Logistics*, 8(1), Article 1. <https://doi.org/10.3390/logistics8010032>
- Akbarzadeh, M., Taleizadeh, A. A., & Esmaeili, M. (2016). Developing an economic production quantity model with scrap, rework and backordering under vendor-managed inventory policy. *International Journal of Advanced Logistics*, 1–16.
- Alfares, H. K., & Afzal, A. R. (2021). An Economic Order Quantity Model for Growing Items with Imperfect Quality and Shortages. *Arabian Journal for Science and Engineering*, 46(2), 1863–1875. <https://doi.org/10.1007/s13369-020-05131-z>

- Alfares, H. K., & Attia, A. M. (2017). A supply chain model with vendor-managed inventory, consignment, and quality inspection errors. *International Journal of Production Research*, 55(19), 5706–5727. <https://doi.org/10.1080/00207543.2017.1330566>
- Anand, A., Mishra, A., Garg, S. K., & Wattal, R. (2021). Vendor Managed Inventory: Issues and Challenges in a Single Vendor Multiple Retailer Supply Chain. In A. Kumar, A. Pal, S. S. Kachhwaha, & P. K. Jain (Eds.), *Recent Advances in Mechanical Engineering* (pp. 955–968). Springer Nature. https://doi.org/10.1007/978-981-15-9678-0_80
- Avinadav, T., Chernonog, T., Meilijson, I., & Perlman, Y. (2022). A consignment contract with revenue sharing between an app developer and a distribution platform. *International Journal of Production Economics*, 243, 108322. <https://doi.org/10.1016/j.ijpe.2021.108322>
- Battini, D., Gunasekaran, A., Faccio, M., Persona, A., & Sgarbossa, F. (2010). Consignment stock inventory model in an integrated supply chain. *International Journal of Production Research*, 48(2), 477–500. <https://doi.org/10.1080/00207540903174981>
- Bazan, E., Jaber, M. Y., Zanoni, S., & Zavanella, L. E. (2014). Vendor Managed Inventory (VMI) with Consignment Stock (CS) agreement for a two-level supply chain with an imperfect production process with/without restoration interruptions. *International Journal of Production Economics*, 157, 289–301. <https://doi.org/10.1016/j.ijpe.2014.02.010>
- Ben-Daya, M., Hassini, E., Hariga, M., & AlDurgam, M. M. (2013a). Consignment and vendor managed inventory in single-vendor multiple buyers supply chains. *International Journal of Production Research*, 51(5), 1347–1365.
- Ben-Daya, M., Hassini, E., Hariga, M., & AlDurgam, M. M. (2013b). Consignment and vendor managed inventory in single-vendor multiple buyers supply chains. *International Journal of Production Research*, 51(5), 1347–1365. <https://doi.org/10.1080/00207543.2012.662725>
- Blackstone, J. H., & Cox, J. F. I. (2008). *APICS Dictionary (12th ed)* (12th ed.). Falls Church, VA: APICS.
- Chiu, S. W., Lin, L.-W., Chen, K.-K., & Chou, C.-L. (2013). Determining production–shipment policy for a vendor–buyer integrated system with rework and an amending multi-delivery schedule. *Economic Modelling*, 33, 668–675.
- Chiu, S. W., Pai, F.-Y., & Wu, W. K. (2013). Alternative approach to determine the common cycle time for a multi-item production system with discontinuous deliveries and failure in rework. *Economic Modelling*, 35, 593–596.
- Choudhary, D., Shankar, R., Dey, P. K., Chaudhary, H., & Thakur, L. S. (2014). Benefits of retailer–supplier partnership initiatives under time-varying demand: A comparative analytical study. *International Journal of Production Research*, 52(14), 4279–4298. <https://doi.org/10.1080/00207543.2013.879615>
- Darwish, M. A., & Odah, O. M. (2010). Vendor managed inventory model for single-vendor multi-retailer supply chains. *European Journal of Operational Research*, 204(3), 473–484.
