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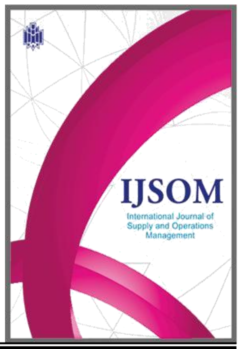

Optimal Control for Inventory System under Uncertainty on Demand and Delivery Using Robust Linear Quadratic Control Approach

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Abstract

The supply chain management comprises many uncertain parameters such as the demand value and delivered product rate as the result of an imperfect delivery process. In this article, therefore, a mathematical model in a linear dynamical state-space equation is formulated for an inventory system with uncertain demand value and imperfect delivery process developed from the existing classical model. The new model is used to determine the optimal decision for this inventory system i.e. to calculate the optimal amount of product that should be ordered from the supplier. Moreover, the optimal decision is calculated for the purpose to control the inventory level as the decision-maker wanted to, in this paper, the inventory level is brought to a set point. The robust linear quadratic control, which is an existing model, is employed to this system with a numerical experiment performed to illustrate the controlling responses. From the obtained results, it achieved the optimal decision with the proper control of the inventory level based on the performed set-point control problem. In addition, the performed computational experiment is compared to some related existing works. The analysis showed that the achieved optimal decision is well enough and is not worse than the other results. In conclusion, the proposed model and the method performed in this research are implementable and therefore can be used by practitioners especially in the supply chain management field.

Keywords: Inventory control; Imperfect delivery; Robust LQR; Uncertain demand.

1. Introduction

People attention to manage supply chain becomes higher and higher not only to gain more benefit by formulating some mathematical model but also maintaining their commitment to protect the environment from the disruption caused by the supply chain activities by developing green supply chain concept (Abdellatif & Graham, 2019). Mathematical model on Logistics and Supply Chain (LSC) plays an important role in the decision-making tool to obtain optimal strategy. An LSC comprises of the upper parts which consist of raw material supplier, manufacturing industries, etc. and the lower parts which consist of distributors, retailers, and the end users. These parts have a common inventory system used to store their product. For example, a manufacturing industry has an inventory system to store raw material used for production and to save the product before it is sold. Furthermore, a retail industry uses an inventory system to store some products before it sold to customers. The stored product is usually used to satisfy customers demand or for future purposes. Therefore, it is important to place adequate control measures in order to avoid stock out. Many approaches have been previously developed by researchers for inventory control purposes, such as a pioneer of mathematical model of inventory system in the form of linear dynamical system which was developed in (Ignaciuk & Bartoszewicz, 2010).

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A more advanced model as a linear hybrid dynamical system was formulated in (Sutrisno et al., 2016) where the system contains discounts on prices. Not only by using mathematical model in dynamical system, but supply chain management is also solved by siphon petri nets which is also interesting to be reviewed (Abdul-Hussin, 2019) and by some developed models from classical models in differential equations form, see e.g. (Pervin et al., 2020).

To control a dynamical system which contains uncertain parameters or disturbances, known as a robust control problem, several approaches can be used as alternatives to solve like robust optimization or robust control method. An alternative of using robust optimization was implemented well in selective newsvendor problem (Abdel-Aal & Selim, 2019). In the other hand, robust control methods have recently been developed. For example, an iterative and Riccati methods were offered in (Hassibi et al., 1999; Xie & Soh, 1993) whereas a robust *H*-infinity control method was implemented to solve switching production-inventory problem (Q. Li et al., 2018). Another approach which was developed from a classical control method is Robust Linear Quadratic Regulator (RLQR) formulated in (Terra et al., 2014). The RLQR control method implemented in our work is the one for controlling linear time invariant state space systems with uncertain parameters. This method finds the optimal control value (or the optimal decision) by minimizing a cost function in quadratic form of the state and input vectors. A more detail explanation of this control technique will be revisited in Section 3.4. From this basic RLQR, some newly methods such as switched RLQR discussed in (Lun et al., 2017) were refined. RLQR performance was illustrated in some research articles related to the mechanical systems (Escárate et al., 2017; Vinodh Kumar & Jerome, 2013; Wang et al., 2018), tracking problem

