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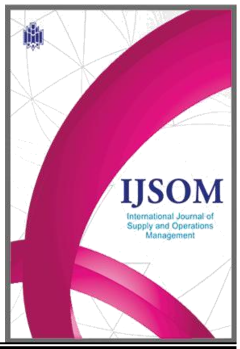

Multi-objective Design of Balanced Sales Territories with Taboo Search: A Practical Case

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Abstract

Sales territory design is an important research field because salesforce allocation within territories impacts sales organization effectiveness and customer service. This work presents a novel multi-objective model for re-designing sales territories with three main objectives: sales balancing, workload balancing, and geographic balancing. To measure sales and workload balancing, the variance among territories was calculated. The metric considered for geographic balancing was the sum of the distances from every salesperson to their assigned customers. A metaheuristic algorithm based on Tabu search was developed to solve a weighted aggregate function that integrates the three objectives. The algorithm is embedded in a procedure to systematically change the weights in the aggregate objective function to produce an approximate Pareto front of solutions. The algorithm was tested with instances based on data from a company in Mexico, providing salesperson-customer assignments that can be projected in territories in geographic information systems. The algorithm converges very fast for the instances studied and produces a Pareto front efficiently. Comparing the current situation of the company to a dominating solution obtained with the algorithm in the Pareto front, a significant improvement in the balance is achieved, in the order of 42.0 - 47.1% on average in the three objective functions. Another managerial benefit achieved by the company was a better understanding for the top managers of the salesforce, the customer preferences, and the challenge of serving a large and dispersed market.

Keywords: Territory design; Multiple criteria;; Metaheuristics; Pareto front; Salesforce; Workload balance.

1. Introduction

The work presented here describes the models and methods used to solve a problem for a company. This company sells hand tools and building materials to hardware stores in Mexico. The market is formed by low volume stores dispersed geographically in the country. To achieve the task of selling to near 3800 customers the company has a salesforce of 40 people. With a high-speed expansion in the last years, the efforts of the sales managers have been aimed at growing the number of customers, paying to the salespeople a base salary plus a commission related to the sales volume. The salesforce had freedom to explore new cities and towns to expand their own customer base. This strategy led to a disorganized sales market, in terms of geography and work balance. This would have been acceptable, but recently the customers claimed poor service from the salespeople, with long periods between visits. Although the customer base was large, that was not converted into higher sales because of the poor service. The analysis of the problem considered that the causes of the poor service were the long distances that some salespeople had to drive, and the high unbalance of the workload.

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Then, a first approximation to solve the problem, described in this work, was to re-organize the salesforce to modify customers' assignment to each salesperson, a problem addressed in the literature with the name of territory design.

Territory design is the problem of grouping small geographic areas called basic areas into larger geographic clusters called territories or districts in a way that they meet relevant planning criteria (Kalcsics et al., 2005). Territory design has been applied in several fields, including politics, education, public services, and logistics (Kalcsics et al., 2005). Recent research has been performed on the territory design problem with commercial approaches focused on product distribution (González-Ramírez et al., 2011; Moreno et al., 2020; Moya-Garcia and Salazar-Aguilar, 2020; Zarrinpoor, 2018; Ríos-Mercado and López-Pérez, 2013; Salazar-Aguilar et al.,2011). Within this context, the sales territory design problem consists of assigning a group of customers or geographic areas to a salesperson. Significant benefits can be achieved by matching the appropriate salesperson to a customer. Among these benefits, the following can be mentioned:

- Reduction of sales expenses;
- Increased market coverage;
- Improvement of both customer satisfaction and customer relations;
- Better performance evaluations;
- Establishment of the salesperson’s responsibilities;

However, the establishment of sales territories is not an easy task. Particularly for commission-based sales organization, the territory allocations impact the individual’s commission potential (Piercy et al., 1999). Also, it may result in an excessive workload for some salespeople. These aspects should be considered within the design of sales territories to reduce the adverse effects on commission-based incomes and sales workload while improving sales organization effectiveness (Gopalakrishna et al., 2016). In this situation, a multi-objective territory design can be considered as a more appropriate approach for the sales territory design problem. In this work, the sales territory design problem is extended from (Correa-Medina et al., 2011) to consider the re-design task.

A mathematical model with three objective functions (multi-objective function) is proposed for designing balanced sales territories. This model incorporates the re-design approach taking into account the fixed locations of the bases of salespeople. The objectives are the geographical balance of the territories, the balance of the workload, and the balance in the total sales amount.

Given the non-linearity of the model, a metaheuristic algorithm was developed based on Tabu Search (TS) because of the difficulty to solve it with commercial optimization software. The TS algorithm considers a weighted aggregate function. The solutions for eight instances were presented. These data come from a real case of a Mexican company that produces and trades hand tools and building materials along with the country.

For a priori approach, where the weights of the objective functions are fixed, the solutions are obtained for the eight instances. In an a posteriori approach, the TS component is embedded in a routine to change the weights of the objective functions systematically, and a Pareto front is constructed with the resulting solutions. The algorithm provides a number of solutions in the Pareto front for each of the instances. The current assignment used by the company is compared to the closest solution in the Pareto front for each instance.

The following contributions are highlighted:

- The mathematical model of the sales territory design problem was modeled with a multi-objective function. This function is a weighted aggregate function that linearly integrates the functions of each identified objective. Based on a real case study, three main objectives were identified for optimization: (1) geographical balance of territories, (2) balance of workload, and (3) balance of sales between salespeople. This model extends the bi-objective approach presented in (Correa Medina et al., 2011) for the optimization of salesforce planning and territory partitioning.
- The model of the sales territory design problem considers an already established salesforce where salespeople have specific locations from where to serve customers. In order to preserve the salesforce, the new

configuration will be constructed around those sale base. Hence, in some way, this problem can be considered as a re-design problem.

- A metaheuristic algorithm based on Tabu Search was developed to solve the multi-objective function. This algorithm was developed because the non-linear components of the multi-objective function make difficult the solving process with commercial optimization software. The algorithm constructs a Pareto front efficiently, and at least one of the solutions obtained dominates the solution corresponding to the current assignment of the company.

The present work is structured and described as follows: a literature review is presented in the second section with special emphasis on multi-objective territory design problems. The third section presents an overview of the characteristics and needs of the company considered for the case study. Then, in the fourth section, the mathematical model's details with the multi-objective function and its metrics are presented and discussed. The solving algorithm based on Tabu Search for the multi-objective model is shown in the fifth section. Results on different instances of the case study are presented and discussed in the sixth section. Finally, conclusions and future directions for the present work are presented in the last section.

2. Literature review

The review by Kalcsics (2015) explains districting applications with their relevant criteria and solution techniques. For the specific case of Sales Territory Design, balance, contiguity, and compactness are identified as the "classical" criteria to be considered. Balance is usually measured and optimized in terms of workload and sales potential, but only a few works consider more than one balancing criterion at the same time. Usually, the number of territories is predetermined, but some flexible approaches are described. Also, Kalcsics (2015) mentions that there is no agreement in the literature as to whether predetermined locations should be kept or be determined by the design process. The balancing functions described by Kalcsics (2015) do not include the ones proposed in our study, similar to a standard deviation computation. The methods applied to solve districting problems are categorized in mathematical programming, heuristics, and metaheuristics. In the case of metaheuristics, Simulated Annealing, GRASP, Genetic Algorithms, and Tabu Search have been implemented. However, only two works were mentioned applying Tabu Search (Bozkaya et al., 2003; Ricca and Simeone, 2008). It is important to mention that Kalcsics (2015) does not make an emphasis on the use of multiobjective approaches.

