Robust Bi-Objective Location-Arc Routing Problem with Time Windows: A Case Study of an Iranian Bank

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Abstract

In Location-Arc Routing Problems (LARP), unlike the well-known locating-routing problems, demand is on the arc and using deadheading arcs is permitted. Few studies have focused on an arc-routing problem. In this research, a complex bi-objective linear mathematical model for the LARP with time windows under uncertainty is presented. Time windows in the arc-routing problem modeling have complexity since the required arc with time windows becomes a deadheading arc without time windows after service. Furthermore, modeling of the vehicle servicing to multiple required arc with the minimum deadheading arc in route is a feature of this work. The proposed LARP is used for modeling of transforming cash in the bank case study. In this case study, demand has uncertainty with unknown probability distributions and we used Bertsimas and Sim approach for it. The case study problem is a node-basing problem with the closest node and time windows for servicing branches. For this purpose, the Multi-Objective Dragonfly Algorithm (MODA) and Non-dominated Sorting Genetic Algorithm (NSGA-II) are used for locating the cash supply centers of a bank in Tehran. Furthermore, comparing results of robust and deterministic LARP models show that the mean and standard deviation of objective function values in the robust model has better performance in realization.

Keywords: Location-arc routing problem; Time windows; Multiple periods; Robust optimization; Demand Uncertainty.

1. Introduction

The aim of a routing problem is to select the optimal routes for vehicles to service the customers considering limitations, equipment, and objectives. In this problem, customers are located on the node or arc/edge of a graph. The Vehicle Routing Problem (VRP) is the node routing problem and the Rural Postman Problem (RPP) is the arc routing problem. Some researchers have focused on arc routing problems recently.

The Capacitated Arc Routing Problem (CARP) is an expansion of the RPP with multi-vehicle and capacity conditions (Golden and Wong, 1981). This problem is defined on an undirected connected graph $G = (V, E)$ that each edge (i.e., customer) has a demand $q_{ij} \geq 0$. For serving required edges, undemanded (deadheading) edges with minimum numbers must be used. After servicing the required edge, it becomes a deadheading edge and can services other required edges.

A Location-Arc Routing Problem (LARP) is a combination of the CARP and the Location-Allocation Problem (LAP) (Ghiani et al., 2001). The LARP is defined as a graph that can be directed, undirected or both. The crucial difference between the CARP and the LARP is that along with routing, the best depot location must be determined. Therefore, this issue is added as a binary decision variable to the mathematical model and makes it even more complicated. The LARP can be used for arc-routing problems (e.g., snow plowing or node-routing problems) with nearby nodes (e.g., the distribution network of supermarkets) (Albareda-Sambola, 2015). Limited studies have been focused on the second application of the LARP (Tavakkoli-Moghaddam et al., 2018).

1.1. Contribution

The main purpose of this paper is to solve transforming the problem of cash in the branches network in a bank. This problem is to select an optimal number and the location of the cash supply center as well as the optimal routing for cash delivery to branches. In the case study, four cash supply centers are carried out delivery and collection of cash to 517 banking units (i.e., 250 branches, 202 counters, and 65 fuel stations) in Tehran. Three cash supply centers have been closed and bank has a challenge for delivery and collection of cash on time. This is a node routing problem with nodes close and can be considered as an arc routing problem for solving it. Therefore, in the case study, branches with a distance of less than 1 km and similar availability can be considered as a required arc with the sum of branches demand. In the case study, delivering and collecting cash on time are necessary, and the definition of time windows constraint is important. Considering the time windows in the arc-routing problem has a different structure than the node-routing problem, since a required arc with time windows after servicing can be used as a deadheading arc without time windows as a path. The waiting time of the vehicle in servicing time is minimized considering the low probability of rubbery, indirectly. The number of cash supply centers, vehicles, and equipment depends on the demand for cash collection or delivery. This demand is uncertain due to uncertainty in the factors affecting the liquidity of the society (Pavlis et al. 2018). Therefore, a robust approach is used for uncertain demand. This paper presents a bi-objective and multi-period LARP model with time windows under uncertainty to minimize the cost and waiting time. The deterministic and robust LARP model are solved by the Multi-Objective Dragonfly Algorithm (MODA) and Non-dominated Sorting Genetic Algorithm (NSGA-II) under nominal data and realization.

