International Journal of Supply and Operations Management

IJSOM

2024, Volume 11, Issue 1, pp. 54-82
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com

A Three-Echelon Retailer-Led Closed Loop Supply Chain with Substitutable Products Coordination: Real-World Beverage Industry Application

Petr Pokorny [a*] and Petr Fiala [a]

[a] *Prague University of Economics and Business, Faculty of Informatics and Statistics, Department of Econometrics*

Abstract

In this paper, we consider a real-world retailer-led three-echelon CLSC with substitutable products in a beverage industry consisting of a manufacturer, a distributor, and a retailer. The manufacturer produces the same product in two different types of packaging, one being a returnable glass bottle (RGB) and the other being a disposable bottle (OW) and the consumer chooses depending on their preference. The RGB is then returned via the specialized distributor, who must ensure the sorting of the returned bottles and their reverse logistics back to the manufacturer. We analyse the CLSC with the dominant role of the retailer and compare the Stackelberg equilibrium with the Nash equilibrium where all CLSC members are equally strong. We prove that the CLSC can make higher profits if the product mix is skewed towards the RGB product. Our analysis also shows that a retailer-led CLSC can be coordinated with an RS contract to achieve a fair share of a double increase in total profits for all partners if the retailer manages its leadership position and maximises its bargaining power. From a game theoretic perspective, we show that our Stackelberg equilibrium is also acceptable in the pure vertical Nash game. Furthermore, the sensitivity analysis explains why it is so difficult to negotiate and coordinate a real CLSC in a Fast Moving Consumer Goods (FMCG) sector, because it is unlikely that the dominant retailer would participate in the coordination negotiations and give up its margins for the benefit of all CLSCs, which is necessary to ensure the effect of reducing prices for the final consumer. This is all set in the real beverage CLSC using actual cost parameters thus providing valuable managerial insights.

Keywords: De-/Centralized; Closed Loop Supply Chain; Coordination; Game Theory; Multi-Echelon; Nash Equilibrium; JEL Classification: C72.

Introduction

Most research on closed-loop supply chains (CLSCs) analyses manufacturer-led chains, where the manufacturer sets prices and controls the margins of its downstream CLSC partners. Similarly, most of the literature deals with a two-echelon CLSC consisting of a manufacturer and a retailer focusing on a single product. In this paper, however, we consider a three-echelon CLSC with substitutable products in a beverage retailer-led CLSC consisting of a manufacturer, a distributor and a retailer. The manufacturer produces the same product in two different types of packaging, a returnable glass bottle (RGB) and a disposable bottle (OW). The consumer has the choice of using the returnable or the disposable packaging, depending on their preference. There is no difference in the sensory perception of the product, it is only the consumer's convenience and willingness to buy RGB along with the price that determines

*Corresponding author email address: pokornyp@vse.cz
DOI: 10.22034/IJSOM.2023.109848.2688

their decision to buy RGB or OW products. The RGB is then returned via the distributor, who must first collect the returned empty bottles from the retailer who receives them from consumers. The distributor then has to sort the returned bottles and return them to the manufacturer. In a real beverage FMCG CLSC, the retailer is the dominant leader, controlling upstream agents and prices due to fierce competition. Logistics is highly specific, both forward and reverse, and plays a significant role in the overall cost of CLSC. The pool of potential logistics service providers is also limited and their overall margins are low and we cannot leave this CLSC tier out of our analysis. From the manufacturer's point of view, the cost pressure from both the distributor and the retailer is immense and puts the manufacturer in an uncomfortable position to control and optimise its costs as much as possible. The beverage industry is a highly competitive environment with relatively low margins and extremely price sensitive consumers. This is further exacerbated by the market power of international key accounts, which resist any price increases proposed by the manufacturer. It is estimated that more than 90% of sales in the retail chain depend on discounts. These discounts are usually given in the form of reduced wholesale prices and, based on the performance of the promotion, so-called variable trade (back) bonuses are paid retrospectively. These rebates then represent the shared revenue between the partners. A similar transaction takes place between the manufacturer and the distributor. If a full truck load is shipped directly from the manufacturer without any additional warehousing or handling costs, this saving is a positive revenue compared to the standard calculated costs. These are the mechanisms we consider to affect the financial flows between partners, and it is on the basis of these observed facts that we model the potential performance improvements of the overall CLSC. Therefore, we analyse the CLSC with the dominant role of the retailer and compare the Stackelberg equilibrium (leader-follower model) with the Nash equilibrium where all CLSC members are equally strong, which provides a good benchmark. We show the trade-offs between producing and selling RGB and OW products and perform an extensive sensitivity analysis, which clearly shows that the CLSC can make higher profits if the product mix is skewed towards the RGB product, allowing the CLSC to benefit from the cost difference between forward and reverse logistics.

The research questions in this paper are as follows:

- How does the consumer's product choice affect the profit of the CLSC and how is the profit distributed among the three CLSC members under different cost scenarios?
- Can the RS contract achieve a fair share profit distribution even under a retailer's leadership and its dominant position?
- How do the different cost structures of the forward and backward chains affect the consumer's choice and vice versa?
- What is required to coordinate a retailer-led CLSC with substitutable products, and what are the parameters of an achievable revenue sharing contract for such a multi-level CLSC?
- How does the retailer need to approach the coordination negotiations?

The contribution of this paper is as follows:

- We derive a fully coordinated RS contract for a three echelon retailer-led CLSC with substitutable products and test it on a constrained demand set for which we derive the exact conditions of applicability. To the best of our knowledge, this combination does not exist in the literature.
- The model is applied to the real industry setting, with cost parameters reflecting structural relationships, albeit simplified to examine the conditions of profit sharing between members. From this perspective, the conclusions we draw from the decentralised models provide useful managerial insights.
- We use the model to explain why it is so difficult to achieve coordination in real multilevel CLSCs in the FMCG sector, where price competition is very fierce. We also show that in real life, the decision making of CLSC members is not always rational, even though there are clear benefits for all stakeholders.

The paper is organised as follows. In chapter one we review the existing research literature, in chapter two we present all the models, followed by detailed assumptions and equilibrium results of the decentralised models in chapter three. Chapter four focuses on sensitivity analysis of the decentralised models, and in chapter five we study and prove the RS coordination models.

1 Literature Review

SC is often analysed in its two extreme cases, i.e. a fully centralised (or integrated) model with a central decision maker aiming to optimise the competing objectives. In the real world, most SCs are made up of independent agents or proprietary firms pursuing their self-interested decisions, some of which are integrated (centralised) to some extent (Fiala, 2005). This case is usually referred to as the decentralised model. Centralised SC is set as the most cost-effective benchmark, as it generates the highest expected profit. On the other hand, the overall profit of a fully decentralised SC suffers from inefficiency, as all agents pursue their individual goals and maximise their own profits regardless of the overall performance of the SC. Between these two extremes, there is a space for performance improvement known as SC coordination. Coordination can be achieved in two ways, either through the cooperative GT approach or through SC contracts such as Revenue Sharing (RS), Buy-back, Two-Part Tariff (TPT) or Quantity Discounts. An introductory overview of all SC contracts is summarised in (Erhun & Keskinocak, 2003), in this paper we will focus on the so-called Revenue Sharing (RS) contract in combination with a TPT contract. For a detailed explanation and discussion of the strengths and limitations of RS, see (Cachon & Lariviere, 2005). RS contracts can improve the performance of the decentralised chain towards the centralised case. (Fiala, 2016) shows a cooperative model on the profit allocation among the SC members through excess profit allocation. In a TPT contract, the supplier charges a fixed fee F and a wholesaler's unit price. The variable F stands for a lump sum payment and represents a flexible contract that determines the profit sharing between the contracting parties. (De Giovanni & Zaccour, 2019) provide an excellent overview of CLSC along with the survey of coordination mechanisms. The topological structure of these networks is analyzed in detail in (Shekarian & Flapper, 2021).

Recent literature on CLSC has placed greater emphasis on the sustainability factor, expanding the notion of CLSC to include carbon footprint reduction and complex social and environmental compliance. The environmental impact of COVID-19 in terms of CO2 emissions and social impact such as the closure of SC are analysed in (Abbasi, 2022). A review of the so-called green (G)CLSC can be found in (Abbasi, Khalili, et al., 2022; Abbasi & Ahmadi Choukolaei, 2023). (Al-Refaie et al., 2021) focuses on the optimal use of end-of-life products and the resulting savings using a multi-objective optimization approach. Similarly, (Abbasi, Mehraban, et al., 2022a) present the Recovery Network (RN) model to capture the elements and trade-offs of the circular economy. In the work of (Abbasi & Erdebilli, 2023), the authors show how the tightening of carbon policies is affecting business decisions. In (Abbasi, Khalili, et al., 2022), The framework is introduced through a composite index to measure the sustainability of the SC during the COVID-19 pandemic. (Abbasi & Erdebilli, 2023) present models that can help managers predict the impact of regulatory changes on emissions. The phenomenon of hygiene costs in the design of GSCLCs following the outbreak of COVID-19 is discussed in (Abbasi, Mehraban, et al., 2022b). CLSC involving subsidy models in herbal medicine can be found in (Garai et al., 2020) and (Garai & Sarkar, 2021) focusing on economic independence of the reverse chain. (B. Sarkar et al., 2017) develop a mixed-integer non-linear programming model to capture the impacts of returnable transport items, similar to our study, on the environment. An interesting CLSC study is conducted in (Dey et al., 2021) and (Dey et al., 2022; Dey & Seok, 2022) on manufacturing self-control system that remanufactures the imperfect products in the same cycle they are detected, thus eliminating the waste.

The literature on 3-echelon SC RS contracts is not very rich, as most of the works have been published on 2-echelon SC structures. Nevertheless, (Giannoccaro & Pontrandolfo, 2004) derive the optimal parameter values for a 3-echelon SC where demand is modelled using a newsvendor model. In (Zhou et al., 2013), a similar closed-loop demand function is used to analyse the OEM-supplier relationship. A two-tier CLSC with a cost-sharing model and retailer market leadership is analysed in (De Giovanni & Zaccour, 2013) and the authors show that the coordination is possible in a specific case. (De Giovanni, 2014) analyze a 2-level CLSC with greening efforts and apply a reverse revenue sharing contract to increase the sales as well as the return rate. (Q. Pang et al., 2014b) build demand disruptions in a 3-echelon SC and apply the RS contract to coordinate it, (Q. H. Pang et al., 2015) modify the model with distributor sales efforts. (Hou et al., 2015) analyse the RS contract in a 3-level SC in terms of R's risk aversion. (Xie et al., 2016) Investigate a revenue sharing agreement for a dual on/offline SC. (Hou et al., 2017) analyses a 3-level SC under different scenarios and derives the critical ratio for a coordinated 3-level manufacurer-led SC with stochastic demand. (De Giovanni, 2017) Studies the impact of endogenous and exogenous incentives in the 2-player game through a profit sharing model. (Giri et al., 2017) inspect the effectiveness of the consignment policy in a 3-level CLSC. A retailer-led retailer–supplier CLSC is presented in (A. Taleizadeh, Soleymanfar, et al., 2017) with different alliance structures when another supplier enters the market with a substitutable product., (A. Taleizadeh, Moshtagh, et al., 2017) analyse a three-stage CLSC and the optimal pricing decision under the price reference assumption and (A. Taleizadeh & Moshtagh, 2018) propose a consignment stock system under realistic assumptions that the perceived quality of remanufactured products may be lower than traditionally expected in the models. (Xie et al., 2018) analyzes a dual chain and combines a RS contract in the forward chain with cost sharing contract in the reverse chain. (B. Sarkar et al., 2019) propose a study on the impact of enahnced returnable packaging materials on the CLSC sustainability. (A.

A. Taleizadeh et al., 2020) provide a study of the simultaneous effect of increased advertising and value consumer surplus using an advertising cost sharing contract. (Chakroborty et al., 2020) also consider a consignment stock and conclude that the chain performs better when the top level is leading. (Huang & Yao, 2020) formulate a 3-level chain and study coordination and behaviour via a TPT contract. (Aazami & Saidi-Mehrabad, 2021) propose a new new multi-period production-distribution planning model for perishable products. In (Gharibi & Abdollahzadeh, 2021), a mixture of multi-objective multilevel MILP and inverse DEA is applied to design CLSC reverse logistics. (Ullah et al., 2021) show that remanufacturing pays off in chains with high ordering and set-up costs. (Zhong et al., 2021) model a distributor's negotiation of the wholesale price in a 3-echelon CLSC with its upstream and downstream partners. (Kennedy et al., 2021) discuss the the importance of a distributor in a 3 tier CLSC based on product types. A study of the impact of inflation on global supply chain performance is provided by (Padiyar et al., 2022). (Zeng & Yang, 2022) model the impact of risk aversion on the profit allocation in a 3-level CLSC. (Dosdogru et al., 2022) proposes an interesting simulation model of CLSC with impact on service levels and overordering cost. (Mondal et al., 2022) study the importance of quality of returned products for remanufacturing and the role of the vendor cooperating and sharing the remanufacturing technology with the manufacturer to control the carbon emissions. In (Dey et al., 2023) study the home delivery service charges conditioned to minimum quantities using fuzzy approach. (M. Sarkar et al., 2023) study the bull-whip effect on in dual chain. (A. A. Taleizadeh et al., 2023) analyze the co-existence of new and remanufactured products in the market with respect to acceptance quality levels.

1.1 Research Gap

The following Table 1 summarises our review of the literature on multi-level CLSC with retailer-led RS coordination and substitutable products. After reviewing the research literature, we identified a research gap on the topic of real FMCG business environment focusing on RS coordination with the dominant retailer in a highly competitive, low-margin industry such as the beverage industry, where the customer would choose between two subsitutable products of indifferent quality and distributors would provide specialised services. In addition, we see an opportunity to analyse the motivation of the players, especially the retailer, and to compare the game-theoretical assumption of rationality with the actual decision-making process in business reality.

