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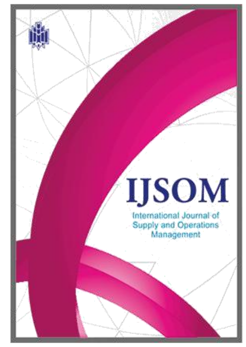

Two Novel Generalized Information Measures for Fuzzy Sets

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Abstract

In the present manuscript, it is our utmost aim and objective to comprehensively and extensively expound upon two highly innovative and remarkably novel information measures that have been adeptly and appropriately extended to encompass fuzzy sets. These measures have undergone a rigorous and meticulous scrutiny and examination with regard to axiomatic principles, and have evinced numerous and manifold advantageous properties, thus providing a compelling and highly persuasive validation of their reliability, credibility, and overall effectiveness. It is of paramount importance to note that the measures in question have been meticulously and painstakingly designed with the express aim and purpose of effectively quantifying the extent of information that is inextricably embedded and enmeshed within fuzzy sets, and are firmly grounded in the foundational and bedrock principles of information theory, which is an indisputably and irrefutably significant and substantial academic discipline. Our contributions to the field of study are truly and incontestably unparalleled, and offer a fresh, innovative, and unprecedented perspective on the study of fuzzy sets and their associated information content, which is of immense and inestimable value to the overall advancement and progress of the academic discipline.

Keywords: Fuzzy Set; Fuzzy Entropy; Membership Function; Measure; Vagueness.

1. Introduction

Through his seminal work, Shannon (1948) contributed a fundamental concept of entropy to communication theory, which laid the foundation for information theory. The formulation of this theory adhered to the second law of thermodynamics, which postulates that the disorder of systems increases with time due to increasing entropy. As a result, information theory has transcended its initial focus on communication systems and has been extensively employed in fields such as statistics, information processing, and computing.
The emergence of fuzzy entropy measures is a consequence of the indeterminacy arising from the vagueness of information, while probabilistic entropy stems from the uncertainty associated with the probabilistic essence of available information. The advancement of probability theory and fuzzy set theory has significantly contributed to the elimination of uncertainty in real-world scenarios and decision-making predicaments caused by imprecision.
Zadeh (1965) introduced the concept of fuzzy set by utilizing membership function. To further enhance the understanding of fuzzy set, Zadeh (1968) proposed a framework that combines both the probability and membership function. To encapsulate the intuitive comprehension of the degree of fuzziness, De Luca and Termini (1972)

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developed an axiomatic framework based on Shannon's (1948) entropy. Kaufmann (1980) introduced the metric distance-based fuzzy entropy, which measures the distance between a fuzzy set and a crisp set. In a survey conducted by Bhandari and Pal (1993) on fuzzy set, Renyi's (1961) entropy was used to introduce fuzzy entropy. Kapur (1997) postulated a measure of fuzzy entropy that employs the concept of structural-entropy, as initially introduced by Havrda and Charvat (1967). Parkash (1998) introduced two novel measures of fuzzy entropy contingent upon trigonometric functions. Tomar and Ohlan (2014 and 2015) as well as Verma and Sharma (2011) have previously introduced a number of entropy measures for fuzzy sets.

In recent years, numerous authors have proposed a variety of entropy measures with diverse applications in fields such as decision-making, pattern recognition, and machine learning. Liang et al. (2018) introduced the Pythagorean fuzzy number (PFN) into decision-theoretic rough sets (DTRSs) and constructed a new model of Pythagorean fuzzy decision-theoretic rough sets (PFDTRSs) based on the Bayesian decision procedure.