Table 1. Related works and their contributions

Reference	Uncertain demand	Imperfect Delivery	Perishable products	Multi-supplier	Multi-product	Multi-storage	Price discount	Shortage	Lead time	Method to solve
(Barman et al., 2021)	semi[1]	no	yes	no	no	no	no	yes	no	Fuzzy-based optimization
(Sutrisno et al., 2017)	no	no	no	yes	yes	no	no	no	no	Fuzzy programming
(Rahdar et al., 2018)	yes	no	no	no	no	no	no	yes	unc.[2]	two-stage tri-level optimization
(Dey et al., 2019)	semi	no	no	no	no	no	no	yes	unc.	Derivative-based optimization
(Transchel & Hansen, 2019)	semi	no	yes	no	no	no	no	no	unc.	simulation-based optimization technique
(Yang et al., 2020)	no	no	yes	no	no	no	no	no	no	Derivative based optimization
(Roy, Pervin, & Weber, 2020)	yes	no	yes	no	no	yes	yes	yes	no	Derivative based optimization
(Roy, Pervin, Weber, et al., 2020)	semi	no	yes	no	no	no	no	yes	no	Derivative based optimization
(Nobil et al., 2020)	no	no	yes	no	no	no	no	no	yes	Derivative-based optimization
(Darmawan et al., 2021)	yes	no	no	yes	no	yes	no	no	no	Genetic Algorithm
(Bhatia et al., 2020)	yes	no	no	no	yes	yes	yes	no	no	Forward Dynamic Programming
(Malladi et al., 2020)	yes	no	no	yes	no	yes	no	no	yes	Mixed-integer programming
(Limansyah & Lesmono, 2020)	semi	no	no	no	no	no	yes	no	no	Derivative based optimization
(Luo et al., 2020)	yes	no	yes	no	no	no	no	no	no	Dynamic programming
(Araya-Sassi et al., 2020)	no	no	no	no	yes	yes	no	no	no	Mixed-integer non-linear programming
(Patriarca et al., 2020)	no	no	yes	no	no	no	no	no	no	Monte Carlo simulation
(Q. K. Li et al., 2020)	yes	no	yes	no	no	no	no	no	yes	H_inf Consensus
This paper	yes	yes	no	no	no	no	no	no	yes	RLQR

[1] the value is stated as a function of other variables
[2] the value is uncertain and approached by either probabilistic or fuzzy or another function

on the 3-DOF lab. Helicopter (Sini et al., 2017), UAV attitude controller (Dezhi & Xiaojun, 2017), stall flutter suppression (Niel et al., 2017), slung load transportation (Lee et al., 2017), Quay-Side Crane controlling (Sun et al., 2019), inverted pendulum control problem (Chawla et al., 2019), benchmark problem (Zhou et al., 2019), diabetic problem (Humaidi et al., 2019), and many other problems also used this refined technique. In addition, the optimal input for a problem using RLQR approach, was calculated using several alternative algorithms such as the linear matrix inequality (Doliya & Bhandari, 2018) and semi-definite programming approaches (Huang et al., 2017). Furthermore, some case studies were reported regarding the inventory system like Parking Space Allocation (L. Zhang & Mu, 2018).

The problems mentioned above only covers one uncertain parameter in the model i.e. demand value. An extended model is needed to be developed if the problem contains more than one uncertain parameter. In this paper, a dynamical system is developed which is the extension by including the demand and delivery uncertainty processes. Also, the RLQR is employed to calculate the optimal amount of the product order in the control system. The advantages of the proposed approach implemented in this paper include the ability of the model to handle two uncertain parameters (demand value and the rate of the damaged product amount during delivery), the ability to bring the inventory level to a set-point reference given by decision-maker, and the optimality of the provided decision in the sense of inventory cost. However, some limitations are still carried out by our approach as following. The mathematical model can only carry one product type that supplied by only one supplier.

