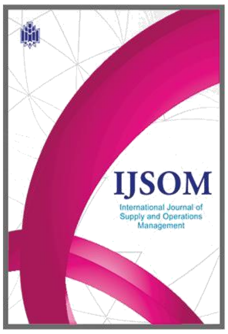
International Journal of Supply and Operations Management

IJSOM

May 2021, Volume 8, Issue 2, pp. 96-113
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com



A Hybrid Genetic-Simulated Annealing-Auction Algorithm for a Fully Fuzzy Multi-Period Multi-Depot Vehicle Routing Problem

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Abstract

In this paper, an integer linear programming formulation is developed for a novel fuzzy multi-period multi-depot vehicle routing problem. The novelty belongs to both the model and the solution methodology. In the proposed model, vehicles are not forced to return to their starting depots. The fuzzy problem is transformed into a mixed-integer programming problem by applying credibility measure whose optimal solution is an (α,β)-credibility optimal solution to the fuzzy problem. To solve the problem, a hybrid genetic-simulated annealing-auction algorithm (HGSA), empowered by a modern simulated annealing cooling schedule function, is developed. Finally, the efficiency of the algorithm is illustrated by employing a variety of test problems and benchmark examples. The obtained results showed that the algorithm provides satisfactory results in terms of different performance criteria.

Keywords. Periodic Routing Problem; Multi-Depot; Hybrid Algorithm; Auction Algorithm; Genetic Algorithm; Simulated Annealing Algorithm.

1. Introduction

Millions of people are affected by natural or man-made disasters thus delivering the aftermath services in the least possible time is of essential importance (Thomas and Kopczak, 2005). This entails a fast but deliberate operational planning for best managing of the humanitarian relief chain. Some studies estimate that the logistics and supply chain management activities include more than 80% of the total humanitarian relief operations (Van Wassenhove, 2006). In this regard, operational research models can be successfully applied (Van Wassenhove and Pedraza Martinez, 2012). The aim is to minimize the time of responding to the damaged areas and maximize the satisfaction level and fairness in the distribution of commodities (Saffarian et al., 2017).

In multi-depot vehicle routing problems, customers are serviced by vehicles located in several depots. This problem was firstly appeared in 1995 (Sumichras and Markham, 1995). Since then, various versions of the vehicle routing problem have been studied in the literature. Giosa et al. (2002) considered a multi-depot vehicle routing problem with time windows. Nagy and Salhi (2005) studied a multi-depot vehicle routing problem with pick up and deliveries. Crevier et al. (2007) proposed a multi-depot vehicle routing model with inter-depot routes.

Since multi-depot vehicle routing problems are NP-Hard, a multitude of studies adopted heuristic and metaheuristic algorithms as solution methodologies. For instance, Wu et al. (2002) provided heuristic solutions for a multi-depot location-routing problem. Genetic algorithm, which is a popular metaheuristic among researchers, was used in a number of studies

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DOI: 10.22034/ijsom.2021.2.1

(Dondo and Cerdá, 2009; Ho et al., 2008; Thangiah and Salhi, 2001). A memetic algorithm was proposed by Ngueveu et al. (2010) for a cumulative capacitated VRP (CCVRP) aiming to minimize total time of visiting customers. Ribeiro and Laporte (2012) devised an adaptive large neighborhood search heuristic for the cumulative capacitated vehicle routing problem and compared it with the memetic algorithm proposed by Ngueveu et al. (2010). Ke and Feng (2013) proposed a two-phase metaheuristic for solving CCVRP and evaluated its performance using a number of test problems.

Periodic routing problems (PRP) are routing problems that aims to provide services for customers over a specified time horizon such that total routing costs be minimized. It was introduced by Beltrami and Bodin (1974) for the first time. There is a wide applicability for periodic routing problems from optimizing periodic maintenance operations for Schindler Elevator Corporation (Blakeley et al., 2003) to periodic milk collection problem (Claassen and Hendriks, 2007). PRP belongs to NP-hard problems for which a wide variety of heuristic and metaheuristic algorithms are developed including tabu search, variable neighborhood search and genetic algorithm (Alonso et al., 2008; Drummond et al., 2001; Hemmelmayr et al., 2009). Multi-depot PRP is a generalized case of PRP in which several depots can be used to serve customers. While it has more applicability in real world, it is not sufficiently addressed in the existing literature (Kang et al., 2005; Angelelli and Speranza, 2002).

In spite of the vast body of the literature studying VRPs, a small proportion has considered uncertainties (Maity et al. 2019). Saffarian et al. (2015) considered a bi-objective model for relief chain logistic in uncertainty condition including uncertainty in traveling time and demand in damaged areas. Roy and Midya (2019) proposed a Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. Furthermore, an intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain is studied by Midya et al. (2020). Rabbani et al. (2018) considered environmental issues for designing a municipal solid waste system and proposed a new mathematical model. Uncertainty and budget constraint in multi-objective multi-item fixed charge solid transportation problem were also studied by Majumder et al. (2019). Also, Mohamadi et al. (2015) introduced a credibility-based chance-constrained transfer point location model for the relief logistics design. The results were applied for earthquake disaster management on region 1 of Tehran city. Das et al. (2020) considered the application of type-2 fuzzy logic to a multi-objective green solid transportation-location problem with dwell time under carbon tax, cap and offset policy. They compared fuzzy vs. non-fuzzy techniques for the problem. Hasan and Mashud (2019) proposed an Economic Order Quantity model for decaying products with the frequency of advertisement, selling price and continuous time-dependent demand under partially backlogged shortage. Mashud et al. (2019) extended their previous work by employing a two-level trade-credit approach to an integrated price-sensitive inventory model with shortages. Table 1 summarizes new researches on vehicle routing problem that have considered uncertainty conditions.

In most vehicle routing problem studies, except for open-routing problems, it is assumed that vehicles are obliged to return to their starting depot after serving a subset of customers. In this study, in order to improve productivity, a new version of multi-period VRP with several depots is considered wherein vehicles are allowed to return to a depot other than the initial one; an idea brought up by Kek et al. (2008). This relaxation can lead to better response times in relief chains, where impacted areas can be rapidly visited by nearby vehicles. Also, due to the inherent uncertainties in the transportation problems, there are uncertainties in precisely calculating the parameters of the vehicle routing problem such as traveling cost and demand of customers. To deal with vague and imprecise data in real transportation networks, in this paper fuzzy multi-depot multi-period vehicle routing problem is considered. The reasons for using fuzzy approach to deal with uncertainties are as follows: Firstly, transportation problems deal with linguistic descriptions like high cost, high demand, etc., that can only be modeled by fuzzy approaches. Secondly, few studies have been conducted in the field of vehicle routing problems by time-varying traveling cost that are not applicable in large-scale networks due to the rapidly increasing the dimension of the problem. Finally, calculations in probabilistic space are very complex. Therefore, this approach cannot always meet the needs of the problem for real and large-scale applications.

The rest of the paper is organized as follows. In section 2, the problem is introduced and modeled by a fully fuzzy integer programming problem. The solution methodology is proposed in section 3. Experimental results are given in section 4. Finally, the paper ends with a brief conclusion and future directions in section 5.