Table 1. Related Literature Gap

Literature source	Multi-Echelon	Retailer's Leadership	CLSC	Product Choice	Revenue Sharing	Non-Cooperative GT
(Giannoccaro & Pontrandolfo, 2004)	*				*	*
(De Giovanni & Zaccour, 2013)		*	*			*
(Q. Pang et al., 2014a)	*				*	*
(Q. Pang et al., 2014b)	*				*	*
(De Giovanni, 2014)			*		*	*
(Q. H. Pang et al., 2015)	*				*	*
(Xie et al., 2016)			*		*	*
(A. Taleizadeh, Moshtagh, et al., 2017)	*	*	*			*
(A. Taleizadeh, Soleymanfar, et al., 2017)		*	*			*
(De Giovanni, 2017)		*	*			*
(Hou et al., 2017)	*				*	*
(Xie et al., 2018)	*		*	*	*	*
(A. Taleizadeh & Moshtagh, 2018)	*	*	*			*
(Zheng et al., 2019)	*		*	*		
(A. A. Taleizadeh et al., 2020)			*		*	*
(Huang & Yao, 2020)	*		*			*
(Chakroborty et al., 2020)	*	*	*			*
(Ullah et al., 2021)	*		*			*
(Zhong et al., 2021)	*		*			*
(Kennedy et al., 2021)	*		*	*		*
(Aazami & Saidi-Mehrabad, 2021)	*		*			
(Padiyar et al., 2022)						
(Mondal et al., 2022)	*		*			*
(A. A. Taleizadeh et al., 2023)	*		*	*		*
This paper	*	*	*	*	*	*

2 Model Description

We are dealing with a beverage CLSC that involves forward and reverse logistics flows. The customer has the choice of participating in a CLSC by purchasing a beverage in a returnable glass bottle (RGB) or using a disposable bottle (OW), which is discarded after use with no guarantee of being recycled and remanufactured. Both products are of the same quality and the customer does not expect to notice any difference. If the customer chooses an RGB, the market evidence shows a return rate close to 100%. The main driver is the deposit that the customer receives when the product is returned. Another driver is the lower price of the finished product compared to an OW bottle. On the other hand, the collection of returned bottles requires some effort, starting with the customer, who has to take the bottles to a collection point, the retailer, to redeem the deposit. The retailer has to take care of collection, storage and logistics with the distributor. The distributor not only optimises the transport of the RGB back to the manufacturer for refilling, but also organises the sorting of the bottles, as different manufacturers use different types of RGB. The task is to ensure that each manufacturer receives its own bottles. Sorting is usually done either manually or (semi)automatically. As a result, reverse logistics generates additional transport costs and sorting/administrative costs. These are partially offset by the lower purchase price of an RGB compared to an OW bottle for the manufacturer. Considering a 3-echelon CLSC, we assume a network sc consisting of a manufacturer m, a distributor d, and a retailer r, indexed j, $j \in \{m, d, r, sc\}$. Below, in Table 2**Error! Reference source not found.**, we give the notation used throughout the paper. all units of measure are hectoliters [hl] and currency [$].

Table 2. Model Notation

$p_o/\ p_r$	Unit retail price of OW/RGB products in [$/hl]
$q_o/\ q_r$	Production quantity of OW/ RGB products in [hl]
$c_{\ o}^{\ j}/c_{\ r}^{\ j}$	Unit production/purchasing cost of agent j related to OW/ RGB in [$/hl]
$w_{\ o}^{\ j}/w_{\ r}^{\ j}$	Unit wholesale price charged by agents $\mathrm{M, D}$ for OW/ RGB in [$/hl]
$m_{\ o}^{\ j}/m_{\ r}^{\ j}$	Unit margin of agents $\mathrm{D, R}$ on OW/ RGB products in [$/hl]
S	Unit cost of sorting the RGB empties in [$/hl]
δ	Consumer willingness to pay for RGB products, an index
α, β	Bargaining power of agents in the Revenue Sharing contracts in [%]
π_j^i	Profit functions of agents $\mathrm{J, J} \in \{\mathrm{M, D, R, SC}\}$ in model I , $\mathrm{I} \in \{\mathrm{C, N, SM, SR, RN, RR}\}$ in [$].

Model C is the integrated or centralised case with one decision maker. Model N is a decentralised vertical Nash equilibrium where agents make decisions simultaneously. Models SM/SD/SR are decentralised models based on a wholesale contract with the manufacturer/distributor/retailer alternating as Stackelberg leader. Finally, the RN/RR models are revenue-sharing contracts of the N/SR models. For clarity of the notation, r in subscript refers to RGB for all variables except profit function π_j^i.

The top-down orientation indicates the order in which the players make their decisions. The only exception is the Vertical Nash Model N, where all players make their decisions simultaneously and independently. In the SM model, the manufacturer first sets the wholesale price to the distributor, who then sets the wholesale price to the retailer, who then sets the final price to the consumer. In the world of beverage manufacturers, this is not so common unless the manufacturer has a dominant market position as a producer with strong brands. In the SR model, the decision making process starts with the retailer and ends with the manufacturer deciding on the wholesale price to the distributor based on the distributor's input. As we are dealing with an FMCG environment in our study, this is the most likely market condition in which most FMCG manufacturers find themselves, dominated by international retail chains and the growing power of local and international distributors.

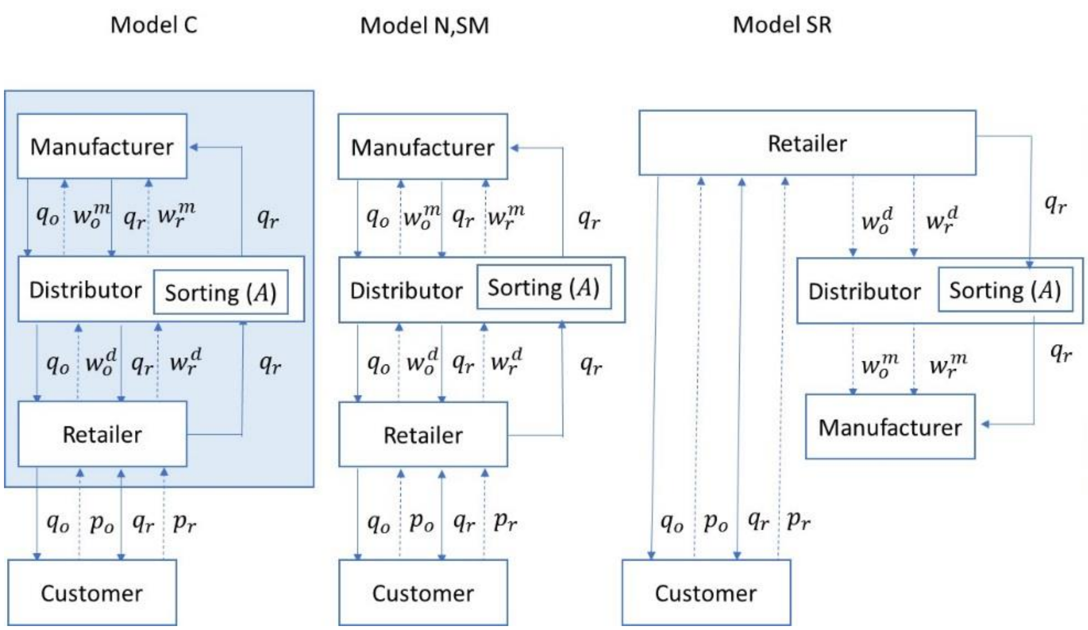

Figure 1. CLSC models under consideration

2.1 Revenue Sharing Contract in a 3-echelon CLSC

In a revenue-sharing (RS) contract between the manufacturer and the distributor, the RS includes a wholesale price and a share of the revenue retained by the distributor $\left(w_{o/r}^{m}, \beta\right)$. Similarly, between the distributor and the retailer, the RS includes the distributor's wholesale price and a share of the revenue retained by the retailer $\left(w_{o/r}^{d}, \alpha\right)$. Figure 2 shows the RS contract scheme in a three-echelon CLSC.

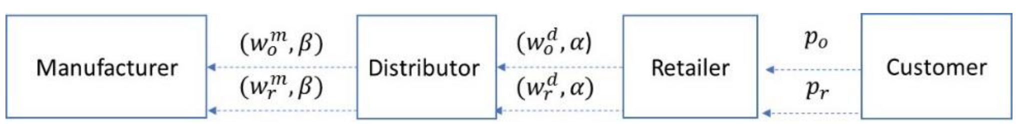

Figure 2. Revenue Sharing contract in 3-echelon CLSC

(Giannoccaro & Pontrandolfo, 2004) analyse the conditions effectiveness criteria for both wholesale prices to ensure that the contract is desirable for all agents. They derive the win-win region conditions for a Vertical Nash (N) model where all agents decide simultaneously. A series of works by (Q. Pang et al., 2014b, 2014a; Q. H. Pang et al., 2015) analyse a three-stage forward SC with stochastic demand under different market conditions, such as sales or distributor efforts, and apply the pure RS contract enhanced with a quantity discount wholesale contract. In the work of (Hou et al., 2017), this model is analysed for a simple forward SC with stochastic demand, and in addition to the N model, the authors analyse the conditions under which the manufacturer-led Stackeberg model can be coordinated with the RS contract. According to their analysis, they conclude that RS can coordinate the three-level model only if the retailer orders exactly the same quantity as in the decentralised uncoordinated model. As mentioned above, in this paper we take a real-world three-level CLSC and analyse the conditions under which the RS contract can coordinate this chain, assuming different marker leadership. To the best of our knowledge, we have not found any study on a 3-echelon CLSC dominated by a retailer and fully coordinated with an RS contract. By fully coordinated we mean that the price/quantity variables are constrained by the performance of the centralised CLSC.

2.2 Customer Demand Functions

In (Ferguson & Toktay, 2006), the assumptions are made for a model that considers a consumer's choice between a remanufactured product and a new product. A new product provides a utility u to the consumer, whereas a remanufactured product provides only a fraction of this utility, δu. The fraction δ is called the willingness-to-pay for a remanufactured product. Since the CLSC includes the so-called 3R principle (Return, Recycle, Re-manufacture), we will consider an RGB packaged product as remanufactured and an OW bottle as new without loss of generality. (Ferguson & Toktay, 2006) assume a uniformly distributed $\delta \in [0,1]$. The customer's choice is then based on his utility, i.e. either u or δu, and the final product price. The inverse demand functions for OW and RGB products are formulated in (1) and (2) respectively.

$$p_o = Q - q_o - \delta q_r, \tag{1}$$
$$p_r = \delta(Q - q_o - q_r), \tag{2}$$

where $Q = 1$ is the normalized market size. Solving (1) and (2) for quantities, we obtain the demand functions for OW and RGB products as

$$q_o(p_o, p_r) = 1 + \frac{(p_o - p_r)}{(\delta - 1)}, \tag{3}$$
$$q_r(p_o, p_r) = \frac{(p_o - p_r)}{(1 - \delta)} - \frac{p_r}{\delta}. \tag{4}$$

Eqs. (3) and (4) show the dynamics of prices and quantities in the sense that q_r increases as δ increases, driving up the total market quantity, $q_r + q_o$ The willingness-to-pay parameter, δ, reduces the price impact of RGB products on total demand as well as the impact of the price difference between OW and RGB products. These functional forms are also used in other analytical studies. In the work of (Zhou et al., 2013), this approach is used to analyse the behaviour of an OEM deciding whether to use new or remanufactured parts. (Örsdemir et al., 2014) extend the analysis by assuming a different perceived quality level of remanufactured products. In our case, as mentioned above, both the perceived and the objective product quality are the same for both end products regardless of the packaging. We also assume, contrary to the model of (Ferguson & Toktay, 2006), that there is already a sufficient supply of RGB products on the market for the customer to choose from. (Zheng et al., 2019) use this demand modelling approach to coordinate the CLSC with remanufactured products through a cooperative game theory approach under the consideration of fairness.

3 Model Assumptions

In the next section, we define the cost structure and present the mathematical formulation of the models presented above. We distinguish the purely centralised model, which serves as a benchmark for the performance and analysis of the fully decentralised models, and later as a success criterion for the coordinated models. The Stackelberg leader-follower models are solved by backward recursion and we show the equilibrium results of all decentralised models and discuss their properties. In the last section of the chapter we set the constraints on the demand quantities.

3.1 CLSC Cost Structure

In our real-world example, we consider the following cost elements, shown in Table 3

Table 3. Detailed Cost Parameters Description in [$/hectoliter(hl)]

c_o^m	Unit var. cost of manufacturer to purchase a new OW bottle for filling
c_r^m	Unit var. cost of manufacturer to purchase and reuse an RGB bottle
c_o^d	Unit var. one-way transportation/warehousing/handling cost of distributor
c_r^d	Unit var. round trip transportation/warehousing/handling cost of distributor
c_o^r	Unit var. warehousing/handling cost of retailer related to OW bottles
c_r^r	Unit var. warehousing/handling cost of retailer related to RGB bottles
$c_o^{sc} = c_o$	Total CLSC variable cost for OW bottles, $C_O = \sum_J C_O^J$
$c_r^{sc} = c_r$	Total CLSC variable cost for RGB bottles, $C_R = \sum_J C_R^J$
S	Unit var. cost of sorting the RGBs

The parameter S includes all the costs/hectolitre associated with the retailer's process of collecting the RTP, including the cost of transport back to the producer's premises, c_o captures the costs/hectolitre of the producer using the new non-refillable packaging, scaled to 100%, and finally c_r represents the costs/hectolitre of using the RGB, including processing, handling, sorting and technical breakage losses, scaled to 100% on a comparable basis to $c_o$and S.

3.2 Centralized CLSC - Model C

Equation (5) shows the centralized model profit function, assuming that one function controls all 3 levels.

$$\max_{p_o, p_r} \pi_{sc}^C = q_r(p_r - S - c_r) + q_o(p_o - c_o). \tag{5}$$

Proposition 1: *π_{sc}^C is a concave function of p_o, p_r and has one maximum.*

Proof: From (5) we prove that π_{sc}^C is concave by checking the minors of the Hessian matrix of the function π_L^C.

$$H(\pi_L^C) = \begin{bmatrix} \dfrac{\partial^2 \pi_{sc}^C}{\partial {p_r}^2} & \dfrac{\partial^2 \pi_{sc}^C}{\partial p_r \partial p_o} \\ \dfrac{\partial^2 \pi_{sc}^C}{\partial p_o \partial p_r} & \dfrac{\partial^2 \pi_{sc}^C}{\partial {p_o}^2} \end{bmatrix} = \frac{2}{1-\delta} \begin{bmatrix} -1 & -1 \\ -1 & -\dfrac{1}{\delta} \end{bmatrix}$$

The principal minors are $|M_1| = -\frac{2}{1-\delta} < 0, |M_2| = \frac{4}{\delta(1-\delta)} > 0$, if $0 < \delta < 1$, hence π_L^C is a concave function and has one maximum.

3.3 Decentralized Models – SM, SR, N

Equations (6)-(8) show the profit functions for each CLSC agent under the assumption of full decentralisation, $k \subset j, k = \{SM, SR, N\}$.

$$\max_{w_o^m, w_r^m} \pi_m^k = q_r(w_r^m - c_r^m) + q_o(w_o^m - c_o^m) \tag{6}$$

$$\max_{w_o^d, w_r^d} \pi_d^k = q_r(w_r^d - S - c_r^d - w_r^m) + q_o(w_o^d - c_o^d - w_o^m) \tag{7}$$

$$\max_{p_o, p_r} \pi_r^k = q_r(p_r - c_r^r - w_r^d) + q_o(p_o - c_o^r - w_o^d) \tag{8}$$

And the total CLSC profit for each model is

$$\pi_{sc}^k = \pi_m^k + \pi_d^k + \pi_r^k \tag{9}$$

We also need to assume the following relationships between margins and the final price to the customer in order to solve these equations $p_o = w_o^m + m_o^d + m_o^r$, $p_r = w_r^m + m_r^d + m_r^r$,$w_o^d = w_o^m + m_o^d$, and $w_r^d = w_r^m + m_r^d$.

