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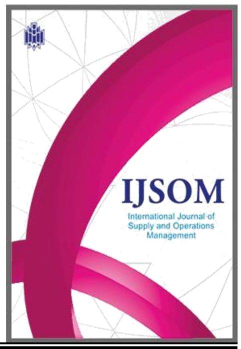

Inventory Policies for Non-instantaneous Deteriorating Items with Advance sales, Advertisement efforts and effect of Static Versus Dynamic Rebate

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Abstract

This paper presents study about the effects of rebates on an inventory model for non- instantaneous deteriorating items with preservation technology investments. Demand of the product is considered to be price-sensitive as well as affected by advertising efforts, advanced booking price discounts and rebates on price once deterioration starts. In the study, a three-phase model has been developed. The first phase is the production phase together with advance booking of the product at a discounted selling price. Second phase is the sales phase at a normal selling price and there is no deterioration of products in this phase. In the third phase products start deteriorating hence a price rebate is offered to the customers on purchase of the product. Effects of static versus dynamic rebates are studied. Aim of the paper is to obtain optimum cycle time, ordering quantity, selling price, amount of advertisement expenditures and amount of preservation expenditures in order to maximize total profit of the retailer. It is observed from the study that offering rebates in the deterioration phase helps retailers to increase demand that results to increase in total profit. By preservation technology investments retailers can reduce deterioration rate and hence he can generate more revenue.

Keywords: Inventory; Deterioration; Advertisement; Advance booking; Rebate; Preservation.

1. Introduction

Promotion of the product has become one of the essential tools for trade in present marketing scenario because proper execution of promotional activities can give new heights to the sales of the product. It helps to introduce the product to a large level and helps to sustain in the competitive market. Promotion of the product can be done in various forms such as advertisement, instant cash discounts, buy-back etc. Advertisement of the product has achieved maximum attention in modern marketing due to its effectiveness of reaching to many people and attracting them to buy the product. Adekoya (2009) submitted his thesis on the impact of advertising on sales volume of a product. Various types of advertisements such as retail advertising, trade advertising, institutional advertising, industrial advertising, product advertising, online advertising etc. have been discussed in the thesis. Li et al. (2013) studied price and advertisement dependent stochastic demand for inventory models in a supply chain. Seyedesfahani (2011) investigated vertical cooperative advertising in a manufacturer retailer supply chain where the retailer runs local advertising and the manufacturer pays for a portion of the entire cost of advertisement. Szmerekowsky et al. (2009) studied inventory model for pricing and two-tier advertising with one manufacturer and one retailer. Xie et al. (2009) studied coordination of advertising and pricing in retailer manufacturer supply chain.

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Advance booking is a handy tool that can be used to generate revenue in advance. It gives probable estimation of product demand in the market. Products booked in advance are delivered to the customers as soon as they arrive to the system and hence the retailer can get rid of the deterioration problem for these bunch of products. It is also an effective tool which is used to be assured of sales of the product. Further, offering products at a discounted rate during the advance booking phase gives boost to demand of the product and helps in generating revenue in advance. Market demand of the product is influenced by many parameters, hence numerous researches have been carried out with various type of product demand.

Dye et al. (2013) investigated inventory model for deteriorating items under joint pricing and ordering policy for advanced booking system with partial order cancellations. Pervin et al. (2019) investigated multi item inventory system for deteriorating products with price sensitive stock dependent demand and two level trade credits. Pervin et al. (2020) studied deteriorating products inventory model with preservation technology investments. In the study they considered price sensitive stock dependent demand and allowed shortages to be partially backlogged. Tsao et al. (2009) studied retailer's optimal ordering and discounting policies under advance sales discount and two echelon trade credits. Selling price plays a key role in sales volume of many products. It has been observed that for many products sales is affected by retail price on a large scale. Higher selling price may increase total revenue but it reduces demand of the product, on the other side lower selling price can increase sales but it may reduce overall profit. Dye et al. (2013) and the many references therein considered the price sensitive demand pattern to obtain optimal policies for inventory models.

Deterioration is defined as decay, spoilage, waste, damage, fungus or evaporation of the product. Many of the researchers have considered instantaneous deterioration in their study. But in practice it is observed that deterioration starts at a certain time after the product is received for sales. E.g. milk products, vegetables, fruits, sweets, pharmaceutical products, medicines, beverages etc. Bardhan et al. (2019) considered an inventory model for non-instantaneous deteriorating items with preservation technology investments under stock dependent demand. Dye et al. (2016) studied the pricing strategies for inventory model with effect of preservation technology investments on deteriorating items. Mishra et al. (2014) developed an inventory model with controllable deterioration rate considering time dependent demand and time varying holding cost. Pal et al. (2018) gave optimal replenishment policy for non-instantaneously perishable products. In the study they considered random start time of the deterioration and also, they used preservation technology investments. Roy et al.(2020) studied two warehouse probabilistic model with two level trade credits for deteriorating items. They considered weibull distribution for deterioration rate and also allowed shortages to be partially backlogged. Shah et al. (2013) gave optimal inventory policies for single supplier single buyer supply chain where they considered price sensitive stock dependent demand. They also considered trade credits from manufacturer to retailer and from retailer to customer. Shah et al. (2018) considered instantaneous deterioration products in the study of inventory model. In the study two level trade credits were taken into account and the demand was considered to be price sensitive time dependent linear in nature. Shah et al. (2018) studied an inventory model for non-instantaneous deteriorating items. They considered market demand to be price sensitive trended with learning effects. Tsao (2016) studied inventory policies for non-instantaneous deteriorating items for an inventory model. When product start deteriorating, retailer faces a decrease in product demand. Moreover, deteriorated products may cause drop down in overall profit and goodwill of the firm. Offering rebates on selling price to sell products in deterioration phase can help the retailer to minimize loss from deteriorated items. Price rebate is unlike the discount which is deducted in advance of the payment. Here, a customer needs to pay full amount for the product and after a certain specified time he gets a price rebate in terms of cash back in his wallet. Once the deterioration phase starts, it is necessary to clear stock as soon as possible for the retailer. Therefore, offering price rebates in terms of cash back helps to increase demand of lower quality products and works as stock clearance sales. Bhaula et al. (2019) gave an optimal inventory model for perishable items under successive price discounts with permissible delay in payments. In the study they considered successive price discounts and variable holding cost considering inflation in the market. Dey et al. (2019) studied the effect of dynamic versus static rebates in their study of investigation on price, displayed stock level and rebate-induced demand using hybrid bat algorithm. Shanshan et al. (2017) studied optimal rebate strategies under dynamic pricing. In their study they developed an inventory model with mail-in-rebate policy.