Table 1. Summary of the researches on vehicle routing problem under uncertainty conditions.

Reference	Description of the problem	Proposed model	Solution method
Das et al. (2020)	Green solid transportation-location problem with dwell time under carbon tax, cap and offset policy	Multi-objective integrated mathematical model	Fuzzy technique and nonfuzzy technique
Mohamadi et al. (2015)	Relief logistics design by credibility-based chance-constrained transfer point location model	Fuzzy mathematical programming model	Global criterion method
Rabbani et al. (2018)	Municipal Solid Waste System Considering Environmentally Issues	Bi-objective model based on location-routing problem	NSGA-II
Roy and Midya (2019)	Fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment	Intuitionistic fuzzy programming	The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)
Saffarian et al. (2015)	Relief chain logistic in uncertainty condition including uncertainty in traveling time and demand	Bi-objective model	Global Criterion method
Majumder et al. (2019)	Multi-objective multi-item fixed charge solid transportation problem with budget constraint	Expected value model Chance-constrained model Dependent chance-constrained model	Linear weighted method, Global criterion method Fuzzy programming method
Midya et al. (2020)	Multi-stage multi-objective fixed-charge solid transportation problem (MMFSTP) with a green supply chain network system under an intuitionistic fuzzy environment.	Multi-objective programming model	Weighted Tchebycheff metrics programming Min-max goal programming
Hasan and Mashud (2019)	Decaying products with the frequency of advertisement, selling price and continuous time-dependent demand under partially backlogged shortage.	Economic Order Quantity model	Generalized reduced gradient (GRG) method.

2. Problem formulation

In this section, a fuzzy integer programming model is proposed for the fully fuzzy multi-depot multi-period vehicle routing problem wherein vehicles are not forced to return to their starting depots. The basic of the model is adapted from Eydi and Abdorahimi (2012). Since in real applications, certainty and precision of data are often illusory, in order to make the proposed model closer to real applications, the objective function and constraints are considered to be fuzzy.

2.1. Assumptions

The formulation is based on the following assumptions:

- A limited number of periods is given.
- Number of depots is fixed.
- A heterogeneous fleet of vehicles is available.
- The capacity of vehicles is predetermined.
- Demand of each customer in each period is specified as a fuzzy parameter.
- The number of customers that should be serviced in each period is defined.
- The customers of each period are different from those of other periods.
- Distance-dependent transportation costs are assumed.
- Each vehicle starts its journey from one depot and ends to another depot, although the starting and ending depots could be also identical.
- A symmetric transportation network is considered.
- The traversing cost, customer’s demand and vehicle’s capacity are considered as fuzzy parameters.

2.2. Indices

I	An index assigned to customers located at the beginning of an edge. (i=1,2,…,N)
J	An index assigned to customers located at the end of an edge. (j=1,2,…,N and j≠i)
T	Index of periods (t=1,2,…,T)
K	Index of vehicles (k=1,2,…,V)
D	Index of depots (d=1,2,…,D)

2.3. Parameters

$\tilde{c}_{i,j,t}$	Fuzzy traversing cost of edge *(i,j)* between customers *i* and *j* in period *t*.
$\tilde{c}'_{d,i,t}$	Fuzzy traversing cost of edge *(i,d)* or edge *(d,i)* between customer *i* and depot *d* in period *t*.
$\tilde{d}_{i,t}$	Fuzzy demand of customer *i* in period *t*.
N_t	Number of customers in period *t*.
$\tilde{c}_k$	Capacity of vehicle *k*.
V	Number of available vehicles in each period.
T	Number of periods in the planning horizon.
D	Number of depots.
M	A big number.

2.4. Sets

B	A subset of customers in each period.
A	Set of depots.
G	Set of all customers and depots in each period.

2.5. Decision variables

$x_{i,j,k,t} \in \{0,1\}$	Equals to 1 if vehicle k traverses edge (i,j) in period t, otherwise 0.
$y_{d,i,k,t} \in \{0,1\}$	Equals to 1 if vehicle *k* traverses edge *(d,i)* in period *t*, otherwise 0.
$z_{i,d,k,t} \in \{0,1\}$	Equals to 1 if vehicle *k* traverses edge *(i,d)* in period *t*, otherwise 0.
$s_{k,d,t} \in \{0,1\}$	Equals to 1 if vehicle *k* is located in depot *d* at the beginning of period *t*, otherwise 0.
$f_{k,d,t} \in \{0,1\}$	Equals to 1 if vehicle *k* is located in depot *d* at the end of period *t*, otherwise 0.

2.6. Mathematical model

The fuzzy integer programming model of the problem is as follows.

$$\text{Min} \sum_{t=1}^{T}\sum_{i=1}^{N_t}\sum_{\substack{j=1 \\ j\neq i}}^{N_t}\sum_{k=1}^{V} x_{i,j,k,t} * \tilde{c}_{i,j,t} + \sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{i=1}^{N_t}\sum_{k=1}^{V} y_{d,i,k,t} * \tilde{c}'_{d,i,t} + \sum_{t=1}^{T}\sum_{i=1}^{N_t}\sum_{d=1}^{D}\sum_{k=1}^{V} z_{i,d,k,t} * \tilde{c}'_{d,i,t} \tag{1}$$

Subjected to:

$$\sum_{d=1}^{D}\sum_{k=1}^{V} y_{d,i,k,t} + \sum_{\substack{j=1 \\ j\neq i}}^{N_t}\sum_{k=1}^{V} x_{j,i,k,t} = 1 \qquad \forall i,t \qquad (2)$$

$$\sum_{\substack{j=1 \\ j\neq i}}^{N_t}\sum_{k=1}^{V} x_{i,j,k,t} + \sum_{d=1}^{D}\sum_{k=1}^{V} z_{i,d,k,t} = 1 \qquad \forall i,t \qquad (3)$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t} + \sum_{j=1}^{N_t}\sum_{d=1}^{D} z_{j,d,k,t} = 0 \qquad \forall k,t \qquad (4)$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t} * \tilde{d}_{i,t} + \sum_{i=1}^{N_t}\sum_{\substack{j=1 \\ j\neq i}}^{N_t} x_{i,j,k,t} * \tilde{d}_{j,t} \leq \tilde{c}_k \qquad \forall k,t \qquad (5)$$

$$\sum_{d=1}^{D} y_{d,i,k,t} + \sum_{\substack{j=1 \\ j\neq i}}^{N_t} x_{j,i,k,t} - \sum_{\substack{j=1 \\ j\neq i}}^{N_t} x_{i,j,k,t} - \sum_{d=1}^{D} z_{i,d,k,t} = 0 \qquad \forall i,k,t \qquad (6)$$

$$\sum_{i=1}^{N_t}\sum_{\substack{j=1 \\ j\neq i}}^{N_t} x_{i,j,k,t} \leq \left(\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t}\right) * M \qquad \forall k,t \qquad (7)$$

$$\sum_{i\in B}^{N_t}\sum_{\substack{j\in B \\ j\neq i}}^{N_t} x_{i,j,k,t} \leq |B| - 1 \qquad \forall k,t,\ \ \forall B \subseteq G \setminus \{A\},\ |B| \geq 2 \qquad (8)$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t} \leq 1 \qquad \forall k,t \qquad (9)$$