3.4 Decentralized Models Equilibrium

In this part we present the closed-loop parameter form of the equilibria of the integrated and decentralised models. These decentralised models will serve as the basis for coordination.
Table 4 summarises the equilibrium results for the key decision variables in the integrated and the four decentralised models.

Table 4. Decentralized Models Variables Equilibria

	C	SM	SR	N
p_o^*	$\frac{1+c_o}{2}$	$\frac{7+c_o}{8}$	$\frac{7+c_o}{8}$	$\frac{7+c_o}{4}$
p_r^*	$\frac{S+c_r+\delta}{2}$	$\frac{S+c_r+7\delta}{8}$	$\frac{S+c_r+7\delta}{8}$	$\frac{S+c_r+7\delta}{4}$
q_o^*	$\frac{\chi}{2}$	$\frac{\chi}{8}$	$\frac{\chi}{8}$	$\frac{\chi}{4}$
q_r^*	$\frac{\psi}{2}$	$\frac{\psi}{8}$	$\frac{\psi}{8}$	$\frac{\psi}{4}$
w_o^{m*}	-	$\frac{1+\ c_o-2(c_o^d+c_o^r)}{2}$	$\frac{1+\ 7c_o-8(c_o^d+c_o^r)}{8}$	$\frac{1+\ 3c_o-4(c_o^d+c_o^r)}{4}$
w_r^{m*}	-	$\frac{\varphi+c_r-2(c_r^d+c_r^r)}{2}$	$\frac{\varphi+7c_r-8(c_r^d+c_r^r)}{8}$	$\frac{\varphi+3c_r-4(c_r^d+c_r^r)}{4}$
w_o^{d*}	-	$\frac{3+\ c_o-4c_o^r}{4}$	$\frac{3+\ 5c_o-8c_o^r}{8}$	$\frac{1+\ c_o-2c_o^r}{2}$
w_r^{d*}	-	$\frac{c_r+S+3\delta-4c_r^r}{4}$	$\frac{5(c_r+S)+3\delta-8c_r^r}{8}$	$\frac{c_r+S+\delta-2c_r^r}{2}$

$\chi=(1-\ c_o+c_r+S-\delta)/(1-\delta)\,;\ \ \psi=(\delta c_o-c_r-S)/\delta(1-\delta);\ \varphi=\delta-S$

We can conclude the following from the results in Table 4 For both prices p_o^*, p_r^* the models show that $C<VN<SM=SR$. Due to the inverse demand functions the quantities of both products q_o^*, q_r^* are sorted in the reverse order, i.e., $C>VN>SM=SR$. It also shows that $q^{VN*}=\frac{q^{C*}}{2}$, $q^{SR*}=q^{SM*}=\frac{q^{C*}}{4}$ for both OW and RGB products. The total quantity is reduced and the prices increase due to the double marginalisation and the introduction of wholesale prices, which we later try to mitigate through the RS contract. From Table 4 we can conclude the following about the wholesale price equilibria:

$$w_{o/r}^{m*}(SM)>w_{o/r}^{m*}(VN)>w_{o/r}^{m*}(SR)$$

$$w_{o/r}^{d*}(SM)>w_{o/r}^{d*}(VN)>w_{o/r}^{d*}(SR)$$

The results correspond to the relative market power. The manufacturer's wholesale prices are highest when m is the leader and then when bargaining power is balanced, as in the N model. When the leader is the retailer, the bargaining power of the manufacturer is significantly weaker. In particular, when r is the leader, d can negotiate the lowest producer price because it is in direct contact with m, and sets its wholesale prices $w_{o/r}^{d*}$ with $w_{o/r}^{m*}$ as the best response to it. We also notice that the wholesale prices of OW products do not depend on δ and are only cost driven. The RGB wholesale prices w_r^* for all models grow with the consumer's willingness-to-pay for RGB products and with the total CLSC costs c_r. Table 4 shows how the downstream costs of the distributor and the retailer constrain the growth of the manufacturer's wholesale price. Similarly, the distributor's wholesale price is negatively affected by the retailer's unit variable cost. Finally we present the profit function equilibria for the respective models and CLSC members.
We also measure the relative performance of the decentralised models by comparing their profits with the CLSC profit of model C in Table 6. The N model is the best performing decentralised model, achieving 75 % of the profit of model C. Model N is followed by SM and SR models performing equally.

In order to keep this paper concise and focused, we will not discuss Model N in the following sections of the paper. Its presence is only to illustrate the difference between the leader-follower model and one where all are equally powerful. As this paper analyses the real world application, we will only consider the leader-follower models, although we will shortly discuss the N model's assumption in the CLSC coordination section to set the scene for the revenue sharing model.

Table 5. Profit functions equilibria

$$\pi_{sc}^{C*} = \frac{(1-c_o)^2}{4} + \frac{(c_r+S-c_o\delta)^2}{4\delta(1-\delta)}$$

$$\pi_r^{N*} = \frac{(1-\delta)(2c_o-1)+\delta(2Sc_o+2c_oc_r-c_o^2)-(c_r+S)^2}{16\delta(1-\delta)} = \pi_d^{N*} = \pi_m^{N*}$$

$$\pi_{SC}^{N*} = \frac{(1-\delta)(6c_o-3)+\delta(6Sc_o+6c_oc_r-3c_o^2)-(3S^2-6Sc_r-3c_r^2)}{16\delta(1-\delta)}$$

$$\pi_m^{SM*} = \frac{(1-c_o)^2}{16} + \frac{(c_r+S-c_o\delta)^2}{16\delta(1-\delta)} = \pi_r^{SR*}$$

$$\pi_d^{SM*} = \frac{(1-c_o)^2}{32} + \frac{(c_r+S-c_o\delta)^2}{32\delta(1-\delta)} = \pi_d^{SR*}$$

$$\pi_r^{SM*} = \frac{(1-c_o)^2}{64} + \frac{(c_r+S-c_o\delta)^2}{64\delta(1-\delta)} = \pi_m^{SR*}$$

$$\pi_{sc}^{SM*} = \frac{7(1-c_o)^2}{64} + \frac{7(c_r+S-c_o\delta)^2}{64\delta(1-\delta)} = \pi_{sc}^{SR*}$$

Table 6. Decentralized CLSC performance vs. integrated model C

[%]	C	SM	SR	N
π_{sc}^*	100	44	44	75

3.5 Customer Demand Constraints

In real-world settings such as ours, we expect the quantities sold q_o, q_r to take positive values, as some customers will choose the effortless consumption of OW bottles and other environmentally and socially responsible consumers will be motivated to use RGB products. We also assume that the total demand $q = q_o + q_r$ is stable and customer choose between the products OW and RGB products. δ is the parameter that captures the customer's preference or willingness to purchase RGB products. Thus, the total quantity of RGBs purchased, q_r, depends on their price and is also influenced by δ, which in turn also influences the quantity of OW bottle products purchased, as the size of the market is limited and these two requirements are balanced. To define the conditions where $q_o \geq, q_r \geq 0$, we first define the extreme cases $q_o = 0, q_r = 0$. From

Table *4* 4, it is clear that $q_r = 0$ if $\psi = 0$, i.e., $\delta = \frac{S+c_r}{c_o}$. In the similar way, we set $q_o = 0$ if $\chi = 0$, i.e., $\delta = S + c_r - c_o + 1$. Now we need to define the regions where both are positive

Proposition 2: *$q_r > 0$, if $\frac{S+c_r}{c_o} < \delta^{LB}$. If the share of total cost of returning and sorting the empty RGB bottles on the purchase cost of the OW bottles is lower than the willingness of the customer to consume RGB products, then it makes economic sense for the producer to produce RGB products, Hence, $\delta^{q_r \geq 0} = \delta^{LB} \geq \frac{S+c_r}{c_o}$ sets the lower bound on δ that ensures positive or minimum zero demand on RGB.*

Proof: We obtain this condition by setting $\Delta_r^i = q_r^{i*} - q_r^{i*} \geq 0$ across all four models $\{C, N, SM, SR\}$ where the nominator of Δ_r^i is $S + c_r - c_o\delta$. Setting $S + c_r - c_o\delta = 0$ and solving for δ, we obtain the lower bound for δ.

Proposition 3: *$q_o > 0$, if $S + c_r - c_o + 1 > \delta$. Hence $\delta^{q_o \geq 0} = \delta^{UB} \leq 1 + S + c_r - c_o$ sets the upper bound on δ that ensures positive or minimum zero demand on OW.*

Proof: By setting $\Delta_o^i = q_o^{i*} - q_o^{i*} \geq 0$ across all four models $\{C, N, SM, SR\}$ we obtain a nominator of Δ_o^i in the form $S + c_r - c_o + 1 - \delta$. Setting $S + c_r - c_o + 1 - \delta = 0$ and solving for δ, we obtain the upper bound for δ.

Proposition 4: *$q_r \geq q_o$, if $(1+\delta)(S+c_r) \leq \delta(\delta + 2c_o - 1)$. This condition determines which products will dominate the market. The value of δ where both product sell equal quantities is set at $\delta^{q_r = q_o} = \delta^{EQ} = \frac{1+S+c_r-2c_o}{2} + \frac{\sqrt{(S+c_r)^2+6(S+c_r)+4c_o(c_o-1-c_r-S)+1}}{2} = \frac{1+S+c_r-2c_o}{2} + \frac{\omega}{2}$*

Proof: It follows directly from Table 4 when we subtract the equilibria $q_o^{i*} - q_r^{i*}$ for all $i \in \{C, N, SM, SR\}$, where $(1+\delta)(S+c_r) - \delta(\delta + 2c_o - 1)$ is the nominator of the subtraction based on ψ and χ. Solving $(1+\delta)(S+c_r) - \delta(\delta + 2c_o - 1) = 0$, we obtain the relationship where $q_r = q_0$.

Given propositions (1)-(3), the CLSC can be sustainable only if the cost of producing from new materials (OW) c_o is higher than the cost of RGB's reverse logistics $S + c_r$, thus

$$S + c_r < c_o \leq 1 \tag{10}$$

Assuming that the cost of reverse logistics is stable over time because it is based on long-term service contracts and infrastructure, while the c_o can be reduced by a one-off procurement initiative or a new supplier. We will investigate the relationship between the cost c_o and δ and how it affects the quantities sold q_o, q_r. Combining propositions (1)-(3), we get the lower and upper bounds on the $\delta \in [\delta^{LB} < \delta^{EQ} < \delta^{UB}]$. Of course, the market share of each product, q_o and q_r, is important to the firms in the chain in meeting their profit targets. Hence, we are interested only in the subset region where both $q_r \geq 0$ and $q_o \geq 0$, which is constrained by the lines $q_r = q_o = 0$.

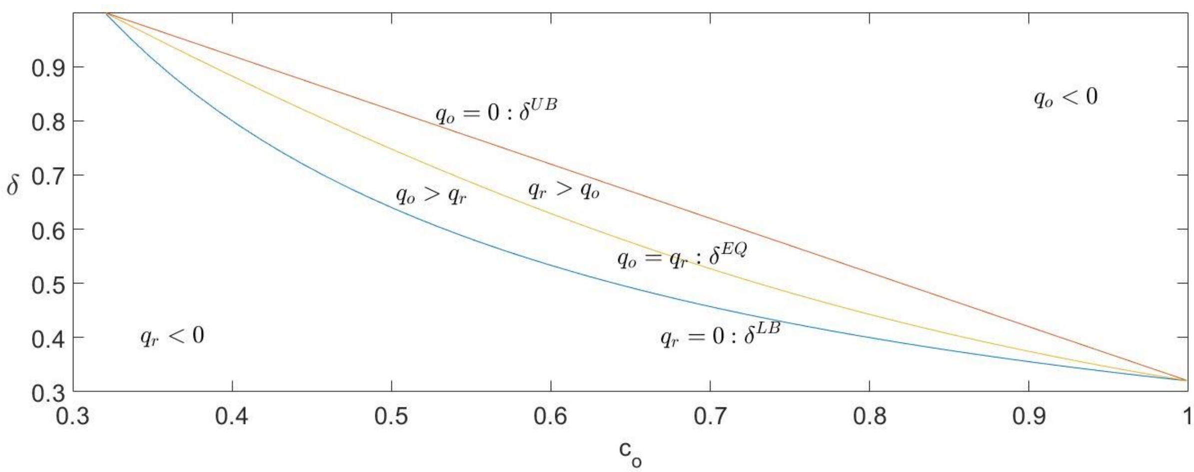

Figure 3. Demand function constraint with $q_r > 0$, $q_o > 0$, and $q_r = q_o$

3.6 Real-World Case Study

The final section discusses our approach to analysing the real drink CLSC. We have defined the set where both products are sold in positive quantities. We will now use the parameters $\delta^{UB}, \delta^{EQ}, \delta^{UB}$ to analyse the properties and sensitivities of the equilibria of the decentralised models. We focus on the limiting case where either $q_r = 0$, meaning that the CLSC is 100% based on RGB and sustainable, or $q_o = 0$, where the market demands only the OW products. To capture the case where both products are sold on the market, we also analyse the case where $q_o = q_\mathrm{r}$.

In our analysis we will focus on the distance between $c_o - (c_r + S)$ constrained by $\delta \in \{\delta^{LB}, \delta^{EQ}, \delta^{UB}\}$. This is critical for any enterprenuer to consider and manage. The lower the cost always favors the profit, however the relative distance of the forward c_o SC costs and the reverse logistics $c_r + S$ is also important because the final consumer prices.

We will also need the real business cost parameters summarized in Table 7. For confidentiality reasons, all the cost parameters presented have been reduced to indices. Nevertheless, the relative distances reveal the true relationships between them. Thus, the parameter S is the cost of sorting to the distributor of 0.02 \$/hectolitre, or \$/hl, etc.

Table 7. Real industry cost parameters

c_r	c_r^m	c_o^d	c_r^d	c_o^r	c_r^r	S
0.3	0.07	0.13	0.22	0.005	0.01	0.02

What we can see from the Table 7 is that according to (13) the $c_o > c_r + S = 0.32$ \$/hl. In our four scenarios c_o is the senstitivity parameter which we use to analyze the equilibria, namely the profits of the CLSC members. The variable logistics costs $c\,_r^d = 0.22$ \$/hl reflect the impact of the cost of the round trip, which is never double the cost of the one-way route, $c\,_o^d = 0.13$ \$/hl. The administrative and additional logistics costs on the retailer's side $c\,_{o/r}^r$ are relatively low compared to the other CLSC members.

4 Performance and Sensitivity Analysis of Decentralized Models

In this chapter, we perform the sensitivity analysis of the main performance metrics to δ and c_o. We show the demand functions and the players' profit functions with respect to the constraints of the demand function and as a function of δ and c_o.