In recent years, metaheuristics have been proposed as solving methods due to the complexity of the sales territory design problem when several constraints are included. Also, metaheuristics have been proposed for implementations when the size of the instances is large. However, in most of the cases, the problems have been studied with single objective models. The analysis of multi-objective problems in the area of districting has attracted increasing interest. Because of the approach presented by this work, a discussion on recent literature related to multi-objective territory design is presented.

A political districting problem was analyzed in Bozkaya et al. (2003), where a weighted function combined five objectives. The first objective balanced the population of the districts with tolerance. The compactness was the second objective, minimizing the sum of the perimeters of the geographic units assigned to the district. The standard deviation of personal incomes of the population assigned to a district was the third objective to be minimized. The fourth objective minimized the changes of the territories with respect to a current partition in terms of the overlapping of areas of the districts. The last objective minimized the changes of the territories with respect to a current partition in terms of the population of the districts. Authors proposed a Tabu Search method embedded into an Adaptive Memory Procedure to solve the problem with a single weighted objective function. Most of the weights in the objective function were fixed, and only one was adapted through the iterations according to feasibility issues. The study was not concerned about constructing a Pareto front of the solutions.

Ricca and Simeone (2008) addressed a political districting problem. Their objectives were to minimize population inequality, non-compactness, and nonconformity to administrative boundaries. The metric for population equality was the absolute difference of the district population with respect to the average population. Compactness was measured as the maximum distance into the districts. Conformity to administrative boundaries was measured by the percentage of units that belonged to different administrative areas in the same district. District centers were not fixed. The aim of the study was to compare four local search heuristics: basic Descent algorithm, Tabu Search, Simulated Annealing, and the Old Bachelor Acceptance algorithm. The approach used a weighted function that combined all the objectives in order to generate a single solution for fixed weights. Their results were favorable to the Old Bachelor Acceptance algorithm.

A bi-objective optimization model was developed by Salazar-Aguilar et al. (2013) for an application in the distribution of bottled beverages. One of the objectives was the minimization of the sum of distances in each territory, for compactness. The other objective was the minimization of the maximum absolute difference with respect to the average of the number of customers in each territory, for balance. The number of territories was fixed, but the centers of the territories were not defined. The balance of sales was considered as a soft constraint. A GRASP scheme with aggregated objective functions was considered. The objective functions were aggregated in a weighted function with variable values through the iterations in order to construct a Pareto front.

Silva de Assis et al. (2014) addressed a re-districting problem to design territories for power-meter reading. In the re-districting approach, the districts centers are fixed, and the number of customers that can change of district is restricted by a constraint. This service problem was approached with two objectives. The first objective minimized the sum of the maximum distance into each territory as a metric for compactness. The second objective minimized the sum of workload deviations from the target values as a metric for balance. The authors solved the problem using GRASP with a greedy function that combined both objectives in a weighted sum. The weights were changed during iterations to generate the Pareto front with the obtained feasible solutions.

A multi-criteria Police districting problem was studied by Camacho-Collados et al. (2015) as a public service problem. One solution to the problem was evaluated considering four attributes. The first attribute was the area assigned to agents. The second attribute was the distance between territories that allowed for rapid support from another district in case of need and emergency. The third attribute was the crime risk associated with the territories. The last attribute was the maximum distance into a district. These attributes were normalized and combined in a weighted function. The decision maker decided the weights of the attributes such that the problem could be solved as a single-objective model once these were defined. Then, a solving method that consisted of a randomized greedy heuristic followed by a local search algorithm was developed. A single solution was obtained for a combination of weights defined a priori by the decision maker.

The problem of districting a health care system in a region of Brazil was tackled by Arns Steiner et al. (2015) with three objectives. One objective maximized the population homogeneity in the districts. Another objective maximized the variety of medical procedures offered in the territories. The last objective minimized the inter-district distances. The first objective described accounts for the balance of the number of inhabitants attended by a district, using an average absolute deviation function. The second objective maximized the maximum number of medical procedures assigned to a district. The third objective minimized the minimal distance between geographical units of the different districts, weighted by the number of inhabitants of the units. The model did not explicitly consider compactness, but it regards an embedding problem that was not reported in previous literature. The authors proposed a multi-objective genetic algorithm based on NSGA-II to construct an approximate Pareto front.

Lei et al. (2016) solved a multi-objective dynamic stochastic districting and routing problem in a distribution context. The problem had four objectives, and constraints related to routing. The dynamic aspect of the problem was treated with a multi-period approach. The stochastic aspect was associated with the presence of stochastic customers in each period added to regular customers. The first objective was focused on balancing the number of districts along the planning horizon. Compactness, computed as the sum of the perimeters assigned to each district, was minimized in the second objective. The third objective function minimized the change of the territories between periods. The last objective minimized the absolute deviation from average profit over the time periods. The routing aspect was addressed by means of an equation for estimation of the solution to the traveling salesman problem involved. The authors proposed a preference-inspired co-evolutionary algorithm (PICEA) to solve the problem. Their results were compared with the implementation of NSGA-II.

Bender et al. (2016) propose a model for territory design, which includes scheduling decisions and with an emphasis on solving that subproblem using a location-allocation heuristic. Later Bender et al. (2018) address a variant of the problem solving with a branch-and-price algorithm. Although in both models, they have a weighted function for two objectives, their computational experiments use fixed values for the weights producing single optimal solutions.

Sudtachat et al. (2020) studied a relocation and districting problem with two objectives to re-design an Emergency medical system. One objective is to maximize the total expected coverage for the EMS system, and the second objective is to maximize the covered calls within a pre-specified response time threshold. They use a matheuristic with a mathematical model solving the districting problem embedded into a tabu search algorithm to solve the relocation. Nevertheless, the problem has two objectives, the structure of the method decomposes hierarchically the problem to obtain single solutions.

Moya-Garcia and Salazar-Aguilar (2020) studied a problem for territory design with the purpose of determining the size of the salesforce. Their model has a weighted combined function with two objectives. One objective minimizes the total number of territories, and the other minimizes the dispersion of the territories. The problem is solved for fixed weights of the objective functions using a heuristic procedure that creates routes first and clusters later.

As described by these recent works, the combination of characteristics of the study presented in this paper is unique. Some novelties are the following:

- There are few studies concerning the application of territory design or districting to the sales activity. Even in those cases, the problem is addressed as a single-objective situation or producing single solutions instead of Pareto fronts.
- The nature of the objectives in multi-objective districting problems is very different in all the literature reviewed. Therefore, the combination of objectives considered in this work is specific.
- The use of a multi-objective version of Tabu Search has not been reported in the literature for the Sales Territory Design problem to produce a Pareto front of solutions.
- Also, although some examples in the literature consider the re-districting approach with fixed territories center, it is not common in the studies described above.

