International Journal of Supply and Operations Management

IJSOM

May 2022, Volume 9, Issue 2, pp. 235-250 ISSN-Print: 2383-1359 ISSN-Online: 2383-2525 www.ijsom.com

Optimization of the Stochastic Home Health Care Routing and Scheduling Problem with Multiple Hard Time Windows

Mohammed Bazirha^{a,*}, Abdeslam Kadrani^a, Rachid Benmansour^a

^a SI2M laboratory, National Institute of Statistics and Applied Economics, Rabat, Morocco

Abstract

Home health care (HHC) aims to assist patients at home and to help them to live with greater independence, avoiding hospitalization or admission to care institutions. The patients should be visited within their availability periods. Unfortunately, the uncertainties related to the traveling and caring times would sometimes violate these time windows constraints, which will qualify the service as poor or even risky. This work addresses the home health care routing and scheduling problem (HHCRSP) with multiple hard/fixed time windows as well as stochastic travel and service times. A two-stage stochastic programming model recourse (SPR model) is proposed to deal with the uncertainty. The recourse is to skip patients if their availability periods will be violated. The objective is to minimize caregivers' traveling cost and the average number of unvisited patients. Monte Carlo simulation is embedded into the genetic algorithm (GA) to solve the SPR model. The results highlight the efficiency of the GA, show the complexity of the SPR model, and indicate the advantage of using multiple time windows.

Keywords: Genetic Algorithm; Simulation; Stochastic Programming Recourse Model; Multiple Time Windows.

1. **Introduction**

Home health care (HHC) allows patients to live safely, independently and in a personal environment in the case of aging, disabling disease or injury. It aims to visit patients in their homes to perform care services, it includes: 1) medical services such as nursing, physical and speech therapy; 2) help seniors with services of daily living, such as dressing, bathing, and eating; 3) assist with cleaning, cooking, and other housekeeping. According to the report on aging and health (WHO (2015)), on average, people would live to age 77; 15 of those years would be lived with disability. The percentage of seniors is increasing in European countries and is expected to increase further in the next years (Tarricone and Tsouros (2008)). HHC services will allow patients to live safely and independently in their homes and avoid moving to hospital to receive care activities. Also, it expected to decrease admissions in hospitals (Cissé et al. (2017)). HHC companies must assign qualified caregivers to patients and define their routes to provide requested services. These companies must consider both patients' preferences such as availability periods to increase their satisfaction and optimize one or more criteria such as the transportation cost. This problem is recognized as HHCRSP, which combine two NP-hard problems known in the literature as the vehicle routing problem (VRP) with time windows (Braysy and Gendreau (2005)) and the nurse rostering problem (Burke et al. (2004)).

Most works only adopt deterministic methods and/or models to address the HHCRSP. The constraints and objectives considered in the literature vary from one study to another: multi-modal transportation (Hiermann et al. (2015)), timedependent travel times (Rest and Hirsch (2016)), temporal dependencies between services (Mankowska et al. (2014); Rasmussen et al. (2012); Redjem and Marcon (2016)), multiple availability periods of patients (Bazirha et al. (2020a)), lunch break requirements (Liu et al. (2017)) and multi-objective optimization (Decerle et al. (2019); Braekers et al. (2016); Duque et al. (2015); Fathollahi-Fard et al. (2020)). However, these models are generally less robust, the predefined schedule should be revised for any change arising in practical situations. Otherwise, services may be provided with tardiness for patients who have not yet been visited, therefore the service will be qualified poor or risky.

Corresponding author email address: m.bazirha@gmail.com DOI: 10.22034/ijsom.2021.109079.2170

Bazirha, Kadrani, Benmansour

Travel and service times are critical elements in the planning and are not always fixed as estimated due to practical reasons (Shi et al. (2019)). Travel time duration between patients could be affected by driving skills, road, and weather conditions. For the service time is depending for example on diagnosing time and parking situations (Shi et al. (2019)).

Most of previous efforts have been focused on studying the VRP with stochastic parameters such as demands, travel and service times (Laporte et al. (1992); Li et al. (2010); Tas et al. (2014a, b); Luo et al. (2016); Errico et al. (2016); Marinaki and Marinakis (2016); Mendoza et al. (2016)). But there are only a few works that have dealt with the HHCRSP with stochastic parameters (e.g., Shi et al. (2018, 2019); Cappanera et al. (2018, 2021); Shahnejat et al. (2021); Yuan et al. (2015); Bazirha et al. (2020b, 2021)) (see Table 1). These studies only consider a single availability period per patient and no study, as far as we know, has used multiple time windows for the stochastic HHCRSP. This work considers multiple time windows for patients that aim to minimize the number of unvisited patients as well as to increase the chance to provide requested services within patients' time windows since are supposed hard/fixed with stochastic parameters (travel and service times). In addition, the proposed model generalizes time windows constraints. It allows to use multiple time windows for some patients and a single availability period for others. The problem with a single availability period for some patients could be considered as a problem with multiple windows by keeping the valid period and setting the others to [*0, 0*].

To deal with the uncertainty, several models have been proposed in the literature. The robust optimization (Ben-Tal et al. (2009)) constructs a feasible solution for any realization of the uncertainty in a given set (Bertsimas et al. (2011)), this uncertainty is not stochastic, but rather deterministic and set-based (Bertsimas et al. (2011)). The chance constrained model (Charnes and Cooper (1959)) seeks a solution for which the failure probability is less than some given threshold and corrective actions are not considered in the failure case (Gendreau et al. (1996)). The SPR model (Bernard (1955)) seeks a solution that minimizes the cost of the first stage solution plus the expected net cost of recourse (second stage) (Gendreau et al. (1996)). The objective function of the SPR model is more meaningful than the chance constrained model (Gendreau et al. (1996)). This work adopts the SPR model to cope with the uncertainty of travel and service times, the recourse is to skip providing a service operation if it will be carried out with a tardiness.