2. Related Works

Inventory control problem has been studied for decades but it still seems as an interesting and happening problem since the development of the problem especially in its complexity is still growing. Many approaches have been developed by many researchers to study this problem resulting in so many reports available in the literature. Some of them are discussed here in order to show the gap that is filled by this paper. In previous studies, several uncertain parameters including delivery aspects in an inventory model were discussed; however, the behavior represented by those uncertain parameters was different, which provides a uniqueness of each study. For example, uncertainty of delivery terms was considered in (Bagshaw, 2015; Kosorukov et al., 2020) where the delivery duration/lead time was random. In reference (Ignaciuk, 2014), an inventory management with unreliable multiple delivery options was discussed; however, the control method was proposed without optimizing an objective function, and hence the effort was not optimized. Other related studies are summarized in Table 1; many references including this paper and their contributions are shown. Ten aspects showed in the table to compare one reference to another. So far, there is no study that considers both uncertain demand and imperfect delivery processes in one model, where the uncertain demand variable is considered as an additive disturbance and the imperfect delivery process variable is considered as a multiplicative disturbance. This is the situation studied in this paper, which has not been studied in any other papers. Moreover, in this paper, the product acceptance rate, which comes from imperfect delivery processes, is treated as an uncertain/random variable which properly represents its natural behavior.

In particular, mathematical models for inventory control problem are effortless to find the literatures. Each model has a uniqueness to the form or the problem solved by. In other words, a mathematical model is formulated by viewing the situation of the faced problem. The pioneer and simplest mathematical model for inventory control purposes was formulated as a linear state space equation with deterministic parameters (Ignaciuk & Bartoszewicz, 2010a). Following the complexity of the problem, some more advance models was formulated to control this inventory system. For a case containing uncertain parameter(s), a linear dynamical model with uncertain parameter was formulated for problems considering uncertain demand. The control method to calculate the optimal decision, many control algorithms can be used. Sliding mode control method was shown a well performance to control the inventory system with uncertain demand shown by that the sliding mode policy was guaranteed the system response and full demand satisfaction (Ignaciuk & Bartoszewicz, 2011). Other model to handle inventory system with uncertain demand was formulated as a linear state space equation with random parameter stating the random demand where the difference from the previous mentioned model is the method to solve i.e. RLQR and R-MPC (Saputra et al., 2017; Sutrisno, Widowati, & Heru Tjahjana, 2019). Another condition of inventory system developed from the simpler problem is a problem with imperfect delivery process that causing the model contains uncertain parameter in the delivery process. To solve this problem, a linear state space model was also developed where the method to solve was RLQR (Luthfi et al., 2018). Another model was appeared to control a perishable inventory systems in a dynamical system and the optimal policy was calculated by using Markov decision process approach (Y. Zhang & Wang, 2019). By developing the models mentioned above, in this research, the proposed mathematical model contains two uncertain conditions i.e. the demand value and imperfect delivery process which differs from other works.

3. Materials and Methods

In order to describe the problem solved in this paper deeply, a Problem Definition section is considered containing the condition of the faced problem and some hold assumptions. Furthermore, the methodology implemented in this research, the proposed mathematical model, and some related works section are following which are completing the material and method used in this research.

3.1 Problem Definition

In this discussion, an inventory system with a product type is analyzed and periodical reviewed either daily, weekly, monthly or yearly by the decision maker. The product stored in the warehouse is supplied with possible damages during the delivery process from the supplier to the warehouse. Therefore, this means that not all ordered product is stored in the warehouse. The rate of damaged poduct during delivery process is uncertain due to rigorous demand from the buyer which should be satisfied. Unfortunately, the demand value is also uncertain as the decision maker (warehouse/inventory owner) determines the number of products that should be ordered for each time review period. Due to the fact that the demand and the delivery process are containing uncertainty, optimal decision should be determined under these uncertain values.

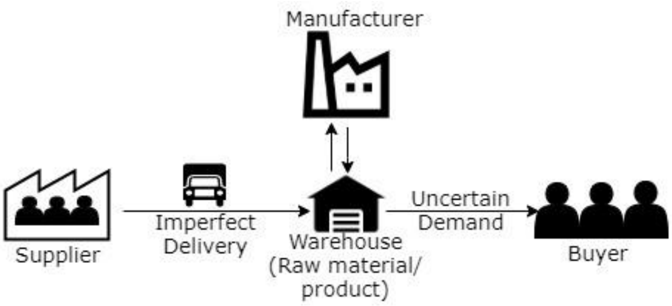

Figure 1. Methodology used in the research.

