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Pricing Models for a Two-echelon Supply Chain with Substitute and Complementary Products Considering Disruption Risk

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Abstract

Nowadays in markets, products are categorized as an independent, substitute, and complementary ones. Being a complementary product could impact the demand for related products, so the pricing of these items could affect the other ones' demand. To investigate these effects, here a two-echelon supply chain with three manufacturers and a distributor for two substitutes and one complimentary product simultaneously has been investigated. The relationships between the manufacturers and the distributer are modeled by both cooperative and non-cooperative games. Due to political discussions, equipment failure, and natural disasters, our supply chain would confront disruption between the distributor and the manufacturers, which brings dissatisfaction, and the orders could not be fulfilled by manufacturers completely. Above mentioned issues bring variations in wholesalers and distributor prices. The optimal prices for both have been determined in our proposed model by taking advantage of game theories as well. Finally, the effects of key parameters on supply chain decisions and profit functions were investigated and numerical examples were developed to show the performance of the model, and sensitivity analysis was performed on important parameters to derive managerial insights.

Keywords: Supply Chain; Pricing; Complementary Product; Substitute Products; Game Theory; Disruption Risk.

1. Introduction

The variety of products and services as well as the uncertainty of customer demand has prompted companies to work together in the form of a supply chain to increase customer satisfaction. But competition and existing conflicts among the chain members' objectives are the key elements for the fact that every action affects the others' interests. One of these key parameters which greatly impacts supply chain profit and the number of demands is the price of the products or services. So, each supply chain may disrupt due to machinery breakdowns, lack of labor and political issues. Therefore, the upstream levels in a supply chain cannot fully satisfy a downstream levels order which has a significant effect on the profitability of a supply chain. By considering the competition, existing conflicts among supply chains' objectives, and the simultaneous reactions of the supply chain (SC) members, game theory was considered as a suitable method for making proper decisions in the proposed supply chain.

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2. Literature Review

In our model, two main subjects are distinguished, namely: a) substitute / complementary products and b) associated disruption risk in supply chain management. So the literature review of this research is inquired into two streams as follows:

a) Determining the optimal price of the products in a supply chain for maximizing the total profit of the SC is one of the favorite issues for researchers recently, and many studies were done in this field of study. Cao et al. (2012), Kuo and Huang (2012), Seyed Esfahani et al. (2011) and Wu et al. (2012)] Huang et al. (2012) developed a two-period pricing and production decision model in a one- manufacturer-one-retailer dual-channel supply chain that experiences a disruption in demand during the planning horizon. Song et al.(2022) explored a game-theoretical model of the green supply chain with a manufacturer and a retailer, the retailer sells two products with different environmental properties and the same functionality. Sun et al.(2021) examined pricing and replenishment decisions for seasonal and nonseasonal products in a three-echelon shared supply chain. We adopt a time-varying demand function and propose two models (off-season and peak season) for seasonal products. Wei and Zhao (2015) considered pricing and remanufacturing decisions in a duopoly market with two competing supply chains, which compete at both manufacturer and retailer levels. There are one manufacturer and one retailer in each supply chain; one of them produces new products directly from raw material, while the other manufacturer incorporates a remanufacturing process to return the used products into the original production system. Xie (2017) developed a profit-maximization problem, in which the revenue and costs for a new product sold under a two-dimensional warranty are affected by the product price and the area of warranty region.

Taleizadeh et al. (2015) developed an economic production quantity model in a three levels supply chain including multiple non-competing suppliers, a single manufacturer, and multiple non-competing retailers for multiple products with a rework process under integrated and non-integrated structures. They used the Stackelberg model for optimizing the supply chain total profit and determining the best price and production policy considering both defined conditions. Dan et al. (2012) provided the best decisions on services and retailer prices in two centralized and decentralized dual-channel supply chains using the Stackelberg model. The results showed that service levels had an enormous impact on the manufacturer and retailer pricing strategies. Hue et al. (2010) proposed a pricing model to help manufacturers for offering their products online, while the retailer implements traditional methods. They showed that delivery time greatly affects demands, profits, and pricing strategy when the proposed dual-channel supply chain works on both centralized and decentralized methods. In the competitive market, most of the products have substitutes or some complementary products. Therefore, a lot of research was carried out on the pricing policies of the substitutable products which can be summarized as follows: Chen et al. (2013) provided a pricing policy in a supply chain with substitutable products when the manufacturer has both direct and online channels for selling his products while the retailer sells another manufacturer's product. Karakul and Chan (2010) presented a single-period model for substitutable products as a combination of product pricing and procurement lead time when the demand for substitutable products is random. In contrast with research about substitutable products, there are a few pieces of research on complementary products. Mukhopadhyay et al. (2011) examined marketing strategies in two different companies for complementary products. In their model, each company predicts the non-deterministic market, and also they can decide whether share the information with the other company or not. They proved that sharing information can bring more profit for the leader in a market. Esmaeilzadeh et al. (2016) presented a pricing model considering two complementary products in a two-echelon supply chain for dual states, as follows:

i. Complementary production cost is the same at both levels and, when
ii. The aforementioned cost is different for levels and is dependent on demand

Arshadi Khamseh et al. (2014) offered a pricing model for complementary products in a fuzzy supply chain considering firms with different market power. Finally, related literature on substitutable and complementary products is summarized in table 2, respectively.

Table 1. Researches on substitutable products

Authors	Product		Deterioration product		Uncertainty		Decision variable		
	Substitute	**Complementary**	**No**	**Yes**	**No**	**Yes**	**Price**	**Amount of order**	**Production**
Karakul and Chan (2010)	*		*			*	*	*	
Zhao and Wei (2012)	*		*			*	*	*	
Zhao et al. (2012)	*		*			*	*		
Chen et al. (2013)	*		*		*		*		
Ai et al. (2012)	*		*				*		
Zhao et al. (2012)	*		*			*	*		

Table 2. Researches on complementary products

Authors	Product		Deterioration product		Uncertainty		Decision variable			
	Substitute	**Complementary**	**No**	**Yes**	**No**	**Yes**	**Price**	**Amount of order**	**Production**	**Others**
Zhang et al. (2012)		*	*		*			*	*	
Zhang et al. (2011)			*			*			*	
Schmitt and Snyder (2012)		*	*			*				*
Li (2010)		*	*		*				*	
Lleras and Miller (2010)		*	*		*				*	
Wei et al. (2013)		*	*		*					*
Shavandi et al. (2012)	*	*		*		*				*
Esmaeilzadeh and Taleizadeh (2016)		*	*		*		*			
Arshadi Khamseh et al. (2014)		*	*			*	*			
Wei et al. (2013)		*	*		*				*	

b) In the supply chain due to the machinery breakdowns, lack of labor, political issues, etc. Disruption risk may occur and bring irrecoverable effects on the supply chain's total profit. So, supply chain disruptions have become a critical concern for businesses all around the world.

Here, we will review some of the related research on this subject: Xanthopoulos et al. (2012) proposed the newsvendor problem with two supply channels where there is a possibility of disruption risk between the retailer and the distributor in each channel. When facing disruption, only a percentage of orders can be fulfilled by the distributor. Mohsenzadeh ledari et al. (2015) defined the newsvendor model in a multi-level supply chain with two-channel considering disruption risks among retailer and distributor when no percentage of an order must be met and the retailer will supply the order amount directly from the manufacturer in a special order. Qi (2013) proposed a model in which the retailer can provide products from two suppliers: the first supplier with low cost and no guarantee (high risk) and the second one with a higher price, and full guarantee by assuming no risk with the conducted survey. Schmitt and Snyder (2012) presented a model considering supply chain risks in two distinct categories: disruptions and yield uncertainty. They demonstrated the importance of analyzing a sufficiently long time horizon regarding inventory systems modeling under supply disruptions.

Ali et al. (2018) examined the effect of potential market demand disruptions on price and service levels for competing retailers. To investigate the effect of potential demand disruptions, they considered both centralized and decentralized supply chain structures. Sangaiah et al. (2020) presented a robust mixed-integer linear programming model for LNG sales planning over a given time horizon aiming to minimize the costs of the vendor. Since the parameter of the manufacturer, supply has an uncertain nature in the real world, this parameter is regarded to be interval-based uncertain. Gheibi and Fay (2021) presented the optimal procurement policy of a multi-product retailer in the presence of possible supply disruptions. Their analysis reveals that, in anticipation of potential supply disruptions, a retailer would typically benefit from ordering more units from a reliable supplier and fewer units from an unreliable one. Liu et al.(2022) presented how firms can mitigate supply chain disruption threats by optimizing pricing decisions and launching proactive actions in the form of effort investment jointly. From the research and studies carried out on pricing, it can be concluded that the most related research in this area was involved in determining the prices for substitutable products in a supply chain, and a few of them focused on the price of complementary products in a supply chain. Consequently, it seems that substitutable products with main complementary product pricing models considering disruption risks were not discussed so far. Therefore, regarding political problems, equipment failure, and natural disasters, a percentage of distributer order (s) may be not met by the manufacturer(s). The possibility of such cases is probable and known as disruption risk. So, when having disruption, just a percentage of orders can be met and this risk has a significant effect on the supply chain's total profit. Generally, most research if considered substituted products would focus on the prices of substituted ones, and research in the field of a simultaneous substitute and complementary products are rare.

Some innovations and contributions proposed in this research paper that distinguish our study from the recent studies can be categorized as follows:

- ✓ A pricing model for substitute and complementary products simultaneously has been proposed,
- ✓ Pricing in Chain-to-Chain competition of two-echelon supply chain is defined,
- ✓ Disruption risk among distributer and manufacturers has been considered and solved using Stackelberg and cooperation game theory.