We use CPLEX solver to solve the deterministic model as well as the GA based heuristic since experiments in the literature show that it has a good ability for global searching (Shi et al. (2018)). We embed Monte Carlo simulation into the GA to solve the SPR model since GA parameters are independent of the parameters of the problem. However, heuristics based on local search strategies, such as Tabu Search (Glover (1986)), Simulated Annealing (Kirkpatrick et al. (1983)) and Variable Search Neighborhood (Mladenovic and Hansen (1997)), are not suitable to be combined with the simulation since its parameters depend on the problem size (Bazirha et al. (2020b)). For each new solution, the simulation is performed to estimate the recourse. The convergence to the expected real value may take more time. Indeed, the more the number of convergence iterations increases, the more the estimated value approaches to the expected real value (Law of large numbers).

In previous studies, on the one hand, exact approaches such as branch-and-price algorithm have been proposed to solve stochastic models, where the service time (Errico et al. (2016); Yuan et al. (2015)) or the travel time (Tas et al. (2014b)) is supposed stochastic and the expected value is calculated by a mathematical formula. On the other hand, in (Shi et al. (2018); Bazirha et al. (2020b)) both travel and service times are supposed stochastic, and the simulation has been used to estimate the expected value. As explained above, the simulation takes time to find a good estimation. In addition, although exact methods give the optimal solution, their computation time increases monotonically with the size of the problem. To solve our problem within a reasonable computational time, a heuristic with the simulation is used rather than exact approaches.

The remainder of the paper is structured as follows: Section 2 describes the problem statement. The SPR model for the HHCRSP and Monte Carlo simulation are presented in section 3. The heuristic method is presented in section 4. Numerical experiments are conducted in section 5. Section 6 concludes the paper.

2. Problem statement

The HHCRSP with multiple time windows and stochastic travel and service times is defined as follows: given a HHC company that provides a set of services $S = \{1, 2, ..., q\}$ to a set of patients $N = \{1, 2, ..., n\}$, which will be visited on a day. The goal is to find a valid daily planning in which available caregivers $K = \{1, 2, ..., c\}$, will be efficiently assigned to patients considering the uncertainty in terms of traveling and caring times that may occur.

Caregivers start from the HHC company center and must return to that center after visiting all assigned patients. They use the same mode of transportation to travel between patients (i.e., cars of the HHC company). They are paid for their regular working time regardless of the amount of care they do. To ensure fairness among them, a maximum number of patients not to exceed is fixed (*max*_*v*).

Caregivers' qualification is indicated in a matrix of binary parameters ∆*ks*. The binary parameter ∆*ks* is set to 1 if caregiver k can perform service *s*, and 0 otherwise. Requested services are indicated in in a matrix of binary parameters δ_{is} , which is set to 1 if patient *i* requires service *s*, and 0 otherwise. Multiple time windows are adopted for each patient $[a_{ik}, b_{il}]$, where a_{il} and b_{il} are, respectively, the earliest and latest possible service times of the time window $l \in L = \{1, 2, ..., p\}$, where *p* is the number of patients' time windows. The decision maker could select any availability period *l* to schedule the requested visit for each patient *i*.

It is more likely that providing some services will not be compatible with patients' preferred availability periods since travel \tilde{T}_{ij} and service \tilde{t}_{is} times are supposed stochastic and patients' time windows are assumed hard/fixed. Therefore, the recourse is defined as skipping a patient when carrying out the service will not be compatible with his availability periods. The goal is to establish a daily planning that minimizes the transportation cost (first stage) as well as the average number of unvisited patients (second stage) with respect to patients' time windows, skills requirements, and the maximum of visits not to exceed by each caregiver.

3. Mathematical formulation

The problem is formulated as a two-stage stochastic programming model recourse with multiple hard/fixed time windows. Caregivers' routes and assignment are defined in the first stage. The second stage aims to introduce the recourse if they arrive lately to patients, which is expressed as skipping patients' visits since their time windows are assumed hard/fixed. The recourse is to minimize the average number of unvisited patients. The notation of sets, decision variables and parameters used in the model are defined as follows:

3.1. Sets

- $N = \{1, 2, ..., n\}$: set of patients;
- N^0 and N^{n+1} : the extended sets of *N* that include nodes *0* and *n+1*, which represent the HHC center;
- $K = \{1, 2, ..., c\}$: set of caregivers;
- $S = \{1, 2, ..., q\}$: set of services and skills;

 $L = \{1, 2, ..., p\}$: set of patients' availability periods. Each patient *i* has *p* time windows: Actually, there are L_i valid periods, the others (i.e., $p - L_i$) are null.

3.2. Deterministic parameters

- *M*: a large positive constant;
- *Max_v*: maximum of patients that a caregiver could visit;
- $-$ [*a_{il}*, *b_{il}*]: the *l*th time window of the patient *i*;
- $-c_{ij}$: transportation cost from patient *i* to patient *j*;
- δ ^{*is*}: equals 1 if patient *i* requests service *s*;
- *∆ks*: equals 1 if caregiver *k* is skilled to perform service *s*.

3.3. Stochastic parameters

- ̃ : travel times between patients' locations*;*
- ̃ *:* processing time of service *s* at patient *i*;
- \tilde{E} (.): the average number of uninvited patients, which expresses the recourse of the second stage.

3.4. Decision variables

- x_{ijk} : equals 1 if caregiver *k* visits patients *j* after patient *i*, 0 otherwise;
- *yiks*: equals 1 if caregiver *k* visits patient *i* to perform service *s*, 0 otherwise;
- ζ *il*: equals 1 if the *l*th time window is selected for patient *i*, 0 otherwise;
- \check{S}_{ik} : caregiver *k* starting time at patient *i*;

3.5. Parameters for Recourse model

- v_i : equals 1 if the service operation requested by patient *i* will be skipped (will not be provided), 0 otherwise;
- α: penalty cost for each unvisited patient.