$$\sum_{d=1}^{D} s_{k,d,t} = 1 \qquad \forall k,t \qquad (10)$$

$$\sum_{d=1}^{D} f_{k,d,t} = 1 \qquad \forall k,t \qquad (11)$$

$$\sum_{i=1}^{N_t} y_{d,i,k,t} \leq s_{k,d,t} \qquad \forall k,d,t \qquad (12)$$

$$\sum_{i=1}^{N_t} z_{i,d,k,t} \leq f_{k,d,t} \qquad \forall k,d,t \qquad (13)$$

$$f_{k,d,t-1} = s_{k,d,t} \qquad \forall k,d \quad \forall t \geq 2 \qquad (14)$$

$$x_{i,j,k,t} \in \{0,1\} \qquad \forall i,k,t, \quad \forall j \neq i \qquad (15)$$

$$y_{d,i,k,t} \in \{0,1\} \qquad \forall d,i,k,t \qquad (16)$$

$$z_{i,d,k,t} \in \{0,1\} \qquad \forall d,i,k,t \qquad (17)$$

$$s_{k,d,t} \in \{0,1\} \qquad \forall k,d,t \qquad (18)$$

$$f_{k,d,t} \in \{0,1\} \qquad \forall k,d,t \qquad (19)$$

The objective function (1) minimizes total routing costs during the planning horizon. Routing costs consist of three parts: cost of traversing between customers, cost of traversing between depots and customers and cost of traversing between customers and depots. Since the traversing cost parameters are considered to be fuzzy, a fuzzy objective function is considered.

Constraints (2) and (3) ensure that each customer is served exactly once in each period. Constraint (4) ensures that each vehicle's route starts from one depot and ends to another, which is not necessarily the same as the initial depot. Fuzzy constraint (5) guarantees that the total demand of all customers in the route of each vehicle must be less than the capacity of that vehicle. Flow conservation constraints are given by constraint (6). Constraint (7) states that vehicle's route must start from a depot. Constraint (8) prevents the creation of subtours. Constraint (9) states that a number of vehicles may be idle at each time period. Constraint (10) and (11) indicate that at the beginning and end of each time period, each vehicle is located in one depot. Constraint (12) and (13) show the relationship between variables $y_{d,i,k,t}$ and $s_{k,d,t}$ and $z_{i,d,k,t}$ and $f_{k,d,t}$, respectively. Also, the relationship between variable $f_{k,d,t}$ and $s_{k,d,t}$ is stated in constraint (14). Constraints (15) – (19) show that all decision variables are binary.

Credibility relation is used to obtain the (α,β)–optimal solution of the fully fuzzy multi-period and multi-depot vehicle routing problem, in which α and β are the satisfaction degrees in the constraints and the objective function of the problem, respectively. For this aim, the following theorem is applied (Niksirat, 2016).

Consider the general fully fuzzy mathematical programming problem

$$\min \ z = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \ldots + \tilde{c}_n x_n$$

s.t.

$$\tilde{a}_{11} x_1 + \tilde{a}_{12} x_2 + \ldots + \tilde{a}_{1n} x_n \geq \tilde{b}_1$$

...

$$\tilde{a}_{m1} x_1 + \tilde{a}_{m2} x_2 + \ldots + \tilde{a}_{mn} x_n \geq \tilde{b}_m$$

$$x_1, \ldots, x_n \geq 0$$

Let $\max\left\{(\tilde{c}_j)^L_\beta, (\tilde{c}_j)^L_{1-\beta}\right\} \leq c_j \leq \min\left\{(\tilde{c}_j)^R_\beta, (\tilde{c}_j)^R_{1-\beta}\right\}, j = 1, \ldots, n$ and $\alpha, \beta \in [0,1]$, the optimal solution of the following problem is an (α,β)–credibility optimal solution for the above problem .

$$\min \ z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

$$(\tilde{a}_{11})^L_{1-\mu_1} x_1 + (\tilde{a}_{12})^L_{1-\mu_1} x_2 + \ldots + (\tilde{a}_{1n})^L_{1-\mu_1} x_n \geq (\tilde{b}_1)^R_{1-\mu_1}$$

$$(\tilde{a}_{11})^R_{2\alpha-\mu_1} x_1 + (\tilde{a}_{12})^R_{2\alpha-\mu_1} x_2 + \ldots + (\tilde{a}_{1n})^R_{2\alpha-\mu_1} x_n \geq (\tilde{b}_1)^L_{2\alpha-\mu_1}$$

...

$$(\tilde{a}_{m1})^L_{1-\mu_m} + (\tilde{a}_{m2})^L_{1-\mu_m} x_2 + \ldots + (\tilde{a}_{mn})^L_{1-\mu_m} x_n \geq (\tilde{b}_m)^R_{1-\mu_m}$$

$$(\tilde{a}_{m1})^R_{2\alpha-\mu_m} x_1 + (\tilde{a}_{m2})^R_{2\alpha-\mu_m} x_2 + \ldots + (\tilde{a}_{mn})^R_{2\alpha-\mu_m} x_n \geq (\tilde{b}_m)^L_{2\alpha-\mu_m}$$

$$x_1, \ldots, x_n \geq 0, \ max\{0, 2\alpha - 1\} \leq \mu_i \leq min\{2\alpha, 1\}, i = 1, \ldots, m$$

Based on the aforementioned theorem, the (α,β)–credibility optimal solution of the considered fully fuzzy multi-depot multi-period vehicle routing problem is obtained by solving the following problem:

$$\mathrm{Min}\sum_{t=1}^{T}\sum_{i=1}^{N_t}\sum_{\substack{j=1\\ j\neq i}}^{N_t}\sum_{k=1}^{V} x_{i,j,k,t} * c_{i,j,t} + \sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{i=1}^{N_t}\sum_{k=1}^{V} y_{d,i,k,t} * c'_{d,i,t} + \sum_{t=1}^{T}\sum_{i=1}^{N_t}\sum_{d=1}^{D}\sum_{k=1}^{V} z_{i,d,k,t} * c'_{d,i,t} \tag{20}$$