The paper is organized as follows: In section three problem definition and model notations have been presented, in section four, a description based on Stackelberg and cooperation models were defined and modeled. Section five is dedicated to parametric sensitivity analysis, Section six is dedicated to a numerical example of our proposed models and sensitivity analysis, final section represents results and future research.

3. Problem Definition and Model Notation

In this model, a two-echelon supply chain is proposed including three manufacturers and a distributor. It is assumed that the two manufacturers (manufacture 2 and 3) produce a substitutable product for the third one (manufacture 1). For example, suppose the products are: tea, coffee, and cube sugar, respectively. Direct sales from the manufacturer to the final customer are not allowed in our model. Demands for all three products are linear affecting his own and other products' prices. Disruption risk is considered between these two echelons. In such cases, just a percentage of the orders are fulfilled.

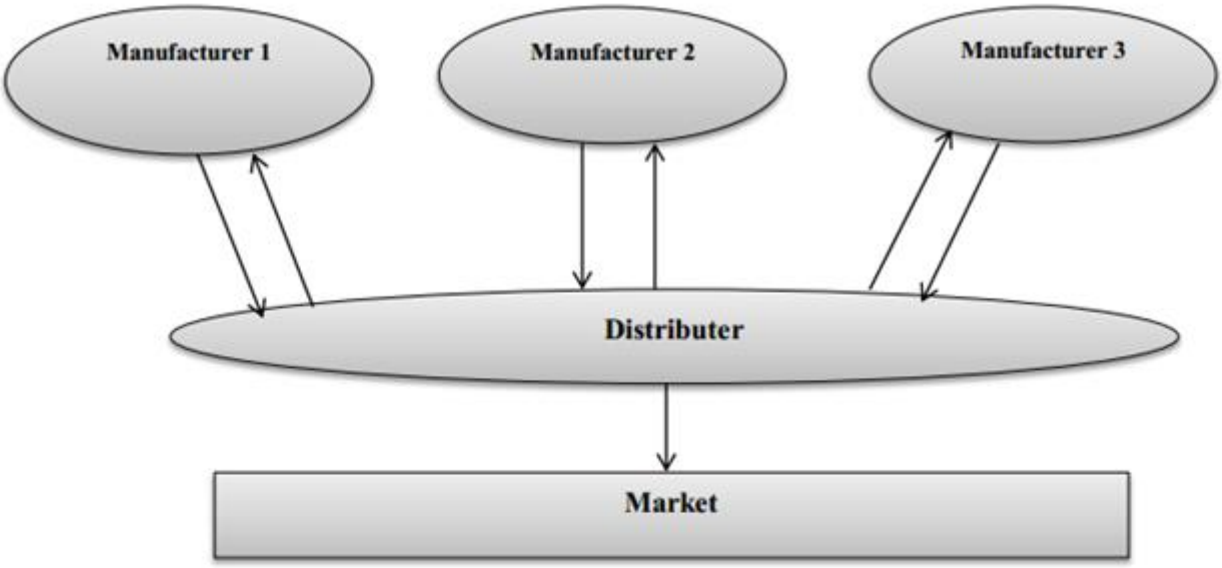

Figure 1. Two-echelon supply chain with three manufacturers and one distributor

As the proposed two-echelon supply chain is defined before, here we focus on establishing a proper game model for the collaboration and interplays among different levels of the supply chain: The first model is the Stackelberg in which the manufacturers are assumed as the leader of the distributer. So, each manufacturer determined products price at first, and then the distributor specifes the price of his products. The second model is a cooperation strategy in which all the levels of the supply chain integrate and act as a single supply chain, consequently. The following parameters and decisions variables are applied through the current paper to model the proposed supply chain:

Parameters:

α_i : Market share for the i^th^, ((i = 2,3) represent for substitutable products and 1 is for complementary product (i = 1)),

β : The effect of price on the product's demand,

γ : The effect of price on the other product's demand,

p_2 : Disruption risk probability among distributor and manufacturer 2,

p_3 : Disruption risk probability among distributor and manufacturer 3,

c_i : Unit production cost in the i^{th} manufacturer,

y_2 : Order percentage can be paid in case of disruption for product 2,

y_3 : Order percentage can be paid in case of disruption for product 3.

Decision variables:

T_i : Selling price of the i^{th} manufacturer product to the distributer,

w_i : The distributer selling price for i^{th} manufacturer product.

Functions:

D_i : Demand function for the i^{th} manufacturer's product,

π_{Mi} : i^{th} manufacturer's profit function,

π_D : Distributer's profit function,

π_{sc} : Supply chain's profit function in the cooperative model.

4. Model Description

As explained in section 2, products 2 and 3 are substitutble and complementry for product 1,respectively.

4.1. Assumptions

Assumption1: $w_i \geq T_i \geq c_i$

Distributer prices must be greater than manufacturers' prices since the distributer profit should be positive. In addition, to have positive profit for manufacturers, manufacturer prices must be greater than the manufacturing cost of products.

Assumption2: $\beta \geq \gamma \geq 0$

The importance of self-price coefficient for each product is greater than the cross-price coefficient of another product. Regarding increasing a ξ unit in price, the number of customers relinquishing to buy a product is greater than the number of another product's customers.

Assumption3: By increasing one unit in the price of a product 2, $\beta - \gamma$ percent of customers relinquish buying the product, and also by increasing one unit in the price of a product 3, $\beta - \gamma$ percent of customers relinquish to buy the product. So it can be concluded that by one unit increasing in the price of a product 1, $2(\beta - \gamma)$ customers relinquish buying that product.

Assumption4: Disruption risk is just considered among the distributor and the manufacturers of substitute products, then, we have Demand functions:

$$D_1 = \alpha_1 - 2(\beta - \gamma)w_1 - (\beta - \gamma)w_2 - (\beta - \gamma)w_3 \tag{1}$$

D_1 Indicates demand for product 1 (the complementary product). The first part indicates the base demand at zero prices. The second part indicates a declination in demand based on a one-unit increase in the price of the complementary product. Since, one unit increases in the price of product 2, $\beta - \gamma$ purchasers are discouraged; similarly, by one unit increase in the price of product 3, $\beta - \gamma$ purchasers will be discouraged. Therefore, increasing a unit of product 1 will result in discouraging $2(\beta - \gamma)$ percent of customers from purchasing a product. The third and the fourth parts indicate the demand declination of product 1 as a result of a one-unit increase in the price of the substituted products 2 and 3.

$$D_2 = \alpha_2 - \beta(w_1 + w_2) + \gamma w_3 \tag{2}$$

D_2 Defines the demand for the product 2. The first part indicates the base demand for the product at zero price. The second part indicates the decrease in demand for the product 2 based on one unit increase in the prices of products 1 and 2, and the third part indicates an increase in the demand for the product 2 as a result of one-unit increase in the price of product 3.

$$D_3 = \alpha_3 - \beta(w_1 + w_3) + \gamma w_2 \tag{3}$$

D_3 Is the demand for the product 3. The first part indicates the base demand for the product at zero price. The second part indicates the decrease in demand for product 3 based on a one-unit increase in the prices of products 1 and 3, and the third part indicates an increase in the demand for product 3 as a result of a one-unit increase in the price of product 2.

4.2. Profit Functions in Stackelberg Model

The profit function of Manufacturer 1 can be written as follows:

$$\pi_{M1} = D_1(T_1 - c_1) \tag{4}$$

The first manufacturer's profit is equal to: the multiplication of each unit profit (the difference between the sale price and production cost) by the total amount of that product's demand.

The profit function of manufacturer 2 can be written as follows:

$$\pi_{M2} = (1 - p_2)D_2(T_2 - c_2) + p_2 y_2 D_2(T_2 - c_2) \tag{5}$$

The second manufacturer's profit is equal to: the multiplication of each unit profit (the difference between the sale price and production cost) by the total amount of that product's demand with considering disruption risk and without considering disruption risk.

The profit function of manufacturer 3 is as follows:

$$\pi_{M3} = (1 - p_3)D_3(T_3 - c_3) + p_3 y_3 D_3(T_3 - c_3) \tag{6}$$

The third manufacturer's profit is equal to: the multiplication of each unit profit (the difference between the sale price and production cost) by the total amount of that product's demand considering disruption risk and without considering disruption risk Now, the distributor profit function can be defined as follows:

$$\pi_D = D_1(w_1 - T_1) + (1 - p_2)D_2(w_2 - T_2) + p_2 y_2 D_2(w_2 - T_2) + (1 - p_3)D_3(w_3 - T_3) + p_3 y_3 D_3(w_3 - T_3) \tag{7}$$

The distributor's profit is equal to the sum of these three products' sales profit, which is equal to the multiplication of each unit profit (the difference between the sale price and buying cost) by the total amount of that product demand with considering disruption risk and without considering disruption risk.

Proposition1. The distributer's profit function is concave.

Proof.