3.6. Mathematical model

The SPR model proposed to address this problem is adapted from our previous work (Bazirha et al. (2020b)) by using another recourse model (Errico et al. (2016)) (skip patients) instead of using a penalty cost for violating time windows (Shi et al. (2018)). In addition, we add other constraints such as multiple time windows and maximum number of patients to visit per caregiver. The model is defined as follows:

$z_{il} \in \{0,1\}$ $\forall i \in N, k \in K$ (19)

The objective function is defined as minimizing the transportation cost and the average number of unvisited patients caused by skipping them if their time windows will be violated. Constraints (1) and (2) impose that each patient is visited exactly by one caregiver to perform his requested service. Constraints (3) and (4) guarantee that the HHC center is the start and the end of caregivers' tours. Constraints (5) impose route continuity for the patients assigned to a caregiver *k*. In doing so, tours will be constructed rather than open paths. Constraints (6) and (7) determine either service operation *s* requested by patient *i* will be provided, or it will be skipped. Indeed, if $(v_i = 0, i.e.,$ Constraints (6) must be verified), the service operation will be provided for patient *i* and the start time for patient *j* must respect completion time of providing the requested service operation for patient *i*. Otherwise $(v_i = 1, i.e.,$ Constraints (7) must be verified), the service operation for patient *i* will be skipped. These constraints ensure that the starting times of services are strictly increasing in each caregiver's route, which remove sub-tours (Mankowska et al. (2014)). Constraints (8) define the variable *yiks*, caregiver *k* provides a service to patient *i* imply that caregiver *k* goes to another location (a patient or the HHC center) after visiting patient *i*. Constraints (9) guarantee that each assigned caregiver is skilled to perform a requested service. Constraints (10) ensure that each caregiver does not exceed the maximum number of visits allowed. Constraints (11) and (12) require that time windows will not be violated. Constraints (13) ensure for each patient that only one time window is selected from her/his availability periods to provide the requested service. Constraints (14)-(19) set the domains of decision variables.

3.7. Expected recourse estimation procedure

Algorithm 1: The expected recourse estimation procedure

```
1 Initialization:
           - Set sum_v = 0;
 \overline{2}- Set T_i = 0;
 \overline{\mathbf{3}}- Set iter = 0 :
 \overline{4}while (condition 1 or condition 2 is not reached) do\overline{\mathbf{5}}for k \leftarrow 1 to K do
 6
                 for j \in V_k do
 \overline{7}Generate randomly \widetilde{T}_{ij};
  s
                       Compute \widetilde{A}_{jk};<br>Generate randomly \widetilde{t}_{js};
  q
10
                       T_i \leftarrow +\infty;11for l \leftarrow 1 to L do
12Compute the tardiness T_l using the period l;
13
                             if T_i < T_j then<br>Set T_j \leftarrow T_l;<br>Set p_j \leftarrow l;
1415
 16
                             end
1718
                       end
                       Set v_j \leftarrow 0;
19if T_j > 0 then
20
                             Set v_i \leftarrow 1;
21else
22Compute \widetilde{S}_{jk} using the period p_j;
2<sup>3</sup>end
2425
                       sum_v \leftarrow sum_v + v_j;
26
                 end
27end
          iter \longleftarrow iter + 1;
28
    end
29
    Set \mathbb{E}(.) \longleftarrow \frac{\alpha \times sum_v}{iter};
30
```
The recourse model in stochastic programming depends on the nature of the problem and its constraints. Constraints containing stochastic parameters are more likely to be violated, therefore a recourse must be used to deal with the uncertainty. In (Shi et al. (2018)), the authors defined the recourse as a penalty cost for a tardiness of a service operation and a remuneration for caregivers' extra working time. This recourse requires to be used with soft/flexible time windows.

In (Errico et al. (2016)), the authors defined the recourse when a route becomes infeasible as: skipping the service at the current customer and skipping the visit at the next customer. This recourse is used with hard/fixed time windows. Since we suppose that patients' time windows must be respected, we define the recourse as skipping providing a service operation for a patient when his availability periods could not be respected. The algorithm 1 is used to estimate the excepted value of recourse. sum_y is the total number of skipped visits. T_i is the minimal tardiness of providing the service operation to patient *j* considering all his availability periods. p_i contains the selected period *l* for the patient *j*. V_k contains patients assigned to the caregiver *k*.

The simulation is running until either *condition 1* or *condition 2* is met. Condition 1 is fixed as a maximum number of iterations, denoted by *MaxIterMCS*. Condition 2 expresses the gap between the estimated values at iterations *t* − 1 and *t*: $gap = \frac{E()_{t-1}-E()_{t}}{E()}$. This formula must exactly hold at a maximum number of iterations, denoted by *MaxIterGap*, and it $E(.)_{t-1}$ computes the gap between $E(.)_{t-1}$ and $E(.)_t$ with an error ε . When the estimated value converges to the real expected value, values of $E(.)_{t-1}$ and $E(.)_t$ will be very close to each other, and the *gap* tends to zero. Reducing to 0 and increasing *MaxIterGap* will ensure that the estimated value tends towards the real expected value. However, the computational time increases significantly since the simulation is carried out for each new generated solution. Condition 1, i.e., *MaxIterMCS*, is used to avoid falling into an infinite loop that could be caused by the condition 2, especially when ε tends towards 0 and *MaxIterGap* tends towards a large number. The condition used in (Shi et al. (2018)), is a particular case of the proposed conditions ($\varepsilon = 0$, *MaxIterGap* =+∞ and *maxIterMCS* = 100).

4. Genetic Algorithm

The Genetic Algorithm (Holland (1992)) is a population-based heuristic, which involves a simulation of Darwinian "survival of the fittest". Parents compete against each other, and the fittest ones will be selected to passe their characteristics to the produced offspring through crossover and mutation operators. This process is applied to an initial population and keeps on iterating until a generation with the highest fitness will be kept.