In this paper, we consider the demand to be price sensitive as well as influenced by promotional expenditures. In first phase, products are booked in advance at a discounted rate together with production of the products. Delivery of products booked in advance is given as soon as the product arrives to the outlet for sale. Products are considered to be non-instantaneously deteriorated. Once deterioration phase starts it is necessary to clear stock as soon as possible because decayed products may harm to good quality products and also to the goodwill of retailer in the market. Keeping this in consideration, during deterioration phase a price rebate in the form of cash-back is offered to the customer on purchase of product. Static rebate stands for a constant price value in the form of cash-back on purchase of product. On the other side, dynamic rebate is time dependent price value in the form of cash back on purchase of product. As the time passes value of dynamic rebate increases. Effect of dynamic versus static rebate has been analyzed. Case with no price rebate is also compared with static and dynamic rebate cases. We considered preservation technology investments in order to reduce the deterioration rate. We aim to maximize total profit of the retailer with respect to cycle time, selling price and advertisement expenditures. Table 1 compares the difference between the proposed model and the previous published models.

Table 1. Comparison between proposed model and some related published models

Authors	Price sensitive demand	Promotion of Product	Advance booking	Deterioration	Preservation technology investment	Rebate
Adekoya (2011)	×	√	×	×	×	×
Bardhan et al. (2019)	×	×	×	Non-instantaneous	√	×
Bhaula et al. (2019)	√	×	×	Non-instantaneous	×	×
Dey et al. (2019)	√	×	×	Instantaneous	√	Static/Dynamic
Dye & Hsieh (2013)	√	×	√	Instantaneous	×	×
Dye & Yang (2016)	√	√	×	Instantaneous	√	×
Li et al. (2013)	√	√	×	×	×	×
Mishra et al. (2014)	×	×	×	Non-instantaneous	√	×
Pal et al. (2018)	×	×	×	Random	√	×
Pervin et al. (2020)	√	×	×	Instantaneous	√	×
Pervin et al. (2019)	√	×	×	Instantaneous	×	×
Saha & Goyal (2014)	√	×	×	×	×	Joint Rebate
Saha et al. (2019)	√	√	×	×	×	Static
Roy et al. (2020)	×	×	×	Instantaneous	×	×
Seyedesfahani et al. (2011)	√	√	×	×	×	×
Shah & Naik (2018)	√	×	×	Non-instantaneous	×	×
Shah & Vaghela (2018)	√	×	×	Instantaneous	×	×
Shanshan et al. (2017)	√	×	×	×	×	Static
Szmerekovsky (2009)	√	√	×	×	×	×
Tsao (2009)	×	×	√	×	×	×
Tsao (2014)	×	×	×	Non-instantaneous	√	×
Xie & Wei (2009)	√	√	×	×	×	×
Proposed model	√	√	√	Non-instantaneous	√	Static/Dynamic

From the survey in Table 1 it can be observed that most of the research work has been carried out either without consideration of rebate strategy or with consideration of only static rebate. In present study authors have studied the effect of two type of rebates: static and dynamic. Since the products are non- instantaneously deteriorating by nature, it is desirable that the stock should vanish as soon as possible to avoid major loss due to completely decayed products. Hence offering rebate on selling price helps retailer to increase product demand, stock clearance and reduce loss due to decayed products during deterioration phase. There are certain products with very high rate of deterioration. Such products deteriorate very fast as the time passes. Static rebate may not suffice for the quick stock clearance of such products. Dynamic rebate is a kind of rebate where the rebate value increases with time. Sometimes, it is imperative to provide higher rebate as the product's quality gets affected. Hence once the deterioration of product starts, the retailer may provide the customers dynamic price rebate according to the product quality.

The paper is organized as follows. Section 1 is introduction. Section 2 includes assumption and notations used to develop the inventory model. Mathematical model is developed in section 3. In section 4 the results are validated through numeric hypothetical inventory parametric values and the managerial issues are discussed through sensitivity analysis, where one inventory parameter is varied by $-20\%, -10\%, 10\%$ and 20%. Section 5 concludes the study.

2. Assumptions and Notations:

For the development of mathematical model we consider following notations and assumptions.