The objective function of the distributor is described as follows:

$$\pi_D = D_1(w_1 - T_1) + (1 - p_2)D_2(w_2 - T_2) + p_2 y_2 D_2(w_2 - T_2) + (1 - p_3)D_3(w_3 - T_3) + p_3 y_3 D_3(w_3 - T_3) \tag{8}$$

$$H_{\text{distributer}} = \begin{bmatrix} \dfrac{\partial^2 \pi_d}{\partial w_1^2} & \dfrac{\partial^2 \pi_d}{\partial w_1 \partial w_2} & \dfrac{\partial^2 \pi_d}{\partial w_1 \partial w_3} \\ \dfrac{\partial^2 \pi_d}{\partial w_2 \partial w_1} & \dfrac{\partial^2 \pi_d}{\partial w_2^2} & \dfrac{\partial^2 \pi_d}{\partial w_2 \partial w_3} \\ \dfrac{\partial^2 \pi_d}{\partial w_3 \partial w_1} & \dfrac{\partial^2 \pi_d}{\partial w_3 \partial w_2} & \dfrac{\partial^2 \pi_d}{\partial w_3^2} \end{bmatrix} \tag{9}$$

$$H_{\text{distributer}} = \begin{bmatrix} -4\beta + 4\gamma & -\beta + \gamma - (1 - p_2)\beta - p_2 y_2 \beta & \begin{array}{l} -\beta + \gamma - (1 - p_3)\beta \\ -p_3 y_3 \beta \end{array} \\ \begin{array}{l} -\beta + \gamma - (1 - p_2)\beta \\ -p_2 y_2 \beta \end{array} & -2(1 - p_2)\beta - 2p_2 y_2 \beta & \begin{array}{l} (1 - p_2)\gamma + p_2 y_2 \gamma + (1 - p_3)\gamma \\ +p_3 y_3 \gamma \end{array} \\ \begin{array}{l} -\beta + \gamma - (1 - p_3)\beta \\ -p_3 y_3 \beta \end{array} & \begin{array}{l} (1 - p_2)\gamma + p_2 y_2 \gamma + (1 - p_3)\gamma \\ +p_3 y_3 \gamma \end{array} & -2(1 - p_3)\beta - 2p_3 y_3 \beta \end{bmatrix} \tag{10}$$

The first minor determinant is equal to $-4\beta + 4\gamma$; as $\beta \geq \gamma$, so the determinant is negative. The second minor determinant is equal to:

$$(-4\beta+4\gamma).(-2(1-p_2)\beta-2p_2y_2\beta)-(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta).(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta) \tag{11}$$

Which is positive such as:

$(-4\beta+4\gamma)(-2(1-p_2)\beta-2p_2y_2\beta)\geq 0$ (because $(-)(-)\geq 0$)

$(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta).(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta)\geq 0$ (because $(-)(-)\geq 0$)

The second minor determinant is the difference between two positive values if we prove that the first part is bigger than the second part, so the difference between the two positive values will be positive; as a result, the second minor determinant is positive.

$4(-\beta+\gamma)$ is smaller than $(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta)$

$(-2(1-p_2)\beta-2p_2y_2\beta)$ is smaller than $(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta)$ because:

$$-2(1-p_2)\beta-2p_2y_2\beta=-2\beta(1-p_2+p_2y_2)\leq-\beta(1-p_2+p_2y_2) \tag{12}$$

$$(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta)=(-\beta+\gamma-\beta(1-p_2+p_2y_2)) \tag{13}$$

So:

$$(-2(1-p_2)\beta-2p_2y_2\beta)\pounds(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta) \tag{14}$$

Briefly, we can have:

$$4(-\beta+\gamma)\leq(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta)\leq 0 \tag{15}$$

$$(-2(1-p_2)\beta-2p_2y_2\beta)\leq(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta)\leq 0 \tag{16}$$

So the second minor determinant is positive.

The third minor determinant is equal to:

$$\begin{aligned}&[(-4\beta+4\gamma).(4\beta^2.((1-p_2+p_2y_2).(1-p_3+p_3y_3))-\gamma^2.(1-p_2+p_2y_2+1-p_3+p_3y_3)^2]+[(2(1-p_3)\beta+2p_3y_3\beta).(-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta)^2)\\&+((-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta).(-\beta+\gamma-(1-p_3)\beta-p_3y_3\beta).(1-p_2)\gamma+p_2y_2\gamma+(1-p_3)\gamma+p_3y_3\gamma))]+[((-\beta+\gamma-(1-p_2)\beta-p_2y_2\beta).(-\beta+\gamma\\&-(1-p_3)\beta-p_3y_3\beta).(1-p_2)\gamma+p_2y_2\gamma+(1-p_3)\gamma+p_3y_3\gamma))+(2(1-p_2)\beta+2p_2y_2\beta).(-\beta+\gamma-(1-p_3)\beta-p_3y_3\beta)^2)]\end{aligned} \tag{17}$$

That the first section was merely negative and the second and the third were positive; to see the concave form of the matrix after summarizing the sentences, the second and the third phrases are:

$$\begin{aligned}&((-\beta+\gamma-\beta(1-p_2+p_2y_2)).(-\beta+\gamma-\beta(1-p_3+p_3y_3)).((\gamma.(1-p_2+p_2y_2+1-p_3+p_3y_3)))+\\&((2\beta.(1-p_2+p_2y_2).(-\beta+\gamma-\beta.(1-p_3+p_3y_3))^2)+((2\beta.(1-p_3+p_3y_3).(-\beta+\gamma-\beta(1-p_2+p_2y_2))^2)\end{aligned} \tag{18}$$

Regarding to the enlargement of the above expression, now we define:

$$a=1-p_2+p_2y_2 \quad ,b=1-p_3+p_3y_3 \tag{19}$$

So the first expression will be equal to:

$$(-4\beta+4\gamma)(4\beta^2ab-4\gamma^2(a+b)^2) \tag{20}$$

And the total sum of the second and the third terms are:

$$((-\beta.(1+a)+\gamma).(-\beta.(1+b)+\gamma).(\gamma.(a+b)))+(2\beta a.(-\beta(1+b)+\gamma)^2+2\beta b(-\beta(1+a)+\gamma)^2) \tag{21}$$

Since a, b has a positive value that is less than one, and β is greater than γ, so it can be proved that the absolute value of the first expression is greater than the sum of the second and the third terms, so the third minor determinant of Hessian matrix is negative and the aforementioned matrix is **concave.**

In this model the manufacturers are considered the leader and the distributer as a follower, so the manufacturers determine the sale prices at first and then the distributor specifies the optimal product selling prices according to the manufacturer's selling prices.

Proposition2. The Selling price in distribution level (w_1, w_2, w_3***) is as follows:***

In the Stackelberg model, the distributor is assumed as a follower and the manufacturers are the leaders. According to this model, the optimal prices values in the distribution level are as follows:

$$w_1 = -(b_1 - a_{11}b_2 - a_{12}b_3 + a_{11}a_{22}b_3 + a_{12}a_{32}b_2 - a_{22}a_{32}b_1)/(a_{11}a_{21} + a_{12}a_{31} + a_{22}a_{32} - a_{11}a_{22}a_{31} - a_{12}a_{21}a_{32} - 1) \tag{22}$$

$$w_2 = -(b_2 - a_{21}b_1 - a_{22}b_3 + a_{12}a_{21}b_3 - a_{12}a_{31}b_2 + a_{22}a_{31}b_1)/(a_{11}a_{21} + a_{12}a_{31} + a_{22}a_{32} - a_{11}a_{22}a_{31} - a_{12}a_{21}a_{32} - 1) \tag{23}$$

$$w_3 = -(b_3 - a_{31}b_1 - a_{32}b_2 - a_{11}a_{21}b_3 + a_{11}a_{31}b_2 + a_{21}a_{32}b_1)/(a_{11}a_{21} + a_{12}a_{31} + a_{22}a_{32} - a_{11}a_{22}a_{31} - a_{12}a_{21}a_{32} - 1) \tag{24}$$

Where b_i, a_{ij} are constants defined in Appendix A.

Proof. See Appendix A.

To determine the optimal price of a product at the manufacturing level, first we replace the optimum values obtained for each of the products in the distribution level considering demand functions. Then, we can use demand for each of the products in the profit function of the same chain.

Proposition 3. The manufacturer objective function is concave and the optimal values of the manufacturers in Stackelberg model are as follows:

$$\begin{aligned} T_1 = {} & (z_3(b_{12}b_{23} - b_{13}b_{22}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - \\ & (z_2(b_{12}b_{33} - b_{13}b_{32}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) + \\ & (z_1(b_{22}b_{33} - b_{23}b_{32}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) \end{aligned} \tag{25}$$

$$\begin{aligned} T_2 = {} & (z_2(b_{11}b_{33} - b_{13}b_{31}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - \\ & (z_3(b_{11}b_{23} - b_{13}b_{21}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - \\ & (z_1(b_{21}b_{33} - b_{23}b_{31}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) \end{aligned} \tag{26}$$

$$\begin{aligned} T_3 = {} & (z_3(b_{11}b_{22} - b_{12}b_{21}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - \\ & (z_2(b_{11}b_{32} - b_{12}b_{31}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) + \\ & (z_1(b_{21}b_{32} - b_{22}b_{31}))/(b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) \end{aligned} \tag{27}$$

Where b_{ij}, z_i are constants defined in Appendix B.

Proof. See appendix B.

4.3. Profit Functions in Cooperation Model

In the cooperation model, the supply chain works as an integrated system and seeks to maximize its own total profit. In this model, the sales prices of the manufacturers offering to the distributor are eliminated and only the production costs of each item in the manufacturing process are considered interchangeably.

Supply chain total profit can be written as follows:

$$\pi_{sc} = D_1(w_1 - c_1) + [(1 - p_2)D_2(w_2 - c_2) + p_2y_2D_2(w_2 - c_2)] + [(1 - p_3)D_3(w_3 - c_3) + p_3y_3D_3(w_3 - c_3)] \tag{28}$$

The supply chain's profit is equal to: the sum of these three products' sales profit which is equal to the multiplication of each unit profit (the difference between the sale price and buying cost) by the total amount of that product demand with considering disruption risk and without considering disruption risk.

Proposition4. The supply chain objective function in the cooperation model is concave.

Proof. See Appendix C.