S.t:

$$\sum_{\substack{j=1\\ j\neq i}}^{N_t}\sum_{k=1}^{V} x_{j,i,k,t} + \sum_{d=1}^{D}\sum_{k=1}^{V} y_{d,i,k,t} = 1 \qquad \forall i,t \tag{21}$$

$$\sum_{\substack{j=1\\ j\neq i}}^{N_t}\sum_{k=1}^{V} x_{i,j,k,t} + \sum_{d=1}^{D}\sum_{k=1}^{V} z_{i,d,k,t} = 1 \qquad \forall i,t \tag{22}$$

$$\sum_{d=1}^{D}\sum_{j=1}^{N_t} y_{d,i,k,t} = \sum_{j=1}^{N_t}\sum_{d=1}^{D} z_{j,d,k,t} \qquad \forall k,t \tag{23}$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t} * \left(\tilde{d}_{i,t}\right)^{R}_{1-\mu_{k,t}} + \sum_{i=1}^{N_t}\sum_{\substack{j=1\\ j\neq i}}^{N_t} x_{i,j,k,t} * \left(\tilde{d}_{j,t}\right)^{R}_{1-\mu_{k,t}} \le \left(\tilde{c}_k\right)^{L}_{1-\mu_{k,t}} \qquad \forall k,t \tag{24}$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t} * \left(\tilde{d}_{i,t}\right)^{L}_{2\alpha-\mu_{k,t}} + \sum_{i=1}^{N_t}\sum_{\substack{j=1\\ j\neq i}}^{N_t} x_{i,j,k,t} * \left(\tilde{d}_{j,t}\right)^{L}_{2\alpha-\mu_{k,t}} \le \left(\tilde{c}_k\right)^{R}_{2\alpha-\mu_{k,t}} \qquad \forall k,t \tag{25}$$

$$\sum_{d=1}^{D} y_{d,i,k,t} + \sum_{\substack{j=1\\ j\neq i}}^{N_t} x_{j,i,k,t} - \sum_{\substack{j=1\\ j\neq i}}^{N_t} x_{i,j,k,t} - \sum_{d=1}^{D} z_{i,d,k,t} = 0 \qquad \forall i,k,t \tag{26}$$

$$\sum_{i=1}^{N_t}\sum_{\substack{j=1\\ j\neq i}}^{N_t} x_{i,j,k,t} \le \left(\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t}\right) * M \qquad \forall k,t \tag{27}$$

$$\sum_{i\in B}\sum_{\substack{j\in B\\ j\neq i}}^{N_t} x_{i,j,k,t} \le |B| - 1 \qquad \forall k,t,\ \forall B\subseteq G\setminus\{A\},\ |B|\ge 2 \tag{28}$$

$$\sum_{d=1}^{D}\sum_{i=1}^{N_t} y_{d,i,k,t} \le 1 \qquad \forall k,t \tag{29}$$

$$\sum_{d=1}^{D} s_{k,d,t} = 1 \qquad \forall k,t \tag{30}$$

$$\sum_{d=1}^{D} f_{k,d,t} = 1 \qquad \forall k,t \tag{31}$$

$$\sum_{i=1}^{N_t} y_{d,i,k,t} \le s_{k,d,t} \qquad \forall k,d,t \tag{32}$$

$$\sum_{i=1}^{N_t} z_{i,d,k,t} \le f_{k,d,t} \qquad \forall k,d,t \tag{33}$$

$$f_{k,d,t-1} = s_{k,d,t} \qquad \forall k,d, \forall t \geq 2 \qquad (34)$$

$$x_{i,j,k,t} \in \{0,1\} \qquad \forall i,k,t, \forall j \neq i \qquad (35)$$

$$y_{d,i,k,t} \in \{0,1\} \qquad \forall d,i,k,t \qquad (36)$$

$$y_{i,d,k,t} \in \{0,1\} \qquad \forall d,i,k,t \qquad (37)$$

$$s_{k,d,t} \in \{0,1\} \qquad \forall k,d,t \qquad (38)$$

$$f_{k,d,t} \in \{0,1\} \qquad \forall k,d,t \qquad (39)$$

$$max\{0, 2\alpha - 1\} \leq \mu_{k,t} \leq min\{2\alpha, 1\} \qquad (40)$$

in which

$$\max\left\{\left(\tilde{c}_{i,j,t}\right)_{\beta}^{L}, \left(\tilde{c}_{I,j,t}\right)_{1-\beta}^{L}\right\} \leq c_{i,j,t} \leq \min\left\{\left(\tilde{c}_{i,j,t}\right)_{\beta}^{R}, \left(\tilde{c}_{i,j,t}\right)_{1-\beta}^{R}\right\}$$

and

$$\max\left\{\left(\tilde{c}'_{d,i,t}\right)_{\beta}^{L}, \left(\tilde{c}'_{d,i,t}\right)_{1-\beta}^{L}\right\} \leq c'_{d,i,t} \leq \min\left\{\left(\tilde{c}'_{d,i,t}\right)_{\beta}^{R}, \left(\tilde{c}'_{d,i,t}\right)_{1-\beta}^{R}\right\}.$$

3. Solution methodology

In this section, the hybrid genetic-simulated annealing-auction (HGSA) algorithm is proposed to solve the considered problem. The structure of genetic algorithm is adapted from Eydi and Abdorahimi (2012). Auction and simulated annealing algorithms are used within the genetic algorithm to improve the speed and efficiency of HGSA algorithm. In the subsequent sub-sections, genetic, simulated annealing and auction algorithms are descried and then the hybrid algorithm is introduced.

3.1. Genetic algorithm

The genetic algorithm, is a stochastic method to solve constrained and unconstrained optimization problems (Thangiah and Salhi, 2001). Genetic algorithm is based on the natural selection, a process that imitates biological evolution. The genetic algorithm repeatedly modifies a population of individuals named chromosomes. Generally, the initial population is selected randomly. At each step, the genetic algorithm selects chromosomes from the current population based on their fitness and combines them to produce the next generation. The population evolves over successive generations. The algorithm ends when the termination conditions of the algorithm are satisfied.

3.2. Auction algorithm

The auction algorithm is developed originally to solve the classical assignment problem in parallel computation (Freling et al., 2001). In this work, auction algorithm is used to generate a near-optimal feasible initial solution for the considered problem. To do this, the problem is transformed into an assignment problem as follows. In a directed path from customer *i* to customer *j*, in fact, customer *i* is forward assigned to customer *j* and customer *j* is backward assigned to *i*. An assignment is feasible if each customer is forward and backward assigned to another customer or a depot. By this transformation, auction algorithm is applied to find a feasible solution to the problem. The generated initial solution is improved by the HGSA algorithm.

3.3. Simulated annealing-based mutation method

Genetic algorithm is naturally stochastic. Therefore, the algorithm may fall in local optima during the search process. To overcome, diversification and intensification strategies should be implemented. Mutation method, by accepting worse solutions, implies diversity to let the algorithm escape from local optima. Intensification is also imposed by reducing the probability of selecting worse solutions.

The mutation method in the genetic algorithm is empowered by employing simulated annealing algorithm due to its capability to escape from local optima. By making a small change in a selected chromosome, a mutated one is produced. The chromosome is selected with respect to the mutation rate p_m. If the fitness of the mutated chromosome is improved, that chromosome is accepted and replaced by the selected chromosome. Otherwise, the algorithm accepts the mutated solution with a probability of $P = e^{\frac{-\Delta E}{T}}$, in which, ΔE is the difference between the fitness of the selected chromosome and the mutated one and T is the temperature. The acceptance of worse solutions decreases during the search process by the appropriate cooling schedule function to provide intensification in the final steps of the algorithm.

3.4. Hybrid genetic-simulated annealing-auction algorithm

In this section, the proposed HGSA algorithm is demonstrated. The flowchart of the algorithm is presented in Figure 1. The procedure is elaborated in detail in the following.