3. Problem Description and the Case Study

This work presents the experience of a research project aimed to solve a consultancy for a Mexican company that manufactures and trades hand tools and building materials. This company has approximately 3800 customers who are grouped into nine regions for sales and distribution throughout Mexico. These regions are covered by 40 salespeople.

Each salesperson has a base that is located in a city or town within the region, and it may be different from the location of another salesperson in the same region. Nevertheless, in a large city, more than one salesperson can be located.

For operational reasons, the customers are grouped into Sales Coverage Units (SCUs), which are usually towns, cities, or sections of a city. Hence, each region has a number of SCUs which are served by a salesperson. The SCUs which are served by a salesperson are grouped into "territories."

Although the company has already performed a territory partitioning, a re-design of the territories is required to improve the service level in the sale activities due to the following changes in the market:

- New customers;
- Customers closing their businesses;
- Customers that do not buy anymore;
- Evolution of the purchasing behavior of the customers;
- Overlapping of territories.

These changes have caused a decline in customer service in some territories because of unbalanced workload and long travel distances for customer's coverage. Although territories and assignments already exist, modifications in each region must be performed to address these changes and to find an optimal assignment of each SCU to each salesperson.

In order to accomplish the optimal assignment, the problem was addressed as a multi-objective optimization problem. Particularly, three objectives were identified:

- The geographical balance of the territories: the sum of the distances for every salesperson in the territory must be minimized.
- The balance of the workload: the sum of the time spent on the SCUs assigned to each salesperson must be balanced to achieve homogeneity.
- The balance of the total sales for each salesperson: the total sales of the SCUs for the new assignments must be balanced to achieve a sense of equality among the salespeople.

The geographical balance was considered in terms of distance. It maybe practical in other cases to examine the use of travel time instead of distance. In the case that time or distance must be selected, it is expected that time will change dynamically according to traffic conditions and vehicle performance, making it necessary to use an expected time, while distance will not change. Also, if both metrics could be included, is expected a positive correlation between time and distance such that few solutions may come up from this conflict. Thus it was preferred to use distance, but it is not an obstacle to using travel time instead.

Homogeneity is an important concept for sales and workload balance because (1) it provides a sense of justice among salespeople, and (2) it improves the possibility of having a good service level for the customers. It is assumed that the parameters for workload and sales are averaged from the historical data for a certain time period (e.g., a semester). The details of the multi-objective problem are presented in the next section.

4. Multi-objective Assignment Model

In each region, salespeople are grouped in set V. Each salesperson $i \in V$ will have a sales territory. Also, each region is formed by the set C of Sales Coverage Units (SCUs) which are indexed by $j \in$ C. The company sold s_j money amount to SCU j in the period studied. The records also indicate the workload in number of hours w_j dedicated to selling to SCU j in the same period. The notation is described following:

Sets:
C: Set of Sales Coverage Units (SCUs) to be assigned
V: Set of Salespeople

Indices:
i: Index for salesperson $i \in V$
j: Index for SCU $j \in C$

Parameters:
d_{ij}: Shortest road distance between salesperson i and SCU j
s_j: Money amount sold to SCU j
w_j: Workload (in hours) required to serve SCU j
μ_w: Workload average (in hours) by salesperson
μ_t: Average sale amount by salesperson
α_k: Weights for each objective function f_k in the aggregate function Z such that $k = \{1, 2, 3\}$

Decision variables:
x_{ij}: Binary variable that takes value of $x_{ij} = 1$ if salesperson i is assigned to SCU j and $x_{ij} = 0$ otherwise

Auxiliary variables:
Tn_i: New total sales for salesperson i
Wn_i: New workload for the salesperson i
D: Sum of distances that every salesperson must travel into their territory
f_1: First objective function related to sales homogeneity
f_2: Second objective function related to geographic homogeneity
f_3: Third objective function related to workload homogeneity
Z: Aggregate function

Some assumptions of the model are the following:

- the parameters are static or valid during a long period. Therefore, the re-design activity can be done each six-months or each year.
- The workload dedicated to a SCU is not related to the sales produced. The salesperson has enough time to serve all the customers assigned, i.e. there are not capacity constraints.
- One SCU can be composed by many dispersed customers, but the local traveling distance is not considered.

Dividing the total sales by the number of salespeople, the average of sales μ_t is obtained as follows, in Equation (1):

$$\mu_t = \frac{\sum_{j \in C} s_j}{|V|} \qquad (1)$$

The same computation can be done for the workload. Dividing the total number of hours by the number of salespeople, the average workload μ_w is obtained as follows, in Equation (2):

$$\mu_w = \frac{\sum_{j \in C} w_j}{|V|} \qquad (2)$$

The territory of salesperson i is defined by all the SCUs $j \in C$ assigned to the salesperson. This decision is expressed by binary variable x_{ij}, such that it takes the value of one if SCU j is assigned to salesperson i, and it takes the value of zero otherwise.

For a given territory, certain metrics can be computed. The total sales Tn_i for salesperson i can be computed as follows, in Equation (3):

$$Tn_i = \sum_{j \in C} s_j x_{ij}, \forall i \in V \tag{3}$$

The total workload Wn_i for salesperson i can be computed as follows, in Equation (4):

$$Wn_i = \sum_{j \in C} w_j x_{ij}, \forall i \in V \tag{4}$$

The sum of distances D that every salesperson must travel into their territory is calculated in Equation (5):

$$D = \sum_{i \in V, j \in C} d_{ij} x_{ij} \tag{5}$$

Then, three objective functions are defined. The first objective function minimizes the standard deviation of the total sales of the new assignments. This is performed to preserve the homogeneity of the sales income among the salesforce. This is expressed as follows in Equation (6):

$$min f_1 = \sqrt{\frac{\sum_{i \in V} (Tn_i - \mu_t)^2}{|V| - 1}} \tag{6}$$

The second objective function minimizes the sum of distances for every salesperson from the sales base, in Equation (7). This, in order to induce the geographic homogeneity of the territories. Since the sum of distances is minimized, it is expected that the optimal configuration will be that of compact territories (Eq. 7):

$$min f_2 = D \tag{7}$$

The third objective function minimizes the standard deviation of the workload of the new assignments to achieve the workload homogeneity, in Equation (8):

$$min f_3 = \sqrt{\frac{\sum_{i \in V} (Wn_i - \mu_W)^2}{|V| - 1}} \tag{8}$$

Finally, it is necessary that each SCU be assigned to just one salesperson, as shown in Equation (9). The single assignment is convenient for customer service and operational reasons, because the customer is loyal to the salesperson, and because the company and the customer are communicated by a single channel (Eq. 9):

$$\sum_{i \in V} x_{ij} = 1 \; \forall j \in C \tag{9}$$

The non-linear integer programming model is defined by equations 1-9. In order to integrate the three main objective functions, an aggregate function Z was defined. This multi-objective function integrates Equations (6), (7), and (8) by means of a weighted sum which linearly combines all three objectives. This weighted combination is based on the procedure presented by Ehrgott (2005) for multi-criteria optimization.