Proposition4. Selling price in the supply chain (w_1, w_2, w_3 ***) are as bellow:***

$$w_1 = -(d_1 - c_{11}d_2 - c_{12}d_3 + c_{11}c_{22}d_3 + c_{12}c_{32}d_2 - c_{22}c_{32}d_1) / (c_{11}c_{21} + c_{12}c_{31} + c_{22}c_{32} - c_{11}c_{22}c_{31} - c_{12}c_{21}c_{32} - 1) \tag{29}$$

$$w_2 = -(d_2 - c_{21}d_1 - c_{22}d_3 + c_{12}c_{21}d_3 - c_{12}c_{31}d_2 + c_{22}c_{31}d_1) / (c_{11}c_{21} + c_{12}c_{31} + c_{22}c_{32} - c_{11}c_{22}c_{31} - c_{12}c_{21}c_{32} - 1) \tag{30}$$

$$w_3 = -(d_3 - c_{31}d_1 - c_{32}d_2 - c_{11}c_{21}d_3 + c_{11}c_{31}d_2 + c_{21}c_{32}d_1) / (c_{11}c_{21} + c_{12}c_{31} + c_{22}c_{32} - c_{11}c_{22}c_{31} - c_{12}c_{21}c_{32} - 1) \tag{31}$$

Where d_i , c_{ij} are constants defined in Appendix D.

Proof. See Appendix D.

5. Parametric sensitivity analysis

In this section, the effects of p_i, y_i on the distributer selling prices are investigated. Sections 4.1 to 4.4 are involving the decentralized model.

5.1. Impact of disruption risk probability (p_2) on distributer selling price (w_2)

First derivate of w_2 respect to parameter p_2 shows the impact of disruption risk probability on distributer selling price for product 2.

If $\frac{dw_2}{dp_2} \geq 0$ Then p_2 has positive impact on w_2.

If below condition established then $\frac{dw_2}{dp_2} \geq 0$.

$$p_2 \leq \frac{A_2 - A_1}{A_1(y_2 - 1) - A_3} \tag{32}$$

The value of A_1 to A_3 are provided in appendix E.

5.2. Impact of payable order percentage in the case of disruption risk (y_2) on distribution selling price (w_2)

First derivate of w_2 respect to parameter y_2 shows the impact of payable order percentage in case of disruption risk on distribution selling price for product 2.

If $\frac{dw_2}{dy_2} \geq 0$ Then p_2 has positive impact on y_2.

Considering the following condition, then $\frac{dw_2}{dy_2} \geq 0$.

$$y_2 \geq \frac{B_1 - B_1 p_2 - B_2}{B_3 - B_1 p_2} \tag{33}$$

The value of B_1 ***to*** B_3 ***are provided in appendix E.***

Corollary1. If the probability of disruption occurrence for product 2(p_2) is as small as possible and the payable order percentage in case of disruption risk for product 2 (y_2) is as large as possible, then the selling prices for product 2 will increase. Since, the product is available, the tendency for buying that product will increase. ***Insight1.*** It can be stated that by increasing the selling prices of product 2, the distribution income will increase, consequently. So, distributors' total profit increases.

5.3. Impact of disruption risk probability (p_3) on distribution selling price (w_3)

First derivate of w_2 respect to parameter p_3 shows the impact of disruption risk probability on distribution selling price of product 3.

If $\frac{dw_3}{dp_3} \geq 0$ then p_2 has positive impact on w_3 .If below condition established then $\frac{dw_3}{dp_3} \geq 0$.

$$p_3 \leq \frac{C_2 - C_1}{C_1(y_3 - 1) - C_3} \tag{34}$$

The value of C_1 to C_3 are provided in appendix E.

5.4. Impact of payable order percentage in case of disruption risk (y_3) on distribution selling price (W_3)

First derivate of w_3 respect to parameter y_3 shows the impact of the payable order percentage in case of disruption risk on distribution selling price of product 3.

If $\frac{dw_3}{dy_3} \geq 0$ Then p_3 has positive impact on y_3 .

If below condition established then $\frac{dw_3}{dy_3} \geq 0$.

$$y_3 \geq \frac{E_1 - E_1 p_3 - E_2}{E_3 - E_1 p_3} \tag{35}$$

The value of E_1 to E_3 are provided in appendix E.

Corollary2. If the probability of disruption occurrence of product 3(p_3) is as small as possible and payable order percentage in case of disruption risk for product 3 (y_3) is as large as possible, then the sales prices of product 3 will increase. Since, that product is available and the tendency for buying that product will increase. ***Insight2.*** It can be stated that by increasing selling prices for product 3, the distribution income will increase, consequently. So, the distributor’s total profit increases.

6. Numerical analysis

In this section, a numerical example is provided to illustrate the feasibility of the proposed problem. Therefore, parameter values are set based on problematic assumptions. The optimal prices in the supply chain levels, the total amount of demand, the profit functions in the Stackelberg model, and the cooperation model are discussed and then the effect of changing important parameters on each defined model is discussed. Table 3 shows the parameter values applied in the numerical example.

Table 3. Numerical examples parameters

Parameter	Value	Parameter	Value
α_1	800	y_3	0.5
α_2	500	β	2.5
α_3	600	γ	1
p_2	0.9	c_1	40
p_3	0.6	c_2	33
y_2	0.8	c_3	30

6.1. Numerical Analysis in Stackelberg Model

In this section, the optimal prices in supply chain levels, the amount of the demand, and the objective functions in the Stackelberg model are shown in Table 4 considering the values of parameters in Table 3.

Table 4. Optimal prices and objective function in Stackelberg model

Decision variables	Value	Decision variables	Value
w_1	138.8711	T_1	112.2612
w_2	82.7547	T_2	79.2167
w_3	102.8730	T_3	92.6659
Objective Function	**Value**	**Demand Function**	**Value**
π_D	3493.7	D_1	105.8628
π_{M1}	7649.8	D_2	48.1967
π_{M2}	256.0253	D_3	79.9239
π_{M3}	3506		

The formulated problem in the Stackelberg form is solved in different sizes in which the optimal selling price and objective function values are presented in Table 4. Moreover, Figure 2 depicts the effect of the product demand on the amount of each manufacturer's profit. As shown in the figure, by increasing the demand amount of each product, the manufacturer's profit will be increased because the selling amount of the product increases.

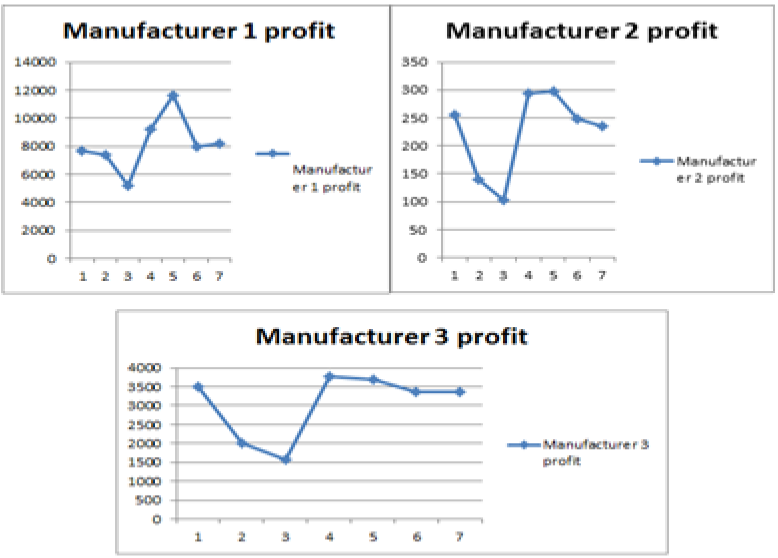

Figure 2. Effect of demand amount on manufacturer profits

6.2. Sensitivity Analysis of the Stackelberg Model

In this model, the important parameters affecting the profitability and the demand function are , α_3, α_2, α_1, p_3, p_2, γ, and the sensitivity analyses were done upon these parameters. β

Table 5. Data for sensitivity analysis

Parameter	Lower level	Middle level	Upper level
α_1	800	850	900
α_2	500	530	550
α_3	600	630	650
p_2	0.7	0.8	0.9
p_3	0.4	0.5	0.6
β	2.5	3	3.5
γ	1	1.3	1.5

Figure 3 shows the effect of changing parameters on each of these three products' demands that is similar to the numerical results derived from solving the model. Fig. 3 reveals that by increasing the price sensitivity while keeping other parameters constant (for cases 2 and 3), each of these three products' demand decreases. Since the demand is assumed as a decreasing function of price so increasing the price will result in decreasing the demand value. In addition, by increasing the base demand of each product while keeping other parameters constant (for cases 4 and 5), each of these three products' demand increases. Since the demand function is assumed as an increasing function of basic demand, by reducing the probability of facing disruption risk in manufacturers 2 and 3 (two substitute products) (for cases 6 and 7), both substitute products' demands reduce. Because the products are more likely to be available and customers can order whenever they need them. Although, the rants will increase complementary products demand.

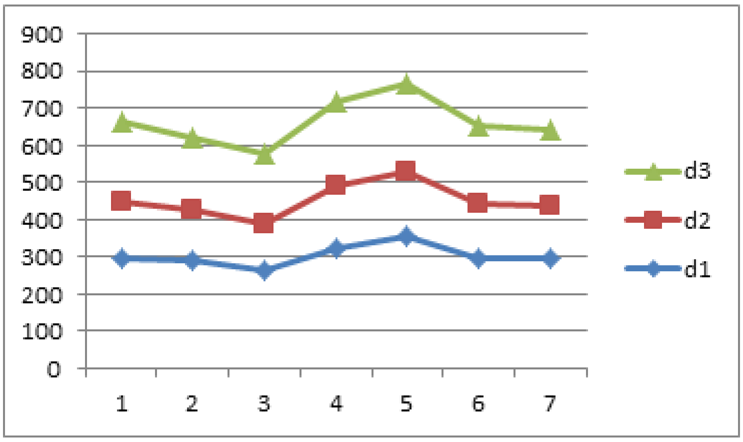

Figure 3. The effect $\beta, \gamma, p_i, \alpha_i$ on demand

Figure 4 depicts the effect of changing parameters on the sale price of the products at the distributor's level. In each case, by decreasing the demand for each product, the sale price at the distributor level reduces because the tendency for buying that product will decrease. So, for attracting customers and sell more products, we should reduce the price. Also, by increasing demand, the sale price will increase, consequently. Because the inclination for buying that product is high and to increase the profit level, one has to increase the sale price. The following results are obtained by comparing Figures 3 and 4.