4.1 Demand and Profit Equilibria

Through sensitivity analysis, we can see in the Figure 4 that q_r increases with increasing δ at each c_o level, as it benefits from the increasing cost difference $c_o - c_r$. On the other hand, q_o increases with increasing δ only when c_o is reasonably low. Given the nature of (3) and (4), the effects balance out and the total market demand, $q_o + q_r$, is the same for any given c_o, as shown in the Figure 4.

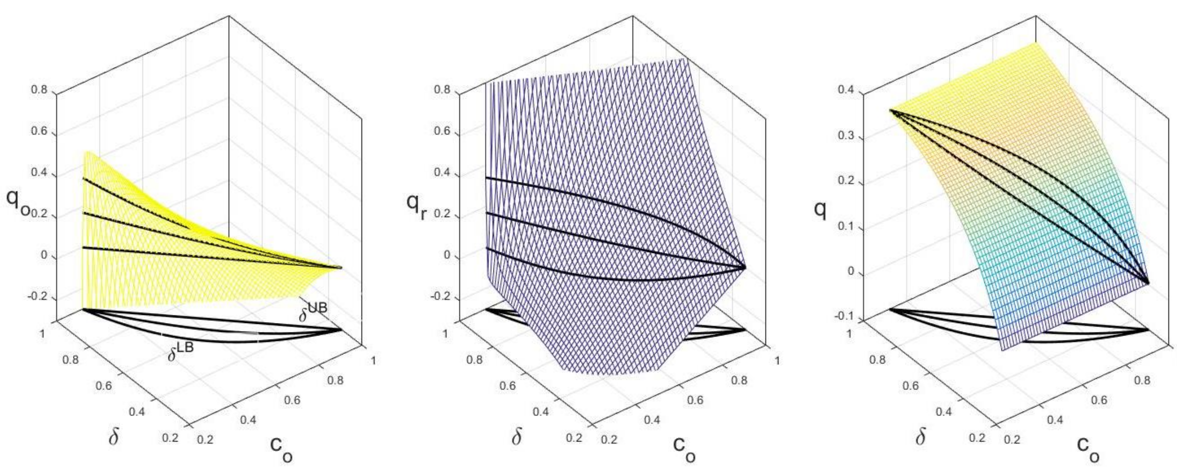

Figure 4. Model C - Optimal sold quantities - sensitivity to δ and c_o

In the Figure 5, we can compare the equilibrium sold quantities according to the models SM/SR with the model C results in Figure 4. The constrained area shows us that the feasible quantities are four times lower for both q_o, q_r.

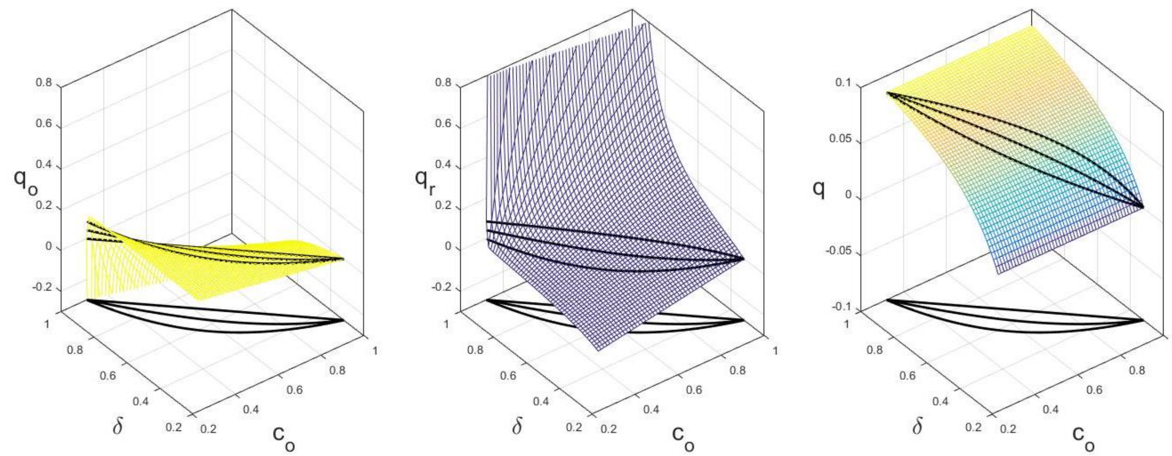

Figure 5. Models SM/SR - Optimal sold quantities - sensitivity to δ and c_o

The final graphs in Figure 6 show the CLSC profit functions of the model C and SM/SR. The profit increases with δ as δ moves from δ^{LB} to δ^{UB} and decreases with the increasing c_o. In other words, it is more profitable for the CSLC to sell more RGB than OW products, and the profitability depends on $c_o - c_r$ and δ.

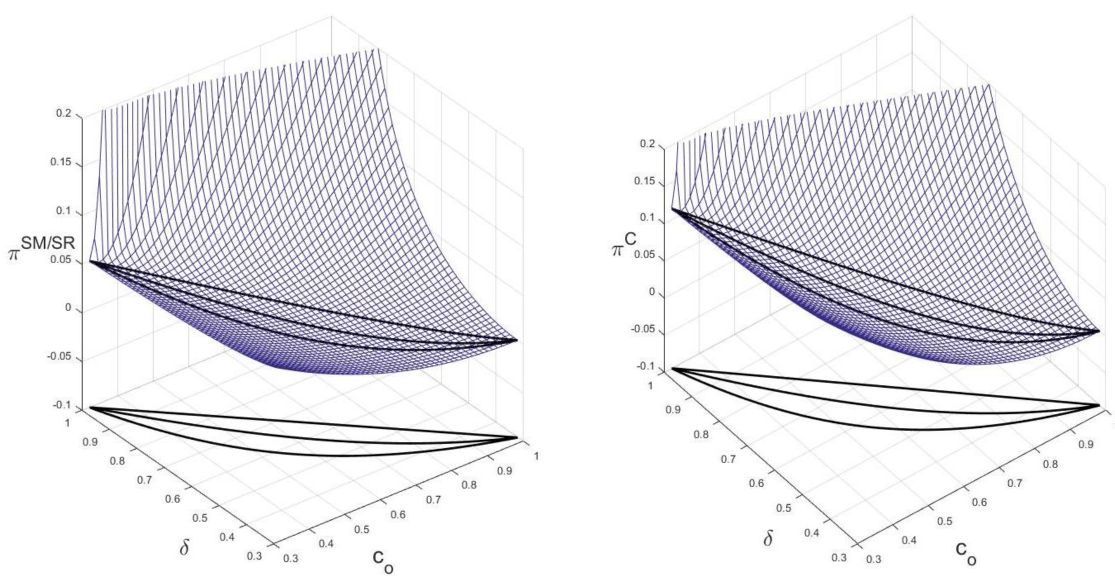

Figure 6. Model C, SM,SR – Profit - sensitivity to δ and c_o

4.2 Decentralized Models Sensitivity Analysis

Next, we carry out the sensitivity analysis to understand what happens to profits and prices in relation to costs and δ. We compare the performance of the key variables in the integrated model C with the decentralised models SM and SR and their sensitivity to cost. The cost parameter chosen for the sensitivity analysis is c_o^m, the manufacturer's purchase cost of new OW bottles. This cost is the main contributor to the total forward cost c_o and is set at ~0.47 \$/hl in our real business environment. Therefore, the c_o parameter, which is the sum of all forward costs, is 0.6 \$/hl when we add the retailer's and distributor's parts. We therefore let this parameter c_o vary from 0.32 \$/hl, where $c_o = c_r$, and thus $q_o = q_r$ to 1 where $q_o = q_r = 0$. Changing the cost parameter also requires a change in consumer behavior as shown in Figure 3 in order to satisfy the non-negative volume constraints, $q_o \geq 0, q_r \geq 0$. Therefore, we show the sensitivity of the key variable within the feasible region and along its boundaries as shown in the Figure 7. For example, the line with the legend entry $q^C = q_o^C$ shows how the purchased volume changes with the relatively increasing cost of the forward vs. reverse chains when the customer only demands OW products. And for the specific boundary case, $q^C = q_o^C$, we see that the CLSC would be better off with a mixed portfolio with a higher share of RGB products, as they would limit the impact of the OW cost increase and the company would have a mitigation plan to maintain volumes. We can also see the performance gap between the decentralised model and the integrated model. The decentralisation combined with the price increase drastically reduces the manoeuvrable space for the CLSC members, in our case by a factor of four, and the double marginalisation takes its toll. In order to show how much this space is reduced, we can compare the vertical distance of the curves at each cost level. In the case of the SM/SR models, the decentralised non-cooperative environment leaves very little room for finding opportunities in the RGB market, as the price of OW rises due to the fight for margins.

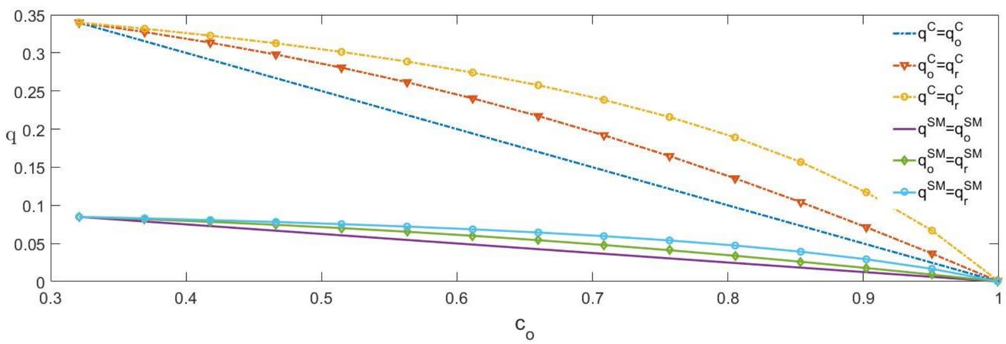

Figure 7. Models C and SM/SR - sensitivity of purchased quantity q to c_o

Figure 8 shows the price dynamics reflecting the volume case. The important fact to note here is the p_o equilibrium, which depends only on c_o and not on δ. Therefore, the increasing cost of new OW bottles for the producer must be reflected in the final price if all actors act rationally. The C/SM/SR models assume that the players have the power to pass on the price increase to the consumer, and we can see the negative effect this has on volumes in Figure 7. Prices are generally set higher in model C than in the decentralized model and we can observe how they converge as the costs increase. Price p_r depends on δ and we can distinguish the cases where different product mixes are sold. And we can observe that increasing the share of RGB products opens space for higher prices and the price is less sensitive to c_o.

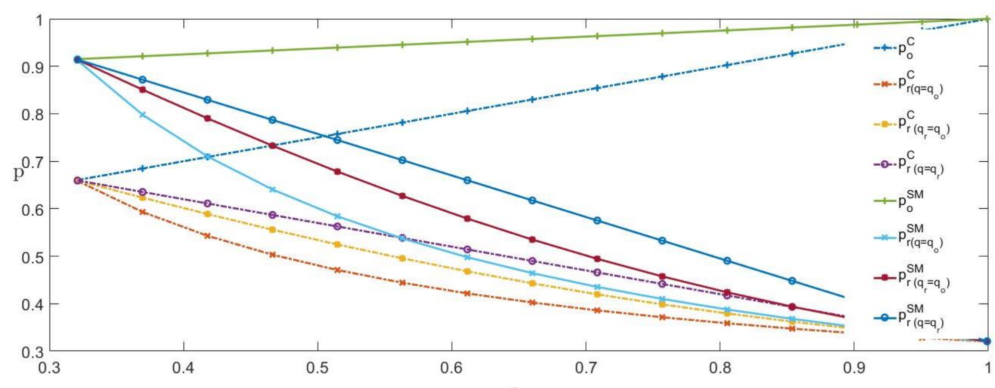

Figure 8. Models C and SM/SR - sensitivity of p to c_o

Figure 9 shows the main performance indicator, profit. It is not surprising that model C performs better than models SM/SR. We can see the gap between the decentralised models and model C, and it is also interesting to observe how quickly profits fall when the market is dominated by OW products. The situation with the profit starts to improve more dramatically when the sales of RGB start to dominate the OW products and the consumers change their behaviour. This change is more visible in model C.

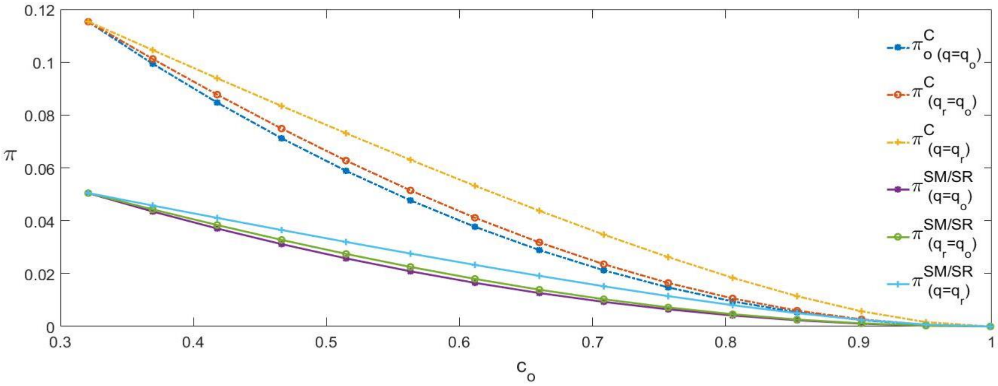

Figure 9. Models C and SM/SR - sensitivity of CLSC profit π to c_o

The distribution of the profits is shown in the Figure 10 for the model M. As shown in
Table *4* 4, shows the manufacturer and the retailer swap their profits in the SM and SR models depending on their leadership, therefore $\pi_m^{SM*} = \pi_r^{SR*}$. All CLSC members benefit more from selling RGB products in their porfolio rather then OW and the leader is always better off than the follower.

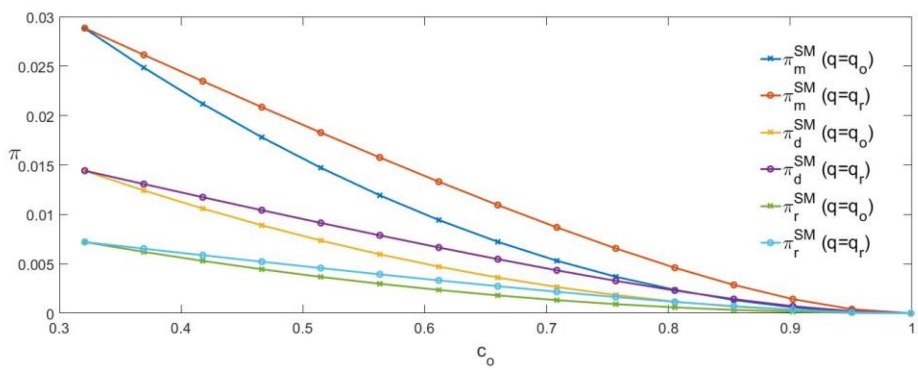

Figure 10. Min and Max individual profits π in SM (SR) model - sensitivity to c_o

The next two graphs focus on wholesale prices. Figure 11 shows the dynamics of the manufacturer's wholesale prices in the two models SM and SR. First, as with p_o, the w_o prices depend only on costs. What stands out is the effect of leadership, where the manufacturer as Stackelberg leader has the power to set a higher w_o, w_r prices than in the case where the retailer as leader pushes down the manufacturer's margins. And in terms of the absolute levels of w_r, the manufacturer can charge more when producing and selling RGBs with the leadership effect.