Although the integration of all objective functions is linear, the objective functions defined by Equations (6) and (8) are not linear. This non-linearity in the computation of the standard deviations and the integrality of the decision variables makes it very hard to solve the multi-objective model using commercial optimization software. Most optimization software are not able to obtain a global optimal solution in a reasonable time, and a true efficient Pareto front cannot be obtained for medium to large-sized instances. Hence, a metaheuristic was developed to solve the model with the associated constraints. The details of this metaheuristic are presented in the next section.

5. The Tabu Search Metaheuristic

The multi-objective function integrates the objective functions f_k defined by Equations (6), (7) and (8) with $k = \{1, 2, 3\}$ respectively by means of weights α_k as defined by the approach presented by Ehrgott (2005) in an aggregate function Z to be minimized, in Equation (10):

$$min Z = \alpha_1 \cdot f_1 + \alpha_2 \cdot f_2 + \alpha_3 \cdot f_3 \tag{10}$$

Hence, the solution of this aggregate function depends on these weights α_k. A metaheuristic based on Tabu Search (TS) was developed to solve the multi-objective problem testing several weight combinations.

The core of the TS algorithm is presented in Algorithm 1, for a fixed combination of weights. An initial solution is generated as follows: every SCU $j \in C$ is assigned to the nearest salesperson $i \in V$ to create an initial advantage with respect to the geographic objective. Thus, the solution structure in the algorithm is an array of length equal to the number of SCUs and each element takes the value of the unique salesperson assigned to the SCU. Fig. 1 illustrates the solution structure, where the shaded red elements are the SCUs flagged as tabu that cannot be replaced. The algorithm is described in detail below.

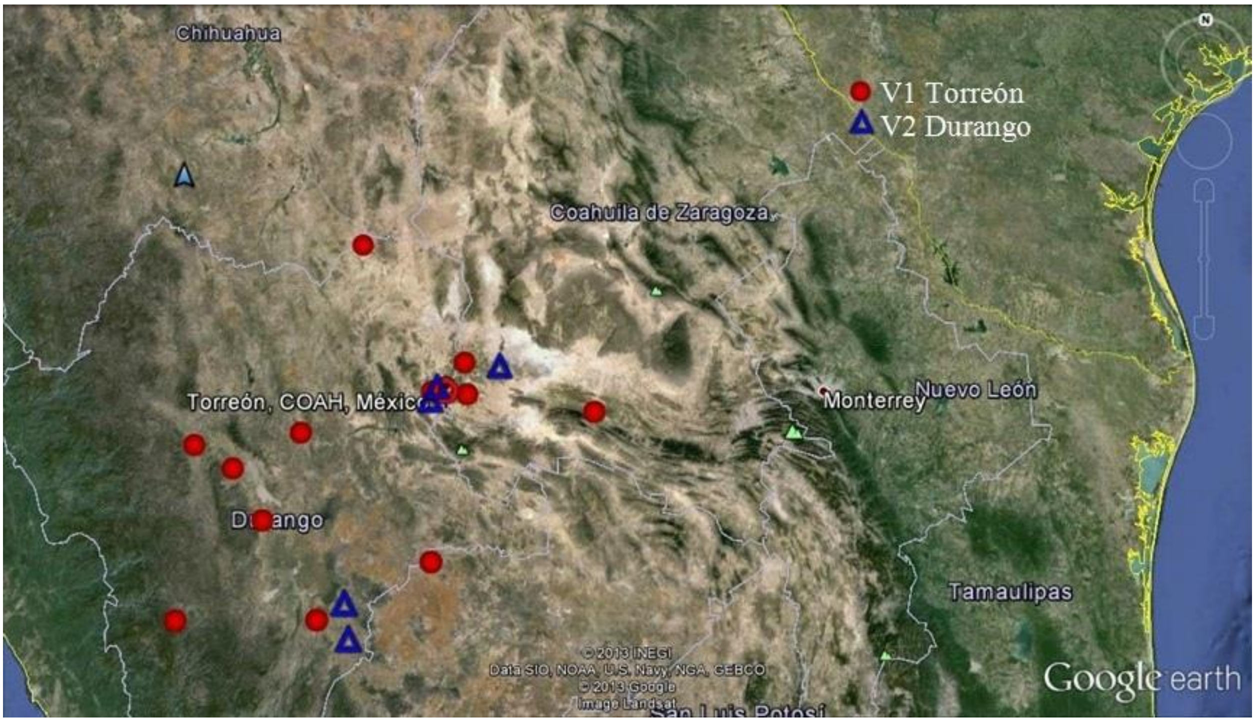

Figure 1. Solution structure in the Tabu Search multi-objective algorithm

The basic steps of the TS multi-objective algorithm are the following:

1. Generate an initial solution such that each SCU is assigned to its nearest salesperson.
2. For a given number of iterations repeat the steps:
    a. Select a random SCU
    b. Replace its assigned salesperson by a different sales person that best minimizes the objective function in Eq. (10).
    c. Add the SCU chosen in step a. to the tabu list so it cannot be chosen again after a number of iterations have passed (less than the total number of SCUs).
    d. If no replacement has been found that improves the objective function for more than a given number of iterations, restart the search from the initial solution.
    e. Remove the SCUs from the tabu list for which its iterations in the tabu list has expired.
3. Return the best solution found

The pseudo-code of the TS algorithm described above is presented in Algorithm 1 below.

Algorithm 1. *Tabu Search Multi-objective algorithm*

1. Initialize set of SCUs C_j $(j=1,\ldots,J)$
2. Initialize set of salespeople V_i $(i=1,\ldots,I)$
3. Generate initial solution p_j $(j=1,\ldots,J)$
4. $p^* = p$
5. $reset=0$
6. for $iter=1$: $Max\_iterations$ $step=1$
7. $\quad j$=Random_SCU(p)

8. i=Find_Best_Salesperson(V)
9. $s_j=i$
10. if $Z(p) < Z(p^*)$
11. $p^* = p$
12. else
13. $reset = reset + 1$
14. End if
15. if $reset$ > $Reset\_iterations$
16. Generate initial solution p_j $(j=1,\ldots,J)$
17. Empty tabu list
18. $reset=0$
19. End if
20. Add SCU j to tabu list
21. Remove SCUs from tabu list with expired tabu tenure
22. End for
23. Output p^*

The improvement strategy is a Perturbation of the current solution x, which takes place in lines 8 through 14 in Algorithm 1. In line 7, a random SCU from the current solution s is chosen. Note that only SCUs that are not in the tabu list can be chosen. Then in line 8 we iterate over all possible salespeople in the set V and chose the one that produces the largest decrease in the cost function in Eq. (10). In line 9 the chosen salesperson is assigned to the SCU chosen in line 7. If the cost the new solution s is smaller than the cost of the best solution s^*, then we replace s^* for s in lines 10 to 14. If the best solution is not improved for a given number of iterations, then we reset the search to the initial solution (lines 15 to 19). The chosen SCU is moved to the tabu list so it cannot be chosen again for reassignment until its tabu tenure expires. We set this tenure to the number of SCUs minus two in order to have at least two valid SCUs to choose when the tabu list is full. When the search is reset to the initial solution all SCUs are removed from the tabu list.