Figure 5 shows the effect of changing parameters on the sale price of each product by the respective manufacturer. In any case, when there is a reduction in each product's total demand, the distributor's sale price in the relevant situations will decrease subsequently. Since the manufacturers are assumed as the distributor's followers in the proposed Stackelberg model. So, when the distributor reduces the related sale price, the sale price offered by the manufacturer to the distributor will decrease, too.

Figure 6 shows the comparison between the profit margin of each supply chain. As it can be seen, manufacturer 3 has the highest profit and regarding the results of the proposed numerical example and aforementioned figures. In addition, it is clear that product 3 has both the highest demand and the highest selling price among the manufacturers in each model.

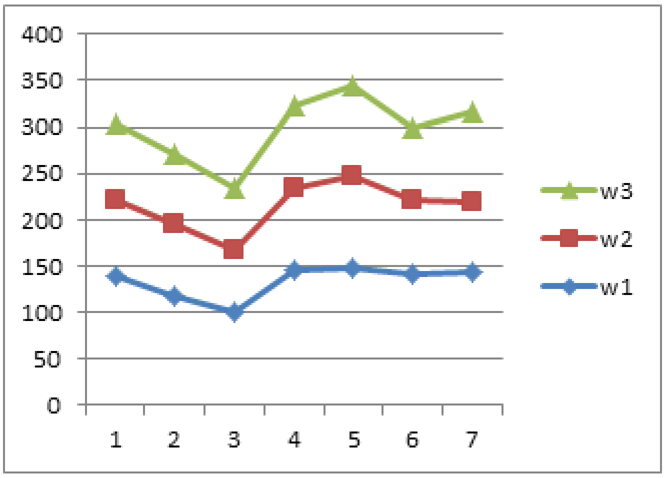

Figure 4. The effect of $\beta, \gamma, p_i, \alpha_i$ on the sale price in the distributor level

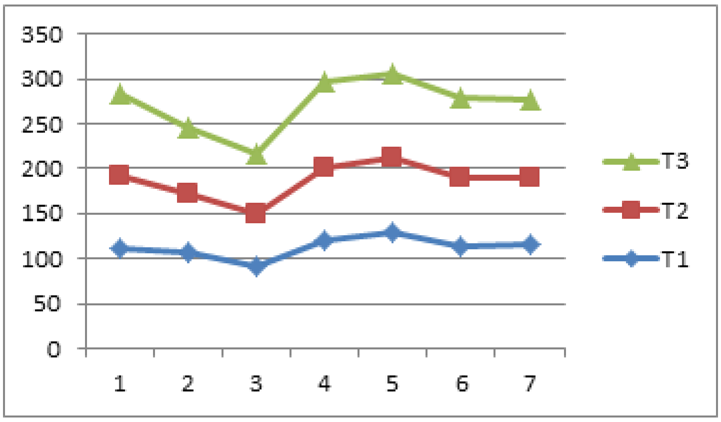

Figure 5. The effect of $\beta, \gamma, p_i, \alpha_i$ on the sale price in the manufacturer's level

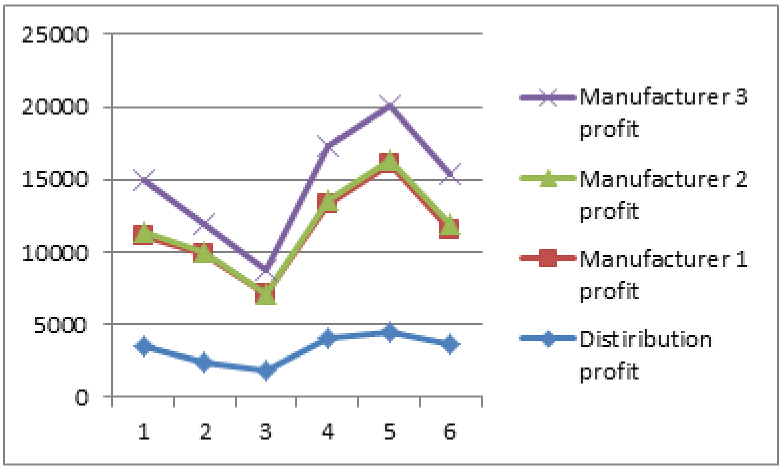

Figure 6. The effect of $\beta, \gamma, p_i, \alpha_i$ on each of the chain levels' profit

6.3. Numerical Analysis of Cooperation Model

In the cooperation model, the whole supply chain is considered a single system. In other words, we seek to maximize the profit of the whole supply chain; therefore, the price of each manufacturer's product offering to the distributor is not considered through this model. Only the product cost and the selling price of the distributor offering to his customers are considered. , The optimal selling price of the products at the distributor's level, each product's demand, and the total supply chain profit are given in Table 6 based on the parameters written in Table 6.

Table 6. The optimal prices and objective function in cooperation model

Decision variables	Value	Demand function	Value
w_1	117.6282	D_1	298.8263
w_2	47.3479	D_2	149.5048
w_3	56.4449	D_3	214.1652
Objective function		**Value**	
π_{sc}		28261	

In the decentralized model (Stackelberg model), the values of profit functions are :

$\pi_{M_1}, \pi_{M_2}, \pi_{M_3}, \pi_D$ in which total profit of supply chain is equal to $\pi_{M_1}, \pi_{M_2}, \pi_{M_3}, \pi_D$, but in the centralized model (Cooperation model), the total profit of supply chain is equal to π_{SC}. As it obvious from table 4 and table 6, the total profit of centralized model is greater than the decentralized model, which is shown in table 7.

Table7. Comparison between the total profit of supply chain in two models

Model	Total profit of supply chain
Decentralized model	14905.5253
centralize model	28261

6.4. Sensitivity Analysis of Cooperation Model

The sensitivity analysis of the cooperation model due to the important parameters of the problem (β , α_3 α_1 ، p_3 ، p_2 ، γ ،) is presented in table 5. α_3 ، α_2 ،

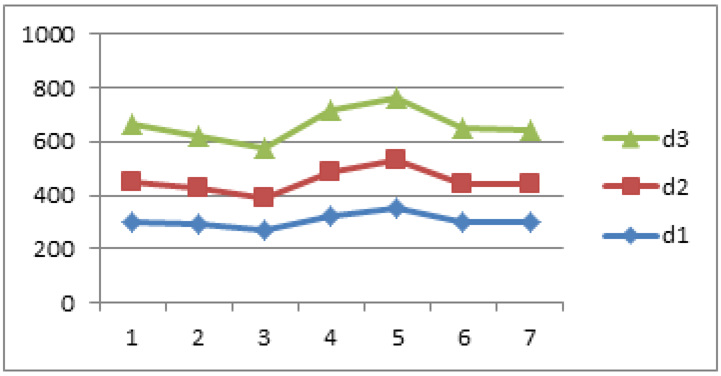

Figure 7. The effect of $\beta, \gamma, p_i, \alpha_i$ on demand

Figure 7 shows the effect of changing parameters on each of these three products' demands. As it can be seen from the results of solving the model and Fig. 7, by increasing the price sensitivity and while considering the other parameters constant (for cases 2 and 3), each of these three products' demand decreases. Since the demand is assumed as a decreasing function of the selling price, it will reduce when the price increases. As the base demand of each product and the other parameters' consistency increase (for cases 4 and 5), each of these three products' demand increases. Since the demand is assumed as an increasing function of the base demand. Also, by reducing disruption risk probabilities in manufacturers 2 and 3 (two substitute products) (for cases 6 and 7), the total amount for each of these three products decreases. But in the Stackelberg model, the total amount of complementary demand will increase whereas in the cooperation model it will decrease because the whole supply chain works as a single system in the cooperation model.

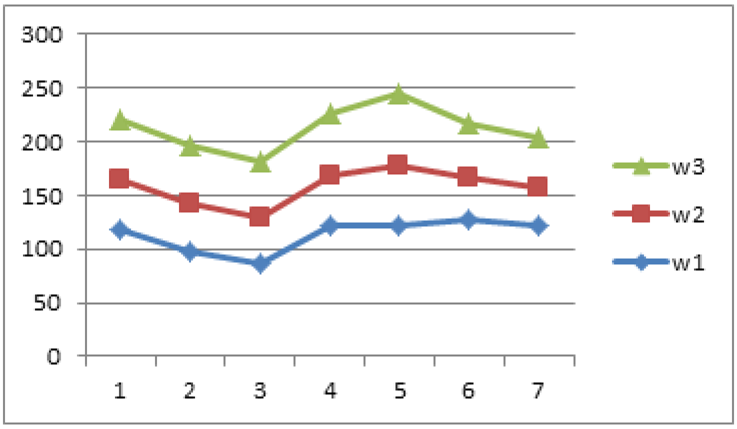

Figure 8. The effect of $\beta, \gamma, p_i, \alpha_i$ on the sales price

Figure 8 shows the effect of changing parameters on the sale price of the products in the proposed supply chain. In each case, the demand for each product decreases as the price of sales decreases. Because the tendency for buying the products decreases and to attract more customers, the price should reduce. Moreover, by increasing the total amount of demand, the sales price increases because the willingness for buying that product is very high, and to increase the profitability, one should increase the sale price. These results are shown in Figures 7 and 8.