Verma and Merigó (2019) propose a new method for decision-making using trigonometric similarity measures for Pythagorean fuzzy sets. They introduce two new generalized similarity measures based on cosine and cotangent functions and discuss their properties and applications in multiple attribute decision-making problems. Peng (2019) proposes new similarity and distance measures for Pythagorean fuzzy sets (PFS) to address the defects of existing measures. These new measures aim to capture indeterminacy more effectively and avoid counterintuitive phenomena. Suman et al. (2022) introduce a superior entropy measure for PFSs compared to previous ones and use it to establish a TOPSIS model for multi-criteria decision-making. Mahmood and Rehman (2022) present a pioneering methodology for bipolar complex fuzzy sets and their implementation in generalized similarity measures.

Building on this previous work, we propose and introduce two novel generalized information measures which are specifically designed for fuzzy sets. These proposed measures are specifically formulated as a means to address the limitations and drawbacks of existing methods of information measures when applied to fuzzy sets. The two proposed measures are designed to be highly effective in characterizing the degree of uncertainty and the level of information content in fuzzy sets, whilst also being able to capture the degree of dependence between different fuzzy sets. Overall, these novel measures are expected to be highly useful in a broad range of applications that involve the analysis and interpretation of fuzzy set-based data, particularly in fields such as decision-making, pattern recognition, and data mining. Section 2 introduces fuzzy sets and reviews some existing entropy measures. Section 3 outlines our proposed new entropy measures for fuzzy sets, which are supported by axiomatic concepts. In Section 4, we provide a numerical example to illustrate the applicability of the measures introduced in Section 3. Finally, Section 5 presents our conclusions and suggestions for future research and the fields of application.

2. Definitions

In this section, we introduce relevant concepts from probabilistic theory and fuzzy set theory for constructing an entropy measure.

Definition1. A fuzzy set F represented as

$$F = \left\{ p_i \in \eta_A\left(p_i\right);\ i = 1, 2, ...n \right\}$$

where $\eta_A\left(p_i\right)$ is membership function defined as

$$\eta_A(p_i) = \begin{cases} 0, p_i \notin F \\ 1, p_i \in F \\ 0.5, p_i \in F \ or \ p_i \notin F \ i.e. most\ uncertain \end{cases}$$

We can also have defined fuzzy set as

$$\eta_A\left(p_i\right) = \left\{ p_1, p_2, ..., p_n \right\} \rightarrow \left[0,1\right]$$

Definition 2: Let F^* be a set which satisfies following conditions

$$\eta_{F^*}\left(p_i\right) \le \eta_F\left(p_i\right) \ for \ 0 \le \eta_F\left(p_i\right) < 0.5 \ for \ all \ p_i \in P$$

$$\eta_{F^*}\left(p_i\right) \ge \eta_F\left(p_i\right) \ for \ 0.5 \le \eta_F\left(p_i\right) < 1 \ for \ all \ p_i \in P$$

for fuzzy set F. The set F^* is said to be sharpened version of fuzzy set.

Definition 3: A real function $K(F)$ which expresses the amount of average vagueness in making decision whether an element belongs to a set or not is called entropy measure if it satisfies four axioms as follows:

$\boldsymbol{B_1(Sharpness):}$ $K(F)$ is maximum if F is crisp set.

$\boldsymbol{B_2(Maximality):}$ $K(F)$ is maximum if F is most fuzzy set.

$\boldsymbol{B_3(Resolution):}$ $K(F^*) \le K(F)$, for any sharpening F^* of F.

$\boldsymbol{B_4(Symmetry):}$ $K(F^C) \le K(F), where\, F^C = \text{Compliment of } F$.

Another measure of entropy was proposed by De Luca and Termini (1972) for fuzzy sets, utilizing the notion of uniform degree of fuzziness, as derived from the aforementioned definitions $\omega_i = \eta_F(p_i)$ and $\varpi_i = 1-\eta_F(p_i)$ as

$$D(F) = -\sum_{i=1}^{n}\left[\omega_i \log\omega_i + (\varpi_i)\log\varpi_i\right]$$