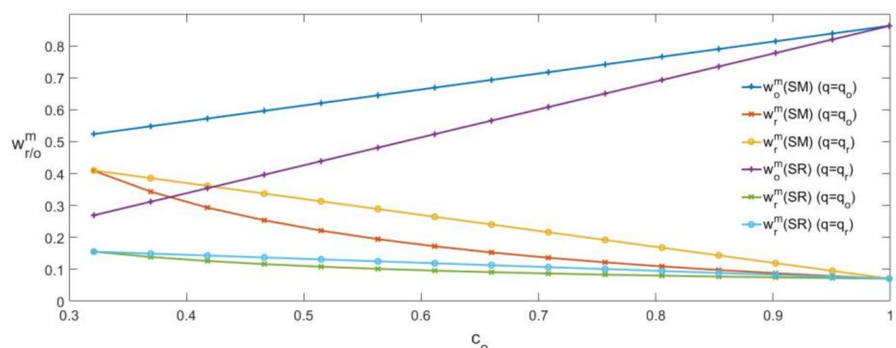

Figure 11. Manufacturers's wholesale prices sensitivity to c_o in Model SM and SR

The distributor's wholesale price shows a similar dynamic to that of the producer. There is a very interesting insight in Figure 12: if the cost of the forward chain increases above 0.65 $/hl, then the distributor would be better off shipping RGB than OW by setting his wholesale price w_r higher. However, this is difficult as the producer is the leader. The situation would improve for the distributor if the retailer led the CLSC.

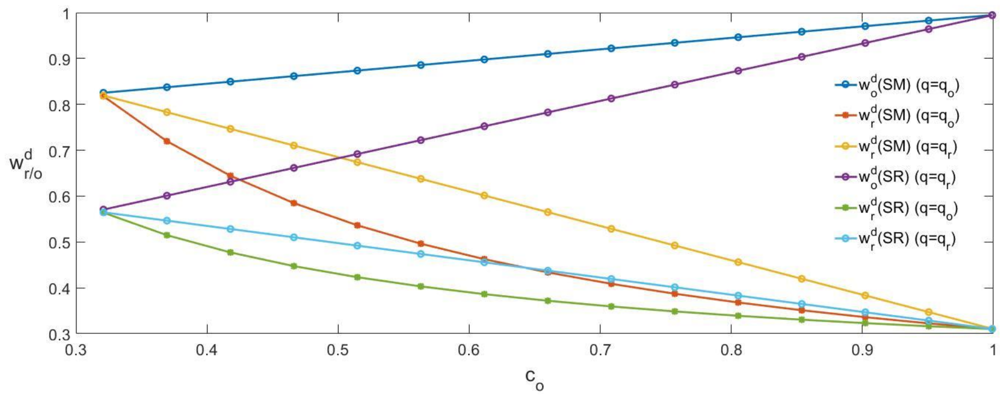

Figure 12. Distributor's wholesale prices sensitivity to c_o in Model SM and SR

Finally, we present in Figure 13 the distributor margins that perform best when the market is dominated by RGB products.

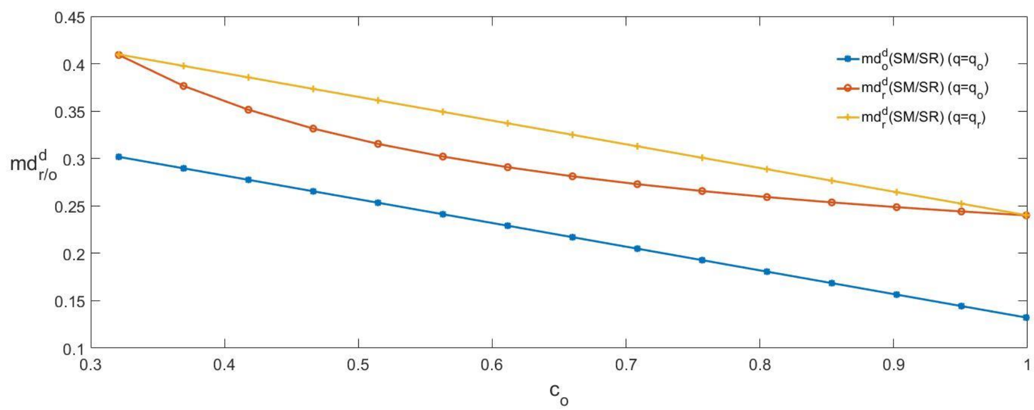

Figure 13. Distributor's margins sensitivity to c_o in Model SM and SR

5 CLSC Coordination - Revenue Sharing Contract

In this section, we apply the RS contract to two decentralised models, trying to find the achievable RS contract parameters $(\alpha, \beta, w^{m*}_{o/r}, w^{d*}_{o/r})$ that would be acceptable to all CLSC members. They will only accept it if their individual profits will be higher than in the decentralised cases. This condition is called individual rationality and can be expressed as $\pi^{RN*}_j \geq \pi^{VN*}_j, \pi^{RR*}_j \geq \pi^{SR*}_j, \forall\, j \in \{m, d, r, sc\}$. The bargaining power of the leader is based on his ability to negotiate such contract terms to obtain a high share of the profit from others in the chain. The parameters (α, β) capture this power within the RS contract so the leader must be able to influence them both and represent the share of revenue that the particular player retains, while the rest is shared with the upstream CLSC partner. Most of the literature shows an RS contract only on a 2-echelon manufacturer-led chain. As mentioned above, in this paper we are motivated to compare the contract terms and feasibility when retailer takes the lead. A retailer-led CLSC is, however, the most common case in the FMCG industry.

5.1 Nash Equilibrium – model RN

Earlier we discussed the decentralised Vertical Nash model N, which assumes that the CLSC members have equal access to information, make decisions independently and have equal market power. As mentioned earlier, we used this model only to illustrate the Nash equilibrium for the case of a three-player CLSC game which we did in Table 4. To coordinate the model N, let us denote it RN, we must solve simultaneously the system of equations (11).

$$\begin{cases} \max\limits_{p_o,p_r} \pi_r^{RN} = (1-\beta)[(1-\alpha)(q_r w_r^m + q_o w_o^m) + q_r w_r^d + q_o w_o^d] - q_r(w_r^d + c_r^r - p_r) - q_o(w_o^d + c_o^r - p_o) \\ \max\limits_{w_o^d,w_r^d} \pi_d^{RN} = \beta[(1-\alpha)(q_r w_r^m + q_o w_o^m) + q_r w_r^d + q_o w_o^d] - q_r(w_r^m + c_r^d + S) - q_o(w_o^m + c_o^d) \\ \max\limits_{w_o^m,w_r^m} \pi_m^{RN} = \alpha(q_r w_r^m + q_o w_o^m) - q_r c_r^m - q_o c_o^m \end{cases}, \tag{11}$$

$$s.t.$$

$p_o^{C*} = p_o^{RN*}$, $p_o^{C*} = p_o^{RN*}$, $q_o^{C*} = q_o^{RN*}$, $q_r^{C*} = q_r^{RN*}$, $\pi_{sc}^{RN*} = \pi_{sc}^{C*}$,

where $p_o = w_o^m + m_o^d + m_o^r$, $p_r = w_r^m + m_r^d + m_r^r$, $w_o^d = w_o^m + m_o^d$, $w_r^d = w_r^m + m_r^d$, and $\pi_{sc}^{RN} = \pi_r^{RN} + \pi_d^{RN} + \pi_m^{RN}$.

To solve (11) we put the first-order-conditions as $\frac{\partial \pi_r^{RN*}(\alpha,\beta)}{\partial m_r^r} = \frac{\partial \pi_r^{RN*}(\alpha,\beta)}{\partial m_o^r} = \frac{\partial \pi_r^{RN*}(\alpha,\beta)}{\partial m_r^d} = \frac{\partial \pi_r^{RN*}(\alpha,\beta)}{\partial m_o^d} = \frac{\partial \pi_r^{RN*}(\alpha,\beta)}{\partial w_r^m} = \frac{\partial \pi_r^{RN*}(\alpha,\beta)}{\partial w_o^m} = 0$.

Model (11) will serve us to compare the impact of the coordination of a Stackelberg leader-follower model with a pure Nash equilibirium. The RN model solution gives us interesting and simple properties provided the absence of the best response functions and complex backwards induction and offer a nice benchmark for the RR model. We begin with the RN model equilibrium and desirability region. Figure 14 shows the Nash equilibrium (model RN) of the RS contract for a three-echelon CLSC, where the area hatched by the three curves represents a bargaining space. It says that the retailer will accept the contract terms if he can retain at least 25% of his revenues in return for lower wholesale prices charged to the distributor. At the same time, both the manufacturer and the distributor will not accept the contract terms if the retailer wants to retain more than 50% of his profit. In a similar way, we arrive at the conditions for the distributor to retain between 1/3 and 2/3 of its profit if it wants to conclude the contract. This conclusion is in line with (Giannoccaro & Pontrandolfo, 2004) and we will deliberately leave out the proof here. Each member maximises its own profit without using the asymmetric market information unavailable to other players which is used in the Stackelberg models. Therefore, the stability region in which the partners are able to negotiate the RS contract terms will be $\alpha \in \langle \frac{1}{4}, \frac{1}{2} \rangle$ and $\beta \in \langle \frac{1}{4}, \frac{3}{4} \rangle$ with parameters constrained by the distributor, manufacturer's and retailer's minimum profit requirements $\beta^{(d)} \geq \frac{1}{4(1-\alpha)}$, $\beta^{(m)} \leq \frac{\alpha - 3/4}{\alpha - 1}$, and $\alpha^{(r)} \geq \frac{1}{4}$, respectively. These relationships are calculated from the RN and N equilibria by calculating the differences $\pi_j^{RN*} - \pi_j^{N*}$ for each member, $j \in \{m, d, r\}$. Therefore the manufacturer will prefer the region around $\alpha \to 0, \beta \to 0$, the distributor will want to move towards $\alpha \to 0, \beta \to 1$ and retailer will insist on $\alpha \to 1$. The point at which all players' profits are equal, $\pi_m^{RN} = \pi_d^{RN} = \pi_r^{RN}$, is $[\alpha, \beta] = \left[\frac{1}{3}, \frac{1}{2}\right]$, which is at the centre of the RN model solution set as shown in Figure 14.

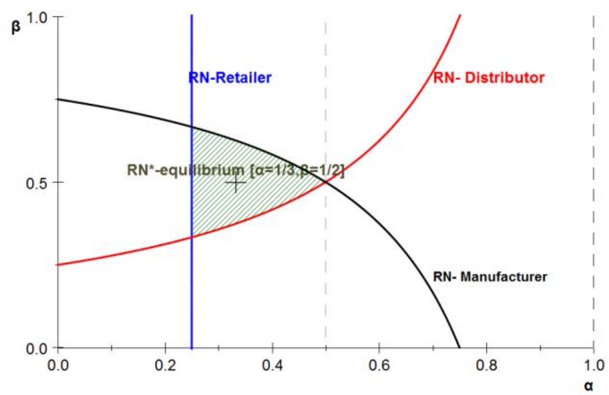

Figure 14. RN Win-Win region

5.2 Retailer-led CLSC Coordination - Model RR

We need to do two things to reflect the retailer's leadership in the RR model. First, we need to rearrange the terms α^{RR}, β^{RR} to ensure that the retailer controls both parameters. Second, we will be modelling a Stackelberg equilibrium, which means we will have to use backward induction instead of solving all the equations at once. The optimisation problem for coordinating a three-echelon retailer-led CLSC with an RS contract is as in (12)

$$\max_{p_o,p_r} \pi_r^{RR} = (1-\beta)[(1-\alpha)(q_r w_r^m + q_o w_o^m) + q_r w_r^d + q_o w_o^d] - q_r(w_r^d + c_r^r - p_r) - q_o(w_o^d + c_o^r - p_o)$$

$$s.t.$$

$$\begin{cases} \max\limits_{w_o^d, w_r^d} \pi_d^{RR} = \beta[(1-\alpha)(q_r w_r^m + q_o w_o^m) + q_r w_r^d + q_o w_o^d] - q_r(w_r^m + c_r^d + S) - q_o(w_o^m + c_o^d), \\ \qquad s.t. \left\{ \max\limits_{w_o^m, w_r^m} \pi_m^{RR} = \alpha(q_r w_r^m + q_o w_o^m) - q_r c_r^m - q_o c_o^m, \right. \end{cases} \quad (12)$$

$$p_o^{C*} = p_o^{RR*}, p_o^{C*} = p_o^{RR*}, q_o^{C*} = q_o^{RR*}, q_r^{C*} = q_r^{RR*}, \pi_{sc}^{RR*} = \pi_{sc}^{C*}.$$

The total CLSC profit is $\pi_{sc}^{RR} = \pi_m^{RR} + \pi_d^{RR} + \pi_r^{RR}$. We model full coordination, which means that we want to improve the overall performance to achieve the profit of model C through coordination. This is called the efficiency criterion and can be formally written as $\pi_{sc}^{C*} = \pi_{sc}^{RN*} = \pi_{sc}^{RR*}$. To achieve this goal, we must constrain either the prices or the quantities to the equilibrium values of model C, i.e. $p_{o/r}^{C*} = p_{o/r}^{RN*} = p_{o/r}^{RR*}$ or $q_{o/r}^{C*} = q_{o/r}^{RN*} = q_{o/r}^{RR*}$. This model is solved by backward recursion with additional constrains embedded in the solution procedure.

Proposition 5: *The RS contract (12) can coordinate the retailer-led CLSC with the set of parameters* $(\alpha^*, \beta^*, w_{o/r}^{m*}, w_{o/r}^{d*})$ *while satisfying*

RR.0: $p_o^{C*} = p_o^{RR*}, p_o^{C*} = p_o^{RR*}, q_o^{C*} = q_o^{RR*}, q_r^{C*} = q_r^{RR*}, \pi_{sc}^{RR*} = \pi_{sc}^{C*}$ *which ensures the performance of RR model at the model C's level, i.e. the efficiency criterion.*

The following set of conditions has to be met to make the RS contract desirable :

RR.1: Individual Rationality:

$\Delta\pi_j^{RR}(\alpha^{RR*}, \beta^{RR*}) = \pi_j^{RR*}(\alpha^{RR*}, \beta^{RR*}) - \pi_j^{SR*} \geq 0,$ *for* $0 \leq \alpha^{RR*}, \beta^{RR*} \leq 1$ *and* $j \in \{m, d, r\}$, *where* $\Delta\pi_j^{RR}(\alpha^{RR*}, \beta^{RR*})$ *captures the individual profit gains for each member under the RS contract parameters* $(\alpha^{RR*}, \beta^{RR*})$.