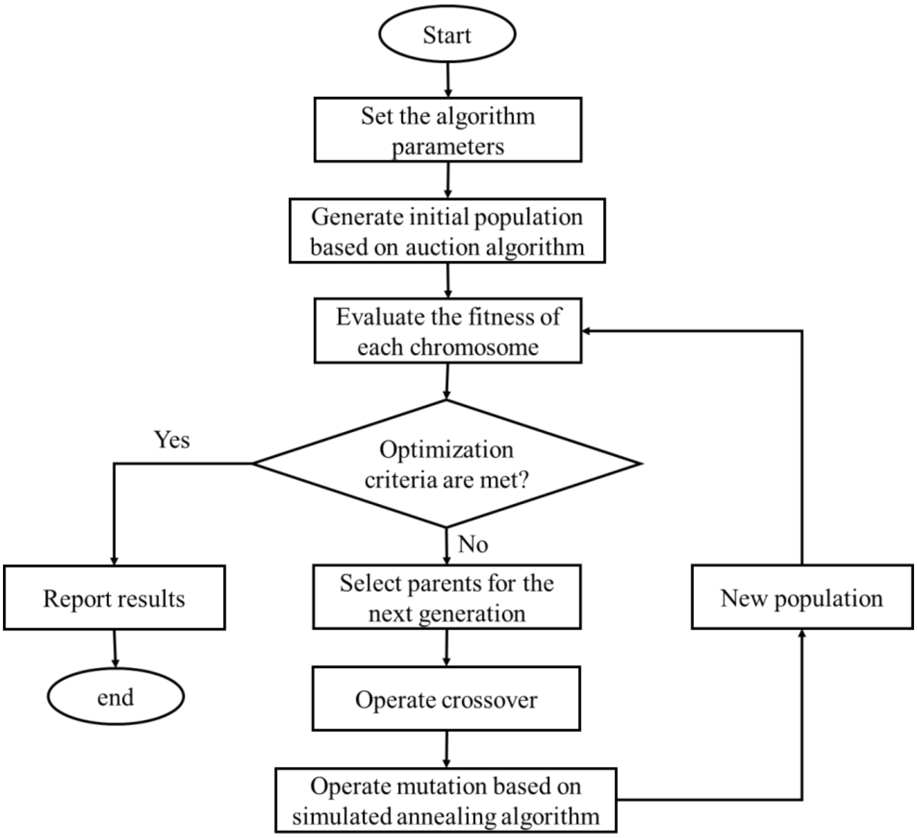

Figure 1. Flowchart of the hybrid genetic-simulate annealing-auction algorithm

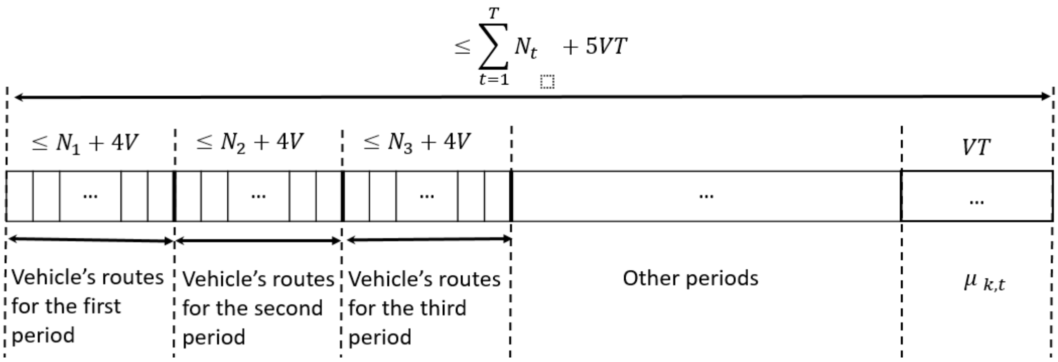

Figure 2. The structure of chromosomes in HGSA algorithm

3.4.1. Chromosome structure

The chromosome structure of HGSA algorithm is depicted in Figure 2. Figure 2 shows that each chromosome contains all the information of one feasible solution. For each period, an array of the vehicle's routes is considered. The length of each chromosome is up to $\sum_{t=1}^{T} N_t + 5VT$. The vehicle's route for each period is encoded as Figure 3. Figure 3 shows the routing information of one period which contains 12 customers, 3 depots and 3 vehicles.

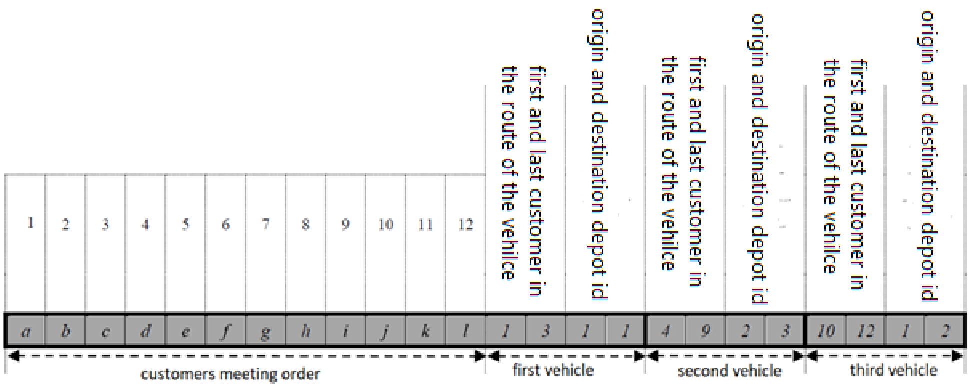

Figure 3. Encoding of the vehicle's routes for each period (Eydi and Abdorahimi, 2012).

In Figure 3, the route of first vehicle is $1 \to a \to b \to c \to 1$, the route of the second vehicle is $2 \to d \to e \to f \to g \to h \to i \to 3$ and the route of the third vehicle is $1 \to j \to k \to l \to 2$.

3.4.2. Initial population

To generate a near-optimal initial population for the genetic algorithm, a two-phase approach is applied for each period. In the first phase, an auction algorithm is used to arrange visits of customers and in the second phase vehicles are assigned to the routes generated in the first phase. In what follows, the details of the aforementioned phases are described.

3.4.2.1. First phase

In the first phase, a set of routes is generated that shows the order of customer visits. To apply the auction algorithm, the problem of finding routes with minimum cost is transformed into a single depot vehicle routing (SDVR) problem. Note that the vehicle's capacity constraints are neglected in this phase and enforced in the second phase. To transform the problem into a SDVR problem, two dummy depots are considered. An arc is drawn from the first dummy depot to other depots as well as from each depot to the second dummy depot. The cost of these arcs are zero. As a result, the auction algorithm is implemented in a network structure, such as the one illustrated in Figure 4. The auction algorithm for SDVR problem is adapted from Freling et al. (2001).