Bhandari and Pal (1993) defined new entropy measure on fuzzy set after the study of previous entropies as

$$D_\alpha(F) = \frac{1}{1-\alpha}\sum_{i=1}^{n}\log\left[(\omega_i)^\alpha + (\varpi_i)^\alpha\right];\ \alpha \ne 1, \alpha > 0$$

$$D_e(F) = \frac{1}{n\left(\sqrt{e}-1\right)}\sum_{i=1}^{n}\left[\omega_i e^{(\varpi_i)} + (\varpi_i)e^{\omega_i} - 1\right]$$

Kapur (1997) proposed entropy as

$$\begin{aligned} D^\alpha(F) = &-\sum_{i=1}^{n}\left[\omega_i \log\omega_i + (\varpi_i)\log(\varpi_i)\right]\log(\varpi_i) \\ &+\frac{1}{\alpha}\sum_{i=1}^{n}\left[\left(1+\alpha\mu_A(x_i)\right)\log\left(1+\alpha\mu_A(x_i)\right)\right]\log\left(1-\alpha\mu_A(x_i)\right) \\ &+\frac{1}{\alpha}\sum_{i=1}^{n}\left[\left(1+\alpha-\alpha\mu_A(x_i)\right)\log\left(1+\alpha-\alpha\mu_A(x_i)\right)\right] \\ &-\frac{1}{\alpha}(1+\alpha)\log(1+\alpha) \end{aligned}$$

3. Main Result on Fuzzy Entropy

We propose two entropy measures on fuzzy set as

$$B_\alpha(F) = \frac{1}{1-\alpha}\log\left[\frac{1}{n}\sum_{i=1}^{n}\left[\omega_i^\alpha + (\varpi_i)^\alpha\right]\right] \tag{1}$$

$$B^\alpha(F) = \frac{1}{1-\alpha}log\left[\frac{1}{n}\sum_{i=1}^{n}(\omega_i^\alpha + (\varpi_i)^\alpha) - \sum_{i=1}^{n}[\omega_i\log\omega_i + (\varpi_i)\log(\varpi_i)]\ \right] \tag{2}$$

$for\ \alpha > 1$

Theorem 1. The measure $B_\alpha(F)$ is a valid measure on fuzzy set.

Proof. To prove the validity of $B_\alpha(F)$ we have to show that measure satisfy four axioms fro $B_1 - B_4$.

$\boldsymbol{B_1(Sharpness):}$ We show that $B_\alpha(F) = 0\ iff\ \omega_i = 0\ or\ \omega_i = 1\quad for\ all\ p_i \in P$

Let $B_\alpha\left(F\right)=0$

$$\Rightarrow \frac{1}{1-\alpha}\log\left[\frac{1}{n}\sum_{i=1}^{n}\left[\omega_i^\alpha+\varpi_i^\alpha\right]\right]=0$$

$$\Rightarrow \log\left[\frac{1}{n}\sum_{i=1}^{n}\left[\omega_i^\alpha+\varpi_i^\alpha\right]\right]=0$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\left[\omega_i^\alpha+\varpi_i^\alpha\right]=1$$

$$\Rightarrow \left[\omega_i^\alpha+\varpi_i^\alpha\right]=1$$

$$\Rightarrow \omega_i=0 \; or \; \omega_i=1 \quad for \; all \; p_i\in P$$

Converse: Let $\omega_i=0 \; or \; \omega_i=1 \; for \; all \; p_i\in P$

$$\Rightarrow \omega_i=0 \; or \; \omega_i=1 \; for \, all \; p_i\in P$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\left[\omega_i^\alpha+\varpi_i^\alpha\right]=1$$

$$\Rightarrow \frac{1}{1-\alpha}\log\left[\frac{1}{n}\sum_{i=1}^{n}\left[\omega_i^\alpha+\varpi_i^\alpha\right]\right]=0$$

$$\Rightarrow B_\alpha\left(F\right)=0$$

Hence, $B_\alpha\left(F\right)=0 \, iff \; F \; is \; crisp \; set$.