Table 2. Summary of mathematical notations

Notation	Type	Interpretation
k	Time variable	Review time that could be hour, day, week, and month, etcetera provided by the decision-maker.
$y(k)$	Output/ response variable	Inventory level (unit) at the end of review time k .
t_d	Parameter	delay time (review time duration from ordering to receiving product)
$a(k)$	Random variable	Acceptance rate of product ordered at review time k . This means that $(1-a)$ is the rate of damaged product during delivery. This parameter is unknown and treated as random variable for every k
$d(k)$	Random variable	Amount of product demanded by buyer at review time k. This parameter is unknown and treated as a random variable for every k .
$u(k)$	Decision variable	Amount of product ordered to supplier at review time k .

3.2 Methodology

The product flow of the problem solved in this research is shown in Figure 1. The demand value which is uncertain is approached as a random variable which is corresponds to a probability distribution function. Furthermore, the simulation is approached as normally distributed and is normalized to a mean value of 0 and variance of 1. The same is applicable to the demand parameter, with the percentage of the received/accepted product delivered from the supplier approached as a random variable. The decision variable in this case, is the product volume (units) purchased from the supplier. Following these assumptions, the dynamics of the inventory level is formulated as a linear dynamical system that contains two random parameters where the input is the purchasing product volume and the output is the inventory level.

This model, is formulated as a discrete time linear equation with uncertain parameters used to calculate the optimal decision by applying the RLQR controller. This is conducted by formulating the cost function containing the input value and reference tracking. By solving the corresponding optimization problem, the optimal input/decision is achieved.

3.3 Mathematical Model

Any mathematical model has some assumptions concerning the situations and limitations. To our model, several assumptions below are applied:

1) The product is not perishable over the storing time in the warehouse. This assumption occurred in many fields of applications, and was also applied in many previous studies, see e.g. (Ignaciuk & Bartoszewicz, 2010b; Nahmias & Pierskalla, 2006; Taparia et al., 2019; H. Zhang et al., 2018)
2) The product ordered to supplier at a time will arrive in the warehouse after some constant delay time. This assumption comes from the fact that, in general, some time is needed to transport products from the supplier to the warehouse. Since we deal with only one fixed supplier, it does make sense that the delay time is constant, which was also applied in some previous works e.g. (Darmawan et al., 2021b; Ignaciuk & Bartoszewicz, 2012; Y. Li & Liu, 2021); however, non-constant or uncertain lead time delay could be considered in the problem, this is one of our future works.
3) The demand value and the amount of product that is damaged during delivery were treated as two uncertain parameters in one model. The former was considered as an additional disturbance whereas the latter was considered

as a multiplicative disturbance in the dynamical model we propose. This has not been considered in existing studies.

4) While solving the optimal control, uncertain parameters were treated as random variables with known probability density functions. This is the most common approach used to treat uncertain values based on historical/trial data as it was implemented in many studies considering uncertain parameters, see e.g. (Ben-Tal et al., 2009; Ignaciuk, 2014; B. Li et al., 2011; Prak et al., 2021; Rahdar et al., 2018b).

Let the amount of product stored in the warehouse at the end of the review time $k \in Z^+$ be $y(k) \in R$ and used to illustrate the output vector of the dynamical system. Let at review time equal k, the decision variable $u(k) \in R$ be the amount of the product ordered at k. In this case, the ordered product at k was not immediately received in the warehouse, but it arrived at review time $(k+t_d)$ where t_d is as the delay or transportation time which is the amount of time needed to transport the product from the supplier to the warehouse. Furthermore, not all ordered product is acceptable to be stored in the inventory due to damages during the transportation process.