2.1 Notations

2.1.1 Decision variables

M	Advertisement expenditure (in $) per cycle
p	Normal selling price (in $) per unit
T	Cycle length of inventory (in months)

ξ	Preservation technology investments (in \$) per cycle $(\xi \geq 0)$

2.1.2 Inventory parameters

A	Ordering cost (in \$) per order
c	Cost price of product (in \$) per unit
g	Discounting factor in advance booking phase $(0 < g < 1)$
h	Holding cost (in \$) per unit per unit time
HC_s	Total holding cost (in \$) per cycle in static rebate case
HC_d	Total holding cost (in \$) per cycle in dynamic rebate case
$I_1(t)$	Production level at time *t* during time interval $[0, t_1]$
$I_e(t)$	Effective inventory level at time *t* during interval $[0, t_1]$
$I_2(t)$	Inventory level at time *t* during time interval $[t_1, t_2]$
$I_{3s}(t)$	Inventory level at time *t* during time interval $[t_2, T]$ in static case
$I_{3r}(t)$	Inventory level at time *t* during time interval $[t_2, T]$in dynamic case
N_1	Number of products sold in first phase
N_2	Number of products sold in second phase
N_{3s}	Number of products sold in third phase – static case
N_{3d}	Number of products sold in third phase – dynamic case
P	Production rate per unit time
P_a	Selling price in \$ / unit in first phase
Q_0	Total production quantity
Q_1	Quantity at the beginning of second phase
Q_2	Quantity at the beginning of third phase
r	Static rebate (in \$) at any time $t \in [t_2, T]$
$r(t)$	Dynamic rebate (in \$) at any time $t \in [t_2, T]$
R_1	Demand rate in first phase
R_2	Demand rate in second phase
$R_3(s)$	Demand rate in third phase in static case
$R_3(d)$	Demand rate in third phase in dynamic case
RC_s	Total rebate cost (in \$) per cycle in static case
RC_d	Total rebate cost (in \$) per cycle in dynamic case
t_1	Production and advance booking phase length
t_2	Time when deterioration begins
θ_0	Natural deterioration rate of product under no preservation $(0 < \theta_0 < 1)$
θ_1	Minimum deterioration rate of product under preservation technology investments $(\theta_1 \leq \theta_0)$
θ	Effective deterioration rate under preservation technology investment ξ ; $(\theta_1 \leq \theta \leq \theta_0)$
γ	Sensitivity factor of rebate in demand of the product at time $t \in [t_2, T]$

2.1.3 Objective function

$\Pr(M, p, T, \xi)$	Average total profit (in \$) of retailer

2.2 Assumptions

(1) Inventory model is considered for a single cycle $[0,T]$, which includes three phases:

(i) $[0,t_1]$: Production as well as advance booking phase with discounted price.

(ii) $[t_1,t_2]$: Sales phase with no deterioration and

(iii) $[t_2,T]$: Deterioration phase with price rebate on selling price.

(2) Demand is influenced by selling price and promotional efforts in first and second phase. In third phase demand is also influenced by rebate. Demand in first, second and third phase is represented by R_1, R_2 and $R_3(s)$ or $R_3(d)$ for static and dynamic respectively.

$R_1 = a - b\cdot P_a + \beta\cdot M$; $0 \le t \le t_1$

$R_2 = a - b\cdot p + \beta\cdot M$; $t_1 \le t \le t_2$ and

$R_3(s) = a - b\cdot p + \beta\cdot M + \gamma\cdot r$; $t_2 \le t \le T$ (For static model) or

$R_3(d) = a - b\cdot p + \beta\cdot M + \gamma\cdot r(t)$; $t_2 \le t \le T$ (For dynamic model).

Here $a > 0$ is constant demand of the product in market, $b > 0, \beta > 0$ and $\gamma > 0$ represent price, promotion efforts and rebate sensitivity factors respectively.

(3) Product is considered to be deteriorated at a constant rate θ_0 during $t_2 \le t \le T$. Deterioration rate of the product can be reduced with effective use of preservation technology investment. The reduced deterioration rate due to preservation technology investment is $\theta = \theta_1 + (\theta_0 - \theta_1)e^{-k\cdot\xi}$ (Dey 2019). Due to decaying nature of the product the deterioration rate cannot vanish therefore θ_1 is considered to be minimum threshold value of deterioration rate. As $\xi \to 0$ i.e. if preservation is not taken into account then $\theta \to \theta_0$. Further if $\xi \to \infty$ then $\theta \to \theta_1$. It shows that after certain limit unnecessary investment in preservation technology results diminishing return from capital investments on preservation.

(4) Product is offered at a discounted price P_a during $0 \le t < t_1$, $g\,\%$ discount is offered on selling price p for the products booked in advance which implies $P_a = (1-g)\cdot p$ where $0 < g < 1$.

(5) Product is sold at a normal selling price p during $t_1 \le t \le T$.

(6) Rebate is offered to customers during deterioration phase, $t_2 \le t \le T$:

(i) For static model: static rebate $= r$; $0 < r < c$.

(ii) For dynamic model: dynamic rebate $r(t) = r + \lambda\cdot t$; $0 < \lambda < 1$, $t_2 \le t \le T$

(7) Shortages are not allowed.

3. Mathematical Model

Graphical structure of the inventory model is shown in (fig. 1).

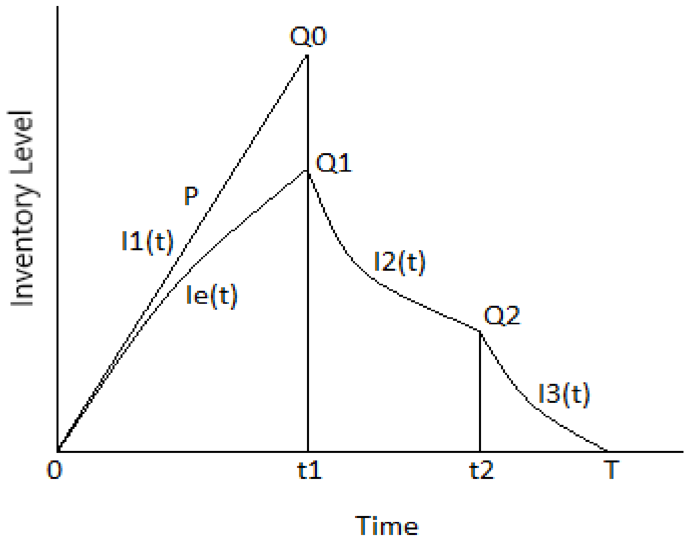

Figure 1. Graphical representation of the Inventory Model

First phase $0 \le t \le t_1$ is production phase as well as advance booking phase. During this phase products are manufactured at rate P together with advance booking of the product at a discounted rate. $I_1(t)$ represents production level at time *t* whereas the effective inventory level due to advance booking of the product is represented by $I_e(t)$. In second phase inventory level decreases only due to demand while in third phase the inventory level decreases due to demand and deterioration. We use preservation technology investments to reduce the rate of deterioration. Further, as soon as the deterioration starts price rebates are given to customers to have quick stock out and to retain the goodwill of the firm. Inventory levels in second and third phase at time t are characterized by $I_2(t)$ and $I_3(t)$ respectively. Products in the system are considered non-instantaneous deteriorating. Production and advance booking phase end at time t_1. Products booked in advance are delivered to customers as soon as the product arrives for sales. Production at t_1 is Q_0 which decreases to Q_1 due to completion of orders received during the advance booking.