RR.2: Market (Stackleberg) Leadership: $\pi_r^{RR*} \geq \pi_d^{RR*} \geq \pi_m^{RR*}$ *provides that the coordinated profit will reflect the market leadership, when retailer has the strongest bargaining power in the negotiations. For more details on the barganing power assumption, see below.*

Proof: RR.0 is proven in the appendix.

5.3 RR model Performance and Sensitivity Analysis

To demonstrate the performance of the RR model in the real world conditions we deliberately choose the following parameters of c_o and δ. The choice of the cost parameter reflects the business reality and thus we are interested in finding out what parameters $(\alpha^*, \beta^*, w_{o/r}^{m*}, w_{o/r}^{d*})$ could coordinate the real world CLSC. Table 8 shows the chosen parameters for the sensitivity and equlibrium analysis. We will also show the different market product mixes and how they impact the coordination, which is visible in the Figure 15.

Table 8. RR model parameters

scenario	q	δ	c_o
A	$q = q_o$	0.5301	0.6036
B	$q_r = q_o$	0.6248	0.6036
C	$q_r \geq q_o$	0.6706	0.6036
D	$q = q_r$	0.7163	0.6036

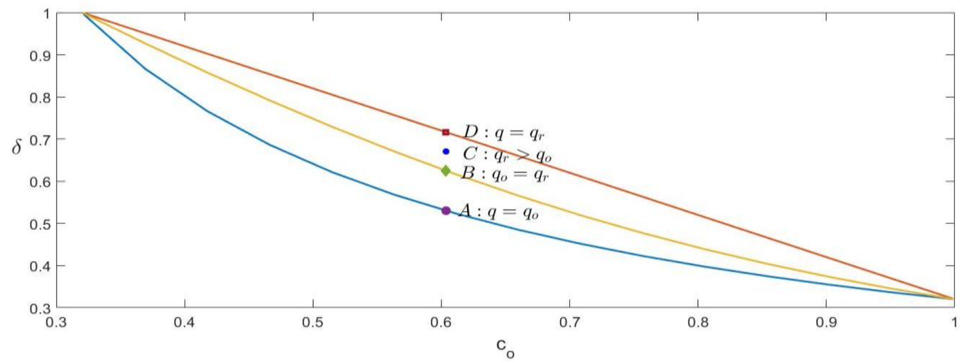

Figure 15. RR model scenarios

The four scenarios are evaluated in the following tables, which have the following structure. The first columns refer to the optimal values of the variables for the decentralised model SM, SR discussed above; we need them, especially the SR one, to validate the *RR.1: Individual Rationality* condition. The three columns labelled RR with either $m, d,$ or r in superscript measure the relative bargaining power of the players. In other words, the retailer is the leader who knows and controls the CLSC. However, when negotiations take place, each player can negotiate the terms that suit them best. They will simply try to get the maximum out of the contract negotiations, i.e. they will maximise their individual profit within the limits of the RS contract controlled by the retailer using their bargaining power. Formally, we solve the maximisation problem as follows $\max_{(\alpha^{RR}, \beta^{RR})} \pi_{j,s}^{RR*}$, s.t. *RR.1*, *RR.2*, $0 \leq \alpha^{RR}, \beta^{RR} \leq 1, \forall\ j \in \{m, d, r\}, s \in \{A, B, C, D\}$. The resulting pairs $(\alpha^{RR*}, \beta^{RR*})$ are shown in the second and third rows. We are mainly interested in the wholesale price reductions, which are different in each scenario, according to the respective $(\alpha^{RR*}, \beta^{RR*})$.

In Scenario A, where the market consists only of OW products, we are interested in the wholesale forward prices. w_o^m are reduced by more than 50 % in exchange for the manufacturer sharing 50 % of their reveue. For the distributor, w_o^d is reduced less, between 20 % - 30 % as a result of them agreeing to share cca 40% of their revenue with the retailer. The CLSC is coordinated, retailer's leadership is met in terms of profit, when they have the biggest bargaining power, i.e. the case of RR^r. This is an interesting finding in all four scenarios. In the RR^r case, the coordinated profits of all players are equal, indicating that the retailer, as the leader, has negotiated a win-win situation for all CLSC members. The equal share of profits also indicates the existence of a Nash equilibrium, as discussed above in the N/RN models.If the distributor or the manufacturer takes the lead in the negotiations, they can obtain higher profits at the expense of the retailer. However, we are still in a Stackelberger equilibrium and all conditions RR.0-2 are satisfied.

Table 9. RR model, Scenario A

Scenario : A	SM	SR	RR^m	RR^d	RR^r*)	$\frac{\pi_r^{RR^m*}-\pi_r^{SR*}}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^d*}-\pi_r^{SR*}}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^r*}-\pi_r^{SR*}}{\pi_r^{SR*}}$
α^{RR*}			0.5000	0.3750	0.3333			
β^{RR*}			0.4083	0.3812	0.3765			
π_{sc}^*	0.0172	0.0172	0.0393	0.0393	0.0393			
π_m^*	0.0098	0.0025	0.0196	0.0147	0.0131	700%	500%	433%
π_d^*	0.0049	0.0049	0.0098	0.0147	0.0131	100%	200%	167%
π_r^*	0.0025	0.0098	0.0098	0.0098	0.0131	0%	0%	33%
p_o^*	0.9505	0.9505	0.8018	0.8018	0.8018			
p_r^*	0.5038	0.5038	0.4250	0.4250	0.4250			
q_o^*	0.0495	0.0495	0.1982	0.1982	0.1982			
q_r^*	-	-	-	-	-			
w_o^{m*}	0.6648	0.5168	0.2423	0.2193	0.2158			
w_r^{m*}	0.1750	0.0963	(0.0284)	(0.0390)	(0.0398)			
w_o^{d*}	0.8959	0.7473	0.6104	0.5409	0.5160			
w_r^{d*}	0.4676	0.3888	0.2888	0.2509	0.2363			

**) The RR.2 condition applies to the RR^r case only*

Scenario B captures the situation when the market product mix of OW and RGB is equal. All the results are the same as in scenario A. w_o^m, w_o^d are reduced by the same amount as they only depend on the cost, which does not change, and we can see what happens to w_r^m, w_r^d when the CLSC sells more RGB products. Specifically, w_r^m enjoys a large reduction, almost to negative values. This can be explained by the fact that the distributor and the retailer agree with the manufacturer to give up their wholesale prices on the RGB product in exchange for a four times higher demand on the total higher demand on q_0, which generates the profit. In other words, the coordination is based on the manufacturer not making a profit on the RGB products by giving up its wholesale prices (margins) w_r^m. In return, the producer increases production of OW at the same wholesale prices w_o^m. And even sharing 30-50% of their profit can generate and excess profit for them. As far as the distributor is concerned, we can see that the rate of revenue sharing is the same as in scenario A, but the reduction in its wholesale price w_r^d is only 50% of that in scenario A. The retailer controls the negotiations.

Table 10. RR model, Scenario B

Scenario : B	SM	SR	RR^m	RR^d	RR^r	$\frac{\pi_r^{RR^m*}-\pi_r^{SR*}}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^d*}-\pi_r^{SR*}}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^r*}-\pi_r^{SR*}}{\pi_r^{SR*}}$
α^{RR*}			0.5000	0.3750	0.3333			
β^{RR*}			0.4090	0.3812	0.3768			
π_{sc}^*	0.0187	0.0187	0.0428	0.0428	0.0428			
π_m^*	0.0107	0.0027	0.0214	0.0160	0.0143	700%	500%	433%
π_d^*	0.0053	0.0053	0.0107	0.0160	0.0143	100%	200%	167%
π_r^*	0.0027	0.0107	0.0107	0.0107	0.0143	0%	0%	33%
p_o^*	0.9505	0.9505	0.8018	0.8018	0.8018			
p_r^*	0.5867	0.5867	0.4724	0.4724	0.4724			
q_o^*	0.0305	0.0305	0.1220	0.1220	0.1220			
q_r^*	0.0305	0.0305	0.1220	0.1220	0.1220			
w_o^{m*}	0.6648	0.5168	0.2431	0.2193	0.2160			
w_r^{m*}	0.2224	0.1081	(0.0010)	(0.0127)	(0.0131)			
w_o^{d*}	0.8959	0.7473	0.6099	0.5409	0.5158			
w_r^{d*}	0.5386	0.4243	0.3705	0.3324	0.3170			

In scenarios C and D, the optimal contract parameters $\alpha^{RR*}, \beta^{RR*}$ remain unchanged or close to those of Scenarios A and B, but the reduction in wholesale prices is smaller because CLSC benefits from selling more RGB than OW products and makes more profit. All contract wholesale prices are positive.

Table 11. RR model, Scenario D

Scenario : B	SM	SR	RR^m	RR^d	RR^r	$\frac{\pi_r^{RR^m*}-\pi_r^S}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^d*}-\pi_r^S}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^r*}-\pi_r^S}{\pi_r^{SR*}}$
α^{RR*}			0.5000	0.3750	0.3333			
β^{RR*}			0.4090	0.3812	0.3768			
π_{sc}^*	0.0187	0.0187	0.0428	0.0428	0.0428			
π_m^*	0.0107	0.0027	0.0214	0.0160	0.0143	700%	500%	433%
π_d^*	0.0053	0.0053	0.0107	0.0160	0.0143	100%	200%	167%
π_r^*	0.0027	0.0107	0.0107	0.0107	0.0143	0%	0%	33%
p_o^*	0.9505	0.9505	0.8018	0.8018	0.8018			
p_r^*	0.5867	0.5867	0.4724	0.4724	0.4724			
q_o^*	0.0305	0.0305	0.1220	0.1220	0.1220			
q_r^*	0.0305	0.0305	0.1220	0.1220	0.1220			
w_o^{m*}	0.6648	0.5168	0.2431	0.2193	0.2160			
w_r^{m*}	0.2224	0.1081	(0.0010)	(0.0127)	(0.0131)			
w_o^{d*}	0.8959	0.7473	0.6099	0.5409	0.5158			
w_r^{d*}	0.5386	0.4243	0.3705	0.3324	0.3170			

Table 12. RR model, Scenario D

Scenario : C	SM	SR	RR^m	RR^d	RR^r	$\frac{\pi_r^{RR^m*}-\pi_r^S}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^d*}-\pi_r^S}{\pi_r^{SR*}}$	$\frac{\pi_r^{RR^r*}-\pi_r^S}{\pi_r^{SR*}}$
α^{RR*}			0.5000	0.3750	0.3333			
β^{RR*}			0.4103	0.3813	0.3771			
π_{sc}^*	0.0207	0.0207	0.0474	0.0474	0.0474			
π_m^*	0.0119	0.0030	0.0237	0.0178	0.0158	700%	500%	433%
π_d^*	0.0059	0.0059	0.0119	0.0178	0.0158	100%	200%	167%
π_r^*	0.0030	0.0119	0.0119	0.0119	0.0158	0%	0%	33%
p_o^*	0.9505	0.9505	0.8018	0.8018	0.8018			
p_r^*	0.6268	0.6268	0.4953	0.4953	0.4953			
q_o^*	0.0174	0.0174	0.0695	0.0695	0.0695			
q_r^*	0.0480	0.0480	0.1919	0.1919	0.1919			
w_o^{m*}	0.6648	0.5168	0.2446	0.2194	0.2164			
w_r^{m*}	0.2453	0.1138	0.0131	0.0000	(0.0000)			
w_o^{d*}	0.8959	0.7473	0.6090	0.5408	0.5156			
w_r^{d*}	0.5729	0.4415	0.4091	0.3717	0.3558			

Across all scenarios, it is important to note the relative improvement in the profits of the CLSC members, as shown in the last three columns of Tables (9)-(12). We can see that the retailer achieves the highest profit in the SR model. The coordination of the chain leads to a profit surplus that can be distributed among the CLSC members according to their bargaining power and conditions. We read that in the RR^m model the manufacturer can negotiate most of the surplus with a large theoretical increase in profits (700%) from a relatively low base. This explains well how difficult it would be for the manufacturer to achieve this in the real world when the retailer controls the CLSC. If the distributor is an excellent negotiator (model RR^d), he can negotiate a 200% increase in his profit compared to the SR model. However, it is only if the retailer maintains its leadership position (model RR^r) that these effects occur.

1. All three members share the same final coordinated profit, i.e., $\pi_m^{RR} = \pi_d^{RR} = \pi_r^{RR}$.
2. The retailer can increase his profit by 33% even above his SR level.
3. This (RR) Stackelberg equilibrium solution is also acceptable as a RS coordinated Vertical Nash (RN) model solution, as shown in the next chapter. This shows that effective coordination of a leader-follower model requires the leader (retailer) to be highly accountable and responsible to the rest of the CLSC it controls to give up any temptation to abuse its position.

5.4 RR and RN Models Comparison

In this chapter we bring the RR and RN models together to see if there is a link between the Nash (RN) and Stackelberg (RR) equilibria. In Figure 16 we see the win-win region of the RN model and the three points where RR-R means RR^r and so on. We start with the producer who maximises his profit at point RR-M. It shows perfectly that this is the maximum profit that the other players would allow m to take and still sign the contract. The retailer as the market leader would have to agree with the distributor to allow manufacturer keep a high profit surplus at their own cost. It is located in the area below the RN-manufacturer curve and its position indicates that m would have to be extremely strong and effective in negotiations. The distributor performs close to the RN-distributor threshold line with the RR-D (RR^d) contract position if he shows bargaining strength and reaches his maximum. Finally, the Stackelberg equilibrium for the case where the retailer demonstrates leadership and bargaining dominance, RR-R, will land on the RN-Nash equilibrium area of the RN model, ensuring $\pi_m^{RR} = \pi_d^{RR} = \pi_r^{RR}$. In other words, the Stackelberg equilibrium is a special case of the Nash equilibrium where the retailer is the Stackelberg leader with the dominant bargaining power, using its dominant power to balance and coordinate the chain as if there were no leader and no followers, and at the point RR-R it holds that, $\pi_m^{RR} = \pi_m^{RN}, \pi_d^{RR} > \pi_d^{RN}, \pi_r^{RR} < \pi_r^{RN}$ with both RN and RR revenue sharing contract attainable and individually rational, because the RR.1 condition holds for both models at the RR-R point.