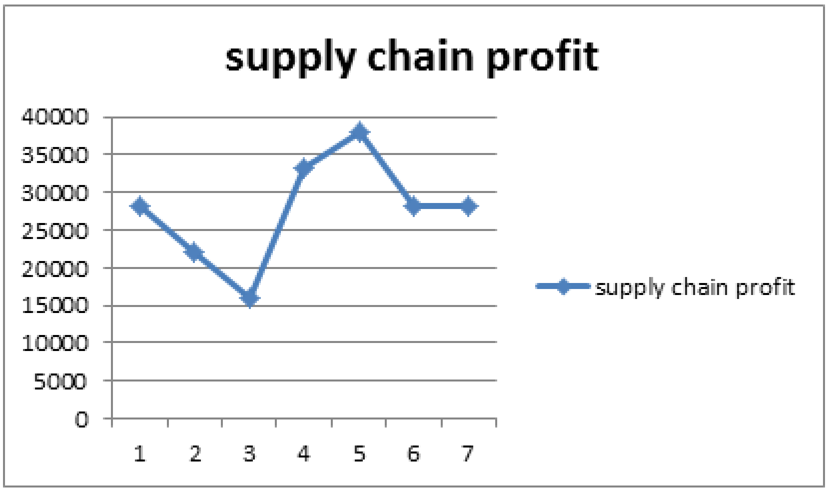

Figure 9. The effect of $\beta, \gamma, p_i, \alpha_i$ on the total chain profit

Figure 9 demonstrates the whole supply chain profit in different models. As it is obvious in the model, the whole supply chain profit is obtained by the multiplication of demand by the difference between production costs and selling price. According to figures 7 and 8, if demand and sales prices decrease, the whole supply chain profit decreases subsequently and vice versa. It should be noted that all insights are visible in figure 9.

7. Conclusion

In this paper, a pricing model is proposed for two substitute and one complimentary product to examine optimal solutions both in a Stackelberg (where the distributor is a follower and the manufacturers are leaders) and a cooperation model considering disruption risks, simultaneously. Here, the probability of disruption risk was considered just for two substitute products while most researchers studied substitute products or complementary products separately. Then, we solve several numerical examples and examine the effect of changing important parameters on decision variables and objective functions in both Stackelberg and cooperation models. In this paper, it has been shown that by increasing price sensitivity, demand will decrease. So, for keeping competitive status in each market, the sale price should be reduced since the tendency to decrease the amount of buying products will reduce the profitability. Moreover, by increasing the tendency for buying products, the sale price can be increased to maximize the whole chain's profit. Also, by reducing disruption risk probability with respect to the product availability, the demand for those products decreases in the Stackelberg model, and the demand for complementary products increase in the cooperation model. Because all of the supply chain members work as an integrated system, the demand for each of these three products demands decreases.

As the results of the numerical solution in this model, due to the disruption risk, the profitability of the supply chain may be reduced, and by using the cooperation model in which all members of the supply chain act as a single system, the profitability of the supply chain can be maintained. The consistency of the profitability in this issue by applying our proposed solution would be a prominent managerial insight and could guarantee the executive of the supply chain and all the stakeholders a persistent way of profitability.

Finally, future research directions can be conducted by considering more levels in a supply chain, some superseded and complementary products, and the existence of the contracts between the supply chain members.

References

Ai, X., Chen, J. , Zhao, H. , and Tang, X.(2012). Competition among supply chains: Implications of full returns policy. *International Journal of Production Economics,* Vol. 139(1), pp. pp. 257-265.

Ali, S. M., Rahman, M. H. , Tumpa, T. J., Rifat, A. A. M., and Paul, S. K.(2018). Examining price and service competition among retailers in a supply chain under potential demand disruption. *Journal of Retailing and Consumer Services,* Vol. 40, pp. 40-47.

Arshadi Khamseh, A., Soleimani , F. , and Naderi, B.(2014). Pricing decisions for complementary products with firm's different market powers in fuzzy environments. *Journal of Intelligent & Fuzzy Systems,* Vol. 27(5), pp.2327-2340.

Cao, P., Li, J., and Yan, H.(2012). Optimal dynamic pricing of inventories with stochastic demand and discounted criterion. *European Journal of Operational Research,* Vol. 217 (3), pp.580-588.

Chen, Y. C., Fang, S. C. , and Wen, U. P.(2013). Pricing policies for substitutable products in a supply chain with Internet and traditional channels. *European Journal of Operational Research,* Vol. 224(3), pp. 542-551.

Dan, B., Xu, G. , and Liu, C.(2012). Pricing policies in a dual-channel supply with retail services. *International Journal of Production Economics,* Vol. 139(1), pp. 312-320.

Esmaeilzadeh, A., and Taleizadeh, A. A.(2016). Pricing in a two-echelon supply chain with different market powers: game theory approaches. *Journal of Industrial Engineering, International,* Vol. 12(1), pp.119-135.

Hua, G., Wang, S. , and Cheng, T. C. E. (2010). Price and lead time decisions in a dual-channel supply chains. *European Journal of Operational Research,* Vol. 205(1), pp.113-126.

Huang, S., Yang, C., and Zhang, X. (2012). price and production decisions in dual-channel supply chains with demand disruption. *Computers & Industrial Engineering,* Vol. 62(1), pp.70-83.

Karakul, M., and Chan, L. M. A. (2010). Joint pricing and procurement of substitutable products with random demands – A technical note. *European Journal of Operational Research,* Vol. 201(1), pp. 324-328.

Kuo, C., and Huang, K.(2012). Dynamic pricing of limited inventories for multi-generation products. *European Journal of Operational Research,* Vol. 217(2), pp. 394-403.

Li, Z. (2010). Regulating risk-averse producers: The case of complementary products.” *Economics Letters,* Vol. 106(3), pp. 230–233.

Lleras, J. S., and Miller, N. H.(2010). The entry incentives of complementary producers: A simple model with implications for antitrust policy. *Economics Letters,* Vol. 110(2), pp. 147-150.

Maity, K., and Maiti, M.(2009). Optimal inventory policies for deteriorating complementary and substitute items. *International Journal of Systems Science,* Vol. 40(3), pp. 267–276.

Mohsenzadeh Ledari, A., Pasandideh, S. H. R., and Nouri Koupaei, M.(2018). A new newsvendor policy model for dual-sourcing supply chains by considering disruption risk and special order. *Journal of Intelligent Manufacturing,* Vol. 29(1), pp. 237-244.

Mukhopadhyay, K., Yue, S. , and Zhu, X.(2011). A stackelberg model of pricing of complementary goods under information asymmetry. *International Journal of Production Economics,* Vol. 134(2), pp. 424-433.

Qi, L.(2013). A continuous-review inventory model with random disruptions at the primary supplier. *European Journal of Operational Research,* Vol. 225(1), pp. 59–74.

Schmitt, A. J., and Snyder, L. V.(2012). Infinite horizon models for inventory control under yield uncertainty and disruptions. *Computers & Operations Research,* Vol. 39(4), pp. 850–862.

Seyed Esfahani , M., Biazaran, M. , and Gharakhani, M.(2011). A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer-retailer supply chain. European *Journal of Operational Research*, Vol. 211(2), pp. 263-273.

Shavandi, H., Mahlooji, H. , and Nosratian, N. E.(2012). A constrained multi-product pricing and inventory control problem. *Applied Soft Computing,* Vol. 12(8), pp. 2454–2461.

Taleizadeh, A. A., and Noori-daryan, M.(2016). Pricing, inventory and production policies in a supplychain of pharmacological products with rework process: a game theoretic approach. *Operational Research,* Vol. 16(1), pp. 89-115.

Wei, J., Zhao, J. , and Li, Y.(2013). Pricing decisions for complementary products with firms different market powers. *European Journal of Operational Research,* Vol. 224(3), pp. 507–519.

Wei,J., and Zhao, J.(2015). Pricing and remanufacturing decisions in two competing supply chains. *International Journal of Production Research,* Vol. 53(1), pp. 258-278.

Wu, C. H. , Chen, C. W. , and Hsieh, C. C.(2012). Competitive pricing decisions in a two-echelon supply chain with horizontal and vertical competition. *International Journal of Production Economics,* Vol. 135 (1), pp. 265-274.

Xanthopoulos, A., Vlachos, D. , and Iakovou, E.(2012).Optimal newsvendor policies for dual-sourcing supply chains: A disruption risk management framework. *Computers & Operations Research,* Vol. 39(2), pp. 350–357.

Xie,W.(2017). Optimal pricing and two-dimensional warranty policies for a new product. *International Journal of Production Research,* Vol. 55(22), pp. 6857-6870.

Zhang, R., Kaku, I. , and Xiao, Y.(2012). Model and heuristic algorithm of the joint replenishment problem with complete backordering and correlated demand. *International Journal of Production Economics,* Vol. 139(1), pp. 33–41.

Zhao, J., Tang, W., Zhao, R. , and Wei, J.(2012). Pricing decisions for substitutable products with a common retailer in fuzzy environments. *European Journal of Operational Research,* Vol. 216(2), pp. 409-419.

Zhang, R.Q., Kaku, I. , and Xiao, Y.Y.(2011). Deterministic EOQ with partial backordering and correlated demand caused by cross-selling. *European Journal of Operational Research,* Vol. 210(3), pp. 573-551.