Phase-1: Production and Advance booking phase $\left(0 \le t \le t_1\right)$

In this phase, Products are manufactured at rate P and hence the inventory level increases by the same rate. Therefore, we have the following equation.

$$\frac{d}{dt} I_1(t) = P \ ; \ 0 \le t \le t_1 \tag{1}$$

Solving and by using boundary conditions $I_1(0) = 0$ and $I_1(t_1) = Q_0$ we get, $P = \frac{Q_0}{t_1}$ (2) and the production inventory at any time *t* is: $I_1(t) = \frac{Q_0 \cdot t}{t_1}$ (3)

Advance booking of the product generates demand $R_1 = a - b \cdot P_a + \beta \cdot M$ (4)

Due to advance booking during production phase, effective inventory level can be obtained from the following differential equation.

$$\frac{d}{dt} I_e(t) = P - R_1 \ ; \ 0 \le t \le t_1 \tag{5}$$

Using the boundary conditions $I_e(0) = 0$ and $I_e(t_1) = Q_1$ for solving equation (5) and using equation (2) we get, $Q_1 = Q_0 - R_1 \cdot t_1$. Therefore, $Q_0 = Q_1 + R_1 \cdot t_1$ (6)

Number of products sold in this phase is: $N_1 = R_1 \cdot t_1$ (7)

Phase-2: Sales period with no deterioration $\left(t_1 \le t \le t_2\right)$

Demand of the product in this phase is $R_2 = a - b\cdot p + \beta\cdot M$ (8)

Inventory level in second phase decreases only due to demand and it is governed by following differential equation,

$$\frac{d}{dt}I_2(t) = -R_2 \ ;\ t_1 \le t \le t_2 \quad (9)$$

Using the boundary conditions $I_2(t_1) = Q_1$ and $I_2(t_2) = Q_2$ for solving equation (9) we get,

$Q_2 = Q_1 - R_2\cdot(t_2 - t_1)$ and $I_2(t) = Q_1 - R_2\cdot(t - t_1)$ on simplifying we obtain Q_1 and $I_2(t)$ as shown below in equation (10) and (11).

$$Q_1 = Q_2 + R_2\cdot(t_2 - t_1) \quad (10)$$

$$I_2(t) = Q_2 + R_2\cdot(t_2 - t) \quad (11)$$

Number of product sold in this phase is: $N_2 = R_2\cdot(t_2 - t_1)$ (12)

Phase-3: Deterioration phase $(t_2 \le t \le T)$

During this phase product start deteriorating at constant rate θ_0. In order to reduce loss due to deteriorated items, retailer offers price rebate on purchase of the product in the form of cash back. Next we proceed to study effect of static rebate versus dynamic rebate in cases Case A and Case B respectively.

Case A: Static rebate

Demand of product in this phase is $R_3(s) = a - b\cdot p + \beta\cdot M + \gamma\cdot r$ (13)

Corresponding inventory level at any point of time *t* is governed by the differential equation,

$$\frac{d}{dt}I_{3s}(t) = -R_3(s) - \theta\cdot I_{3s}(t) \ ;\ t_2 \le t \le T \quad (14)$$

Using the boundary conditions $I_{3s}(t_2) = Q_2$ and $I_{3s}(T) = 0$ for solving equation (14) we get,

$$I_{3s}(t) = \frac{R_3(s)}{\theta}\cdot\left[e^{\theta\cdot(T-t)} - 1\right] \quad (15)$$

and $Q_2 = \dfrac{R_3(s)}{\theta}\cdot\left[e^{\theta\cdot(T-t_2)} - 1\right]$ (16)

Substituting value of Q_2 from equation (16) in equation (10) we get,

$Q_1 = \dfrac{R_3(s)}{\theta}\cdot\left[e^{\theta\cdot(T-t_2)} - 1\right] + R_2(t_2 - t_1)$ and substituting this value in equation (6) we get,

$$Q_0 = \frac{R_3(s)}{\theta}\cdot\left[e^{\theta\cdot(T-t_2)} - 1\right] + R_2(t_2 - t_1) + R_1\cdot t_1 \quad (17)$$

Number of product sold in this case during third phase is: $N_{3s} = R_3(s)\cdot(T - t_2)$ (18)

Total revenue generated in $[0,T]$ for the static rebate case is as shown below.

$$TR_s = P_a\cdot N_1 + p\cdot N_2 + p\cdot N_{3s} \quad (19)$$

Due to offering static rebate in third phase retailer incurs rebate cost $RC_s = r\cdot N_{3s}$ (20)

Total holding cost for cycle is: $HC_s = h\cdot\left(\int_0^{t_1} I_1(t)\,dt + \int_{t_1}^{t_2} I_2(t)\,dt + \int_{t_2}^{T} I_{3s}(t)\,dt\right)$ (21)

Total profit per unit cycle, $\Pr_s = \dfrac{1}{T}\left(TR_s - c\cdot Q_0 - A - \xi - M - RC_s - HC_s\right)$ (22)