Let $a(k) \in [0,1] \subset R$ denotes the acceptance rate which is uncertain for each review time. Therefore, the actual product received in the warehouse is $a(k) \cdot u(k)$. At any review time period k, some products where brought out from the warehouse upon request by the buyer (or for other purposes). Let the demand value be $d(k) \subset R$. Table 2 summarize the mathematical notations used in the model. Note that, in real application, all variables in this model should be nonnegative. However, without omitting this situation, these are provided as real numbers by the fact that the value of these variables could be transformed by a coordinate transformation into a big enough value in the calculation so that the smallest possible value could not be negative.

For initial value, at the review time k=0, the inventory is $y(0) = y_0$. If $t_d = 0$, then the dynamics of y is formulated as follows:

$$
\begin{aligned}
&y(0) = y_0 \\
&y(1) = y_0 + a(1)u(1) - d(1) \\
&y(2) = y(1) + a(2)u(2) - d(2) = y_0 + a(1)u(1) + a(2)u(2) - d(1) - d(2) \\
&y(3) = y(2) + a(3)u(3) - d(3) = y_0 + a(1)u(1) + a(2)u(2) + a(3)u(3) - d(1) - d(2) - d(3) \\
&y(k) = y_0 + a(1)u(1) + a(2)u(2) + \cdots + a(k)u(k) - d(1) - d(2) - \cdots - d(k).
\end{aligned}
$$

If $t_d > 0$, then the dynamics of the inventory level $y(k)$ is formulated as follows:

$$
y(k) = y_0 + a(1)u(1) + a(2)u(2) + \cdots + a(k-t_d)u(k-t_d) - d(1) - d(2) - \cdots - d(k) \tag{1}
$$

Where $0 = [a,u](1-t_d) = [a,u](2-t_d) = \cdots = [a,u](0)$

If there is no product order before the initial review time or they can be provided with the product order at the corresponding review time before the initial review time and its corresponding acceptance rate if the decision maker ordered some amount of product before the initial review time. This dynamical equation of the inventory level y is then will be rewritten in the general form of linear state space equation (5) so that it will suit the RLQR. To achieve this, the equation of the demand $d(k)$ is redefine as the function of auxiliary state $x_1(k) \in R$ satisfying $x_1(k+1) = x_1(k), x_1(0) = 1$. This leads to the solution $x_1(k) = 1, \forall k$. Then the dynamical equation of the demand value can now be redefined as $d(k) := d(k)x_1(k)$. Now, let $x_2(k) = y(k)$ and $x_i(k) = u(k - t_d + i)$ for every $i = 3, \ldots, n$ where $-t_d + i$ is so called delay time of input signal u. Therefore, equation (1) may be expressed as

$$\left.\begin{aligned} x_1(k+1) &= x_1(k) \\ x_2(k+1) &= x_2(k)+x_3(k)-d(k)x_1(k)=x_2(k)+x_3(k)-d(k)x_1(k) \\ x_3(k+1) &= x_4(k) \\ &\vdots \\ x_{n-1}(k+1) &= x_n(k) \\ x_n(k+1) &= a(k)\cdot u(k) \\ y(k) &= x_2(k). \end{aligned}\right\} \quad (2)$$

Now, consider $x(k)=[x_1(k),x_2(k),\ \dots\ ,x_n(k)]^T$ that so called the state vector of the system. Then, system (2) can be rewritten as the following equation

$$x(k+1)=\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_n(k+1) \end{pmatrix}=\begin{pmatrix} x_1(k) \\ x_2(k)+x_3(k)-d(k)x_1(k) \\ x_4(k) \\ \vdots \\ x_n(k) \\ a(k)u(k) \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -d(k) & 1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}\begin{pmatrix} x_1(\mathrm{k}) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ \vdots \\ x_n(k) \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ a(k) \end{pmatrix}u(k) \quad (3)$$

$$y(k)=\left(0,1,0,0,\dots,0\right)x(k). \quad (4)$$

We could see the system above as a discrete-time linear state space system with parametric uncertainties as the following

$$x(k+1)=\left\{F+\delta F\right\}x(k)+\left\{G+\delta G\right\}u(k),\ k=0,1,2,\dots \quad (5)$$

where

$$F=\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix},\ \delta F=\begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ -d(k) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix},\ G=\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix},\delta G=\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ a(k) \end{pmatrix},$$