The first step is how to represent solutions to apply crossover and mutation operations, then the decoding method to compute the fitness of individuals. A solution is represented by two chromosomes where their sizes equal to the number of patients. The first chromosome (patients' chromosome) contains patients and their requested services (included in parenthesis), and the second chromosome (caregivers' chromosome) contains assigned caregivers.

Example: Table 2 shows an example of a solution encoding that involve 2 caregivers, 3 types of services and 6 patients. Caregiver 1 is assigned to patients 4, 5, and 2 to perform respectively services 2,1 and 2 while caregiver 2 is assigned to patients 6, 1 and 3 to carry out respectively services 1,3 and 3.

To compute the fitness of an offspring, for each subset of patients assigned to a caregiver, arrival and starting times will be iteratively calculated in the same order as they appear at patients' chromosome. The earliest availability period *l* ∈ *L* that minimizes the tardiness of providing the requested service operation *s* will be chosen for each patient.

4.1. Crossover operator

The crossover operation is the main genetic operator in genetic algorithm used to pass parents' genes to their children. Several crossover operations have been proposed in the literature. In this study, we use the 2-point crossover operator to reproduce an offspring from two parents. It will be independently applied for each chromosome:

- Generate two random crossover points p_1 and p_2 in the parent;
- Copy the segment between points p_1 and p_2 from the first parent to the first offspring;
- Copy the segment before p_1 and the segment after p_2 from the second parent to the first offspring;
- Repeat for the second offspring with the parent's role reversed.

Steps 1 and 2 are similar for both chromosomes while the third step must be adapted to patients' chromosome. Indeed, the repetition of caregivers does not affect the infeasibility of a solution when exchanging genes between parents since each caregiver appears many times in a solution (see figures 1 and 3). In contrast, each patient is allowed to appear in a solution only once, so step 3 will be adapted to sort genes of the segment before p_1 and the segment after p_2 in the same order as they appear at each opposite parent. This adaptation avoids deleting or duplicating a patient in a solution (see figures 1 and 2).

4.2. Mutation operator

The mutation is performed at random with a small probability to increase the diversity of solutions, to avoid local optimums and to diversify the search directions. In this work, two mutation operations are used. The first one (patients' mutation) is to swap the positions of two patients, which avoid deleting or duplicating a patient. The second one (caregivers' mutation) is to switch an assigned caregiver at random. Suppose that the conditions of applying mutation operators are verified. The randomly generated positions are 3 and 6 for patients and 1 for caregivers. Table 3 shows the mutated solution from the solution in Table 2.

4.3. Fitness and selection

In order to apply crossover and mutation operators, two parents must be selected from the population. Many selection methods have been proposed in the literature to choose the fittest individuals for reproduction. The most commonly used selection methods include roulette wheel selection, rank selection, and tournament selection. The first one suffers from problem of premature convergence due to the possible presence of a dominant individual that always wins the competition and is selected as a parent. The second suffers from the slower convergence and the sorting must be done to each chromosome to assign ranks, which increase the computational time. We use the tournament selection to choose the individuals for reproduction. *k* individuals are selected from a large population of size *Psize* to compete against each other. The fittest one is selected to participate in the crossover operation.

For each solution, three components are computed: caregivers' transportation cost (see equation 22); number of patients visited after exceeding the maximum number to visit per caregiver (see equation 20); and the expected value for the SPR model (see equation 21). For the deterministic model, F_2 is equivalent to the number of unvisited patients. Solutions are first compared according to the value of F_1 , the individual with small value will be selected. If constraints 10 are satisfied $(F_1 = 0)$, solutions are compared according to the value of F_2 . For the deterministic model, this value must converge to 0, while it could be greater than 0 for the SPR model. If solutions have the same values of F_1 and F_2 , the solution with small transportation cost will be selected. We use this lexicographic order to ensure convergence to feasible solutions $(F_1 = 0)$ and to avoid using aggregation techniques for F_2 and F_3 since fixing weights is confusing and units are not the same. In addition, for any skipping a patient without providing the requested service operation, a wasting cost will be occurred. For example, if a caregiver will visit patients p_1 , p_2 and p_3 and it happens that he will skip the patient p_2 , the wasting cost that may occur is $c_{p_1p_2} + c_{p_2p_3} - c_{p_1p_3}$ since he could be visit patient p_3 directly after patient p_1 . Therefore, it interesting to minimize first the number of uninvited patients to ensure patients' satisfaction and avoid wasting costs.

$$
F_1 = \sum_{k=1}^{c} Max \left(\sum_{i=1}^{n} \sum_{s=1}^{q} y_{iks} - Max_v, 0 \right) \tag{20}
$$

$$
F_2 = E\left[\sum_{i=1}^n v_i\right]
$$

\n
$$
F_3 = \sum_{k=1}^c \sum_{i=0}^n \sum_{j=1}^{n+1} c_{ij} x_{ijk}
$$
\n(21)

4.4. Genetic algorithm procedure

Algorithm 2 needs an initial population of the size P_{size} and to define mutation and crossover probabilities (p_m and p_c). Whenever crossover and mutation operations are performed to generate an offspring, a repair procedure is used to ensure skills requirements constraints are satisfied. It browses each patient in the solution and checks if a qualified caregiver is assigned through crossover and mutation operators, otherwise the unqualified caregiver is randomly replaced by a skilled one. After the transportation cost is computed, the simulation is carried out to calculate the average number of unvisited patients for the current offspring. This process keeps on iterating for several iterations until no improvements in the best solution found could be achieved.