B_2(Maximality): $B_\alpha\left(F\right)$ is maximum iff $\omega_i=0.5 \; for \; all \; p_i\in P$

Now,

$$\frac{\partial B_\alpha\left(F\right)}{\partial\omega_i}=\frac{\alpha}{1-\alpha}\left[\frac{\sum_{i=1}^{n}\left[\omega_i^{\alpha-1}-\varpi_i^{\alpha-1}\right]}{\sum_{i=1}^{n}\left[\omega_i^{\alpha-1}+\varpi_i^{\alpha-1}\right]}\right]$$

Case – 1. Let $0\le\omega_i<0.5$ we have

$$\frac{\partial B_\alpha\left(F\right)}{\partial\omega_i}>0 \quad \forall \; \alpha>1$$

$B_\alpha\left(F\right)$ is an increasing function.

Case – 2. Let $0.5<\omega_i\le1$ we have

$$\frac{\partial B_\alpha\left(F\right)}{\partial\omega_i}>0 \quad \forall \; \alpha>1$$

$B_\alpha\left(F\right)$ is decreasing function.

Case – 3. Let $\omega_i=0.5$ we have

$$\frac{\partial B_\alpha(F)}{\partial \omega_i} = 0 \;\; \forall \; \alpha > 1$$

We observer from Table -1 and figure-1 that $B_\alpha(F)$ is concave function which has maximum value at $\omega_i = 0.5, \;\alpha > 1.$

Table 1. Values of $B_\alpha(F)$ for $0.1 < \alpha < 4$ *and* $0 < \omega_i < 1$.

α	ω_i	0	0.1	0.3	0.4	0.5	0.6	0.7	0.9	1
0.1	$B_\alpha(F)$	0	-0.033	-0.067	-0.074	-0.077	-0.074	-0.067	-0.033	0
0.9		0	-0.304	-0.603	-0.671	-0.693	-0.671	-0.603	-0.304	0
1.5		0	0.0609	0.1206	0.1342	0.1386	0.1342	0.1206	0.0609	0
2		0	0.0304	0.0603	0.0671	0.0693	0.0671	0.0603	0.0304	0
2.5		0	0.0203	0.0402	0.0447	0.0462	0.0447	0.0402	0.0203	0
3		0	0.0152	0.0301	0.0335	0.0346	0.0335	0.0301	0.0152	0
3.5		0	0.0121	0.0241	0.0268	0.0277	0.0268	0.0241	0.0121	0
4		0	0.0101	0.0201	0.0223	0.0231	0.02236	0.0201	0.0101	0

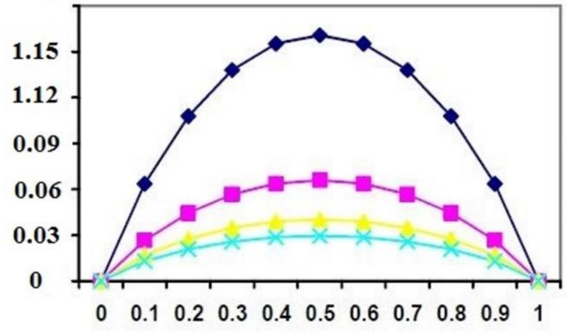

Figure 1. $B_\alpha(F)$

B_3(Resolution): To prove $B_\alpha(F^*) \le B_\alpha(F)$ where F^* is sharpened version of F. On the basis of previous axiom we know that $B_\alpha(F)$ is increasing function for $0 \le \omega_i < 0.5$ and decreasing function for $0.5 < \omega_i \le 1$. We set,

$$\eta_{F^*}(p_i) \le \eta_F(p_i) \;\Rightarrow\; B_\alpha(F^*) \le B_\alpha(F) \text{ and}$$

$$\eta_{F^*}(p_i) \ge \eta_F(p_i) \;\Rightarrow\; B_\alpha(F^*) \le B_\alpha(F)$$

$$\Rightarrow \quad B_\alpha(F^*) \le B_\alpha(F) \text{ in both cases.}$$

B_4(Symmetry): Let F^C be the compliment of F . We take $\omega_i = \varpi_i$, due to membership function. Then clearly $B_\alpha(F^C) = B_\alpha(F)$.