$\left[\delta F,\delta G\right]=H\Delta\left[E_F,E_G\right]$, and matrices δF and δG are denoting matrices of uncertainty with known matrices $H \epsilon R^{n\times 1}$, $E_F \epsilon R^{1\times n}$, $E_G \epsilon R^{1\times m}$ and $-1\le\Delta\le 1$ is random. Note that d and a are contained in matrices δF and δG respectively and that the randomness of demand value d and product acceptance rate a is now represented by random matrices δF and

δG respectively. Furthermore, they represented by random parameter Δ from equation $\left[\delta F, \delta G\right] = H\Delta\left[E_F, E_G\right]$. The mathematical modeling process of the inventory level y is now complete. Next, the RLQR control method is revisited to control the derived dynamical equation.

3.4 Robust Linear Quadratic Regulator (Revisiting)

Consider the linear discrete time state space system with uncertain parameters in the general form(6). The controlling purpose in this case is regulating the state vector x i.e. bringing the state x to zero as soon as possible with minimal effort. This effort is represented by the cost function defined by the controller. Moreover, a regulator can be utilized as a set-point controller by transforming the original state vector x to $\hat{x} = x - x_r$ where x_r is the set-point or reference point that wanted to achieve. This is a basic concept in control system theory. Furthermore, trajectory tracking purpose can also be utilized by defining the trajectory as set of reference points. According to this concept, in this paper, we only consider set-point control problem. One can surely develop the trajectory tracking control problem based on the results produced in this paper.

Robust Linear Quadratic Regulator, a very popular control method in control systems, is then implemented in this paper. The superiorities of RLQR compared to other methods include the simplicity of it's concept i.e. it works with a quadratic objective function that is easier to solve the corresponding optimization problem. We adopt the RLQR from \cite{bib10}.

It is implemented to control the system (7) under condition that assumption of $rank\left(\begin{bmatrix} E_F & E_G \end{bmatrix}\right) = rank\left(E_G\right)$ is applied.

The optimal input value of input vector u is achieved by solving the following optimization problem

$$\min_{u(k)} \max_{\delta F, \delta G} \left\{ J^{\mu}\left(x\left(k+1\right), u(k), \delta F, \delta G\right) \right\} \tag{8}$$

s.t. (5) where the objective function J is defined as

$$\begin{aligned} J^{\mu}\left(x\left(k+1\right), u\left(k\right), \delta F, \delta G\right) = & \begin{bmatrix} x\left(k+1\right) \\ u(k) \end{bmatrix}^T \begin{bmatrix} P(k+1) & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} x\left(k+1\right) \\ u(k) \end{bmatrix} + \left\{ \left(\begin{bmatrix} 0 & 0 \\ I & -G \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\delta G \end{bmatrix} \right) \begin{bmatrix} x\left(k+1\right) \\ u(k) \end{bmatrix} \right. \\ & \left. - \left(\begin{bmatrix} -I \\ F \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ \delta F \end{bmatrix} x(k) \right) \right\}^T \begin{bmatrix} Q & 0 \\ 0 & \mu I \end{bmatrix} \left\{ \left(\begin{bmatrix} 0 & 0 \\ I & -G \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\delta G \end{bmatrix} \right) \begin{bmatrix} x\left(k+1\right) \\ u(k) \end{bmatrix} \right. \\ & \left. - \left(\begin{bmatrix} -I \\ F \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ \delta F \end{bmatrix} x(k) \right) \right\}, \end{aligned} \tag{9}$$

where $Q \succ 0, R \succ 0$ and $P(k+1) \succ 0$ are weighting matrices which are definite positive (presumed so that the objective function is convex and that the minimizer is guaranteed to exists) and $\mu > 0$ is so called penalty parameter. This optimization problem is solved for $\mu \to +\infty$ by defining the initial value for $x\left(0\right)$ and $P\left(N+1\right) \succ 0$ using RLQR algorithm in (Terra et al., 2014). The optimal solution for $u^*(k)$, $k = N, \ldots, 0$ is achieved from

$$\begin{pmatrix} x^*(k+1) \\ u^*(k) \end{pmatrix} = \begin{pmatrix} L \\ K \end{pmatrix} x^*(k) \tag{10}$$