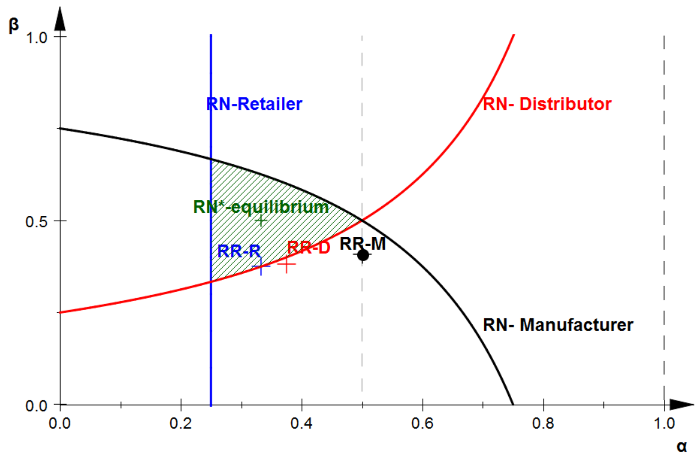

Figure 16. RR and RN models equilibria comparison

5.5 Managerial Insights

This paper aimed to shed light on the topic of multi-level CLSC negotiations using a real data-based CLSC, and there are some key findings that emerge

- The CLSC offering the consumer a choice between sustainable reusable products like RGB or a disposable product like OW opens up new opportunities for economic growth and building a sustainable CLSC.
- The win-win situation occurs when the retailer (a CLSC leader) leads the negotiations and maintains its leadership by trying to achieve a fair share rule in the final coordinated profit allocation. This is of course a model situation, but it reflects the aspiration that the retailer should have, although it may seem irrational for the leader to secure the same profit for its CLSC partners as it actually controls. See also condition RR.2 above. This is only met if the retailer dominates the negotiations, which also supports our conclusion.
- The more RGB products the chain sells, the higher the profit, which is mainly driven by the cost difference between the forward and reverse chains. The win-win situation is exactly when the cost difference is smaller and the CLSC can supply both products at similar costs. In this case, the consumer's preference to use RGB is the only variable that the CLSC should try to influence in order to achieve a sustainable CLSC.
- The coordination of revenue-sharing contracts and their optimal impact on wholesale price setting in a three-echelon CLSC is a complex issue. Nevertheless, RS contracts are regularly concluded in a modified form. In the real beverage business they are called Variable trade (back) bonus (VBB) and represent the share of revenue that the manufacturer credits back to the distributor/retailer if sales plans are met. In the model we

assume that the leader are equipped with all the knowledge of the best reaction of the follower. This is also, in fact, not far from reality of the real business. Specifically in the case of the so-called private labels manufacturing, the CLSC follower, in this case the manufacturer, has to disclose the cost structure of their products, otherwise the distributor or retailer will not negotiate on the contract.

- So the question is, why does it not work in real life to the extent we show here? Logically, the RS contract should provide the scope for a retail price reduction that would compensate for the lower unit margins of m and d that are lost in the RS contract negotiations. In real life, this is not the case, as the price reduction space created by the negotiations is often retained by the dominant leader in order to maintain its margins, so that consumer demand does not change. The CLSC members, on the other hand, return to the decentralised way of working by fighting to recover their margins by increasing their wholesale price, i.e. moving from RR/RN models to SM/SR/N models. If the dominant retailer abuses its position during the negotiations, the CLSC can never enjoy all the benefits, including the fair share of the coordinated profit.

Conclusion

In this paper, we have studied a three-echelon CLSC with substitutable products under real-life conditions in the beverage industry. We have demonstrated the benefits of a CLSC that produces and sells a sustainable product alongside a disposable product, resulting in a positive profit increase for all CLSC members. One of the key issues has been to prove that a real three echelon retailer-led CLSC can be coordinated to perform as if all members were managed by a single decision maker. We developed and proved the coordination model and showed the link between a CLSC with a leader and one with equally strong partners. We concluded that the CLSC can still be coordinated despite the dominant role of the retailer. If the RS contract is the most efficient (i.e., model RR^r), the manufacturer has to reduce its w_r^m, w_o^m significantly, even charging negative wholesale prices, and share 2/3 of its revenues with the partners. For the distributor the wholesale price w_r^d, w_o^d reduction is about 40% and it shares 37% of its revenue. In return, both receive a fourfold increase in production volume, which more than compensates for their lost margins. In fact, our conclusion is that if the leader acts responsibly and uses its bargaining power for the benefit of all, it can serve to treat the surplus profit sharing achieved in the RS contract fairly. From a game theory perspective, we have also shown that the coordinated leader-follower RR model equilibrium is a feasible solution in the coordinated vertical Nash RN model. The most important managerial insight is that it takes the leader retailer to realise that all CLSCs will eventually get a fair share of the coordinated CLSC's profit, which may seem uncomfortable. This extreme case can, of course, be treated as the ultimate goal, with the real CLSC trying to get close to it. Possible extensions or modifications of the model could be to embed the incentive for consumers or CLSC members to prefer the consumption of RGB through advertising or green investments in order to manipulate the willingness to buy parameter of the remanufactured product in the model. The future limitations of such models is that they are computationally demanding and it would be beneficial to combine a non-cooperative game theory approach with a cooperative game theory or optimisation technique.

Acknowledgements

The research project was supported by Grant No. F4/42/2021 of the Internal Grant Agency, Faculty of Informatics and Statistics, Prague University of Economics and Business.

References

Aazami, A., & Saidi-Mehrabad, M. (2021). A production and distribution planning of perishable products with a fixed lifetime under vertical competition in the seller-buyer systems: A real-world application. *Journal of Manufacturing Systems*, *58*, 223–247. https://doi.org/10.1016/j.jmsy.2020.12.001

Abbasi, S. (2022). *The Sustainable Supply Chain of CO2 Emissions During the Coronavirus Disease (COVID-19) Pandemic*.

Abbasi, S., & Ahmadi Choukolaei, H. (2023). A systematic review of green supply chain network design literature focusing on carbon policy. *Decision Analytics Journal*, *6*, 100189. https://doi.org/10.1016/j.dajour.2023.100189

Abbasi, S., & Erdebilli, B. (2023). Green Closed-Loop Supply Chain Networks' Response to Various Carbon Policies during COVID-19. *Sustainability*, *15*, 3677. https://doi.org/10.3390/su15043677

Abbasi, S., Khalili, H., Mehraban, M., & Hajiaghaei-Keshteli, M. (2022). Performance Measurement of the Sustainable Supply Chain During the COVID-19 Pandemic: A real-life case study. *Foundations of Computing and Decision Sciences*, *47*, 327–358. https://doi.org/10.2478/fcds-2022-0018

Abbasi, S., Mehraban, M., & Kanafi, A. (2022a). Designing Sustainable Recovery Network of End-of-Life Product during the COVID-19 Pandemic: A Real and Applied Case Study. *Discrete Dynamics in Nature and Society*, *2022*. https://doi.org/10.1155/2022/6967088

Abbasi, S., Mehraban, M., & Kanafi, A. (2022b). Green Closed-Loop Supply Chain Network Design During the Coronavirus (COVID-19) Pandemic: A Case Study in the Iranian Automotive Industry. *Environmental Modeling & Assessment*, *28*. https://doi.org/10.1007/s10666-022-09863-0

Al-Refaie, A., Jarrar, Y., & Lepkova, N. (2021). Sustainable Design of a Multi-Echelon Closed Loop Supply Chain under Uncertainty for Durable Products. *Sustainability*, *13*, 11126. https://doi.org/10.3390/su131911126

Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, *51*(1), 30–44. https://doi.org/10.1287/mnsc.1040.0215

Chakroborty, A., Maiti, T., & Giri, B. (2020). Consignment stock policy in a closed-loop supply chain. *RAIRO - Operations Research*, *55*. https://doi.org/10.1051/ro/2020064

De Giovanni, P. (2014). Environmental collaboration in a closed-loop supply chain with a reverse revenue sharing contract. *Annals of Operations Research*, *220*(1), 135–157. https://doi.org/10.1007/s10479-011-0912-5

De Giovanni, P. (2017). Closed-loop supply chain coordination through incentives with asymmetric information. *Annals of Operations Research*, *253*(1), 133–167. http://dx.doi.org/10.1007/s10479-016-2334-x

De Giovanni, P., & Zaccour, G. (2013). Cost–Revenue Sharing in a Closed-Loop Supply Chain. *Annals of the International Society of Dynamic Games*, *12*, 395–421. https://doi.org/10.1007/978-0-8176-8355-9_20

De Giovanni, P., & Zaccour, G. (2019). A Selective Survey of Game-Theoretic Models of Closed-Loop Supply Chains. *4OR: A Quarterly Journal of Operations Research*, *17*(1), 1–44.

Dey, B., Park, J., & Seok, H. (2022). Carbon-emission and waste reduction of a manufacturing-remanufacturing system using green technology and autonomated inspection. *RAIRO - Operations Research*, *56*, 2801–2831. https://doi.org/10.1051/ro/2022138

Dey, B., Sarkar, B., & Seok, H. (2021). Cost-effective smart autonomation policy for a hybrid manufacturing-remanufacturing. *Computers & Industrial Engineering*, *162*, 107758. https://doi.org/10.1016/j.cie.2021.107758

Dey, B., Sarkar, M., Chaudhuri, K., & Sarkar, B. (2023). Do you think that the home delivery is good for retailing? *Journal of Retailing and Consumer Services*, *72*, 103237. https://doi.org/10.1016/j.jretconser.2022.103237

Dey, B., & Seok, H. (2022). Intelligent inventory management with autonomation and service strategy. *Journal of Intelligent Manufacturing*. https://doi.org/10.1007/s10845-022-02046-4

Dosdogru, A., Boru İpek, A., Gosken, M., & Ozceylan, E. (2022). Simulation optimization approach for dynamic and stochastic closed loop supply chain network. *Scientia Iranica*. https://doi.org/10.24200/sci.2022.58224.5626

Erhun, F., & Keskinocak, P. (2003). *Game Theory in Business Applications*. 65.

Ferguson, M. E., & Toktay, L. B. (2006). The Effect of Competition on Recovery Strategies. *Production and Operations Management*, *15*(3), 351–368.

Fiala, P. (2005). Information Sharing in Supply Chains. *Omega*, *33*, 419–423. https://doi.org/10.1016/j.omega.2004.07.006

Fiala, P. (2016). Profit allocation games in supply chains. *Central European Journal of Operations Research: CEJOR*, *24*(2), 267–281. https://10.1007/s10100-015-0423-6

Garai, A., Chowdhury, S., Sarkar, B., & Roy, T. (2020). Cost-effective subsidy policy for growers and biofuels-plants in closed-loop supply chain of herbs and herbal medicines: An interactive bi-objective optimization in T-environment. *Applied Soft Computing*, *100*. https://doi.org/10.1016/j.asoc.2020.106949

Garai, A., & Sarkar, B. (2021). Economically independent reverse logistics of customer-centric closed-loop supply chain for herbal medicines and biofuel. *Journal of Cleaner Production*, *334*, 129977. https://doi.org/10.1016/j.jclepro.2021.129977

Gharibi, K., & Abdollahzadeh, S. (2021). A mixed-integer linear programming approach for circular economy-led closed-loop supply chains in green reverse logistics network design under uncertainty. *Journal of Enterprise Information Management*, *ahead-of-print*. https://doi.org/10.1108/JEIM-11-2020-0472

Giannoccaro, I., & Pontrandolfo, P. (2004). Supply chain coordination by revenue sharing contracts. *International Journal of Production Economics*, *89*(2), 131–139. https://doi.org/10.1016/S0925-5273(03)00047-1

Giri, B., Chakroborty, A., & Maiti, T. (2017). Effectiveness of consignment stock policy in a three-level supply chain. *Operational Research*, *17*. https://doi.org/10.1007/s12351-015-0215-8

Hou, Y., Wei, F., Li, S. X., Huang, Z., & Ashley, A. (2017). Coordination and performance analysis for a three-echelon supply chain with a revenue sharing contract. *International Journal of Production Research*, *55*(1), 202–227. https://doi.org/10.1080/00207543.2016.1201601

Hou, Y., Wei, F., Tian, X., & Liu, X. (2015). Two Revenue Sharing Contracts in a Three-echelon Supply Chain with a Risk-neutral or a Risk-averse Retailer. *Journal of Industrial Engineering and Management-Jiem*, *8*(5), 1428–1474. https://doi.org/10.3926/jiem.1556

Huang, R., & Yao, X. (2020). An analysis of sustainability and channel coordination in a three-echelon supply chain. *Journal of Enterprise Information Management*, *ahead-of-print*. https://doi.org/10.1108/JEIM-12-2019-0413

Kennedy, A., Sethi, S., Siu, C. C., Chi, S., & Yam, P. (2021). Co-op advertising in a dynamic three-echelon supply chain. *Production and Operations Management*, *30*. https://doi.org/10.1111/poms.13487

Mondal, A., Pareek, S., Chaudhuri, K., Bera, A., Bachar, R. K., & Sarkar, B. (2022). Technology license sharing strategy for remanufacturing industries under a closer-loop supply chain management bonding. *RAIRO - Operations Research*, *56*. https://doi.org/10.1051/ro/2022058

Örsdemir, A., Kemahlıoğlu-Ziya, E., & Parlaktürk, A. K. (2014). Competitive Quality Choice and Remanufacturing. *Production & Operations Management*, *23*(1), 48–64. https://doi.org/10.1111/poms.12040

Padiyar, S., Gupta, V., Singh, S. R., Singh, D., Sarkar, M., Dey, B., & Sarkar, B. (2022). Three-Echelon Supply Chain Management with Deteriorated Products under the Effect of Inflation. *Mathematics*, *11*, 104. https://doi.org/10.3390/math11010104

Pang, Q., Chen, Y., & Hu, Y. (2014a). Coordinating Three-Level Supply Chain by Revenue-Sharing Contract with Sales Effort Dependent Demand. *Discrete Dynamics in Nature and Society*, *2014*, 561081. https://doi.org/10.1155/2014/561081

Pang, Q., Chen, Y., & Hu, Y. (2014b). Three-Level Supply Chain Coordination under Disruptions Based on Revenue-Sharing Contract with Price Dependent Demand. *Discrete Dynamics in Nature and Society*, *2014*, 1–11. https://doi.org/10.1155/2014/464612

Pang, Q. H., Wu, X. Y., Tan, M. L., & Cao, X. Y. (2015). Supply Chain Coordination Using Revenue-Sharing Contract with Distributor's Effort Dependent Demand. *International Journal of Simulation Modelling*, *14*(2), 335–348. https://doi.org/10.2507/IJSIMM14(2)CO8

Sarkar, B., Tayyab, M., Kim, N., & Habib, S. (2019). Optimal production delivery policies for supplier and manufacturer in a constrained closed-loop supply chain for returnable transport packaging through metaheuristic approach. *Computers & Industrial Engineering*, *135*. https://doi.org/10.1016/j.cie.2019.05.035

Sarkar, B., Ullah, M., & Kim, N. (2017). Environmental and economic assessment of closed-loop supply chain with remanufacturing and returnable transport items. *Computers & Industrial Engineering*, *111*. https://doi.org/10.1016/j.cie.2017.07.003

Sarkar, M., Dey, B., Ganguly, B., Saxena, N., Yadav, D., & Sarkar, B. (2023). The impact of information sharing and bullwhip effects on improving consumer services in dual-channel retailing. *Journal of Retailing and Consumer Services*, *73*, 103307. https://doi.org/10.1016/j.jretconser.2023.103307

Shekarian, E., & Flapper, S. D. (2021). Analyzing the Structure of Closed-Loop Supply Chains: A Game Theory Perspective. *Sustainability (Basel, Switzerland)*, *13*(1397), 32. https://doi.org/ 10.3390/su13031397

Taleizadeh, A. A., Govindan, K., & Ebrahimi, N. (2020). The effect of promotional cost sharing on the decisions of two-level supply chain with uncertain demand. *Annals of Operations Research*, *290*(1–2), 747–781. https://doi.org/10.1007/s10479-018-3062-1

Taleizadeh, A. A., Moshtagh, M. S., Vahedi-Nouri, B., & Sarkar, B. (2023). New products or remanufactured products: Which is consumer-friendly under a closed-loop multi-level supply chain? *Journal of Retailing and Consumer Services*, *73*, 103295. https://doi.org/10.1016/j.jretconser.2023.103295

Taleizadeh, A., & Moshtagh, M. S. (2018). A consignment stock scheme for closed loop supply chain with imperfect manufacturing processes, lost sales, and quality dependent return: Multi Levels Structure. *International Journal of Production Economics*, *217*. https://doi.org/10.1016/j.ijpe.2018.04.010

Taleizadeh, A., Moshtagh, M. S., & Moon, I. (2017). Optimal decisions of price, quality, effort level and return policy in a three-level closed-loop supply chain based on different game theory approaches. *European J. of Industrial Engineering*, *11*, 486.