This shows the validity of measure define by $B_\alpha(F)$.

Theorem 2: The measure $B^{\alpha}(F)$ is a valid measure on fuzzy set.

Proof. To prove the validity of $B^{\alpha}(F)$ we show that measure satisfies four axioms from $B_1 - B_4$.

B_1(Sharpness): We show that

$B^{\alpha}(A)=0 \ iff \ \omega_i = 0 \ or \ \omega_i = 1 \quad for \ all \ p_i \in P$

Let $B^{\alpha}(A)=0$

$$\Rightarrow \frac{1}{1-\alpha} log\left[\frac{1}{n}\sum_{i=1}^{n}(\omega_i^{\alpha}+(\varpi_i)^{\alpha}) - \sum_{i=1}^{n}[\omega_i \log\omega_i + (\varpi_i)\log(\varpi_i)] \quad\right] = 0$$

$$\Rightarrow \frac{1}{1-\alpha}\log\left[\frac{1}{n}\sum_{i=1}^{n}\left(\omega_i^{\alpha}+\varpi_i^{\alpha}\right)\right]=0$$

$$and \ \sum_{i=1}^{n}\left[\omega_i\log\omega_i+\varpi_i\log\varpi_i\right]=0$$

$$\Rightarrow \omega_i = 0 \ or \ \omega_i = 1 \quad for \ all \ p_i \in P$$

Converse: Let $\omega_i = 0 \ or \ \omega_i = 1 \quad for \ all \ p_i \in P$

$$\Rightarrow \frac{1}{1-\alpha}\log\left[\frac{1}{n}\sum_{i=1}^{n}\left(\omega_i^{\alpha}+\varpi_i^{\alpha}\right)\right]=0$$

$$and \ \sum_{i=1}^{n}\left[\omega_i\log\omega_i+\varpi_i\log\varpi_i\right]=0$$

$$\Rightarrow B^{\alpha}(F)=0$$

Hence we say that $B^{\alpha}(F)=0 \ iff \ \ F \ is \ crisp \ set$.

B_2(Maximality): $B^{\alpha}(F)$ is maximum iff $\omega_i = 0.5 \quad for \ all \ p_i \in P$

Now,

$$\frac{\partial B^{\alpha}(F)}{\partial\omega_i} = \begin{bmatrix} \dfrac{\alpha}{1-\alpha}\left(\dfrac{\sum_{i=1}^{n}\left[\omega_i^{\alpha-1}-\varpi_i^{\alpha-1}\right]}{\sum_{i=1}^{n}\left[\omega_i^{\alpha-1}+\varpi_i^{\alpha-1}\right]}\right) \\ -\sum_{i=1}^{n}\left[\log\omega_i-\log\varpi_i\right] \end{bmatrix}$$

Case – 1. Let $0 \le \omega_i < 0.5$ we have

$$\frac{\partial B^{\alpha}(F)}{\partial\omega_i} > 0 \quad \forall \ \alpha > 1$$

$\Rightarrow B^{\alpha}(F)$ is an increasing function.

Case – 2. Let $0.5 < \omega_i \le 1$, we have

$$\frac{\partial B^{\alpha}(F)}{\partial\omega_i} < 0 \ \forall \ \alpha > 1.$$

$B^{\alpha}\left(F\right)$ is decreasing function.

Case – 3. Let $\omega_i = 0.5$, we have

$$\frac{\partial B^{\alpha}\left(F\right)}{\partial \omega_i} = 0 \;\; \forall \; \alpha > 1$$

Thus we notice from the table -2 and figure-2 that $B^{\alpha}\left(F\right)$ is concave function which has maximum value at $\omega_i = 0.5, \;\; \alpha > 1$.