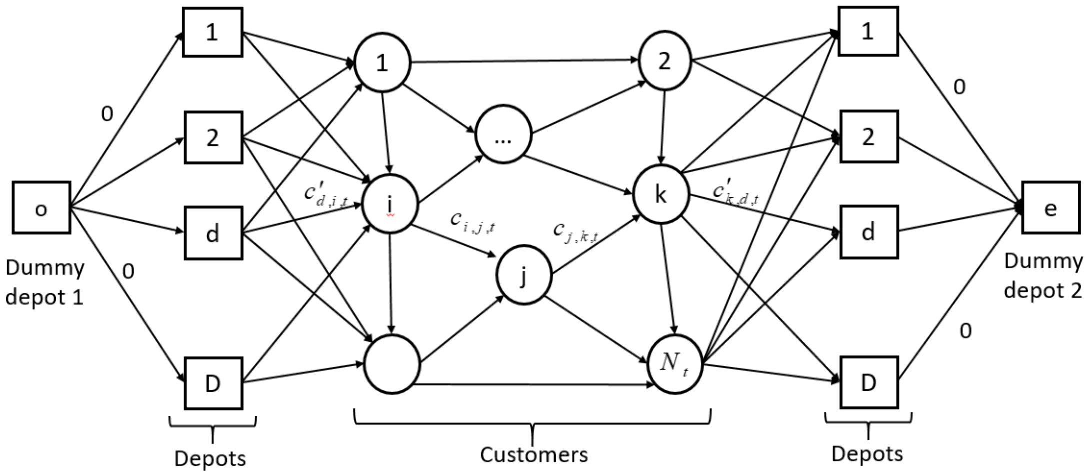

Figure 4. network structure to finding routes in the first population

By removing dummy depots in the first phase, the output of the first phase are a set of routes started from a depot and ends in another.

3.4.2.2. Second phase

In the second phase, the generated routes in the first phase are assigned to vehicles as follows.

Step1: For each depot, consider the subset of routes originated from that depot.

Step 2: A set of vehicles available at the depot is selected so that total capacity of these vehicles exceeds the total demand of customers in the routes considered in Step 1.

Step 3: Based on the total demand of customers and the capacity of the vehicles, each route is assigned to the appropriate vehicle.

3.4.3. Fitness function

The fitness of each chromosome is equal to total routing cost for all periods which is equal to the value of the objective function of problem.

3.4.4. Selection process

The selection process in the genetic algorithm is based on the roulette wheel method (Eydi and Abdorahimi, 2012).

3.4.5. Crossover and mutation method

In the crossover operation, a pair of chromosomes are selected from the population and combined to form two new chromosomes. To perform the single-point crossover method, the crossover parameter (p_c) is set. Then, a random number is selected from $\{1,...,T-1\}$. The information of the periods after the random number are switched between two parents and two children are born; see Figure 5. Also, the proposed simulated annealing-based mutation method is applied in each iteration of HGSA algorithm.

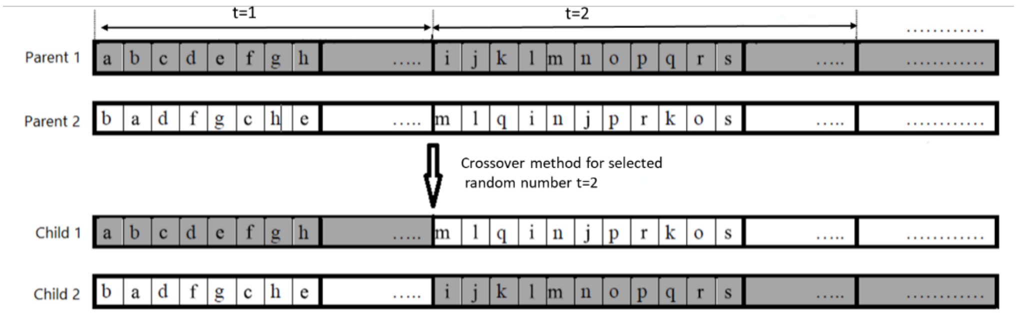

Figure 5. The single-point crossover method

4. Experimental results

In this section, several numerical tests are designed to show the efficiency of the proposed HGSA algorithm. A small example is solved to demonstrate the main concepts and results. Then, sensitivity analysis is done on the cooling schedule functions in simulated annealing-based mutation method. Finally, benchmark examples are solved to illustrate the performance of the proposed algorithm to solve large-scale and real problems. To implement the algorithm, MATLAB software is used on a computer with 8 GB RAM and 1.6-1.8 GHz CPU.

4.1. Small example

A fuzzy multi-depot and multi-period vehicle routing problem is considered with two periods and two depots. In each period, there are seven customers and three vehicles. It is assumed that fuzzy parameters are triangular fuzzy numbers. The demand of customers is presented in Table 2. Also, the fuzzy numbers for the capacity of vehicles are (100,105,110), (110,115,120) and (100,105,110), respectively. The values of parameters $\tilde{c}_{i,j,t}$ and $\tilde{c}'_{d,i,t}$ for $i, j = 1,...,7,\ d = 1,2$ and $t = 1,2$ are given in Table 3, Table 4, Table 5 and Table 6.

Table 2. Fuzzy demand of customers in two periods.

Customers / Periods	1	2	3	4	5	6	7
1	(6,11,15)	(17,20,21)	(8,16,17)	(14,15,16)	(8,16,18)	(8,11,16)	(13,14,21)
2	(8,18,19)	(16,16,17)	(7,11,17)	(6,14,15)	(15,24,25)	(11,18,19)	(16,16,23)

Table 3. The values of parameters $\tilde{c}_{i,j,t}$ for $t = 1$.

ustomers / customers	1	2	3	4	5	6	7
1	-	(10,12,21)	(10,19,29)	(27,31,32)	(15,22,28)	(16,17,26)	(13,22,32)
2	(10,12,14)	-	(14,15,18)	(29,37,39)	(17,21,26)	(21,28,37)	(30,34,4)
3	(13,19,20)	(7,15,16)	-	(41,49,51)	(30,36,37)	(34,35,45)	(30,35,40)
4	(23,31,32)	(29,37,41)	(42,49,59)	-	(20,21,31)	(31,37,38)	(15,23,27)
5	(13,22,24)	(16,21,31)	(33,36,39)	(13,20,29)	-	(32,40,41)	(29,30,36)
6	(14,16,22)	(20,28,33)	(30,35,45)	(20,21,31)	(22,25,29)	-	(37,45,49)
7	(12,22,32)	(31,34,35)	(25,35,37)	(32,37,38)	(30,40,47)	(9,16,24)	-

Table 4. The values of parameters $\tilde{c}_{i,j,t}$ for $t = 2$.

customers / customers	1	2	3	4	5	6	7
1	-	(35,36,40)	(43,45,46)	(17,23,33)	(28,30,40)	(42,45,46)	(26,27,32)
2	(34,35,38)	-	(10,12,20)	(6,12,18)	(11,21,22)	(42,47,55)	(40,41,42)
3	(38,45,54)	(6,12,13)	-	(19,21,25)	(14,22,26)	(37,46,47)	(43,44,54)
4	(8,12,14)	(17,21,28)	(6,14,15)	-	(24,30,35)	(21,31 ,33)	(35,37,38)
5	(20,21,22)	(18,22,25)	(13,14,16)	(20,26,30)	-	(14,22,25)	(12,21,28)
6	(44,47,53)	(44,46,56)	(39,40,45)	(25,26,27)	(8,18,21)	-	(27,28,33)
7	(31,41,42)	(35,44,50)	(22,30,39)	(20,21,31)	(13,18,20)	(14,22,27)	-

Table 5. The values of parameters $\tilde{c}'_{d,i,t}$ for $t=1$.