Algorithm 2: GA procedure

1 Initialization:

 $\overline{2}$ - **Define** population size (P_{size}) ;

- **Define** crossover (p_c) and mutation (p_m) probabilities; $\overline{\mathbf{3}}$

 $\overline{4}$ - **Define** population (*P*) and Offspring (*Of f*);

while (the stopping condition is not reached) do $\overline{\mathbf{5}}$

for $i \leftarrow 1$ to P_{size} do $\boldsymbol{6}$ **Select** randomly P_1 and P_2 from P ; $\overline{7}$ **Generate** randomly p and q ; 8 if $p_c < p$ then $\ddot{\mathbf{Q}}$ Set $Off \leftarrow Crossover(P_1, P_2)$; 10 end 11 if $p_m < q$ then 12 Set $Off \longleftarrow \text{Mutation}(Off);$ 13 end 14 **Repair** Off_1 ; 15 Compute transportation cost; 16 **Run** the simulation; 17 end 18 Set $P \leftarrow \text{Off}$; 19 20 end

4.5. Initial population

Given a population size P_{size} , the initial population is randomly generated as follows:

- For $i = 1$ to P_{size} do:
- Generate patients' visiting order at random;
- Browse each patient in the visiting order and randomly assign a skilled caregiver;
- Compute components F_1 , F_2 and F_3 (see equations 20, 21 and 22).

5. Numerical experiments

The tests are performed on the computer with Intel i7-7600U 2.80-GHz CPU and 16 GB of RAM under windows 10. CPLEX solver version 12.8 is used to implement and test the deterministic version of the SPR model. The language C++ is used to code and test the GA based heuristic.

5.1. Test instances

The test instances were generated using reference instances from the literature. Deterministic and stochastic parameters were, respectively, randomly generated as described in (Mankowska et al. (2014)) and in (Shi et al. (2018)). The HHC center and patients' positions are randomly placed in a 100x100 unit distance area. Transportation cost *cij* and travel times *Tij* are equal to the Euclidean distance *dij* between patient' locations truncated to an integer. The number of services that the HHC center is supposed to provide is fixed to 6 ($S = \{1, ..., 6\}$). For each patient, a single service operation is supposed to be requested and is randomly selected from *S*, its duration *tis* is randomly drawn from the interval [*15, 20*]. A daily planning period of 10 hours is considered in which patients' time windows are randomly placed for instances

with a single time window. In the case of double time windows per patient, the daily planning period is divided into two periods, each one has a length of 5 hours. The first time window is randomly placed in the first 5 hours ([0, 5]) and the second one in the next 5 hours ([5, 10]). Each time window is of length 120 minutes. Caregivers' qualifications are grouped into two subsets {1,2,3} and {4,5,6}. Each caregiver is skilled to provide some services randomly selected, at most three, either in $\{1,2,3\}$ or in $\{4,5,6\}$.

For the SPR model, we randomly generate travel and service times as described in (Shi et al. (2018)). Service times duration follows a normal distribution $\tilde{t}_{is} \sim N(t_{is}, (\frac{t_{is}}{\tau}))$ $\frac{\pi}{5}$ ²) and travel times between patients also follows a normal distribution $\tilde{T}_{ij} \sim N(T_{ij}, \left(\frac{T_{ij}}{2}\right))$ $\frac{1}{3}$)²). Parameters t_{is} and T_{ij} are, respectively, the average processing time of the service *s* and the average traveling time from patient *i* to patient *j*. Parameters $\frac{t_{is}}{5}$ and $\frac{T_{ij}}{3}$ are, respectively, the standard deviation values of the service time *s* for patient *i* and the driving time from patient *i* to patient *j*.

Tuning parameters were defined as follows: $P_c = 0.4$, $P_m = 0.08$, $P_{size} = n \times 20$, GA based heuristic stops when there is no improvement over the best solution found for a number of iterations, which is set to $n \times 5$, and the size of the tournament selection is set to 2.

Two sets of instances are generated. The first set is used with a single time window per patient and contains three subsets (A*, B* and *C*). The instance *A1_1* refers to the instance 1 of the category *A* with single time window. Accordingly, the second set has the same instances as the first set that are used with two availability periods (see Table 4). The instance *A1_2* refers to the instance 1 of the category *A* with double time windows. The same instances are used with single and double availability periods to study their impact on solutions quality.

Set	Subset	Size	Max_v		K
STW	$Ai-$	$i \in \{1, 2, , 7\}$		10	
	Bi_1			25	
	Ci 1		10	50	10
MTW	Ai ₂	$i \in \{1, 2, , 7\}$		10	
	Bi ₂			25	
				50	10

Table 4. Tested instances details

5.2. Computational Results

Instances described above are solved within a time limit of 4 hours. *"LB"* and *"Z"* are, respectively, the lower bound (*LB*) and the objective function value of the deterministic model given by CPLEX. *"Gap"* is computed by the formula *100%× (Z – LB)/Z)* and *"CPU"* is the computing time. For GA based heuristic, we solve each instance 10 times then the best, the worst and the average solutions are retained. "CPU" expresses the total computing time elapsed for 10 runs. *"GAP"* is calculated between the average solution and the lower bound of CPLEX by the same above-mentioned formula. The proven optimal solutions are in boldface.

CPLEX solved to optimality almost all instances of subsets *A, B* and *C* with single time window except instances *C*2 and *C*3 for which a feasible solution was found with, respectively, a gap of 8*.*68% and 6*.*85%. For multiple time windows, the optimum solutions of all instances of the subset *A* were found, while only instances *B*1*, B*3*, B*4 and *B*6 were solved to optimality. For the instances *B*2*, B*5 and *B*7, a feasible solution was found with, respectively, a gap of 10.78%, 13.18% and 14.04%. Instances of the subset *C* are hard to solve, CPLEX was not able to resolve these instances within the time limit. This complexity is due to the multiple time windows, which is exponential. For *n* patients and *p* availability periods for each one, we have $pⁿ$ possibilities to select for each one a time window to receive care services.