Table 2. Values of $B^{\alpha}(F)$ for $0.1 < \alpha < 20 \; and \; 0 < \omega_i < 1$.

α	ω_i	0	0.1	0.3	0.4	0.5	0.6	0.7	0.9	1
0.1	$B^{\alpha}\left(F\right)$	0	0.2912	0.5438	0.5984	0.6161	0.5984	0.5438	0.2912	0
0.9		**0**	**0.0204**	**0.0074**	**0.0019**	**0**	**0.0019**	**0.0074**	**0.0204**	**0**
1.1		0	0.6296	1.2142	1.3440	**1.3862**	1.3440	1.2142	0.6296	0
2			0.3555	0.6712	0.7401	**0.7624**	0.74011	0.6712	0.3555	
2.5		0	0.3453	0.6510	0.7177	**0.7393**	0.7177	0.6510	0.3453	0
3		0	0.3403	0.6410	0.7065	**0.7278**	0.7065	0.6410	0.3403	0
3.5		0	0.3372	0.6350	0.6998	**0.7208**	0.6998	0.6350	0.3372	0
4		0	0.3352	0.6309	0.6953	**0.7162**	0.6953	0.6309	0.3352	0
20		0	0.3266	0.6140	0.6765	**0.6967**	0.6765	0.6140	0.3266	0

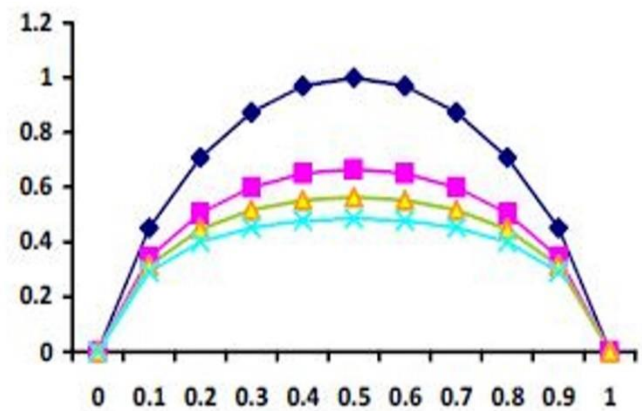

Figure 2. $B^{\alpha}(F)$

B_3(Resolution): To prove $B^{\alpha}\left(F^*\right) \le B^{\alpha}\left(F\right)$ where F^* is sharpened version of F. On the basis of previous axiom we know that $B^{\alpha}\left(F\right)$ is increasing function for $0 \le \omega_i < 0.5$ and decreasing function for $0.5 < \omega_i \le 1$. We thus have

$$\omega_i^* \le \omega_i \;\; \Rightarrow \;\; B^{\alpha}\left(F^*\right) \le B^{\alpha}\left(F\right) \text{ and}$$

$$\omega_i^* \ge \omega_i \;\; \Rightarrow \;\; B^{\alpha}\left(F^*\right) \le B^{\alpha}\left(F\right)$$

$$\Rightarrow \;\; B^{\alpha}\left(F^*\right) \le B^{\alpha}\left(F\right) \text{in both cases.}$$

B_4(Symmetry): Let F^C be the compliment of A. Therefore, we take $\omega_i = \varpi_i$ due to membership function. Then $B^\alpha\left(F^C\right) = B^\alpha\left(F\right)$.

This shows that $B^\alpha\left(F\right)$ is valid entropy measure on fuzzy set.