ustomers / depots	1	2	3	4	5	6	7
1	(29,36,37)	(41,42,48)	(49,54,61)	(1,6,13)	(17,22,29)	(22,27,37)	(35,43,44)
2	(5,13,17)	(18,21,24)	(30,32,36)	(14,17,24)	(8,14,15)	(4,11,15)	(18,26,28)

Table 6. The values of parameters $\tilde{c}'_{d,i,t}$ for $t=2$.

customers / depots	1	2	3	4	5	6	7
1	(41,43,46)	(29,34,35)	(22,31,32)	(27,30,35)	(8,16,21)	(6,14,15)	(22,23,29)
2	(20,22,28)	(23,24,29)	(18,28,29)	(11,12,17)	(7,8,10)	(25,29,35)	(8,18,24)

In Table 7, the credibility (α,β)-optimal solutions for different values of parameter α and β=0.8 are reported. Table 7 shows that by increasing the satisfaction degree α, the total cost is increased. In fact, by increasing parameter α, the generated solution is more robust to changes in the problem's parameters and the degree of confidence of the decision maker in the produced solution increases. Thus, more cost is paid for more reliability and robustness.

Table 7. The credibility (α,β)-optimal solutions for different values of parameter α and β=0.8

α	B	Period	Vehicle's route	Total cost
0.7	0.8	1	$8 \xrightarrow{2} 5 \to 6 \to 0 \to 2 \to 1 \to 4 \to 8$ $7 \xrightarrow{1} 3 \to 7$	275.6
		2	$8 \xrightarrow{2} 3 \to 1 \to 2 \to 4 \to 8$ $7 \xrightarrow{1} 5 \to 6 \to 0 \to 8$	
0.9	0.8	1	$8 \xrightarrow{2} 0 \to 1 \to 2 \to 6 \to 5 \to 8$ $7 \xrightarrow{3} 3 \to 4 \to 8$	285.3
		2	$8 \xrightarrow{1} 0 \to 6 \to 5 \to 7$ $8 \xrightarrow{2} 4 \to 2 \to 1 \to 3 \to 8$	

For each $\left(\tilde{c}_{i,j,t}\right)^L_\beta \le c_{i,j,t} \le \left(\tilde{c}_{i,j,t}\right)^R_\beta$ and $\left(\tilde{c}'_{d,i,t}\right)^L_\beta \le c'_{d,i,t} \le \left(\tilde{c}'_{d,i,t}\right)^R_\beta$, a credibility (α,β)-optimal solution of the problem can be obtained. Different ranking functions can be used to select the values of $c_{i,j,t}$ and $c'_{d,i,t}$. The credibility (α,β)-optimal solution of the problem by using Adamo's and Yager's ranking functions (Brunelli and Mezei, 2013) are reported in Table 8 in which $\alpha = 0.7$ and $\beta = 0.8$.

Table 8 indicates that, small changes are made in the vehicle's routes by applying different ranking functions. However, Changes in the cost are significant. The reason is that the idea proposed in this paper, produces a robust solution. Therefore, with a small change in the traversing cost parameters, the optimal plan does not change significantly, but the value of the objective function changes.

Also, AMPL (A Mathematical Programming Language) software is used to verify the accuracy of the generated solution. AMPL, is a mathematical modeling language to explain and solve optimization problems, developed in 1985 by Robert Fourer, David Gay, and Brian Kernighan at Bell Laboratories (https://ampl.com/). For the small example, the optimal solution produced by HGSA algorithm and AMPL software are the same. The time required to solve the problem in AMPL software is almost three times the time required for the HGSA algorithm.

Table 8. Credibility (α,β)-optimal solutions by using different ranking functions

Ranking function	Period	Vehicle's route	Total cost
Adamo's approach 1 $c_{i,j,t} = (\tilde{c}_{i,j,t})^R_\beta$ $c'_{d,i,t} = (\tilde{c}'_{d,i,t})^R_\beta$	1	$8 \xrightarrow{2} 4 \to 1 \to 2 \to 0 \to 6 \to 5 \to 8$ $7 \xrightarrow{3} 3 \to 7$	293.2
	2	$8 \xrightarrow{2} 4 \to 2 \to 1 \to 3 \to 8$ $7 \xrightarrow{3} 5 \to 6 \to 0 \to 8$	
Adamo's approach 2 $c_{i,j,t} = c_{i,j,t} + (1-\beta)(\tilde{c}_{i,j,t})^R_\beta$ $c'_{d,i,t} = c'_{d,i,t} + (1-\beta)(\tilde{c}'_{d,i,t})^R_\beta$	1	$8 \xrightarrow{2} 4 \to 1 \to 2 \to 0 \to 6 \to 5 \to 8$ $7 \xrightarrow{3} 3 \to 7$	336.64
	2	$8 \xrightarrow{2} 4 \to 2 \to 1 \to 3 \to 8$ $7 \xrightarrow{3} 5 \to 6 \to 0 \to 8$	
Yager's approach 1 $c_{i,j,t} = \frac{1}{2}\left((\tilde{c}_{i,j,t})^R_\beta + (\tilde{c}_{i,j,t})^L_\beta\right)$ $c'_{d,i,t} = \frac{1}{2}\left((\tilde{c}'_{d,i,t})^R_\beta + (\tilde{c}'_{d,i,t})^L_\beta\right)$	1	$8 \xrightarrow{2} 5 \to 6 \to 0 \to 2 \to 1 \to 4 \to 8$ $7 \xrightarrow{1} 3 \to 7$	275.6
	2	$8 \xrightarrow{2} 3 \to 1 \to 2 \to 4 \to 8$ $7 \xrightarrow{1} 5 \to 6 \to 0 \to 8$	
Yager's approach 2 $c_{i,j,t} = c_{i,j,t} + \frac{1}{3}\left((\tilde{c}_{i,j,t})^R_\beta - (\tilde{c}_{i,j,t})^L_\beta\right)$ $c'_{d,i,t} = c'_{d,i,t} + \frac{1}{3}\left((\tilde{c}'_{d,i,t})^R_\beta - (\tilde{c}'_{d,i,t})^L_\beta\right)$	1	$8 \xrightarrow{2} 4 \to 1 \to 2 \to 0 \to 6 \to 5 \to 8$ $7 \xrightarrow{3} 3 \to 7$	288.4
	2	$7 \xrightarrow{1} 5 \to 6 \to 0 \to 8$ $8 \xrightarrow{2} 3 \to 1 \to 2 \to 4 \to 8$	

4.2. Sensitivity analysis on cooling schedule functions

In the next experiment, the effect of different cooling schedule functions on the convergence of the simulated annealing-based mutation method is investigated. Cooling schedule functions have an important role in the convergence speed of the simulated annealing algorithm. Several cooling schedule functions for the simulated annealing-based mutation method are examined. The information of these cooling schedule functions has been collected in Table 9. Cooling schedule function T_6 is proposed in this paper. In this function, temperature reduction is proportional to the fitness of the generated chromosome. Therefore, in iterations where the value of the fitness is improved, the temperature decreases further. Hence, an improved process is proposed to update the parameter T .

Table 9. Comparing different cooling schedule functions for simulated annealing-based mutation method.