In each iteration of the TS algorithm, the improvement strategy in line 8 iterates over the total number of salespeople. Thus, the time complexity of the algorithm is O(N*M), where N is the maximum number of iterations of the TS algorithm and M is the number of salespeople in the data.

Finally, in Algorithm 2 we describe in pseudo-code the process we follow to find solutions with the TS algorithm described in Algorithm 1, for different values of the weights a_1, a_2 and a_3. In Algorithm 2 the weights are changed systematically in a defined number of combinations. Note that the weight values must satisfy $\alpha_1+\alpha_2+\alpha_3$ = 1.0 or 100%. In the algorithm, a fixed variation *delta* of the weights is defined to calculate all the combinations between 0 and 1 for each weight. It starts with a combination (0, 0, 1) for (α_1, α_2, α_3), follows with (0, 0+*delta*, 1-*delta*), and in the last cycle finishes with (1, 0, 0). For each combination of weights, Algorithm 1 (Tabu Search) is called multiple times to produce multiple solutions. All solutions are added to a set of solutions. The set of solutions obtained after all the weights combinations are solved is analyzed to eliminate dominated solutions, according to the definition by Ehrgott (2005), which is the Pareto front for the instance.

Algorithm 2. *Multi-Objective Pareto Generator*

1. $a_1=0$
2. $a_2=0$
3. $a_3=1$
4. $P=List()$
5. while $a1<1$
6. x=Tabu_Search()
7. P.insert(p)
8. if $\alpha_3 < delta$
9. $\alpha_2 = \alpha_2 + \alpha_3$
10. $\alpha_3 = 0$
11. Else
12. $\alpha_2 = \alpha_2 + delta$
13. $\alpha_3 = \alpha_3 - delta$
14. Endif
15. if $\alpha_2 = 1 - \alpha_1$

if $\alpha_2 < delta$
$\alpha_1 = \alpha_1 - \alpha_2$
Else
$\alpha_1 = \alpha_1 + delta$
Endif
$\alpha_2 = 0$
$\alpha_3 = 1 - \alpha_1$
End if
End while
Output P

6. Results

The algorithm was coded in Java, and it was run in a laptop with an Intel Core i7-3687U CPU, at 2.6 GHz, with 8.0 GB RAM. The instances tested are shown in Table 1. Each instance includes several towns and cities from different states of Mexico.

Table 1. Instances codes and sizes

Symbols	Definition and description
C	Set of Sales Coverage Units (SCUs) to be assigned
V	Set of Salespeople
i	Index for salesperson $i \in V$
j	Index for SCU $j \in C$
x_{ij}	Binary variable that takes value of $x_{ij} = 1$ if salesperson i is assigned to SCU j and $x_{ij} = 0$ otherwise
Tn_i	New total sales for salesperson i
Wn_i	New workload for the salesperson i
D	Sum of distances that every salesperson must travel into their territory
d_{ij}	Shortest road distance between salesperson i and SCU j
s_j	Money amount sold to SCU j
w_j	Workload (in hours) required to serve SCU j
μ_w	Workload average (in hours) by salesperson
μ_t	Average sale amount by salesperson
f_1	First objective function related to sales homogeneity
f_2	Second objective function related to geographic homogeneity
f_3	Third objective function related to workload homogeneity
Z	Aggregate function
α_k	Weights for each objective function f_k in the aggregate function Z such that $k = \{1, 2, 3\}$

Considering that objective functions are measured in different scales, a decimal scaling normalization scheme was used (Ogasawara et al., 2010). Some initial runs provided the range of values achievable for each objective function. Therefore, the values used for the aggregate function were computed by dividing the value over the maximum absolute value in each range.

Two types of experiments were conducted with the TS algorithm. The first set of experiments considered fixed weights for the objective functions, and several iterations of the algorithm were performed. This provided insightful information regarding the stability of the generated solutions and the convergence of the Tabu Search component. In the second set of experiments, the weights were changed to obtain a Pareto front for one instance.

6.1. First Set of Experiments

For these experiments, the weights were fixed at 1/3 (33.333%) for each objective function. All the instances were solved with 30 runs (executions) of the TS algorithm. Since solutions are improved by randomly selecting SCUs to re-assign their salespeople, a different result is expected from each run. The parameters of the TS algorithm used in each run are 20000 maximum iterations and 100 iterations of no improvement to reset the search from the initial solution. One example of the results is presented in Table 2 for the AGS instance.

Almost all the instances were solved within a computing time of 70.0 seconds. Table 3 presents the statistics computed for the aggregate objective functions of all instances. The last column presents the coefficient of variation (CV), which is defined as the standard deviation divided by the average. As it can be observed, the TS algorithm delivers very stable results, with a standard deviation of less than 7.0% with respect to the average.

Table 2. Results for the AGS instance with fixed weights of 33.333% for each objective function

Instance Code	Number of SCUs	Number of Salespeople
AGS	97	6
CHI	30	4
GDL	99	6
MER	36	2
MEX	111	7
MTY	50	7
TOR	19	2
VCZ	141	8

Table 3. Statistics for 30 runs with fixed weights

Run	Objective Function Eq. (6)	Objective Function Eq. (7)	Objective Function Eq. (8)	Aggregate Function (Z)
1	0.076	0.131	0.046	0.084
2	0.082	0.125	0.036	0.081
3	0.088	0.126	0.029	0.081
4	0.081	0.124	0.008	0.071
5	0.082	0.135	0.053	0.09
6	0.078	0.179	0.006	0.087
7	0.075	0.175	0.022	0.09
8	0.087	0.124	0.064	0.091
9	0.078	0.171	0.011	0.087
10	0.077	0.192	0.003	0.091
11	0.083	0.106	0.053	0.081
12	0.114	0.11	0.025	0.083
13	0.076	0.169	0.011	0.085
14	0.078	0.177	0.016	0.09
15	0.113	0.113	0.028	0.084
16	0.075	0.174	0.025	0.091
17	0.085	0.122	0.052	0.086
18	0.091	0.104	0.086	0.094
19	0.081	0.114	0.032	0.076
20	0.082	0.102	0.088	0.091
21	0.086	0.182	0.011	0.093
22	0.086	0.115	0.029	0.077
23	0.084	0.128	0.014	0.075
24	0.084	0.11	0.046	0.08
25	0.077	0.181	0.007	0.088
26	0.087	0.127	0.047	0.087
27	0.073	0.189	0.004	0.088
28	0.082	0.171	0.016	0.09
29	0.079	0.18	0.012	0.09
30	0.079	0.163	0.013	0.085

The assignments achieved with the TS algorithm can be placed on a map in a geographic information system. Figure 1 presents an assignment for the TOR instance. Triangles represent the SCUs assigned to one salesperson, and circles represent the SCUs assigned to the other salesperson. The assignments for the MER instance are presented in Figure 2, where triangles represent the SCUs assigned to one salesperson, and circles represent the assignments for the other salesperson.