Where $L \in R^{n\times n}$, $K \in R^{m\times n}$, and $P \in R^{n\times n}$ are derived from

$$\begin{pmatrix} L \\ K \\ P \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -I \\ 0 & 0 & F \\ 0 & 0 & E_F \\ I & 0 & 0 \\ 0 & I & 0 \end{pmatrix}^T \begin{pmatrix} P(k+1)^{-1} & 0 & 0 & 0 & 0 & I & 0 \\ 0 & R^{-1} & 0 & 0 & 0 & 0 & I \\ 0 & 0 & Q^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & -G \\ 0 & 0 & 0 & 0 & 0 & 0 & -E_G \\ I & 0 & 0 & I & 0 & 0 & 0 \\ 0 & I & 0 & -G^T & -E_G^{-1} & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ -I \\ F \\ E_F \\ 0 \\ 0 \end{pmatrix}.$$

4. Computational Experiment

To illustrate the model and how the decision is calculated, we perform computational experiment here. The data were randomly taken but reasonable based on our practical experience on inventory management.

4.1 Parameter Setting

Consider the inventory system described in the problem definition section which is modeled as (5) with the following parameters:

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, G = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

where δF and δG are randomly generated. Let the lead time delay is 3 periods and the initial inventory level is 50 units. In this experiment, the weighting matrices $P = I_4$, $Q = I_4$, $R = 1$ were utilized. Let the reference inventory level which the decision maker be 25 units, by solving (8), substituting the value of matrices L and K into (10), the optimal decision for u and the response value for y shown in Figure 2 and 3 are derived.

4.2 Results and Discussions

MATLAB 2018b has been employed to do all calculations in this experiment whereas the computer's specification used in this research has Dual Core 3.0 GHz of processor and 4 GB of memory. This is importance since a computational calculation really depends on the specification of the computer operated in the experiment and thus different specification may consume different sources such as computational time. The optimal input of the system that also represented the optimal decision of the given inventory system i.e. the product volume (units) which should be purchased to the supplier is shown in Figure 2. This was produced by the RLQR's optimal input value from (8). By following this decision, the response or the output of the given inventory system i.e. the product stock level is achieved as illustrated in Figure 3. This shows the evolution of the inventory level i.e. product units stored in the warehouse from the initial to the final simulation time. It can be seen from this figure that the initial inventory level is 50 units and decreased following demand satisfying and reference tracking. At period 20, the inventory level was sufficiently close to the reference point, but it fluctuated through period 20 to 60 due to uncertain demand and received product from the supplier. After period 60, the demand value of the model was simulated as zero, the response to the purchased product became zero, while the inventory level was close to 25 units.

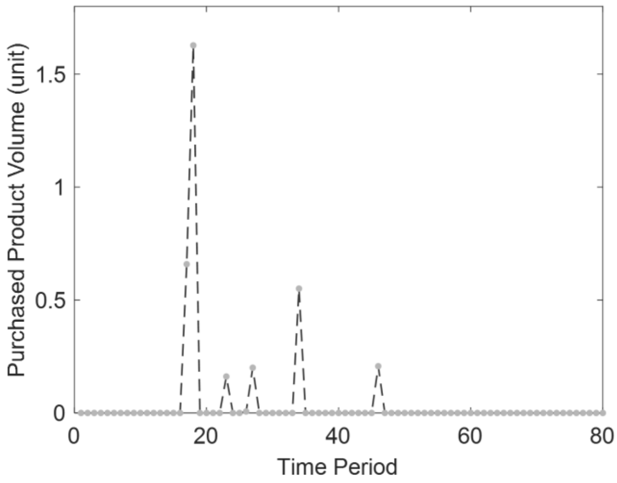

Figure 2. Purchased product volume.