Reference	Cooling schedule function	Description
Geman and Geman, (1984)	$T_1(r) = \frac{T_0}{\log(t+1)}$	T_0 is the initial temperature and t is the number of iterations
Jalali and Boyce, (1995)	$T_2(t) = \frac{T_0}{\log(t+1)^3}$	T_2 is similar to T_1 With the difference that in early iterations the temperature is reduced faster.
Kirkpatrick et al., (1983)	$T_3(t) = \beta^t T_0$	β is a constant between 0.8 and 0.99.
Lin et al., 2000	$T_4(t) = \frac{1}{\rho+1}\left[\rho + \tanh(\beta)^t\right] T_{t-1}$	$\beta \in [0.8, 0.99]$ and $\rho = 4$.
Ghatee and Niksirat, 2013	$T_5(t) = \frac{1}{\rho+1}\left[\rho + \frac{1}{1+\tanh(\beta)^t}\right] T_{t-1}$	$\beta \in [0.8, 0.99]$ and $\rho = 4$.
This paper	$T_6(t) = \frac{1}{\rho+1}\left[\rho + \frac{1}{fit(c_t)}\right] T_{t-1}$	$fit(c_t)$ is the fitness of the chromosome c_t and $\rho = 4$.

To compare the effect of different cooling schedule functions on the performance of the simulated annealing algorithm, a performance measure is considered as the product of the number of iterations multiplied by the CPU-time. The results are shown in Table 10.

Table 10. The values of performance measure for different cooling schedule functions.

Cooling schedule function	Initial Temperature			
	20	50	200	400
T_1	635098/76	∞	∞	∞
T_2	560/32	334193/1	∞	∞
T_3	70/23	90/14	111/65	178/9
T_4	57/36	78/43	105/26	140/76
T_5	49/92	70/99	104/12	135/7
T_6	43/93	64/79	78/44	128/04

Based on the results of Table 10, the convergence speed of the algorithm is very slow for the cooling schedule functions T_1 and T_2. Also, these functions are sensitive to the initial temperature. At higher initial temperatures, convergence does not occur. The convergence speed for the cooling schedule functions T_3, T_4, T_5 and T_6 has been improved and the sensitivity of the algorithm to select the initial temperature has been reduced. The value of performance measure for the cooling schedule function T_6 is the least. Therefore, T_6 has been used in the proposed HGSA algorithm.

4.3. Benchmark examples

In this section, some benchmark examples are used to investigate the efficiency of the proposed algorithm in large-scale problems. Since there are no benchmark examples for the proposed fuzzy problem in the literature, a number of benchmark examples of the multi-depot vehicle routing problem are considered and converted to the fuzzy one in the manner described below. The topology structure of these benchmark examples are preserved. The benchmark examples are combined to create a multi-period (each benchmark example for a period) problem. Fuzzy parameters are considered as triangular fuzzy numbers (r_1, r_2, r_3) in which r_2 is the corresponding parameter in the related classical multi-depot vehicle routing problem and two parameters r_1 and r_3 are random numbers in intervals $[r_2 - 10, r_2]$ and $[r_2, r_2 + 10]$, respectively. The benchmark examples of the multi-depot vehicle routing problem can be downloaded from http://www.bernabe.dorronsoro.es/vrp/.

To compare the results, a fuzzy simulation method is applied as follows. A scenario-based method is used to select the scenario that meets the highest customer demand with the least number of vehicles. So, the generated solution is more stable to changes in the parameters of the problem. To generate this scenario, let $\tilde{d} = (\tilde{d}_{i,t})_{i=1,\ldots,N,t=1,\ldots,T}$ be the vector of demand and $\tilde{c} = (\tilde{c}_k)_{k=1,\ldots,V}$ be the vector of the capacity. M independent random vectors are generated for $d = (d_{i,t})_{i=1,\ldots,N,t=1,\ldots,T}$ and $c = (c_k)_{k=1,\ldots,V}$. Then, the scenario with the minimum value of $\left\| d - \left(\tilde{d}\right)_\alpha^R \right\|$ and $\left\| c - \left(\tilde{c}\right)_\alpha^L \right\|$ is selected. Also, M random values are generated for each parameter $\tilde{c}_{i,j,t}$ and $\tilde{c}'_{d,i,t}$ and the values with the maximum membership value are selected. The classical multi-period multi-depot vehicle routing problem is solved with the selected scenario and the optimal solution is compared with credibility (α,β)-optimal solution. Benchmarks examples specifications and results are reported in Table 11.

Table 11. The credibility (α,β)-optimal solution and simulated optimal solution for the benchmark examples (α=β=0.8).

Instances	# Vehicles		# Customers		# Depots	Optimal Solution		Error rate
	t=1	t=2	t=1	t=2		Credibility (α,β)-optimal solution	Simulated optimal solution	
P01,P01	4	4	50	50	4	1144.104	1163.43	1.6611
P01,P02	4	2	50	50	4	1043.231	1059.681	1.5525
P01,P03	4	3	50	75	5	1213.681	1204.414	0.7694
P02,P02	2	2	50	50	4	917.002	893.438	2.6374
P02,P03	2	3	50	75	5	1079.525	1115.214	3.2002
P03,P03	3	3	75	75	5	1226.596	1268.121	3.2745

Table 11 shows that the error rate between the credibility (α,β)-optimal solution and the simulated optimal solution in all cases is less than 3.5 in which

$$\text{Error rate} = 100 * \frac{\left|\text{credibility } (\alpha,\beta)\text{-optimal solution-simulated optimal solution}\right|}{\text{simulated optimal solution}}.$$

These results show that the solution obtained from the proposed HGSA algorithm is a relatively good approximation for the optimal solution of the problem.

5. Conclusions

In this paper, fully fuzzy multi-depot and multi-period vehicle routing problem with flexibility in specifying the last depot was investigated to handle the imprecise traveling cost and inexact customer's demand. A fuzzy integer programming model was proposed for the problem. Credibility relation was used to deal with fuzzy objective function and fuzzy constraints. A new solution method called hybrid Genetic-Simulated Annealing-Auction algorithm was developed to obtain the credibility (α,β)-optimal solution for the fuzzy problem. Experimental results were reported to demonstrate the efficiency of the proposed model and solution approach. For the small example, AMPL software was used to solve the transformed non-fuzzy problem. Comparison of the results showed that the solution produced by the hybrid algorithm was sufficiently accurate. In addition, the time taken to generate the solution by the hybrid algorithm was one third of the time required in the AMPL software. Moreover, benchmark examples were used to investigate the performance of the algorithm in large-scale problems. The results were compared with a fuzzy simulation method. The maximum error percentage of 3.5 indicated desired accuracy of the proposed method. In the future researches, authors are going to consider other fuzzy relations and/or implement other metaheuristic algorithms to compare the results. The relief chain can be reconfigured with three echelons: central disaster facilities of the city as main suppliers, local emergency facilities as distributors, and urban areas as customer. Also, location and routing decisions can be considered simultaneously. Furthermore, considering a heterogeneous fleet of vehicles, multi-commodity feature, delay penalties and speed rewards is strongly recommended for creating a more generalized form of the model. Finally, a multi-objective optimization approach can be employed in future researches.

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