The GA based heuristic solves instances in short CPU running times compared to CPLEX solver. GA is able to reach optimal solutions for some instances and to provide near-optimal solutions for the others (see figures (4,5) and tables (5, 6)). The complexity due to the multiple time windows faced by CPLEX did not affected the GA since we select for each patient the best time window independently of the other patients' time windows, which avoid the exponential complexity. Instance C5, with double time windows, shows the worst CPU running time, which is on average 17.3 seconds, while CPELX solver cannot find a feasible solution within 4 hours for subset C with double time windows. The worst gaps found for subsets with single time widows *A, B* and *C* are, respectively, 1.13% (*A*6), 6.27% (*B*2) and 19.33% (*C*2) (see Table 5). For instances with multiple time windows, the worst gaps found are, respectively, 1.16% (*A*1), 21.01 %(*B*2) and 47.44% (*C*2) (see Table 6).

The SPR model is solved by Monte Carlo simulation embedded into the GA. Caregivers' transportation cost and the average number of unvisited patients is considered for each instance. The CPU time running time is significantly increasing for the SPR model compared the deterministic version due to simulation that must be carried out for each new offspring to compute the expected value (see Table 7). In addition, the expected value equals to zero for all instances except instances *A*3, *A*7, *B*2 and *B*6 used with single time window, which shows the robustness of the SPR model.

Bazirha, Kadrani, Benmansour

A comparison is performed between instances with single and multiple time windows for both models. Figures 6 and 7 clearly show the advantage of adopting multiple time windows since the transportation cost is higher for instances with single availability period. Figures 8 and 9 show that caregivers' transportation cost is higher for the stochastic model because solutions with lower expected value are prioritized to avoid skipping patients without providing services operations and increase patients' satisfaction. Solutions with higher difference between patients' latest service times (*bil*) and caregivers' completion C_{ik} are prioritized to increase the chance of respecting patients' time windows since travel and service times are supposed stochastic. The more the margin $(b_{i} - C_{ik})$ is higher the more the fluctuation of caregivers' arrival times remains robust and compatible with patient's time windows. To illustrate that, we solved instance *A*5 using both models, the two solutions found are: for the deterministic model, caregivers' transportation cost is 662 (see tables 5 and 8) and for the stochastic model is 744 (see tables 7 and 9). We computed starting and completion times for patients visited by the caregiver 1 for both models stochastic and deterministic. The minimum difference between the latest service time (b_i) and the completion time C_i is 12 for the stochastic model and 4 for the deterministic model (see tables 10 and 11).

Instances		CPLEX			GA based heuristic (10 runs)				
STW	LB	Z	Gap	CPU	Best	Worst	Average	Gap	CPU
A1 1	525.00	525	0.00%	1.43	525	566	529.10	0.77%	< 1
AI_2	754.00	754	0.00%	1.28	754	754	754.00	0.00%	< 1
A1_3	588.00	588	0.00% 1.31		588	588	588.00	0.00%	≤ 1
AI_4	817.00	817	0.00%	1.43	817	817	817.00	0.00%	< 1
A1_5	662.00	662	0.00% 1.62		662	677	665.40	0.51%	< 1
$A1_6$	439.00	439	0.00% 1.37		439	464	444.00	1.13%	< 1
A1_7	539.00	539	0.00% 1.36		539	539	539.00	0.00%	< 1
$B1_1$	1165.00	1165	0.00% 2.64			1165 1300	1232.50	5.48%	8.34
$B1_2$	993.00	993	0.00% 2.40		993	1108	1059.40	6.27%	8.71
$B1_3$	1089.00	1089	0.00% 3.58		1089	1162	1124.40	3.15%	8.76
B1_4	928.00	928	0.00% 3.56		928	1001	965.00	3.83%	9.12
$B1_5$	1064.00		1064 0.00% 2.27			1064 1246	1110.60	4.20%	10.51
B1_6	1196.00		1196 0.00% 3.04			1196 1276	1226.40	2.48%	10.84
BI_7	1099.00	1099	0.00%	1.80	1099	1172	1116.40	1.56%	9.02
Cl 1	1490.00			1490 0.00% 411.76	1555	1719	1644.80	9.41%	92.70
C1 2	1529.64 1675 8.68%			14400		1799 1996	1896.20	19.33%	120.54
Cl_3	1425.26		1530 6.85%	14400	1633	1853	1723.00	17.28%	80.62
Cl_4	1603.00		1603 0.00%	5389.00	1803	1998	1889.70	15.17%	102.29
CI 5	1482.00		1482 0.00%	388.24		1614 1974	1752.70	15.44%	141.29
Cl_6	1658.00		1658 0.00%	172.16		1736 1999	1848.30	10.30%	92.03
C1 7	1492.00 1492 0.00% 232.13					1608 1742	1675.00	10.93%	91.38

Table 5. Numerical results of instances with single time window and deterministic travel and service times

Table 6. Continued													
Instances	CPLEX					GA based heuristic (10 runs)							
MTW	LB	Z	Gap	CPU	Best	Worst	Average	Gap	CPU				
$B2_4$	739.00	739	0.00%	512.62	743	852	782.50	5.56%	9.60				
$B2_5$	726.67	837	13.18%	14400	853	941	894.30	18.74%	12.36				
$B2_6$	944.00	944	0.00%	1065.68	950	1043	998.20	5.43%	11.06				
$B2_7$	810.57	943	14.04%	14400	948	1047	993.00	18.37%	10.93				
$C2_l$	800.04			14400	1330	1549	1406.80	43.13%	125.68				
$C2_2$	834.54	$\overline{}$		14400	1467	1644	1587.80	47.44%	162.22				
$C2_3$	999.36	$\overline{}$		14400	1437	1601	1519.60	34.24%	124.98				
C24	860.14	\overline{a}		14400	1474	1721	1622.20	46.98%	138.26				
$C2_5$	812.12	$\overline{}$		14400	1367	1582	1453.30	44.12%	173.21				
$C2_6$	842.53	$\overline{}$		14400	1417	1598	1505.03	44.02%	138.54				
$C2-7$	842.50	$\qquad \qquad -$		14400	1361	1517	1436.30	41.34%	143.47				