1.2. Organization

The rest of this paper is structured as follows. Section 2 is associated with the literature review, including the LARP, time windows in an arc routing problem, and robust approach. In Section 3, a robust mathematical model for the LARP is presented. The case study is described in Section 4. In Section 5, the MODA and solution representation are explained. To evaluate the performance of the proposed deterministic and robust models are solved by the MODA and NSGA-II for the bank case study in Section 6. Finally, this paper is concluded in Section 7.

2. Literature review

The number of LARP researches are increased, recently. However, LARP has a higher complexity versus to Location-Routing Problem (LRP), and many features such as time window, uncertainty needs to be surveyed in this problem.

2.1. Review of location-arc routing problems

The LARP was proposed by Levy and Bodin (1989) to solve the routing problem at the post office in the USA. They used a Location-Allocation-Routing method to solve this model. Ghiani and Laporte (2001) presented a linear multi-depot model for the LARP. They converted it to RPP model and solved it by a branch-and-cut. Liu et al. (2014) presented a survey of recent researches on LARP. The specifications of the LARP are addressed in Table 1.

Doulabi and Seifi (2013) presented two MILP models for single and multi-depot problems with the flow variables and solved them by simulated annealing. Lopes et al. (2014) proposed a LARP mathematical model and solved it by several heuristics. They tested different constructive heuristics combining variable neighborhood search, greedy randomized adaptive search procedure, and tabu search. Their model implies each customer has required an individual vehicle that is not practical. Laporte et al. (2015) developed LARP models presented by Doulabi and Seifi (2013) and Lopes et al. (2014). They solved their model with the exact method. Essink and Wagelmans (2015) used Lopes et al. (2014) model and solved it by metaheuristic TS-GRASP. Riquelme-Rodríguez et al. (2016) proposed a non-linear model for a periodic LARP and used a heuristic method for solving their model. Huber (2016) presented a bi- objectives model without formulating constraints. He used heuristic method for solving benchmark instances. Amini et al. (2017) addressed an uncertain LARP and employed two scenario-based approaches. They evaluated the performance of models with the results of the numerical example.

Tavakkoli-Moghaddam et al. (2018) presented a multi-product LARP by developing two mathematical models under two assumptions. All types of products of a required arc must be supplied by a tour and each type of a required arc product must be supplied by a tour. Fernández et al. (2019) modeled several LARPs on an undirected graph and solved them with branch and cut. Amini et al. (2019) developed the LARP when there are transportation decisions from suppliers to established depots. They used augmented ε-constraint, NSGA-II, and Multi-Objective Late Acceptance Hill-Climbing

algorithm for solving this problem. Mirzaei-Khafri et al. (2019) considered a CLARP with deadlines and a fleet of capacitated heterogeneous vehicles. They showed that the result of the sensitivity analysis of their proposed model can meet arc routing timing requirements more precisely compared to the CARP.

2.2. Time windows in an arc routing problem

In this paper, the time window is important for servicing branches of the bank case study. The CARP researches address this concept. Vansteenwegen et al. (2010) used a meta-heuristic method to solve the ARP problem with soft time windows for a case study of the mobile mapping van problem. They did not present any mathematical programming model. Macedo et al. (2011) are not modeled time window and solved it with metaheuristic methods. Lystlund et al. (2012) presented two linear models for the CARP problem with hard and soft time windows and solved it with the heuristic. Black et al. (2013) define a CARP model with prize-collecting. The proposed model was non-linear that was not possible to linearize. Therefore, a heuristic method has been used for solving the model. Çetinkaya et al. (2013) claim that it is not possible to model time windows in arc routing problems and should be transformed into a node routing problem. They did not present a model for the ARP with time windows and prize-collecting and used a meta-heuristic to solve it. Çetinkaya et al. (2018) proposed a model for the LRP by defining time windows for arcs with military applications. While in the proposed model, the time window is defined for arcs, the customers are located on the nodes and the model is node-based.