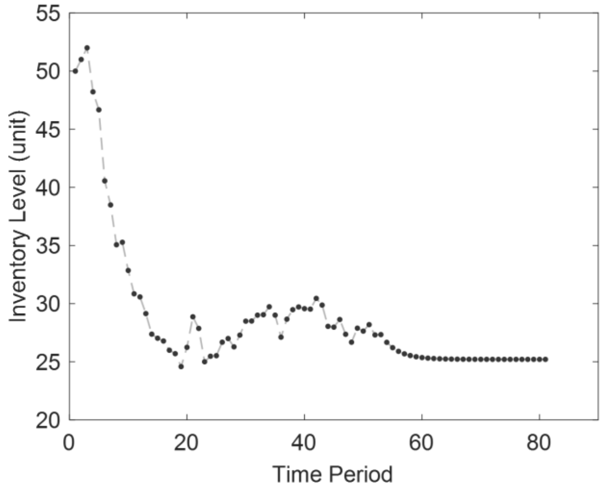

Figure 3. Inventory level

First, for validation purposes, the calculation results shown in Figure 3 is compared to some existing results by previous works. In many inventory control problems, the purpose of controlling is bringing the stock level to some desired/reference point which also conducted in this paper. By referring to Figure 4 in (Ignaciuk & Bartoszewicz, 2011), Figure 1 in (Luthfi et al., 2018), Figure 2 in (Saputra et al., 2017), Figure 3 in (Sutrisno et al., 2018), Figure 2 in (Sutrisno, Widowati, & Tjahiana, 2019), the performance of the used control method is analyzed by observing the inventory level where all those results were shown well performance i.e. the inventory level followed the given set/reference point. Next, the superiority of the proposed model in this paper is that the model can handle two uncertain parameters that are demand value and the acceptance rate of the delivered product amount. Some newly published papers are also provided two uncertain parameters (see e.g. (Dey et al., 2021; Mishra et al., 2021; Sarkar et al., 2020; Sett et al., 2020)), however, the provided parameters were not the demand and the acceptance rate of the delivered product amount and hence the situations discussed in those papers are different. Then, in managerial insights point of view, the model proposed in this paper actually can be implemented for all kind of products as long as the product will not expire during storing periods. Besides that, the reference point may differ for each review time period. Finally, before the optimal decision is implemented, the decision maker can simulate first the response of the system computationally because the computational time is sufficiently short. In the simulation with eighty review time performed in this paper, the computation took only around five seconds. By knowing the computational time, the decision-maker can decide whether he/she has enough time or not to run the computational simulation before the action is taken.

By the fact that the proposed model in this paper has some limitations, but nevertheless, future researchers may use the results achieved here e.g., for developing new improved models and for searching new method to calculate the optimal decision and then comparing the new results to results achieved in this paper. In particular, practitioners can still employ the model formulated in this paper and implemented the achieved decision even though they can still do some improvisations according to their instincts or based on the managerial insight points in the following.

4.3 Managerial Insights

Some managerial insights can be drawn from the results. Those include:

1) This an inventory model that can be used to control the inventory level by bringing the inventory level to a certain level referred by the decision-maker. This reference point is suitable keep the inventory level "safe" that can be possibly used to satisfy unpredicted demand.
2) At any time, the decision-maker can change his/her reference point. By re-calculating the model, the new optimal decision will be achieved.
3) The decision-maker could actually extend the number of future review time in the computation. The longer the future review time included the better the decision would be achieved but the longer the computational time needed.
4) The better the computer's specification used in the calculation, the faster the computational time. However, computer with better specification costs more expensive.

5. Conclusions

A linear state space model containing two uncertain parameters was considered in this paper to solve an optimal control problem of inventory system with uncertain demand and imperfect delivery. The optimal decision was calculated with the RLQR approach for set-point tracking purposes of the inventory level. Computational simulations were illustrated on how to solve the problem using the proposed model. The optimal decision was achieved and the inventory level was followed the reference level given by the decision-maker. From these results, we conclude that our proposed model has been worked well even though it still has limitations including it works only with one product type and with one supplier. It also contains only one warehouse. Therefore, further researches are still needed to develop the model. By concerning to these limitations, there are some future research prospects regarding the results achieved in this paper. Our future research will develop a model which capable to handling a case of multiple supplier and multiple product type. Furthermore, other control methods such as robust MPC will be performed and compared in order to find the best method used to solve the problem.

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