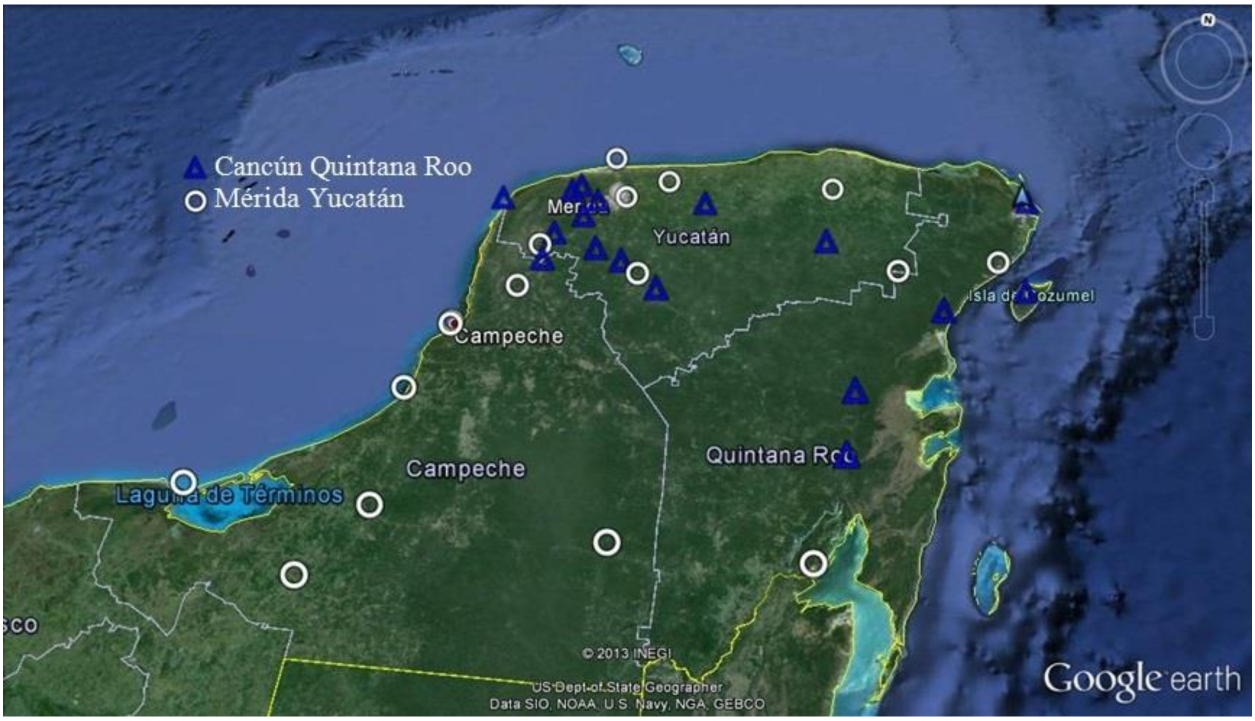

Figure 2. Assignments for the TOR instance

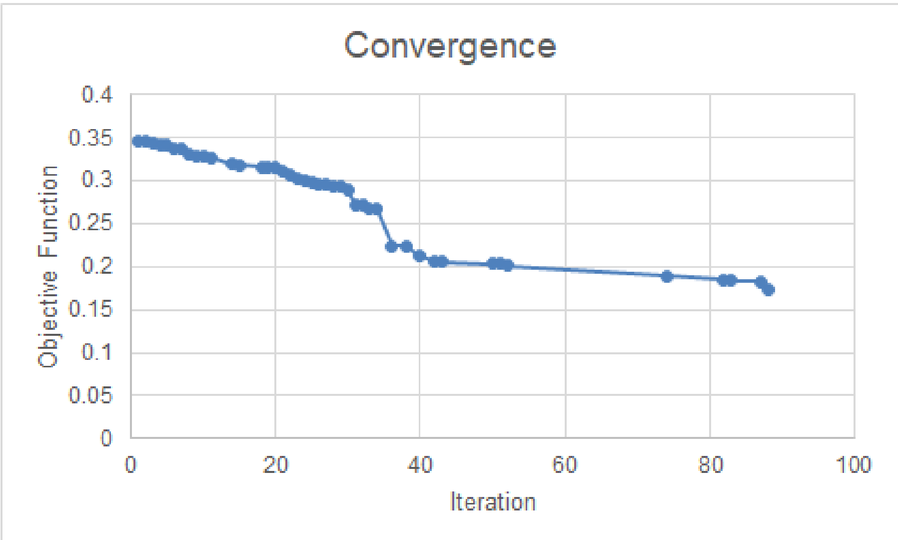

Figure 3. Assignments for the MER instance

Specifically, to understand the convergence speed of the Tabu Search component, the behavior was registered for 19455 iterations with fixed weights in the aggregated function for the VCZ instance. The result is presented in Figure 3. The best result was obtained very early at iteration 88. A final improvement of less than 1.0% with respect to the best solution (not included in Figure 3) was obtained at iteration 10436.

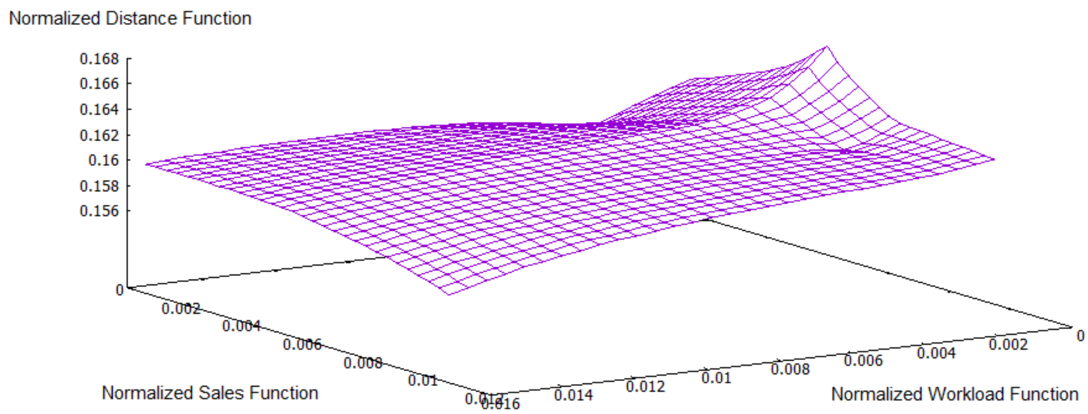

Figure 4. Convergence of the Tabu Search component

6.2. Second Set of Experiments

For a multi-objective problem, from an a priori approach, the weights of the aggregate function are fixed because of a preference of the decision maker. However, in an a-posteriori approach, the weights are not defined initially, and it is necessary to generate the Pareto front. In the last case, the decision maker selects a solution from the front according to preferences not included in the model.

Many metaheuristics were designed for single objective problems, like Simulated Annealing (SA) and Tabu Search (TS). The necessity to address more complex, realistic, and in many cases multi-objective problems, urges to adapt those methods. In our proposal, a TS algorithm is presented, embedded in a routine where the weights of the aggregate function are changed systematically. The TS algorithm finds a solution for each combination of weights, and with these solutions, the Pareto front is constructed once the dominance of each solution is verified.

For the VCZ instance, the routine created the combinations changing 5.0% of the weights of each objective function at each iteration. The solutions from the TS algorithm for all the weights combinations were obtained in 2365 seconds. From these solutions, the Pareto front was finally constructed with 286 non-dominated solutions. This Pareto front can be represented as a surface in a 3-D graph, where each objective is an axis, as shown in Figure 4. For a deeper analysis, 2-D graphs are shown, comparing each pair of objectives.