Table 1. Comparison of LARP features

Reference	Depot				Customer				Vehicle			Material		Model Features					Solving Method		
	Multi depot	Capacity	Heterogeneous	Inventory	Capacity	Splitting allowed	Time Windows	Determine the number	Capacity	Heterogeneous	Time available	Multi Produce	Risk	Multi-objective	Multi-period	Linear	Game theory	Uncertain	Exact	heuristic	Metaheuristic
(Doulabi & Seifi, 2013)	×	×							×	×											×
(Lopes et al., 2014)	×																				×
(Essink & Wagelmans, 2015)	×											×			×					×	
(Riquelme-Rodríguez et al., 2016)	×			×																	×
(Huber, 2016)	×			×										×						×	
(Amini et al., 2017)	×	×															×			×	
(Tavakkoli-Moghaddam et al., 2018)	×	×							×			×								×	
(Javanmardi & Hafezalkotob, 2018)	×	×												×			×				×
(Fernández et al., 2019)	×	×																	×		
(Amini et al., 2019)	×	×							×					×							×
(Mirzaei-Khafri et al., 2019)	×	×								×											×
(Mirzaei-Khafri et al., 2020)	×	×								×							×			×	
This paper	×	×	×		×		×	×	×	×	×	×		×				×	×	×	×

2.3. Uncertainty in arc routing problem

The main approaches to deal with data uncertainty in optimization are stochastic, fuzzy and robust programming. These approaches have different applications (Teimoori et al., 2014; Fazli-Khalafa, Hamidieh, 2017; Hamidieha et al., 2018). Stochastic programming i.e. Scenario-based stochastic programming can be applied for probabilistic randomness data. Fuzzy programming is like stochastic programming and the difference is in the way uncertainty is modeled. Robust optimization approaches can be used for data with unknown probability distributions. A robust approach is a method for solving problems of linear optimization under uncertain conditions. To ensure that the solution remains near-optimal and feasible when the data changes, this approach has accepted a sub-optimal solution for nominal data values. Optimal solutions that are less sensitive to uncertainty is called a robust solution (Bertsimas & Sim, 2004). This method is an alternative for stochastic programming and sensitivity analysis that is used in discrete problems. Different models have been developed to formulate linear optimization problems with robust methods. Three primary models in a robust approach based on interval uncertainty are Soyster (1973), Ben-Tal and Nimrovsky (1998), and Bertsimas and Sim (2004). The Soyster approach is the most conservative method among others, such that the objective value is much worse than the objective value of nominal linear optimization problem. Ben-Tal and Nimrovsky approach is a quadratic conical

model with $n + 2k$ variable and $m + 2k$ constraint. Since this model is non-linear, it is not efficient solution for discrete optimization models. The Bertsimas and Sim (2004) approach change the linear programming (a) to a robust model (b) as follows:

Min $c^T x$

s.t.

$$\sum_j \tilde{a}_{ij} x_j \leq b_i \qquad \forall i \qquad \text{(a)}$$

$$l \leq x \leq u$$

Min $\quad c^T x + q_0 \Gamma_0 + \sum_{j \in J_0} r_{0j}$

s.t.

$$\sum_j a_{ij} x_j + q_i \Gamma_i + \sum_{j \in J_i} r_{ij} \leq b_i \qquad \forall i \qquad \text{(b)}$$

$$q_i + r_{ij} \geq \hat{a}_{ij} E_j \qquad \forall i \neq 0, j \epsilon J_i$$

$$q_0 + r_{0j} \geq e_j E_j \qquad \forall j \epsilon J_i$$

$$l_j \leq x_j \leq u_j \qquad \forall j \epsilon J_i$$

$$q_i, r_{ij}, y_j \geq 0 \qquad \forall i, j \epsilon J_i$$

where coefficients $\tilde{a}_{ij}$ are uncertainty and random variable in $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$, a_{ij} and $\hat{a}_{ij}$ are the nominal values and the variation magnitude of the uncertain parameter, respectively. J_i denote the set of uncertain coefficients of row *i*. For the *i*-th constraint, a control parameter Γ_i, is called the price of robustness. Parameters $\Gamma_i \epsilon [0, |j_i|]$ and $|j_i|$ are the cardinality of the set j_i.