Case B: Dynamic rebate

Demand of product in this phase is $R_3(d) = a - b\cdot p + \beta\cdot M + \gamma\cdot r(t)$ (23)

Corresponding inventory level at any point of time *t* is governed by the differential equation,

$$\frac{d}{dt}I_{3d}(t) = -R_3(d) - \theta\cdot I_{3d}(t) \ ;\ t_2 \le t \le T \quad (24)$$

Using the boundary conditions $I_{3d}(t_2)=Q_2$ and $I_{3d}(T)=0$ for solving equation (24) we get,

$$I_{3d}(t)=\frac{R_3(d)}{\theta}\cdot\left[e^{\theta\cdot(T-t)}-1\right]+\frac{\gamma\cdot\lambda}{\theta}\cdot\left[T\cdot e^{\theta\cdot(T-t)}-t\right]-\frac{\gamma\cdot\lambda}{\theta^2}\cdot\left[e^{\theta\cdot(T-t)}-1\right] \tag{25}$$

And $$Q_2=\frac{R_3(d)-\frac{\gamma\cdot\lambda}{\theta}}{\theta}\cdot\left[e^{\theta\cdot(T-t_2)}-1\right]+\frac{\gamma\cdot\lambda\cdot T}{\theta}\cdot e^{\theta\cdot(T-t_2)}-\frac{\gamma\cdot\lambda\cdot t_2}{\theta} \tag{26}$$

Substituting value of Q_2 from equation (26) in equation (10) we get,

$$Q_1=\frac{R_3(d)-\frac{\gamma\cdot\lambda}{\theta}}{\theta}\cdot\left[e^{\theta\cdot(T-t_2)}-1\right]+\frac{\gamma\cdot\lambda\cdot T}{\theta}\cdot e^{\theta\cdot(T-t_2)}-\frac{\gamma\cdot\lambda\cdot t_2}{\theta}+R_2(t_2-t_1)$$

and substituting this value in equation (6) we get,

$$Q_0=\frac{R_3(d)-\frac{\gamma\cdot\lambda}{\theta}}{\theta}\cdot\left[e^{\theta\cdot(T-t_2)}-1\right]+\frac{\gamma\cdot\lambda\cdot T}{\theta}\cdot e^{\theta\cdot(T-t_2)}-\frac{\gamma\cdot\lambda\cdot t_2}{\theta}+R_2(t_2-t_1)+R_1\cdot t_1 \tag{27}$$

Number of product sold in this case during third phase is: $N_{3d}=\int_{t_2}^{T}R_3(d)\cdot dt$ (28)

Total revenue generated in $[0,T]$ for the dynamic rebate case is as shown below.

$$TR_d=P_a\cdot N_1+p\cdot N_2+p\cdot N_{3d} \tag{29}$$

Due to offering dynamic rebate in third phase retailer incurs rebate cost as shown below. $RC_d=\int_{t_2}^{T}(r+\lambda\cdot t)R_3(d)dt$

(30)

Total holding cost for cycle is: $HC_d=h\cdot\left(\int_0^{t_1}I_1(t)dt+\int_{t_1}^{t_2}I_2(t)dt+\int_{t_2}^{T}I_{3d}(t)dt\right)$ (31)

Total profit per unit cycle, $\Pr_d=\frac{1}{T}(TR_d-c\cdot Q_0-A-\xi-M-RC_d-HC_d)$ (32)

The objective is to maximize total profit of the retailer which can be obtained by differentiating equation (22) for static case or equation (32) for dynamic case, with respect to decision variables M, p, T and ξ. By setting them zero in order to get solution. This is shown in following procedure.

Step 1: Allocate values to all inventory parameters other than decision variables.

Step 2: Work out $\frac{\partial \Pr}{\partial M}=0, \frac{\partial \Pr}{\partial p}=0$, $\frac{\partial \Pr}{\partial T}=0$ and $\frac{\partial \Pr}{\partial \xi}=0$ to get optimum values of decision variables, M , p , T and ξ respectively.

Step 3: Substitute values of decision variables obtained above in equation (22) and equation (17) for static case and in equation (32) and equation (27) for dynamic case to get optimum values of production quantity and profit respectively.

4. Numerical Example

In this section we illustrate with numerical example in order to validate the inventory model.
We consider following example to justify the mathematical formulation.

Example: Consider $A=\$100$ per order, $a=300$, $b=7$, $c=\$12$ per unit, $h=\$0.5$ /unit/month, $t_1=0.6$ month, $t_2=1$ month, $\beta=0.02$, $g=0.05$, $k=0.015$, $\gamma=10$, $r=3$, $\lambda=0.1$, $\theta_0=0.2$ and $\theta_1=0.06$.

We use these hypothetical values to analyze managerial aspects in various cases mentioned in the table 1.On following the procedure mentioned above to get the optimal values of advertisement cost, selling price, cycle time, preservation cost, order quantity and total profit of retailer for static rebate, dynamic rebate and no rebate with use of preservation technology investments and without using preservation technology investments are as shown in table2.

Table 2: Optimal values of decision variables, ordering quantity and total profit

Case	PTI	Adv. Cost (M)	Selling price (p)	Cycle time (T)	Preservation Cost (ξ)	Order Quantity (Q_0)	Profit (Pr)
Static Rebate	√	$ 435.52	$ 31.58	3.166 months	$ 140.59	371.71 units	$ 1506.96
	×	$ 1539.4	$ 33.67	2.973 months	-	404.85 units	$ 1392.79
Dynamic Rebate	√	$ 417.81	$ 31.73	3.165 months	$ 141.08	371.77 units	$ 1510.42
	×	$ 1721.1	$ 34.75	3.034 months	-	407.72 units	$ 1259.75
No Rebate	√	$ 591.69	$ 29.28	3.220 months	$ 137.73	371.92 units	$ 1431.27
	×	$ 1680.7	$ 31.45	3.026 months	-	403.59 units	$ 1325.18

*PTI = Preservation Technology Investment, Adv. Cost = Advertisement Cost.