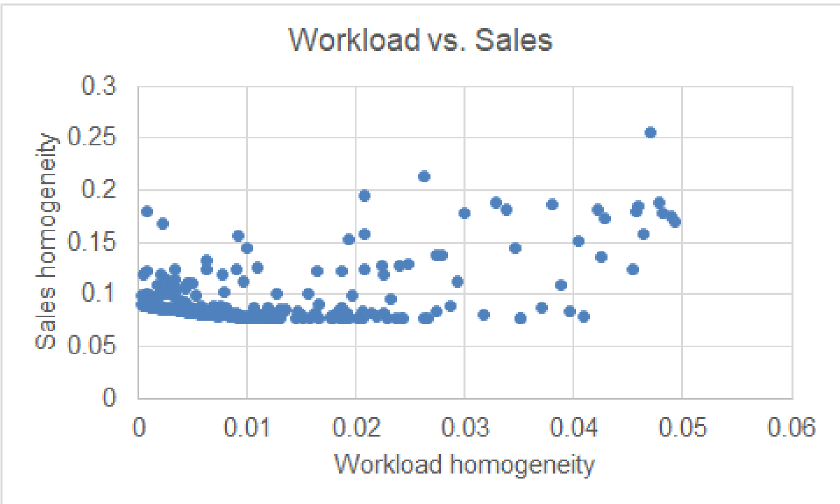

Figure 5. Pareto front for VCZ instance

Figure 5 presents the front for Workload and Sales. The meaning of the axis is Workload heterogeneity for the X-axis and Sales heterogeneity for the Y-axis. It is important to remember that the objective functions in Equations (6) and (8) account for the dispersion with respect to an average. In this way, a greater value means that the assignments produce larger differences between salespeople. Figure 4 presents all the 286 solutions, considering the three objectives. In a strict sense, a "partial" Pareto front can be obtained for Workload and Sales selecting only those solutions in the lower edge of the cloud of points, which minimizes those two objectives only.

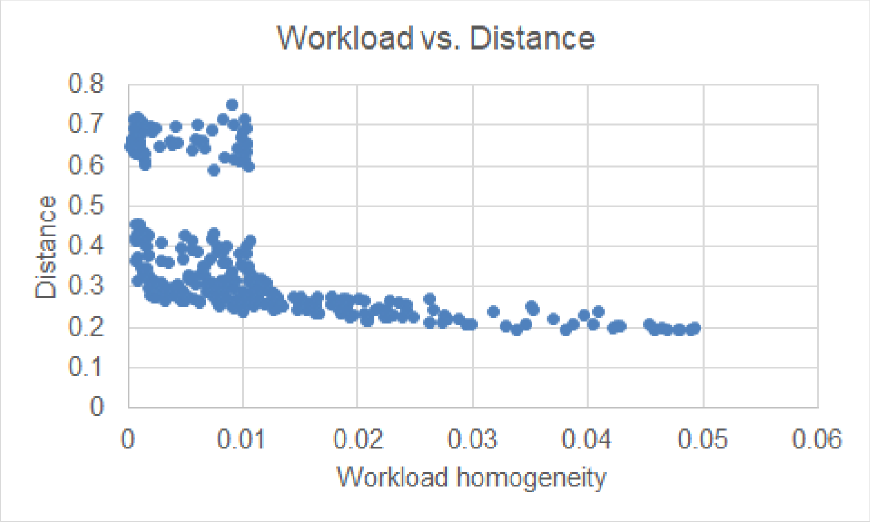

Figure 6. Pareto front for Workload and Sales

Figure 6 shows the front for Workload and Distance. While a dispersion near to zero can be obtained for the Workload heterogeneity, the Distance cannot be minimized beyond the necessary distance between a salesperson and its closer SCU.

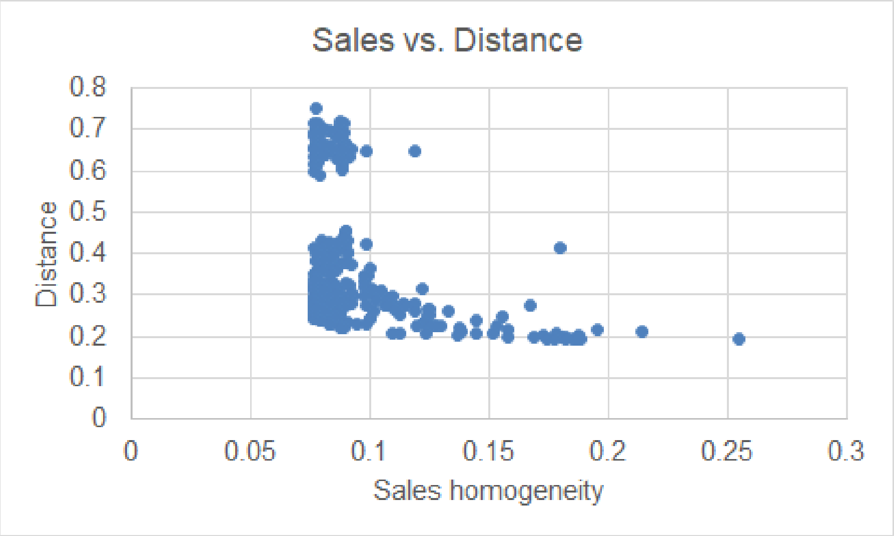

Figure 7. Pareto front for Workload and Distance

The front for Sales and Distance is shown in Figure 7. It is remarkable that the Sales heterogeneity, such as Workload heterogeneity, cannot achieve a minimum of zero. This is because each SCU has a number of sales (and workload) which is assigned in a discrete fashion to a salesperson. Therefore, to achieve assignments equal to the average, a perfect combination for each salesperson must be found, which in most of the cases is not possible.