Babaee Tirkolaee et al. (2018) presented a robust periodic CARP for urban waste collection considering drivers and crew's working time. Babaee Tirkolaee et al. (2019) proposed a robust bi-objective multi-period CARP for urban waste collection using a Multi-Objective Invasive Weed Optimization (MOIWO). Babaee Tirkolaee et al. (2020) consider the multi-trip CARP under fuzzy demands for urban solid waste management. Mirzaei-Khafri et al. (2020) consider a location-arc routing problem (LARP) on an undirected network. The proposed robust model was less sensitive to demand variations and was validated through Monte-Carlo simulation and Relative Extra Cost (REC) measures with promising results. The results of sensitivity analysis showed that by increasing the degrees of conservatism, planners might employ more vehicles. Also, more depots might be opened to service all required roads.

The literature review in this section shows that the models for LARPs do not address time windows. The review of arc routing researches with time windows shows that a few researches focus on presenting a mathematical model, and all models except Lystlund et al. (2012) are non-linear. Besides, the models for LARPs are not used for the financial case study. The scenario-based stochastic programming and fuzzy approach to deal with data uncertainty in the models for LARPs are used and robust programming is not used so far.

3. Mathematical model

The LARP is defined as a graph $G = (V, E \cup R)$, which $V = \{v_i, \ldots, v_n\}$ is a set of vertex, $E = \{[v_i, v_j]: v_i, v_j \in V, i \neq j\}$ is a set of edges, and $R = \{(v_i, v_j): v_i, v_j \in V, i \neq j\}$ is a set of arcs, which can be directed, undirected, or a combination of both. Here, the LARP problem is a directional graph and the connection of two vertexes is called an arc. The vertex set V contains a non-empty subset J of n potential depot locations ($J \subseteq V$). Every arc $(i, j) \in R$ has a non-negative traversal cost and a non-negative demand for service. The arcs with positive demand form the subset A of the arcs required to be serviced, only once, by a vehicle K with capacity Q in a determined time window. Vehicles start and end their route in the same depot, and each new vehicle. The deadheading arc is movement from the end *i* of one required arc to the start *j* of another required arc without servicing the traversed arcs.

The assumptions, sets, parameters, uncertain parameters, decision variables for robust bi-objective LARP model are presented in the following subsections.

3.1. Assumptions

The main assumptions are as follows:

- Every required arc is traversed by one vehicle
- The vehicle is returned to the depot that was started
- The total demand for the arcs selected on tour is less than the vehicle capacity
- The number of deadheading arc selected in every tour is minimized
- Each required arc is serviced once that can be used as a deadheading arc without a time windows
- Waiting time is allowed for each required arc at the start of service
- The distribution of products is delivery and collection
- The number of deadheaded traverses in each arc is not limited

3.2. Sets, parameters, and decision variables

Sets, parameters, and decision variables used in mathematical models are as follow:

Sets

I	Set of all vertices ($I = \{1, ..., i\}$, which includes customers and depots)
J	Set of depots ($J = \{1, ..., j'\}$) that j' is the maximum number of depots)
K	Set of vehicle $K = \{1, ..., k\}$
P	Set of period $P = \{1, ..., p\}$
S	Set of steps that every vehicle travels $S = \{1, ..., s\}$
A	Set of required arcs that has to be visited
V_A	Set of vertices that are extremities of the arcs in set A
$\hat{V}$	Union of the set J and V_A ($\hat{V} = V_A \cup J$)
$\hat{A}$	Set of arcs forming a complete graph with $\hat{V}$
$\delta^+(J)(\delta^-(J))$	Set of arcs leaving (entering) on the set of vertices J. When S contains a single vertex v, $\delta^+(v)$ is a simplification for $\delta^+(\{v\})$