Table 7. Numerical results of instances with stochastic travel and services times

Figure 4. CPLEX and GA solutions comparison for instances of the set STW with deterministic parameters

Figure 5. CPLEX and GA solutions comparison for instances of the set MTW with deterministic parameters

Figure 6. Comparison of the best-found solutions for the deterministic model according to time windows

Figure 7. Comparison of the best-found solutions for the SPR model according to time windows

Figure 8. Comparison of the best-found solutions with single time window according to type of model

Figure 9. Comparison of the best-found solutions with multiple time windows according to type of model

Table 10. Caregiver 1 starting and completion times for assigned patients considering the solution found by the deterministic model for instances A5 with single time window

Patients		5(6)		8(5)		3(5)		
Time windows $[a_{i1}, b_{i1}]$	200	320	275	395	222	342	425	545
Start and completion times $[S_{i1}, C_{i1}]$	200	220	277	294	318	338	425	444
$b_{i1} - C_{i1}$		100		101				101

Table 11. Caregiver 1 starting and completion times for assigned patients considering the solution found by the SPR model for instances A5 with single time window.

6. Conclusion

Home health care companies seek to respect patients' time windows, which it becomes challenging when stochastic travel or/and service times are considered. Two possible recourses could be used in this case, either accepting providing services with a tardiness and time windows must be soft/flexible, or skipping a visit when a route becomes infeasible. In this study, the second recourse is adopted to deal stochastic service and travel times with multiple hard/fixed time windows. This multiplicity will increase the chance to minimize unvisited patients and give the decision maker more choices to schedule visits. A two-stage stochastic programming model with recourse is proposed to minimize the transportation cost as well as the average number of unvisited patients. The first stage is to find patients' visiting order and caregivers' assignment with respect to patients' availability periods, skill requirements and the maximum number of patients not to exceed per caregiver. The second stage is to introduce the recourse defined as skipping patients when routes become infeasible according to their availability periods. The deterministic model is solved using CPLEX solver and the GA. Monte Carlo simulation is embedded into the GA to solve the stochastic model. The tests prove the high performance of the GA to deal with large instances in a little amount of time, GA can reach optimal solutions for some instances and yield nearoptimal solutions for others. The SPR model shows high CPU running times while solving instances due the simulation that is carried out for each new solution to compute the average number of unvisited patients. Using multiple time windows helped to more optimize the transportation cost since more possibilities arise to schedule visits. Future works could be addressed to extend the SPR model to deal with multiple services since patients need several care activities per day. In addition, it would be interesting to compare the performance of the GA based heuristic with some other heuristics.

Data Availability

Datasets related to this article can be found at http://dx.doi.org/10.17632/9i52mtm737.1, an open-source online data repository hosted at Mendeley Data.

References

Bazirha, M., Kadrani, A., Benmansour, R., (2020a). Daily scheduling and routing of home health care with multiple availability periods of patients. In Variable Neighborhood Search. ICVNS 2019. Lecture Notes in Computer Science, Springer International Publishing, Cham, pp. 178–193.

Bazirha, M., Kadrani, A., Benmansour, R., (2020b). Scheduling optimization of the home health care problem with stochastic travel and care times. In 2020 5th International Conference on Logistics Operations Management (GOL), IEEE. pp. 1–8.

Bazirha, M., Kadrani, A., Benmansour, R., (2021). Stochastic home health care routing and scheduling problem with multiple synchronized services. *Annals of Operations Research*, pp. 1–29.

Ben-Tal, A., El Ghaoui, L., Nemirovski, A., (2009). Robust optimization. Vol. 28. Princeton University Press.

Bernard, D.G., (1955). Linear programming under uncertainty. *Management Science,* Vol*.* 1, pp. 97–206.

Bertsimas, D., Brown, D.B., Caramanis, C., (2011). Theory and applications of robust optimization. *SIAM review*, Vol*.* 53, pp. 464–501.

Braekers, K., Hartl, R.F., Parragh, S.N., Tricoire, F., (2016). A bi-objective home care scheduling problem: Analyzing the trade-off between costs and client inconvenience. *European Journal of Operational Research*, Vol*.* 248, pp. 428–443.

Braysy, O., Gendreau, M., (2005). Vehicle routing problem with time windows, part I: Route construction and local search algorithms. *Transportation science,* Vol*.* 39, pp. 104–118.

Burke, E.K., De Causmaecker, P., Berghe, G.V., Van Landeghem, H., (2004). The state of the art of nurse rostering. *Journal of scheduling*, Vol*.* 7, pp. 441–499.

Cappanera, P., Scutellà, M.G., (2021). Addressing consistency and demand uncertainty in the home care planning problem. *Flexible Services and Manufacturing Journal,* pp*.* 1–39.

Cappanera, P., Scutellà, M.G., Nervi, F., Galli, L., (2018). Demand uncertainty in robust home care optimization. *Omega,* Vol*.*80, pp. 95–110.

Charnes, A., Cooper, W.W., (1959). Chance-constrained programming. *Management science,* Vol*.* 6, pp. 73–79.

Cissé, M., Yalçındağ, S., Kergosien, Y., Şahin, E., Lenté, C., and Matta, A., (2017). Or problems related to home health care: A review of relevant routing and scheduling problems. *Operations Research for Health Care*, Vol*.* 13, pp. 1–22.

Decerle, J., Grunder, O., El Hassani, A.H., Barakat, O., (2019). A memetic algorithm for multi-objective optimization of the home health care problem. *Swarm and evolutionary computation*, Vol. 44, pp. 712–727.

Duque, P.M., Castro, M., Sörensen, K., Goos, P., (2015). Home care service planning. The case of Landelijke Thuiszorg. *European Journal of Operational Research*, Vol*.* 243, pp. 292–301.