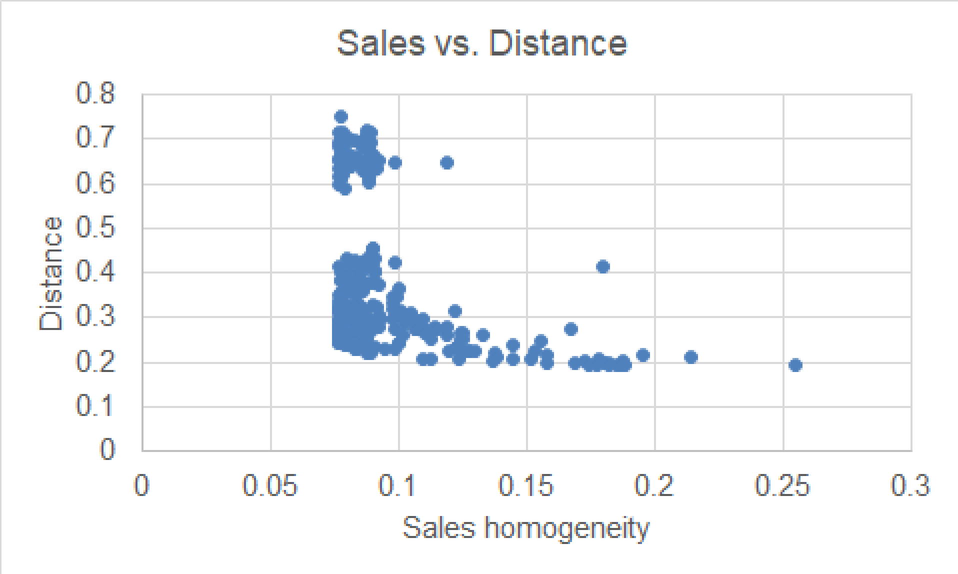

Figure 8. Pareto front for Sales and Distance

Another single run of the TS algorithm produced Pareto fronts for each one of the instances. The number of non-dominated solutions obtained is shown in Table 4. The ability to produce a set of alternative non-dominated solutions is of high importance to the company because the decision maker can select any of these alternatives considering their own preferences. Also, the structure of the solution can be analyzed, and the decision maker can select one based on qualitative criteria, not modeled into the problem. One example is the assignment of certain SCU to a specific salesperson because of family ties of the salesperson with people living in that location. Other reason is that some SCU can be assigned to an ambitious salesperson that can develop the market in that location.

Table 4. Number of non-dominated solutions

Instance	Minimum	Maximum	Average	CV
AGS	0.0712	0.0937	0.0856	0.0676
CHI	0.0276	0.0316	0.0293	0.0402
GDL	0.1256	0.1296	0.1274	0.0081
MER	0.0257	0.0312	0.0287	0.048
MEX	0.0512	0.0567	0.0542	0.0269
MTY	0.0345	0.0398	0.0379	0.0327
TOR	0.0541	0.0609	0.0601	0.0278
VCZ	0.1411	0.17	0.1654	0.0298

From the solutions obtained in Table 4, the last comparison is done. The current assignment of the company was evaluated using objective functions in Eqs. (6-8). The results were normalized using the number of digits of the solutions in Table 4, to make them comparable with those solutions. One of the solutions, the closest to the current solution in the Pareto front, was selected for comparison. These comparisons, for each of the instances, are shown in Table 5. The first column in Table 5 describes the code of the instance; the second column indicates a header to describe if the row is associated with the current solution of the company, or the proposed one obtained with the TS algorithm, and the saving obtained. The last three columns indicate the values for each one of the objective functions.

Table 5. Comparison of current situation vs the closest non-dominated solution

Instance Code	Solution	Eq. (6) sales	Eq. (7) distance	Eq. (8) workload
AGS	Current	0.11983772	0.10207230	0.13400
	Proposed	0.07625296	0.10109249	0.10825
	Saving (%)	36.4	1.0	19.2
CHI	Current	0.03059892	0.07249000	0.16725
	Proposed	0.00042256	0.02311000	0.00725
	Saving (%)	98.6	68.1	95.7
GDL	Current	0.16341102	0.19111900	0.21525
	Proposed	0.16341102	0.08003300	0.01725
	Saving (%)	0.0	58.1	92
MER	Current	0.04524344	0.05847990	0.05250
	Proposed	0.00275308	0.00397062	0.04200
	Saving (%)	93.9	93.2	20.0
MEX	Current	0.07985991	0.10950200	0.37864
	Proposed	0.06930292	0.09157401	0.33021
	Saving (%)	13.2	16.4	12.8
MTY	Current	0.08203884	0.06877110	0.05715
	Proposed	0.07418093	0.06877110	0.04341
	Saving (%)	9.6	0.0	24.0
TOR	Current	0.05655852	0.17738300	0.01125
	Proposed	0.00022440	0.00183580	0.00075
	Saving (%)	99.6	99.0	93.3
VCZ	Current	0.17728125	0.038214117	0.36563
	Proposed	0.13183881	0.038214117	0.35025
	Saving (%)	25.6	0.0	4.2

As expected, the current solution of the company was dominated in all the cases by at least one solution in the Pareto front obtained with the TS algorithm. The savings are in the range of 0-99.6% in the sales unbalance, with an average of 47.1%. In the case of the distance unbalance, the savings are in the range of 0-99% with an average of 42.0%. And for the workload unbalance, the savings are in the range of 4.2-95.7%, being an average of 45.2%. This represents an important improvement for the company when the current assignment is compared to the solutions obtained with the proposed algorithm. The current assignment of the company is highly unbalanced, and the proposed solution demonstrates a better balance in at least two of the objective functions in most of the cases.

Another important benefit achieved is the fact that the study helped to the company to have their data clear and organized, and that the process produced a better understanding for the top managers of the salesforce, the customers, and the challenge of serving a large and dispersed market. The process of gathering the information of sales, locations, customer preferences, and organizational culture helped to the company to identify opportunities and challenges beyond the scope of this paper.

7. Conclusion and Future Work

The work presented here describes the models and methods used to solve a problem for a company that sells hand tools and building materials to hardware stores in Mexico. First, a mathematical model with three objective functions (multi-objective function) is proposed for designing balanced sales territories. The objectives are the geographical balance of the territories, the balance of the workload, and the balance in the total sales amount.

The non-linearity of the model motivated the design of a metaheuristic algorithm based on Tabu Search (TS) to solve the problem. The solutions for eight instances were presented, corresponding to different regions in the country.

The statistical results show a fast convergence of the TS component and good stability of the solutions. The algorithm provides a number of solutions in the Pareto front for each of the instances. The current assignment used by the company is compared to the closest solution in the Pareto front for each instance. The proposed solutions improve the current situation in 47.1% in the sales balance, in 42.0% in the geographical balance, and in 45.2% in the workload balance, in average. These results represent a big improvement with respect to the current situation of the company. The proposed model solves a managerial problem for the territory alignment of the salesforce, and the proposed algorithm solves this strategic problem efficiently.

Anyway, the Pareto front obtained for each instance allow to select other solutions that may be convenient for the operation and management of the salesforce. These alternatives are useful for the decision maker when compared to the current situation where territories are unbalanced in one or several metrics: geography, sales volume, or workload.

For future work, geographic analysis of the solutions must be performed to avoid overlapping of the territories. This overlapping is especially undesirable when it implies that one salesperson moves along the same road used by another salesperson while covering the sales territory because it reduces operative efficiency and service level. The overlapping is usually addressed by introducing contiguity constraints into the model. The classical form of these constraints expands the formulation exponentially (Kalcsics, 2015), making it harder to be solved optimally even for small instances. However, other formulations might be explored to include the contiguity issue (Shirabe, 2009).

Also, out of the scope of this work, some studies make a comparison of performance between several metaheuristics. This comparison might be made with recent metaheuristics used with success like preference-inspired co-evolutionary algorithms, PICEAs (Lei et al., 2016) and hybrids with NSGA-III (Sangaiah et al., 2020).

The territory re-design can be integrated into the routing problem to determine the optimal sequences for visiting the sales coverage units into each territory. In some cases, the related problem can be treated as a location-routing problem (Tirkolaee et al., 2021). For the case of the problem presented in this study, the version named Periodic Vehicle Routing Problem, PVRP, better describes the nature of the activities of salespeople. Thus this extension is an important challenge in territory design.

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