Parameters

$f_{j'p}$	Costs of creating a depot j' in period p
c_{ijp}	Costs of traversing arc (i, j) in period p, if arc $(i, j) \in A$ cost of servicing is equal to $\hat{c}_{ij}$ and if the arc is deadheading (i.e. arc $(i, j) \notin A$) is equal $\acute{c}_{ij}$
$b_{j'p}$	The capacity of depot j in period p
t_{ijp}	Travel time on arc $(i, j) \in A$ in period p, if arc $(i, j) \in A$ time of traveling is equal to $\hat{t}_{ijp}$ and if the arc is deadheading (i.e., arc $(i, j) \notin A$) is equal arcs $\acute{t}_{ijp}$
T_k	The maximum time allowable for vehicle k in period p
F_k	Fixed costs of using vehicle k
N_p	The number of customers in period p
Q_{kp}	The capacity of vehicle k in period p
M	A very large number

Uncertain Parameter

$\tilde{d}_{ijp}$ Demand for service of arc $(i,j) \in A$

Decision variables

x_{ijksp} 1 if arc $(i,j) \in \hat{A}$ is traveled by vehicle k in period p at step s; and 0, otherwise

y_{ijksp} 1 if arc $(i,j) \in A$ is serviced by vehicle k in period p at step s; and 0, otherwise

$w_{ijj'p}$ 1 if arc $(i,j) \in A$ is allocated to depot $j^{'}$ in period p

ω_{ijp} The time that service of the arc $(i,j) \in \hat{A}$ starts

φ_{iksp} The time that vehicle k arrives in node i at step s

ρ_{ijksp} The time that vehicle k starts traversing arc $(i,j) \in \hat{A}$ at step s

u_{ikp} Slack variable for eliminating sub-tours

3.3. Robust bi-objective LARP model

The first proposed LARP model is bi-objective, multi-period, and mixed-integer non-linear programming with uncertain demand. Finally, the proposed model is converted into a linear and robust model.

3.3.1. Objective functions

The objective function (1) minimizes the sum of the costs of depots creation, serviced arcs, and selected vehicles, respectively. The second objective (2) minimizes the waiting time of the vehicle.

$$\text{Min} \quad Z_1 = \sum_{j' \in J} \sum_{p \in P} f_{j'p} w_{ijj'p} + \sum_{(i,j) \in \hat{A}} \sum_{k \in K} \sum_{s \in S} \sum_{p \in P} c_{ijp} x_{ijksp} + \sum_{k \in K} \sum_{(i,j) \in \delta^+(J)} \sum_{s \in S} \sum_{p \in P} F_k x_{ijksp} \tag{1}$$

$$\text{Min} \quad Z_2 = \sum_{k \in K} \sum_{(i,j) \in A} \sum_{s \in S \setminus \{1\}} \sum_{p \in P} (\rho_{ijksp} - \varphi_{iksp}) \tag{2}$$

3.3.2. Basic Constraints

Standard LARP includes basic Constraints (3-11). Constraint (3) ensures that each required arc is serviced once by precisely one vehicle. Constraint (4) guarantees that the flow is preserved i.e., the number of arrivals at any node is equal to the number of leaving. Constraint (5) implies that an arc is serviced by a particular vehicle only if it is traversed by the same vehicle. Constraint (6) is a sub-tour elimination constraint. A tour is started from a depot and is ended at the same depot. The constraint of allocated of the required arcs to a depot is defined by $\sum_{m \in V_A} x_{j'mksp} y_{ijksp} \leq M w_{ijj'p}$. This constraint is non-linear. Therefore, $x_{j'mkp} \times y_{ijkp}$, replaced by $h_{j'mijkp}$. Besides, this constraint is replaced with Constraints (7-9). The constraints make sure all required arcs that are in the same route are devoted to depot j that route is started from it and ended. This constraint forced the vehicle to service to multiple required arc. These constraints are necessary to establish Constraint (10) that ensures the capacity of the depots is not violated. Constraint (11) ensures that the vehicle capacity is not exceeded.