(Table2) Three different scenarios are taken into consideration: offering static rebate versus dynamic rebate versus no rebate in the third phase of inventory cycle when products start deteriorating. Each scenario is further divided into two cases: with preservation technology investment and without preservation technology investment. Results obtained in table1 indicate that if retailer does not use preservation technology investments then he need to invest very excessive amount in advertisement expenditures in order to get maximum profit. However, from the results it is clear in each case it is worth to invest in preservation technology because it is more beneficial to the retailer in order to get higher profit. Moreover offering 'no rebate' on purchase of the product result in lower profit in comparison to other two scenarios and even it may be harmful to firm's goodwill in the market. Out of other two scenarios dynamic rebate suits better to retailer than static rebate for given numeric example. However, depending on rebate value, cycle period and values of other parameters retailer should wisely decide to choose among static rebate and dynamic rebate. The concavity of profit function can be seen from graphs shown in figure 2 and figure 3.

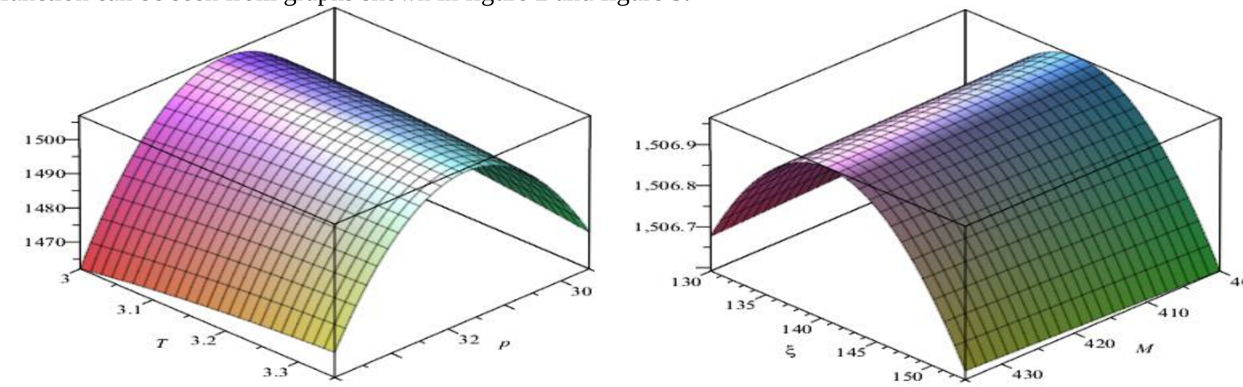

Figure 2.1 Figure 2.2

Figure 2 Concavity of Profit function with respect to T, p and ξ, M (Static rebate)

Figure 2 Concavities of profit function with respect to selling price p and cycle time T is shown in figure 2.1 and concavity of profit function with respect to advertisement expenditure cost M and preservation technology cost ξ is shown in figure 2.2 for static rebate case.

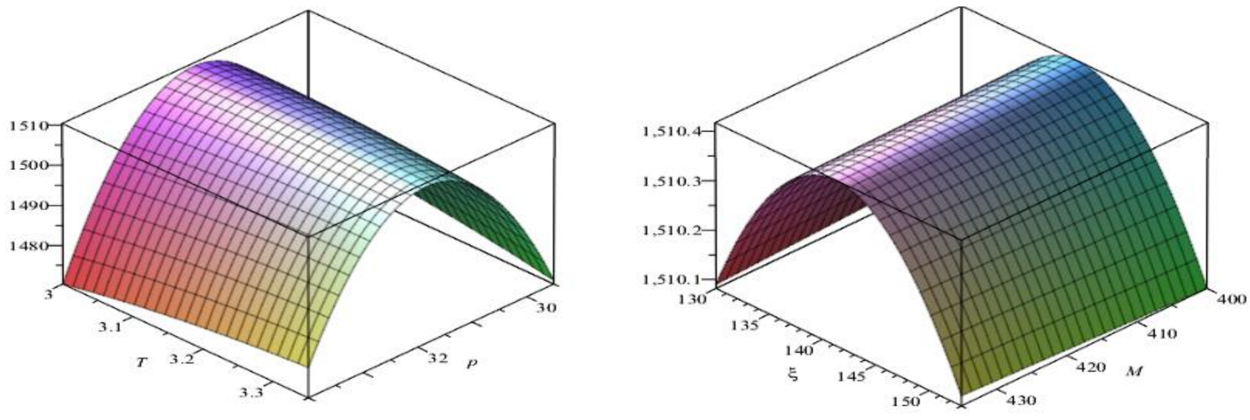

Figure 3.1 Figure 3.2

Figure 3 Concavity of Profit function with respect to T, p and ξ, M (Dynamic rebate)

Figure3 Concavities of profit function with respect to selling price p and cycle time T is shown in figure 3.1 and concavity of profit function with respect to advertisement expenditure cost M and preservation technology cost ξ is shown in figure 3.2 for dynamic rebate case.

Next, we proceed to determine the sensitivity of cycle time, advertisement cost, selling price, order quantity and total profit with respect to change in other inventory parameters by $-20\%, -10\%, 10\%$ and 20%.