- Dey, B. K., Pareek, S., Tayyab, M., & Sarkar, B. (2021). Automation policy to control work-in-process inventory in a smart production system. *International Journal of Production Research*, 59(4), 1258–1280. <https://doi.org/10.1080/00207543.2020.1722325>
- Dey, B. K., & Seok, H. (2024). Intelligent inventory management with automation and service strategy. *Journal of Intelligent Manufacturing*, 35(1), 307–330. <https://doi.org/10.1007/s10845-022-02046-4>
- Ganesh Kumar, M., & Uthayakumar, R. (2019). Modelling on vendor-managed inventory policies with equal and unequal shipments under GHG emission-trading scheme. *International Journal of Production Research*, 57(11), 3362–3381. <https://doi.org/10.1080/00207543.2018.1530471>
- Gharaei, A., Karimi, M., & Hoseini Shekarabi, S. A. (2019). An integrated multi-product, multi-buyer supply chain under penalty, green, and quality control policies and a vendor managed inventory with consignment stock agreement: The outer approximation with equality relaxation and augmented penalty algorithm. *Applied Mathematical Modelling*, 69, 223–254. <https://doi.org/10.1016/j.apm.2018.11.035>
- Gharaei, A., Karimi, M., & Hoseini Shekarabi, S. A. (2023). Vendor-managed inventory for joint replenishment planning in the integrated qualitative supply chains: Generalised benders decomposition under separability approach.

- International Journal of Systems Science: Operations & Logistics*, 10(1), 1962428. <https://doi.org/10.1080/23302674.2021.1962428>
- Giri, B. C., Chakraborty, A., & Maiti, T. (2017). Consignment stock policy with unequal shipments and process unreliability for a two-level supply chain. *International Journal of Production Research*, 55(9), 2489–2505. <https://doi.org/10.1080/00207543.2016.1233362>
- Gümüş, M., Jewkes, E. M., & Bookbinder, J. H. (2008). Impact of consignment inventory and vendor-managed inventory for a two-party supply chain. *International Journal of Production Economics*, 113(2), 502–517.
- Hariga, M., As'ad, R., & Ben-Daya, M. (2022). Vendor Managed Inventory Coordination Under Contractual Storage Agreement and Carbon Regulation Policies. *IEEE Access*, 10, 66098–66112. <https://doi.org/10.1109/ACCESS.2022.3183988>
- Hariga, M., As'Ad, R., & Ben-Daya, M. (2023). A Single-Vendor Multi-Retailer VMI Partnership Under Individual Carbon-Cap Constraints. *IEEE Access*, 11, 45504–45519. <https://doi.org/10.1109/ACCESS.2023.3272903>
- Hariga, M., As'ad, R., & Khan, Z. (2017). Manufacturing-remanufacturing policies for a centralized two stage supply chain under consignment stock partnership. *International Journal of Production Economics*, 183, 362–374.
- Hariga, M., Gumus, M., Daghfous, A., & Goyal, S. K. (2013). A vendor managed inventory model under contractual storage agreement. *Computers & Operations Research*, 40(8), 2138–2144.
- Hemmati, M., Fatemi Ghomi, S. m. t., & Sajadieh, M. S. (2017). Vendor managed inventory with consignment stock for supply chain with stock- and price-dependent demand. *International Journal of Production Research*, 1–18. <https://doi.org/10.1080/00207543.2017.1296203>
- Hong, X., Chunyuan, W., Xu, L., & Diabat, A. (2016). Multiple-vendor, multiple-retailer based vendor-managed inventory. *Annals of Operations Research*, 238(1–2), 277–297.
- Hoque, M. A. (2011). An optimal solution technique to the single-vendor multi-buyer integrated inventory supply chain by incorporating some realistic factors. *European Journal of Operational Research*, 215(1), 80–88.
- Hu, W., Li, Y., & Govindan, K. (2014). The impact of consumer returns policies on consignment contracts with inventory control. *European Journal of Operational Research*, 233(2), 398–407.
- Jaber, M. Y., Zanoni, S., & Zavanella, L. E. (2014a). A consignment stock coordination scheme for the production, remanufacturing and waste disposal problem. *International Journal of Production Research*, 52(1), 50–65. <https://doi.org/10.1080/00207543.2013.827804>
- Jaber, M. Y., Zanoni, S., & Zavanella, L. E. (2014b). Economic order quantity models for imperfect items with buy and repair options. *International Journal of Production Economics*, 155, 126–131. <https://doi.org/10.1016/j.ijpe.2013.10.014>
- Jamshidpour Poshtahani, S., & Pasandideh, S. H. R. (2020). Optimizing a bi-objective vendor-managed inventory of multi-product EPQ model for a green supply chain with stochastic constraints. *Journal of Industrial and Systems Engineering*, 13(1), 1–34.