$$\sum_{k \in K} y_{ijksp} = 1 \qquad\qquad \forall\, (i,j) \epsilon A, s \epsilon S, p \epsilon P \tag{3}$$

$$\sum_{(i,j)\,\epsilon \delta^+(i)} x_{ijksp} - \sum_{(i,j)\epsilon\, \delta^-(i)} x_{ijksp} = 0 \qquad\qquad \forall\, i \epsilon V_A, k \epsilon K, s \epsilon S, p \epsilon P \tag{4}$$

$$x_{ijksp} \geq y_{ijksp} \qquad\qquad\qquad \forall\, (i,j) \epsilon A, k \epsilon K, s \epsilon S, p \epsilon P \qquad\qquad (5)$$

$$u_{ikp} - u_{jkp} + N_p x_{ijksp} \leq N_p - 1 \qquad\qquad \forall\, (i,j) \epsilon V_A, i \neq j, k \epsilon K, s \epsilon S, p \epsilon P \qquad (6)$$

$$h_{j'mijkp} \leq M w_{ijj'p} \qquad\qquad\qquad \forall j' \epsilon J, m \epsilon V_A, (i,j) \epsilon A, k \epsilon K, p \epsilon P \qquad (7)$$

$$x_{j'mkp} + y_{ijkp} \leq 1 + h_{j'mijkp} \qquad\qquad \forall j' \epsilon J, m \epsilon V_A, (i,j) \epsilon A, k \epsilon K, p \epsilon P \qquad (8)$$

$$x_{j'mkp} + y_{ijkp} \geq 2 * h_{j'mijkp} \qquad\qquad \forall j' \epsilon J, m \epsilon V_A, (i,j) \epsilon A, k \epsilon K, p \epsilon P \qquad (9)$$

$$\sum_{(i,j) \epsilon R} \tilde{d}_{ijp} w_{ijj'p} \leq b_{j'p} \qquad\qquad \forall\, j' \epsilon J, k \epsilon K, s \epsilon S, p \epsilon P \qquad\qquad (10)$$

$$\sum_{(i,j) \epsilon A} \tilde{d}_{ijp} y_{ijksp} \leq Q_{kp} \qquad\qquad \forall\, k \epsilon K, s \epsilon S, p \epsilon P \qquad\qquad\qquad (11)$$

3.3.3. Time windows constraint

Time window constraints are summarized in Constraints (12) – (19). The time windows in the arc routing problem have a different structure compared to the node routing problem. After servicing a required arc, we can use it as a deadheading arc without time windows. Constraint (12) denotes that the start time of the journey on arc $(i,j) \epsilon \hat{A}$ cannot be before the arrival time in node i at step s. Index s is steps that every vehicle travels. This index is necessary for Constraints (12) – (19), since arrival time in node j is calculated based on arrival time in node i in previous step of vehicle. In standard LARP, this index is not necessary.

Constraints (13) – (14) implies that if vehicle k services to the required arc $(i,j) \epsilon A$, the arrival time in node j will equal to sum of the start time and service time. Constraints (15) and (16) demonstrates that if vehicle k traverses the deadheading arc $(i,j) \epsilon \hat{A}$, the arrival time in node j will equal the sum of the start time and travel time. Constraints (13) – (16) are functional constraints if the vehicle selects the arc for service or travel. Constraint (17) ensures that the service time variable ω_{ijp} is equal to start time of servicing the vehicle. Constraint (18) ensures that required arc cannot be deadheading arc earlier than finishing the service time. Constraints (12) – (18) are functional constraints if the required arc $(i,j) \epsilon A$ is selected in optimum routes. Constraint (19) ensures that the vehicles, at the first step, will start in the depot at time zero. Constraint (20) ensures that the vehicle usage time in each tour does not exceed the maximum available time.