Errico, F., Desaulniers, G., Gendreau, M., Rei, W., Rousseau, L.M., (2016). A priori optimization with recourse for the vehicle routing problem with hard time windows and stochastic service times. *European Journal of Operational Research*, Vol*.* 249, pp. 55–66.

Fathollahi-Fard, A.M., Ahmadi, A., Goodarzian, F., Cheikhrouhou, N., (2020). A bi-objective home healthcare routing and scheduling problem considering patients' satisfaction in a fuzzy environment. *Applied soft computing*, Vol*.* 93*,* pp*.* 106385.

Gendreau, M., Laporte, G., Séguin, R., (1996). Stochastic vehicle routing. *European Journal of Operational Research,* Vol*.* 88, pp. 3–12.

Glover, F., (1986). Future paths for integer programming and links to artificial intelligence. *Computers & operations research*, Vol*.* 13, pp. 533–549.

Hiermann, G., Prandtstetter, M., Rendl, A., Puchinger, J., Raidl, G.R., (2015). Metaheuristics for solving a multimodal home-healthcare scheduling problem. *Central European Journal of Operations Research*, Vol*.* 23, pp. 89–113.

Holland, J.H., (1992). Genetic algorithms. *Scientific American*, Vol*.* 267, pp. 66–73.

Kirkpatrick, S., Gelatt, C.D., Vecchi, M.P., (1983). *Optimization by simulated annealing. Science*, Vol*.* 220, pp. 671– 680.

Laporte, G., Louveaux, F., Mercure, H., (1992). The vehicle routing problem with stochastic travel times. *Transportation science*, Vol*.* 26, pp. 161–170.

Li, X., Tian, P., Leung, S.C., (2010). Vehicle routing problems with time windows and stochastic travel and service times: Models and algorithm. *International Journal of Production Economics*, Vol*.* 125, pp. 137–145.

Liu, R., Yuan, B., Jiang, Z., (2017). Mathematical model and exact algorithm for the home care worker scheduling and routing problem with lunch break requirements. *International Journal of Production Research*, Vol*.* 55, pp. 558–575.

Luo, Z., Qin, H., Zhang, D., Lim, A., (2016). Adaptive large neighborhood search heuristics for the vehicle routing problem with stochastic demands and weight-related cost. *Transportation Research Part E: Logistics and Transportation Review*, Vol*.* 85, pp. 69–89.

Mankowska, D.S., Meisel, F., Bierwirth, C., (2014). The home health care routing and scheduling problem with interdependent services. *Health care management science*, Vol*.* 17, pp. 15–30.

Marinaki, M., Marinakis, Y., (2016). A glowworm swarm optimization algorithm for the vehicle routing problem with stochastic demands. *Expert Systems with Applications*, Vol. 46, pp. 145–163.

Mendoza, J.E., Rousseau, L.M., Villegas, J.G., (2016). A hybrid metaheuristic for the vehicle routing problem with stochastic demand and duration constraints. *Journal of Heuristics*, Vol*.* 22, pp. 539–566.

Mladenovic, N., Hansen, P., (1997). Variable neighborhood search. *Computers & operations research*, Vol*.* 24, pp. 1097– 1100.

Rasmussen, M.S., Justesen, T., Dohn, A., Larsen, J., (2012). The home care crew scheduling problem: Preference-based visit clustering and temporal dependencies. *European Journal of Operational Research*, Vol*.* 219, pp. 598–610.

Redjem, R., Marcon, E., (2016). Operations management in the home care services: a heuristic for the caregivers' routing problem. *Flexible Services and Manufacturing Journal*, Vol*.* 28, pp. 280–303.

Rest, K.D., Hirsch, P., (2016). Daily scheduling of home health care services using time-dependent public transport. *Flexible Services and Manufacturing Journal*, Vol*.* 28, pp. 495–525.

Saffarian, M., Barzinpour, F., Eghbali, M.A., (2015). A robust programming approach to bi-objective optimization model in the disaster relief logistics response phase. *International Journal of Supply and Operations Management,* Vol*.* 2, pp. 595–616.

Shahnejat-Bushehri, S., Tavakkoli-Moghaddam, R., Boronoos, M., Ghasemkhani, A., (2021). A robust home health care routing-scheduling problem with temporal dependencies under uncertainty. *Expert Systems with Applications*, Vol*.* 182, pp. 115209.

Shi, Y., Boudouh, T., Grunder, O., (2019). A robust optimization for a home health care routing and scheduling problem with consideration of uncertain travel and service times. *Transportation Research Part E: Logistics and Transportation Review*, Vol*.* 128, pp. 52–95.

Shi, Y., Boudouh, T., Grunder, O., Wang, D., (2018). Modeling and solving simultaneous delivery and pick-up problem with stochastic travel and service times in home health care. *Expert Systems with Applications*, Vol*.* 102, pp. 218–233.

Solomon, M.M., Desrosiers, J., (1988). Survey paper—time window constrained routing and scheduling problems. *Transportation Science*, Vol*.* 22, pp. 1–1[3.](http://dx.doi.org/10.1287/trsc.22.1.1)

Tarricone, R., Tsouros, A.D., (2008). Home care in Europe: the solid facts. WHO Regional Office Europe.

Tas, D., Dellaert, N., van Woensel, T., De Kok, T., (2014a). The time-dependent vehicle routing problem with soft time windows and stochastic travel times. *Transportation Research Part C: Emerging Technologies*, Vol. 48, pp. 66–83.

Tas, D., Gendreau, M., Dellaert, N., Van Woensel, T., De Kok, A., (2014b). Vehicle routing with soft time windows and stochastic travel times: A column generation and branch-and-price solution approach. *European Journal of Operational Research*, Vol*.* 236, pp. 789–799.

Yuan, B., Liu, R., Jiang, Z., (2015). A branch-and-price algorithm for the home health care scheduling and routing problem with stochastic service times and skill requirements. *International Journal of Production Research*, Vol*.* 53, pp. 7450–7464.