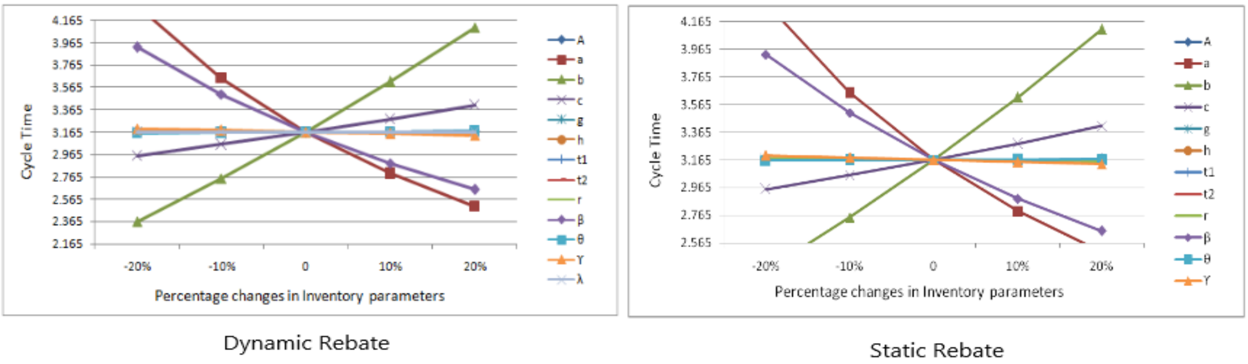

Figure 4. Sensitivity analysis for cycle time (T) in static and dynamic cases

(Figure 4) The graphs in the figure represent sensitivity of cycle time T with respect to change in other inventory parameters. It can be seen from graphs that cycle time gradually increases with increase in ordering cost A, price sensitivity index b and cost price c. Moreover increase in demand factor a, rebate r, advertisement sensitivity index β and rebate sensitivity index γ result a decrease in cycle time. Change in holding cost and deterioration rate have negligible effect on cycle time. Increase in deterioration rate is not affecting noticeably to cycle time due to presence of preservation technology investments.

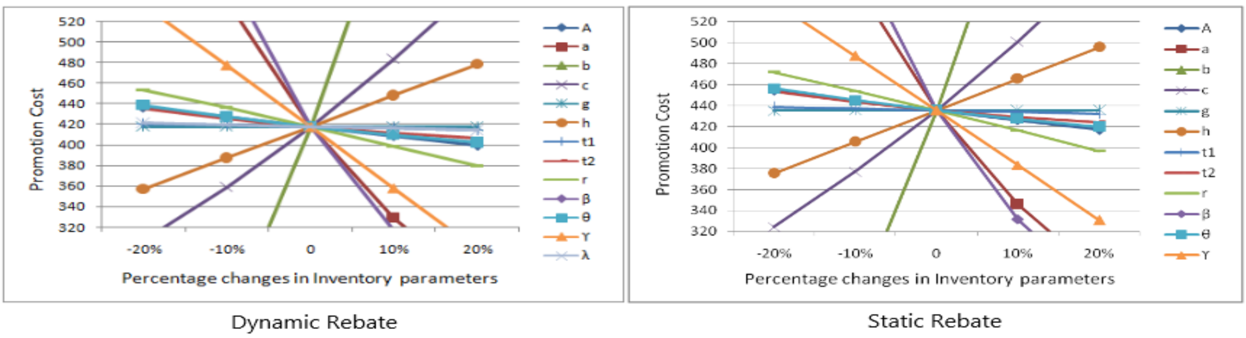

Figure 5 Sensitivity analysis for Advertisement cost (M) in static and dynamic cases

(Figure 5) From the graphs one can conclude that advertisement cost is very sensitive to many parameters such as $A, a, b, c, h, r, \beta, \theta$ and γ .Other parameter effects very less to the advertisement cost. M is very responsive to β as it is directly associated to the component of advertisement cost. From the figure it can be observed that if we offer more discount in the advance booking phase then we need to invest less in promotional efforts. An increase is noticed in the advertisement with respect to increase in c, b and h. On the other side, increase in ordering cost, demand factor a, advance booking time t_1, deterioration start time t_2, deterioration rate θ and rebate factor γ results into decrease in advertisement cost.

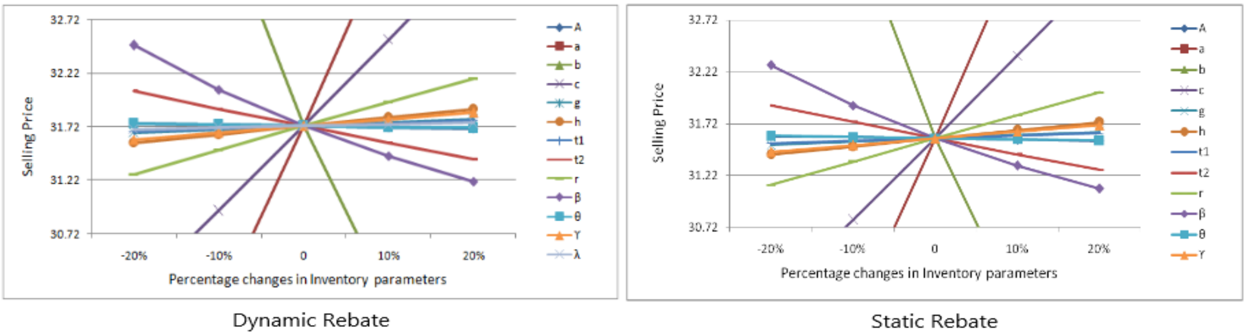

Figure 6 Sensitivity analysis for Selling price (p) in static and dynamic cases

(Figure 6) Sensitivity of selling price p is presented in the graphs. We can observe that increase in various costs associated to the inventory model such as cost price of the product, ordering cost, inventory holding cost and rebate cost results into increase in selling price. Further, increase in discounting factor g, advance booking time t_1, rebate factor γ and demand factor a also out turn an increase in selling price. The selling price decreases with increase in b, β and deterioration start time t_2 .