Zhao, J., and Wei, J.(2012). Pricing game of two substitutable products supply chains under centralized and decentralized decisions. *Journal of the Chinese Institute of Industrial Engineers*, Vol. 29(7), pp. 435-443.

Zhao, J., Tang, W., and Wei, J.(2012). Pricing decision for substitutable products with retail competition in a fuzzy environment. *International Journal of Production Economics*, Vol. 135(1) , pp. 144-153.

Sangaiah,A.K., Babaee Tirkolaee,E., Goli,A., and Dehnavi-Arani,S.(2020). Robust ptimization and mixed-integer linear programming model for LNG supply chain planning problem. *Soft Computing*, Vol.24, pp. 7885-7905.

Song, H., Chu,H., Yue,H., and Chen,Y.(2022). Green supply chain coordination with substitutable products under cost sharing contract. *Procedia Computer Science*, Vol. 199, pp.1112-1119.

Sun,Z., Xu,Q., & Liu, J.(2021). Pricing and replenishment decisions for seasonal and nonseasonal products in a shared supply chain. *International Journal of Production Economics*, Vol. 233(7), pp. 1-17.

Gheibi, S. and Fay,S.(2021). The impact of supply disruption risk on a retailer's pricing and procurement strategies in the presence of a substitute product. *Journal of Retailing,* Vol. 97(3), pp. 359-376.

Liu, Z., Li,M., and Zhai,X.(2022). Managing supply chain disruption threat via a strategy combining pricing and self-protection. *International Journal of Production Economics*, Vol. 247, https://doi.org/10.1016/j.cie.2021.107433.

Appendix A

Proposition2. The optimal value of the sales price in distributer level in Stackelberg mode in which the distributer is follower and manufacturers are leaders.

Proof.

The Optimal value of the sales price in distributer level in Stackelberg mode in which distributer is follower and manufacturers are leaders will be attached with the bellow equations and solving these equations.

$$\begin{cases} \dfrac{\partial \pi_d}{\partial w_1} = 0 \\ \dfrac{\partial \pi_d}{\partial w_2} = 0 \\ \dfrac{\partial \pi_d}{\partial w_3} = 0 \end{cases} \tag{36}$$

$$w_1 - \frac{1}{4}\frac{1}{(-\beta+\gamma)} w_2(2\beta - \gamma - \beta p_2 + p_2 y_2 \beta) - \frac{1}{4}\frac{1}{(-\beta+\gamma)} w_3(2\beta - \gamma - \beta p_3 + p_3 y_3 \beta)$$

$$= \frac{1}{4}\frac{1}{(-\beta+\gamma)}(-2\beta T_1 + 2\gamma T_1 - \alpha_1 - \beta T_2 - \beta T_2 + \beta p_2 T_2 - p_2 y_2 \beta T_2 - \beta T_3 + \beta p_3 T_3 - p_3 y_3 \beta T_3) \tag{37}$$

$$w_2 + \frac{1}{2}\frac{1}{\beta(1 - p_2 + p_2 y_2)} w_1(2\beta - \gamma - p_2\beta + p_2 y_2 \beta) + \frac{1}{2}\frac{1}{\beta(1 - p_2 + p_2 y_2)} w_3(-p_3 y_3 \gamma - 2\gamma + p_2\gamma - p_2 y_2 \gamma + \gamma p_3) =$$

$$\frac{-1}{2}\frac{1}{\beta(1 - p_2 + p_2 y_2)}(-\beta T_1 + \gamma T_1 - \beta T_2 + \beta p_2 T_2 + p_3 y_3 \gamma T_3 - \alpha_2 + p_2\alpha_2 - p_2 y_2 \beta T_2 - p_2 y_2 \alpha_2 + \gamma T_3 - \gamma p_3 T_3) \tag{38}$$

$$w_3 + \frac{1}{2}\frac{1}{\beta(1 - p_3 + p_3 y_3)} w_1(2\beta - \gamma - p_3\beta + p_3 y_3 \beta) + \frac{1}{2}\frac{1}{\beta(1 - p_3 + p_3 y_3)} w_2(-2\gamma + \gamma p_2 - p_2 y_2 \gamma + p_3\gamma - p_3 y_3 \gamma) =$$

$$-\frac{1}{2}\frac{1}{\beta(1 - p_3 + p_3 y_3)}(-\beta T_1 + \gamma T_1 + \gamma T_2 - \gamma p_2 T_2 + p_2 y_2 \gamma T_2 - \beta T_3 + \beta p_3 T_3 - \alpha_3 + p_3\alpha_3 - p_3 y_3 \beta T_3 - p_3 y_3 \alpha_3) \tag{39}$$

By solving the above equations, the optimal values of distributer selling price will be attached. Due to the enlargement of the equations, we must transform them into smalleer ones as follows:

$$\begin{cases} w_1 + a_{11}w_2 + a_{12}w_3 = b_1 \\ w_2 + a_{21}w_1 + a_{22}w_3 = b_2 \\ w_3 + a_{31}w_1 + a_{32}w_2 = b_3 \end{cases} \qquad (40)$$

$$w_1 = -(b_1 - a_{11}b_2 - a_{12}b_3 + a_{11}a_{22}b_3 + a_{12}a_{32}b_2 - a_{22}a_{32}b_1) / (a_{11}a_{21} + a_{12}a_{31} + a_{22}a_{32} - a_{11}a_{22}a_{31} - a_{12}a_{21}a_{32} - 1) \qquad (41)$$

$$w_2 = -(b_2 - a_{21}b_1 - a_{22}b_3 + a_{12}a_{21}b_3 - a_{12}a_{31}b_2 + a_{22}a_{31}b_1) / (a_{11}a_{21} + a_{12}a_{31} + a_{22}a_{32} - a_{11}a_{22}a_{31} - a_{12}a_{21}a_{32} - 1) \qquad (42)$$

$$w_3 = -(b_3 - a_{31}b_1 - a_{32}b_2 - a_{11}a_{21}b_3 + a_{11}a_{31}b_2 + a_{21}a_{32}b_1) / (a_{11}a_{21} + a_{12}a_{31} + a_{22}a_{32} - a_{11}a_{22}a_{31} - a_{12}a_{21}a_{32} - 1) \qquad (43)$$

Appendix B

Proposition 3. The manufacturers objective function are concave and **t**he Optimal value of each product's sale price for the manufacturer in stackelberg mode.

Proof.

Considering the competition between the distributer and the manufacturers of that product in each chain is Stackelberg, the optimal solution is obtained by placing prices.

For the first manufacturer, the second order derivative can be calculated as follow:

$$\frac{\partial \pi_{M_1}^2}{\partial^2 p_1} = -2(\beta - \gamma)(\frac{1}{2}\frac{-2\beta + 2\gamma}{-\beta + \gamma} - \frac{1}{2}\frac{-\beta + \gamma}{\beta(1 - p_2 + p_2 y_2)} - \frac{1}{2}\frac{-\beta + \gamma}{\beta(1 - p_3 + p_3 y_3)}) \le 0 \qquad (44)$$

So first manufacturer's profit is <u>**concave.**</u>

For the second manufacturer, the second order derivative can be calculated as follow:

$$\frac{\partial^2 \pi_{M2}}{\partial p_2^2} = 2(-\beta(\frac{1}{4}\frac{-\beta + \beta p_2 - p_2 y_2 \beta}{-\beta + \chi} - \frac{1}{2}\frac{-\beta + \beta p_2 - p_2 y_2 \beta}{\beta(1 - p_2 + p_2 y_2)}) - \frac{1}{2}\frac{\gamma(\gamma - \gamma p_2 + p_2 y_2 \gamma)}{\beta(1 - p_3 + p_3 y_3)})(1 - p_2 + p_2 y_2) \le 0 \qquad (45)$$

So second manufacturer's profit is <u>**concave.**</u>

For the third manufacturer, the second order derivative can be calculated as follow:

$$\frac{\partial^2 \pi_{M3}}{\partial p_3^2} = 2(-\beta(\frac{1}{4}\frac{-\beta + \beta p_3 - p_3 y_3 \beta}{-\beta + \gamma} - \frac{1}{2}\frac{-\beta + \beta p_3 - p_3 y_3 \beta}{\beta(1 - p_3 + p_3 y_3)}) - \frac{1}{2}\frac{\gamma(\gamma - \gamma p_3 + p_3 y_3 \gamma)}{\beta(1 - p_2 + p_2 y_2)})(1 - p_3 + p_3 y_3) \le 0 \qquad (46)$$

So third manufacturer's profit is <u>**concave.**</u>

The Optimal value of each product's sale price for the manufacturer by solving the following equations will be attached.

$$\begin{cases} \dfrac{\partial \pi_{M1}}{\partial T_1} = 0 \\ \dfrac{\partial \pi_{M2}}{\partial T_2} = 0 \\ \dfrac{\partial \pi_{M3}}{\partial T_3} = 0 \end{cases} \qquad (47)$$

The above equations have been solved with MATLAB software, in which the dimensions of equations are very large because of this reason we should transform them into the following equations:

$$\begin{cases} b_{11}T_1 + b_{12}T_2 + b_{13}T_3 = z_1 \\ b_{21}T_1 + b_{22}T_2 + b_{23}T_3 = z_2 \\ b_{31}T_1 + b_{32}T_2 + b_{33}T_3 = z_3 \end{cases} \qquad (48)$$

In which by solving the above equations, the optimal value are as bellow:

$$T_1 = (z_3(b_{12}b_{23} - b_{13}b_{22})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - (z_2(b_{12}b_{33} - b_{13}b_{32})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) + (z_1(b_{22}b_{33} - b_{23}b_{32})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) \tag{49}$$

$$T_2 = (z_2(b_{11}b_{33} - b_{13}b_{31})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - (z_3(b_{11}b_{23} - b_{13}b_{21})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - (z_1(b_{21}b_{33} - b_{23}b_{31})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) \tag{50}$$

$$T_3 = (z_3(b_{11}b_{22} - b_{12}b_{21})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) - (z_2(b_{11}b_{32} - b_{12}b_{31})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) + (z_1(b_{21}b_{32} - b_{22}b_{31})) / (b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31}) \tag{51}$$

Appendix C.