Taleizadeh, A., Soleymanfar, V. R., & Choi, T.-M. (2017). Optimal Pricing and Alliance Strategy in a Retailer-Led Supply Chain with the Return Policy: A Game-Theoretic Analysis. *Information Sciences*, *420*. https://doi.org/10.1016/j.ins.2017.08.054

Ullah, M., Asghar, I., Zahid, M., Omair, M., Alarjani, A., & Sarkar, B. (2021). Ramification of remanufacturing in a sustainable three-echelon closed-loop supply chain management for returnable products. *Journal of Cleaner Production*, *290*, 125609. https://doi.org/10.1016/j.jclepro.2020.125609

Xie, J., Liang, L., Liu, L., & Ieromonachou, P. (2016). Coordination Contracts of Dual-channel with Cooperation Advertising in Closed-loop Supply Chains. *International Journal of Production Economics*, *183*. https://doi.org/10.1016/j.ijpe.2016.07.026

Xie, J., Zhang, W., Liang, L., Xia, Y., Yin, J., & Yang, G. (2018). The revenue and cost sharing contract of pricing and servicing policies in a dual-channel closed-loop supply chain. *Journal of Cleaner Production*, *191*, 361–383. https://doi.org/10.1016/j.jclepro.2018.04.223

Zeng, T., & Yang, T. (2022). Unfair and Risky? Profit Allocation in Closed-Loop Supply Chains by Cooperative Game Approaches. *Applied Sciences*, *12*, 6245. https://doi.org/10.3390/app12126245

Zheng, X.-X., Liu, Z., Li, K. W., Huang, J., & Chen, J. (2019). Cooperative game approaches to coordinating a three-echelon closed-loop supply chain with fairness concerns. *International Journal of Production Economics*, *212*, 92–110. https://doi.org/10.1016/j.ijpe.2019.01.011

Zhong, F., Zhou, Z., & Leng, M. (2021). Negotiation-Sequence, Pricing, and Ordering Decisions in a Three-Echelon Supply Chain: A Coopetitive-Game Analysis. *European Journal of Operational Research*, *294*. https://doi.org/10.1016/j.ejor.2021.02.020

Zhou, Y., Xiong, Y., Li, G., Xiong, Z., & Beck, M. (2013). The bright side of manufacturing–remanufacturing conflict in a decentralised closed-loop supply chain. *International Journal of Production Research*, *51*(9), 2639–2651. https://doi.org/10.1080/00207543.2012.737956

Appendix:

Proof of proposition 5, part RR.0:

First, we list again all the profit functions of all the CLSC members from (12)

$$\max_{w_o^m, w_r^m} \pi_m^{RR} = \alpha(q_r w_r^m + q_o w_o^m) - q_r c_r^m - q_o c_o^m \quad (13)$$

$$\max_{w_o^d, w_r^d} \pi_d^{RR} = \beta[(1-\alpha)(q_r w_r^m + q_o w_o^m) + q_r w_r^d + q_o w_o^d] - q_r(w_r^m + c_r^d + S) - q_o(w_o^m + c_o^d) \quad (14)$$

$$\max_{p_o, p_r} \pi_r^{RR} = (1-\beta)[(1-\alpha)(q_r w_r^m + q_o w_o^m) + q_r w_r^d + q_o w_o^d] - q_r(w_r^d + c_r^r - p_r) - q_o(w_o^d + c_o^r - p_o) \quad (15)$$

where,

$$q_o(p_o, p_r) = 1 + \frac{(p_o - p_r)}{(\delta - 1)}, \quad (16)$$

$$q_r(p_o, p_r) = \frac{(p_o - p_r)}{(1 - \delta)} - \frac{p_r}{\delta}. \quad (17)$$

For the sake of completeness, we will repeat the coordination conditions.

$p_o^{C*} = p_o^{RR*}$, $p_o^{C*} = p_o^{RR*}$, $q_o^{C*} = q_o^{RR*}$, $q_r^{C*} = q_r^{RR*}$

This all should result in $\pi_{sc}^{RR*} = \pi_m^{RR*}(\alpha,\beta) + \pi_d^{RR*}(\alpha,\beta) + \pi_r^{RR*}(\alpha,\beta) = \pi_{sc}^{C*}$. (18)

We also need the identity equations that capture the price formation process using margins.

$$p_o = w_o^m + m_o^d + m_o^r \quad (19)$$

$$p_r = w_r^m + m_r^d + m_r^r \quad (20)$$

$$w_o^d = w_o^m + m_o^d \quad (21)$$

$$w_r^d = w_r^m + m_r^d \quad (22)$$

First, we substitute (19)-(22) to (16) and (17) giving us

$$q_o = 1 + (m_o^d - m_r^d + m_o^r - m_r^r + w_o^m - w_r^m)/(\delta - 1) \quad (23)$$

$$q_r = \frac{m_o^d - m_r^d + m_o^r - m_r^r + w_o^m - w_r^m}{1-\delta} - \frac{(m_r^d + m_r^r + w_r^m)}{\delta} \quad (24)$$

Next, we substitute (23) and (24) to all profit function (13)-(15). In a retailer-led chain, leader knows the reactions of the upstream members to the prices they set. We have to begin with the manufacturer who is trying to maximize their own profit from wholesale prices. w_r^m, w_o^m. Hence, the first-order-condition (FOC) of m is

$\frac{\partial \pi_m^{RR}}{\partial w_r^m} = \frac{\partial \pi_m^{RR}}{\partial w_o^m} = 0$, which gives us the m's best response functions to the d's margins

$$BR_o^m(m_o^d, m_o^r) = \frac{c_o - c_o^d - c_o^r + \alpha(1 - m_o^d - m_o^r)}{2\alpha}, \quad (25)$$

$$BR_r^m(m_r^d, m_r^r) = \frac{c_r - c_r^d - c_r^r + \alpha(1 - m_r^d - m_r^r)}{2\alpha}. \quad (26)$$

In order to work in the price/quantity conditions, we will use the equations (19) and (20) and set the conditioned w_r^d, w_o^d as

$$w_o^d(m_o^r) = p_o^{C*} - m_o^r = \frac{1 + c_o - 2m_o^r}{2}, \quad (27)$$

$$w_r^d(m_r^r) = p_r^{C*} - m_r^r = \frac{c_r + \delta + S - 2m_r^r}{2}. \quad (28)$$

Next step is to express m_o^r and m_r^r using the m's BR functions (25), (26) and coordination conditions (27),(28) by solving the equations

$$BR_o^m(m_o^d, m_o^r) - w_o^d(m_o^r) + m_o^d = 0, \tag{29}$$

$$BR_r^m(m_r^d, m_r^r) - w_r^d(m_r^r) + m_r^d = 0, \tag{30}$$

Solving (29) and (30) for m_o^r, m_r^r we obtain the retailer's best response functions to the margins chosen by the distributor.

$$m_o^r(m_o^d) = \frac{c_o^d + c_o^r - c_o + \alpha(c_o - m_o^d)}{\alpha} \tag{31}$$

$$m_r^r(m_r^d) = \frac{c_r + \delta + S - 2m_r^d + [c_r^d + c_r^r - c_r + \alpha(\delta + m_r^d)]}{\alpha} \tag{32}$$

Last variables we need to express in terms of $m_o^d, m_r^d, m_o^r, m_r^r$ are the end consumer prices p_o, p_r. We do it by substituting (25), (26), (27), (28), (31), (32) to (19) and (20) and obtain,

$$p_o(m_o^d, m_o^r) = \frac{\alpha + c_o^d + c_o^r - c_o + \alpha(2c_o - m_o^d - m_o^r)}{2\alpha} \tag{33}$$

$$p_r(m_r^d, m_r^r) = \frac{c_r^d + c_r^r - c_r + 2\alpha(c_r + \delta + S) - \alpha(m_r^d + m_r^r)}{2\alpha} \tag{34}$$

Now, we need to update the quantity eq. (16) and (17) with (33) and (34) to obtain $q_o(m_o^d, m_o^r)$ and $q_r(m_r^d, m_r^r)$ and then substitute (25), (26), (27), (28), (30), (31), (33), (34) and the updated $q_o(m_o^d, m_o^r)$ and $q_r(m_r^d, m_r^r)$ to the distributors profit function (14) and receive the distributor's profit function in terms of their and retailer's profits

$$\pi_d^{RR}(m_o^d, m_o^r, m_r^d, m_r^r) = \beta(\alpha - 1)\left[\frac{\Delta_1\left(\frac{1+c_o-2m_o^r}{2}\right)}{2(\delta-1)} - \frac{\Delta_1\left(\frac{c_r+\delta+S-2m_r^r}{2}\right)}{\Delta_3}\right] + \frac{\Delta_2\left(\frac{1+c_o+2c_o^d-2m_o^r}{2}\right)}{2(\delta-1)} - \frac{\Delta_1\left(\frac{c_r+\delta+S+2c_r^d-2m_r^r}{2}\right)}{\Delta_3} - \frac{\beta\Delta_2\left(\frac{1+c_o+2m_o^d-2m_o^r}{2}\right)}{2(\delta-1)} - \frac{S\Delta_1}{\Delta_3} + \frac{\beta\Delta_1\left(\frac{c_r+\delta+2m_r^d-2m_r^r}{2}\right)}{\Delta_3}, \quad \text{where} \tag{35}$$

$\Delta_1 = S + c_r + \delta + 2m_r^d - \delta(c_o + 2m_n^d)$, $\Delta_2 = 1 + S - c_o + c_r - \delta + 2(m_r^d - m_o^d)$, and $\Delta_3 = 2\delta(\delta - 1)$.

Now, d is maximizing their profit by setting the FOD $\frac{\partial \pi_d^{RR}}{\partial m_o^d} = \frac{\partial \pi_d^{RR}}{\partial m_r^d} = 0$ and this gives the d's best response function to the prices (margins) set by the retailer in the form

$$BR_o^d(m_o^r) = \frac{1 + c_o + 2c_o^d - 2m_o^r + \beta(\alpha - 1 - 3c_o + 4m_o^r) + \alpha\beta(c_o - 2m_o^r)}{4\beta}, \tag{36}$$

$$BR_r^d(m_r^r) = \frac{3S + c_r + 2c_r^d + \delta - 2m_r^r + \beta(4m_r^r - 3S - 3c_r - \delta) + \alpha\beta(S + c_r + \delta - 2m_r^r)}{4\beta}. \tag{37}$$

If we put $BR_o^d(m_o^r) = m_o^d$ and $BR_r^d(m_r^r) = m_r^d$, we can substitute (36) to (31) and (37) to (32), whereby we get the equilibrium wholesale margins of the retailer m_o^{r*}, m_r^{r*}

$$m_o^{r*} = \frac{\alpha\beta[1 + \alpha + c_o(\alpha - 7)] + 4\beta(c_o - c_o^d - c_o^r) + 2\alpha(c_o + c_o^d + 1)}{2\alpha(1 + \alpha\beta - 4\beta)} \tag{38}$$

$$m_r^{r*} = \frac{\alpha\beta[S\alpha + \delta(\alpha - 1) - 7(S + c_r)] + 4\beta(c_r - c_r^d - c_r^r) + \alpha(c_r + 2c_r^d + 3S + \delta)}{2\alpha(1 + \alpha\beta - 4\beta)}, \tag{39}$$

which we substitute back to (36) and (37) and obtain the equilibrium wholesale margins of the distributor m_o^{d*}, m_r^{d*}. Substituting (38), (39) , m_o^{d*}, m_r^{d*} back to (25), (26), we finally obtain the manufacturer's (w_o^{m*}, w_r^{m*})/distributor's (w_o^{d*}, w_r^{d*}) and equilibrium wholesale prices that coordinate the CLSC in the form.

$$w_o^{m*} = \frac{\alpha(1 - c_o) + 2(c_o - c_o^d - c_o^r)}{2\alpha} \tag{40}$$

$$w_r^{m*} = \frac{\alpha(\delta - c_r - S) + 2(c_o - c_r^d - c_r^r)}{2\alpha} \tag{41}$$

$$w_o^{d*} = \frac{3\alpha\beta(c_o-1)+4\beta(c_o^d+c_o^r-c_o)-2\alpha c_o^d}{2\alpha(1+\alpha\beta-4\beta)} \tag{42}$$

$$w_r^{d*} = \frac{3\alpha\beta(S-\delta+c_r)+4\beta(c_r^d+c_r^r-c_r)-2\alpha(c_r^d+S)}{2\alpha(1+\alpha\beta-4\beta)} \tag{43}$$

It is easy now to prove that substituting (38)-(43) to (19)-(22) will gives us end consumer prices p_o, p_r which meet the coordination conditions RR.0, i.e., $p_o^{C*} = p_o^{RR*}$, $p_o^{C*} = p_o^{RR*}$, hence $\pi_{sc}^{RR*} = \pi_{sc}^{C*}$. This completes the proof.