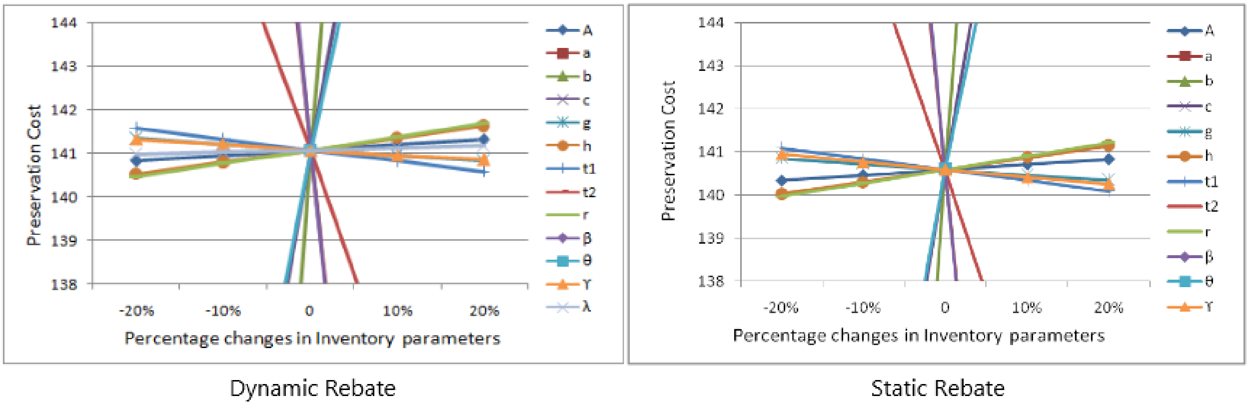

Figure 7 Sensitivity analysis for preservation technology cost (ξ) in static and dynamic cases

(Figure 7) As shown in the graphs we have fluctuations in preservation cost with respect to change in other parameters. With increase in deterioration rate naturally we can see increase in preservation cost. Additionally, increase in price sensitivity factor b and cost price c also admires an increase in preservation cost. As the deterioration start time t_1 increase the deterioration phase become short and hence preservation cost decreases. Other inventory parameters have negligible effect on preservation cost.

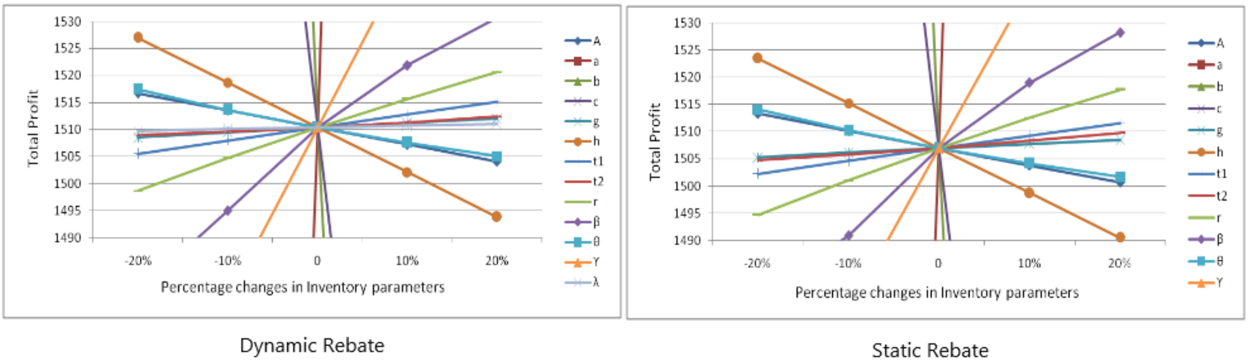

Figure 8. Sensitivity analysis for Total Profit (Pr) in static and dynamic cases

(Figure 8) It represents sensitivity of total profit of retailer with respect to other inventory parameters. As might be expected with increase in the costs associated to the inventory model such as, cost price of product, ordering cost, inventory holding cost and rebate cost the overall profit decreases. Also increase in deterioration rate and price sensitivity factor *b* decreases total profit of retailer. On the other side increase in inventory parameters like demand factor a, discounting factor g, rebate factor β and deterioration start time t_2 admires increase in the total profit.

The inventory model is designed for non-instantaneous deteriorating items. The purpose of giving rebate in third phase is not only to reduce the cycle time, but it is to preserve goodwill of the firm and generate more revenue. For example, an organic mango vendor at the end of season when mangoes start deteriorating provides some cash-back to customers so as to clear stock as soon as possible. Here providing price rebate increase the demand which is useful in generating revenue at a big scale. Providing price rebate for a product which is slightly poor in quality than the fresh one helps to preserve goodwill of the retailer. The retailer should wisely decide the investment amount to be spent for the promotion of the product and also should take care of the discount provided for deteriorated items so that the overall profit can be maximized.

5. Conclusion

We have studied an inventory model for non-instantaneous deteriorating products with advance booking at a discounted price together with production of product as well as price rebate on purchase of the product once the deterioration starts and products start losing quality. We consider demand of the product to be price sensitive as well as affected by promotional efforts. In third phase demand is also influenced by price rebate offered to customers. The study is applicable to products which are non-instantaneous deteriorating in nature. Numerical example has been given to validate the model. Sensitivity analysis has been carried out to check the effect of different parameters on decision variables. It is observed from the study that preservation technology investments are useful to reduce the deterioration rate and increase the profit. Providing rebates on the purchase of product once deterioration starts also helps the retailer to increase the overall profit. In the numerical example with hypothetical data it is observed that dynamic rebate case is more advantageous to retailer compared to static rebate case. However depending on rebate value, cycle time, deterioration rate and other inventory parameters retailer should wisely decide about adoption of the strategy between static and dynamic rebates. Further, providing higher price rebate may help to clear stock quickly but may be harmful to overall profit of the retailer. The retailer needs to take care of investments made in promotion of the product and advance booking discount. Present research model can be extended by allowing shortages. In present study we don’t discuss about redemption of rebate money obtained by customers, therefore an extended model can be developed considering utilization conditions of price rebates. One can extend the model with trade credit policy to have more realistic inventory model.

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