4. NUMERICAL ILLUSTRATION

Let $\Omega = \left(y_1, y_2, ..., y_n\right)$ be a finite universe of discourse and $H = \left\{< y_i, \mu_A\left(y_i\right) > / \, y_i \in \Omega\right\}$ be fuzzy set. We defied fuzzy set on universal set Ω as

$$H^n = \left\{< y_i, \left[\mu_A\left(y_i\right)\right]^n > / \, y_i \in \Omega\right\} \tag{3}$$

We consider fuzzy set H on Ω defined as

$$H = \left\{\left(y_{1,}\ 0.1\right), \left(y_{2,}\ 0.3\right), \left(y_{3,}\ 0.4\right), \left(y_{4,}\ 0.9\right), \left(y_{5,}\ 1\right)\right\}$$

Now with the help of operations defined in equation (3), following are the created fuzzy sets

$$H^{\frac{1}{2}}, H, H^2, H^3, H^4$$

Which are defined as

$H^{\frac{1}{2}}$ may be supposed as “More or less LARGE”

H may be supposed as “LARGE”

H^2 may be supposed as “very LARGE”

H^3 may be supposed as “quite very LARGE”

H^4 may be supposed as “very very LARGE”

and the corresponding set for above notations is given as

$$H^{\frac{1}{2}} = \left\{\left(y_1, 0.316\right), \left(y_2, 0.548\right), \left(y_3, 0.632\right), \left(y_4, 0.949\right), \left(y_5, 1\right)\right\}$$

$$H = \left\{\left(y_1, 0.1\right), \left(y_2, 0.3\right)\left(y_3, 0.4\right), \left(y_4, 0.9\right), \left(y_5, 1\right)\right\}$$

$$H^2 = \left\{\left(y_1, 0.010\right), \left(y_2, 0.090\right), \left(y_3, 0.0160\right), \left(y_4, 0.810\right), \left(y_5, 1\right)\right\}$$

$$H^3 = \left\{\left(y_1, 0.001\right), \left(y_2, 0.027\right), \left(y_3, 0.064\right), \left(y_4, 0.729\right), \left(y_5, 1\right)\right\}$$

$$H^4 = \left\{\left(y_1, 0.0001\right), \left(y_2, 0.008\right), \left(y_3, 0.026\right), \left(y_4, 0.656\right), \left(y_5, 1\right)\right\}$$

We calculate some entropy on above fuzzy sets

Entropy defined by B. Kosko (1986) is given by

$$D_K\left(A\right) = \frac{d_p\left(A, A_{near}\right)}{d_p\left(A, A_{far}\right)}$$

Entropy defied by P. Li & B. Liu (2008)

$$D_{LL}\left(A\right) = \sum_{i=1}^{n} S\left[C_r\left(\xi_A = x_i\right)\right]$$

We define the following entropy

$$B_\alpha\left(F\right) = \frac{1}{1-\alpha}\log\left[\frac{1}{n}\sum_{i=1}^{n}\left[\omega_i^\alpha + \varpi_i^\alpha\right]\right]$$

$$B^{\alpha}(F) = \frac{1}{1-\alpha} log\left[\frac{1}{n}\sum_{i=1}^{n}(\omega_i^{\alpha} + (\varpi_i)^{\alpha}) - \sum_{i=1}^{n}[\omega_i \log \omega_i + (\varpi_i)\log(\varpi_i)]\right]$$

According to Li, Lu & Cai (2003), the entropy values of above defined fuzzy sets should be in following pattern

$$B\left(H^{\frac{1}{2}}\right) > B(H) > B\left(H^{2}\right) > B\left(H^{3}\right) > B\left(H^{4}\right)$$

Table 3. Values of $D_k(A)$ *and* $D_{LL}(A)$ *vs* $B_{\alpha}(A)$ *and* $B^{\alpha}(A)$ for $\alpha = 1.1, 2, 10$ *and* 50.