Proposition4. The supply chain Objective function in cooperation model is **concave**.

Proof.

To prove the concavity of distributor profits of Hessian matrix we have:

$$H_{SC} = \begin{bmatrix} \frac{\partial^2 \pi_d}{\partial w_1^2} & \frac{\partial^2 \pi_d}{\partial w_1 \partial w_2} & \frac{\partial^2 \pi_d}{\partial w_1 \partial w_3} \\ \frac{\partial^2 \pi_d}{\partial w_2 \partial w_1} & \frac{\partial^2 \pi_d}{\partial w_2^2} & \frac{\partial^2 \pi_d}{\partial w_2 \partial w_3} \\ \frac{\partial^2 \pi_d}{\partial w_3 \partial w_1} & \frac{\partial^2 \pi_d}{\partial w_3 \partial w_2} & \frac{\partial^2 \pi_d}{\partial w_3^2} \end{bmatrix} \tag{52}$$

$$H_{SC} = \begin{bmatrix} -4\beta + 4\gamma & -\beta + \gamma - (1-p_2)\beta - p_2 y_2 \beta & -\beta + \gamma - (1-p_3)\beta - p_3 y_3 \beta \\ -\beta + \gamma - (1-p_2)\beta - p_2 y_2 \beta & -2(1-p_2)\beta - 2p_2 y_2 \beta & (1-p_2)\gamma + p_2 y_2 \gamma + (1-p_3)\gamma + p_3 y_3 \gamma \\ -\beta + \gamma - (1-p_3)\beta - p_3 y_3 \beta & (1-p_2)\gamma + p_2 y_2 \gamma + (1-p_3)\gamma + p_3 y_3 \gamma & -2(1-p_3)\beta - 2p_3 y_3 \beta \end{bmatrix} \tag{53}$$

Similar to Stackelberg mode, we can show that the above Hessian matrix is **concave**

Appendix D.

Proposition5. Selling price in the supply chain (w_1, w_2, w_3).

Proof.

The optimal value of the sale price in cooperation mode will be attached with the equations bellow and solving these equations.

$$\begin{cases} \frac{\partial \pi_{sc}}{w_1} = 0 \\ \frac{\partial \pi_{sc}}{w_2} = 0 \\ \frac{\partial \pi_{sc}}{w_3} = 0 \end{cases} \tag{54}$$

$$w_1 - \frac{1}{4(-\beta+\gamma)} w_2 (2\beta - \gamma - \beta p_2 + p_2 y_2 \beta) - \frac{1}{4(-\beta+\gamma)} w_3 (2\beta - \gamma - \beta p_3 + p_3 y_3 \beta) = \frac{1}{4(-\beta+\gamma)} (-2\beta c_1 + 2\gamma c_1 - \alpha_1 - \beta c_2 + \beta p_2 c_2 - p_2 y_2 \beta c_2 - \beta c_3 + \beta p_3 c_3 - p_3 y_3 \beta c_3) \tag{55}$$

$$w_2 + \frac{1}{2\beta(1-p_2+p_2y_2)} w_1(2\beta-\gamma-p_2\beta+p_2y_2\beta) + \frac{1}{2\beta(1-p_2+p_2y_2)} w_3(-p_3y_3\gamma-2\gamma+p_2\gamma-p_2y_2\gamma+\gamma p_3) =$$

$$\frac{-1}{2\beta(1-p_2+p_2y_2)}(-\beta c_1+\gamma c_1-\beta c_2+\beta p_2c_2+p_3y_3\gamma c_3-\alpha_2+p_2\alpha_2-p_2y_2\beta c_2-p_2y_2\alpha_2+\gamma c_3-\gamma p_3c_3) \quad (56)$$

$$w_3 + \frac{1}{2\beta(1-p_3+p_3y_3)} w_1(2\beta-\gamma-p_3\beta+p_3y_3\beta) + \frac{1}{2\beta(1-p_3+p_3y_3)} w_2(-2\gamma+\gamma p_2-p_2y_2\gamma+p_3\gamma-p_3y_3\gamma) =$$

$$\frac{-1}{2\beta(1-p_3+p_3y_3)}(-\beta c_1+\gamma c_1+\gamma c_2-\gamma p_2c_2+p_2y_2\gamma c_2-\beta c_3+\beta p_3c_3-\alpha_3+p_3\alpha_3-p_3y_3\beta c_3-p_3y_3\alpha_3) \quad (57)$$

By solving the apparatus, three non-trivial equations w_1, w_2, w_3 are obtained. Similarly, by changing the following variable to obtain the w_1, w_2, w_3 values, the state is obtained before the problem dimensions are enlarged.

$$\begin{cases} w_1 + c_{11}w_2 + c_{12}w_3 = d_1 \\ w_2 + c_{21}w_1 + c_{22}w_3 = d_2 \\ w_3 + c_{31}w_1 + c_{32}w_2 = d_3 \end{cases} \quad (58)$$

$$w_1 = -(d_1 - c_{11}d_2 - c_{12}d_3 + c_{11}c_{22}d_3 + c_{12}c_{32}d_2 - c_{22}c_{32}d_1) / (c_{11}c_{21} + c_{12}c_{31} + c_{22}c_{32} - c_{11}c_{22}c_{31} - c_{12}c_{21}c_{32} - 1) \quad (59)$$

$$w_2 = -(d_2 - c_{21}d_1 - c_{22}d_3 + c_{12}c_{21}d_3 - c_{12}c_{31}d_2 + c_{22}c_{31}d_1) / (c_{11}c_{21} + c_{12}c_{31} + c_{22}c_{32} - c_{11}c_{22}c_{31} - c_{12}c_{21}c_{32} - 1) \quad (60)$$

$$w_3 = -(d_3 - c_{31}d_1 - c_{32}d_2 - c_{11}c_{21}d_3 + c_{11}c_{31}d_2 + c_{21}c_{32}d_1) / (c_{11}c_{21} + c_{12}c_{31} + c_{22}c_{32} - c_{11}c_{22}c_{31} - c_{12}c_{21}c_{32} - 1) \quad (61)$$

Appendix E.

$$A_1 = \beta T_1 + \alpha_2 - \beta w_1 + \gamma w_3 - y_2\beta T_2 - y_2\alpha + y_2\beta w_1 - y_2w_3\gamma \quad (62)$$

$$A_2 = (2\beta w_1 - \beta T_1 - \gamma w_1 + \gamma T_1 - p_3y_3w_3 + p_3y_3\gamma T_3 - \alpha_2 - 2\gamma w_3 + \gamma T_3 + \gamma p_3w_3 - \gamma p_3T_3)(-1+y_2) \quad (63)$$

$$A_3 = p_2(\beta T_2 + \alpha_2 - \beta w_1 + w_3\gamma - y_2\beta T_2 - y_2\alpha_2 + y_2\beta w_1 - y_2w_3\gamma) \quad (64)$$

$$B_1 = -\beta p_2T_2 - p_2\alpha_2 + p_2\beta w_1 - p_2w_3\gamma \quad (65)$$

$$B_2 = 2\beta w_1 - \beta T_1 - \gamma w_1 + \gamma T_1 - \beta T_2 + \beta p_2T_2 - p_3y_3\gamma w_3 + p_3y_3\gamma T_3 - \alpha_2 - 2\gamma w_3 + p_2\alpha_2$$
$$-p_2\beta w_1 + p_2w_3\gamma + \gamma T_3 + \gamma p_3w_3 - \gamma p_3T_3)p_2 \quad (66)$$

$$B_3 = (-p_3\beta T_2 - p_2\alpha_2 + p_2\beta w_1 - p_2w_3\gamma)p_2 \quad (67)$$

$$C_1 = \beta T_1 + \alpha_3 - \beta w_1 + \gamma w_2 - y_3\beta T_3 - y_3\alpha + y_3\beta w_1 - y_3w_2\gamma \quad (68)$$

$$C_2 = (2\beta w_1 - \beta T_1 - \gamma w_1 + \gamma T_1 - p_2y_2w_2 + p_2y_2\gamma T_2 - \alpha_3 - 2\gamma w_2 + \gamma T_2 + \gamma p_2w_2 - \gamma p_2T_2)(-1+y_3) \quad (69)$$

$$C_3 = p_3(\beta T_3 + \alpha_3 - \beta w_1 + w_2\gamma - y_3\beta T_3 - y_3\alpha_3 + y_3\beta w_1 - y_3w_2\gamma) \quad (70)$$

$$E_1 = -\beta p_3T_3 - p_3\alpha_3 + p_3\beta w_1 - p_3w_2\gamma \quad (71)$$

$$E_2 = 2\beta w_1 - \beta T_1 - \gamma w_1 + \gamma T_1 - \beta T_3 + \beta p_3T_3 -$$
$$p_2y_2\gamma w_2 + p_2y_2\gamma T_2 - \alpha_3 - 2\gamma w_2 + p_3\alpha_3 - p_3\beta w_1 + p_3w_2\gamma + \gamma T_2 + \gamma p_2w_2 - \gamma p_2T_2)p_3 \quad (72)$$

$$E_3 = (-p_3\beta T_3 - p_3\alpha_3 + p_3\beta w_1 - p_3w_2\gamma)p_3 \quad (73)$$