- Juman, Z. A. M. S., M'Hallah, R., Lokuhetti, R., & Battaïa, O. (2023). A multi-vendor multi-buyer integrated production-inventory model with synchronised unequal-sized batch delivery. *International Journal of Production Research*, 61(2), 462–484. <https://doi.org/10.1080/00207543.2021.2009586>
- Khakbaz, A., Alfares, H. K., Amirteimoori, A., & Tirkolae, E. B. (2024). A novel cross-docking EOQ-based model to optimize a multi-item multi-supplier multi-retailer inventory management system. *Annals of Operations Research*. <https://doi.org/10.1007/s10479-023-05790-9>
- Khan, M., Jaber, M. Y., & Ahmad, A.-R. (2014). An integrated supply chain model with errors in quality inspection and learning in production. *Omega*, 42(1), 16–24.
- Khan, M., Jaber, M. Y., & Bonney, M. (2011). An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics*, 133(1), 113–118. <https://doi.org/10.1016/j.ijpe.2010.01.023>

- Khan, M., Jaber, M. Y., Zanoni, S., & Zavanella, L. (2016). Vendor managed inventory with consignment stock agreement for a supply chain with defective items. *Applied Mathematical Modelling*, 40(15), 7102–7114.
- Lee, W., Wang, S.-P., & Chen, W.-C. (2017). Forward and backward stocking policies for a two-level supply chain with consignment stock agreement and stock-dependent demand. *European Journal of Operational Research*, 256(3), 830–840.
- Masanta, M., Giri, B. C., & Das, P. (2023). Green consideration in a closed-loop supply chain model with imperfect inspection under learning impact. *Journal of Cleaner Production*, 428, 139201. <https://doi.org/10.1016/j.jclepro.2023.139201>
- Owusu Kwateng, K., Fokuoh, B., & Tetteh, F. K. (2022). The relationship between vendor managed inventory and operational performance. *Modern Supply Chain Research and Applications*, 4(3), 202–222. <https://doi.org/10.1108/MS CRA-03-2022-0009>
- Rouibi, S., & Burlat, P. (2010). The impact of the Vendor Managed Inventory on supply chain performance. *Computers and Industrial Engineering (CIE), 2010 40th International Conference On*, 1–6. http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=5668286
- Ru, J. (2024). Retailer- vs. Vendor-managed inventory in the presence of consumer response to retail stockouts. *International Journal of Production Research*, 0(0), 1–19. <https://doi.org/10.1080/00207543.2024.2313098>
- Salameh, M. K., & Jaber, M. Y. (2000). Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, 64(1), 59–64.
- Salas-Navarro, K., Romero-Montes, J. M., Acevedo-Chedid, J., Ospina-Mateus, H., Florez, W. F., & Cárdenas-Barrón, L. E. (2023). Vendor managed inventory system considering deteriorating items and probabilistic demand for a three-layer supply chain. *Expert Systems with Applications*, 218, 119608. <https://doi.org/10.1016/j.eswa.2023.119608>
- Sana, S. S. (2011). A production-inventory model of imperfect quality products in a three-layer supply chain. *Decision Support Systems*, 50(2), 539–547.
- Utama, D. M., Kusuma, I. R., Amallynda, I., Baroto, T., & Jauhari, W. A. (2024). A single-vendor multi-buyer inventory model with multiple raw material and quality degradation: A case study on agri-food industry. *Results in Control and Optimization*, 14, 100353. <https://doi.org/10.1016/j.rico.2023.100353>
- Wettasinghe, J., & Luong, H. T. (2020). A vendor managed inventory policy with emergency orders. *Journal of Industrial and Production Engineering*, 37(2–3), 120–133. <https://doi.org/10.1080/21681015.2020.1736657>
- Woo, Y.-B., Moon, I., & Kim, B. S. (2021). Production-Inventory control model for a supply chain network with economic production rates under no shortages allowed. *Computers & Industrial Engineering*, 160, 107558. <https://doi.org/10.1016/j.cie.2021.107558>
- Zahran, S. K., Jaber, M. Y., & Zanoni, S. (2016a). Comparing different coordination scenarios in a three-level supply chain system. *International Journal of Production Research*, 0(0), 1–21. <https://doi.org/10.1080/00207543.2016.1249431>
- Zahran, S. K., Jaber, M. Y., & Zanoni, S. (2016b). The consignment stock case for a vendor and a buyer with delay-in-payments. *Computers & Industrial Engineering*, 98, 333–349. <https://doi.org/10.1016/j.cie.2016.06.010>
- Zanoni, S., & Jaber, M. Y. (2015). A two-level supply chain with consignment stock agreement and stock-dependent demand. *International Journal of Production Research*, 53(12), 3561–3572. <https://doi.org/10.1080/00207543.2014.980012>