Fuzzy Set	$D_K(A)$	$D_{LL}(A)$	$B_{\alpha}(A)$				$B^{\alpha}(A)$			
			$\alpha=1.1$	$\alpha=2$	$\alpha=10$	$\alpha=50$	$\alpha=1.1$	$\alpha=2$	$\alpha=10$	$\alpha=50$
$H^{\frac{1}{2}}$	1.389	3.200	0.424	0.353	0.124	0.032	2.596	2.525	2.296	2.203
H	1.331	2.720	0.373	0.290	0.117	0.032	2.307	2.224	2.051	1.966
H^2	1.202	1.655	0.262	0.185	0.076	0.023	1.643	1.566	1.457	1.404
H^3	1.151	1.236	0.179	0.120	0.045	0.016	1.133	1.075	1.000	0.970
H^4	1.136	1.116	-2.07	-0.11	0.008	0.006	-1.265	0.698	0.820	0.818

Some existing information measures have the following pattern as summarized in table 3

$$D_K(H) > D_K\left(H^{\frac{1}{2}}\right) > D_K\left(H^{2}\right) > D_K\left(H^{3}\right) > D_K\left(H^{4}\right)$$

$$D_{LL}\left(H^{3}\right) > D_{LL}\left(H^{\frac{1}{2}}\right) > D_{LL}(H) > D_{LL}\left(H^{4}\right) > D_{LL}\left(H^{2}\right)$$

The above pattern does not satisfy the requirement of Li, Lu & Cai (2003). The new defined entropy measures for different value of α for these summarized fuzzy set are in table3. From the value of entropies given in table, we can conclude that the entropy measures follows, following pattern

$$B_{\alpha}\left(H^{\frac{1}{2}}\right) > B_{\alpha}(H) > B_{\alpha}\left(H^{2}\right) > B_{\alpha}\left(H^{3}\right) > B_{\alpha}\left(H^{4}\right)$$

$$B^{\alpha}\left(H^{\frac{1}{2}}\right) > B^{\alpha}(H) > B^{\alpha}\left(H^{2}\right) > B^{\alpha}\left(H^{3}\right) > B^{\alpha}\left(H^{4}\right)$$

The proposed measures have been shown to fulfill the requirement of Li, Lu & Cai (2003) and exhibit favorable behavior with respect to structured variables. As the current measure fails to meet the desired criteria, the newly defined measures serve as a viable alternative for decision-making purposes and offer increased reliability within the field of information theory.

The present study seeks to explicate the practical as well as theoretical implications of the novel findings that have emerged from the development of not one, but two, new generalized information measures pertaining to the domain of fuzzy set. It is important to underscore that the development of these information measures represents a significant contribution to the field of fuzzy set theory, since they allow for more nuanced and precise modeling of fuzzy information. By providing a more robust and sophisticated framework for analyzing and manipulating fuzzy data, these measures have the potential to advance our understanding of complex real-world phenomena and facilitate the development of more effective decision-making strategies. Overall, the implications of these measures are far-reaching, and hold promise for a wide range of applications across a diverse array of fields.

5. Conclusion

We have put forth novel and innovative fuzzy entropy metrics and have established their soundness. Employing the proposed parametric measures can lead to a decrease in ambiguity and a commensurate enhancement in process efficiency. A more extensive analysis involving numerical data can be carried out in order to compare the total ambiguity of various entropy measures with differing parameter values. The measures that are in place to evaluate and determine the extent to which information is encapsulated within fuzzy sets are established on the foundation of the theory of information. Our innovations and novel ideas make a significant and noteworthy contribution to the field of research pertaining to fuzzy sets and their corresponding information content, by offering an original and innovative viewpoint.

Further, the two newly introduced generalized information measures within the realm of fuzzy set theory for logistics are of noteworthy significance to the field of logistics. The primary objective of these measures is to capture the inherent uncertainty and vagueness that is often prevalent in logistics, with the ultimate aim of improving the decision-making processes and operational outcomes. The practical implementation of these measures holds great promise for the enhancement of supply chain management strategies. Moreover, by integrating these measures into the pre-existing operations management framework, organizational outcomes can be significantly improved, through the augmentation of decision-making capacity, which enables informed and optimized decisions to be made.

Conflict of Interest

We certify that we have no affiliations with or involvement in any organization or entity with any financial interest (such as honoraria, educational grants, participation in speakers' bureaus, membership, employment, consultancies, stock ownership, or other equity interest, and expert testimony or patent-licensing arrangements) or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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