$$\varphi_{iksp} - \rho_{ijksp} \leq M(1 - x_{ijksp}) \qquad\qquad \forall\, (i,j) \epsilon \hat{A}, k \epsilon K, s \epsilon S, p \epsilon P \qquad (12)$$

$$\rho_{ijksp} + \acute{t}_{ijp} - \varphi_{jks+1p} \leq M(1 - y_{ijksp}) \qquad \forall\, (i,j) \epsilon A, k \epsilon K, s \epsilon S, p \epsilon P \qquad (13)$$

$$\varphi_{jks+1p} - (\rho_{ijksp} + \acute{t}_{ijp}) \leq M(1 - y_{ijksp}) \qquad \forall\, (i,j) \epsilon A, k \epsilon K, s \epsilon S, p \epsilon P \qquad (14)$$

$$\rho_{ijksp} + t_{ijp} - \varphi_{jks+1p} \leq M\left(1 - (x_{ijksp} - y_{ijksp})\right) \qquad \forall\, (i,j) \epsilon \hat{A}, k \epsilon K, s \epsilon S, p \epsilon P \qquad (15)$$

$$\varphi_{jks+1p} - (\rho_{ijksp} + t_{ijp}) \leq M\left(1 - (x_{ijksp} - y_{ijksp})\right) \qquad \forall\, (i,j) \epsilon \hat{A}, k \epsilon K, s \epsilon S, p \epsilon P \qquad (16)$$

$$\rho_{ijksp} - \omega_{ijp} \leq M(1 - y_{ijksp}) \qquad\qquad \forall\, (i,j) \epsilon A, k \epsilon K, s \epsilon S, p \epsilon P \qquad (17)$$

$$\omega_{ijp} + \acute{t}_{ijp} - \rho_{ijksp} \leq M\left(1 - (x_{ijksp} - y_{ijksp})\right) \qquad \forall\, (i,j) \epsilon \hat{A}, k \epsilon K, s \epsilon S, p \epsilon P \qquad (18)$$

$$\varphi_{0k1p} = 0 \qquad\qquad \forall\, k\epsilon K, p\epsilon P \qquad\qquad (19)$$

$$\sum_{(i,j)\epsilon A} t_{ijp} x_{ijksp} \leq T_{kp} \qquad\qquad \forall\, k\epsilon K, p\epsilon P \qquad\qquad (20)$$

3.3.4. Decision variables

Constraints (21) make sure that the decision variables are binary and non-negative.

$$x_{ijksp}, y_{ijksp}, w_{ijj'p}, u_{ikp}, h_{j'mijksp} \in \{0,1\}, \;\; \varphi_{iksp}, \rho_{ijksp}, \omega_{ijp} \geq 0 \qquad\qquad (21)$$

If demand in Constraints (10) and (11) is considered determined, objectives function of (1) and (2) and Constraints (3) to (21) are made the deterministic LARP model. In the bank case study, demand is uncertain with unspecified distribution function. In the next subsection, we present a robust model for the LARP by using the approach proposed by Bertsimas and Sim (2004).

3.3.5. Robust LARP model

In the proposed model for the LARP, $\tilde{d}_{ijp} \in \left[d_{ijp} - \hat{d}_{ijp}, d_{ijp} + \hat{d}_{ijp}\right]$ in Constraints (10) and (11) has uncertainty. Accordingly, based on Section 2.3, by introducing variables q_{kp}^1, r_{ijkp}^1 and Γ_{kp}^1, Constraint (10) is replaced by the following constraints:

$$d_{ijp} y_{ijkp} + q_{kp}^1 \Gamma_{kp}^1 + \sum_{(i,j)\epsilon R} r_{ijkp}^1 \leq Q_{kp} \qquad\qquad \forall\, k\epsilon K, p\epsilon P \qquad\qquad (22)$$

$$q_{kp}^1 + r_{ijkp}^1 \geq \hat{d}_{ijp} y_{ijkp} \qquad\qquad \forall\, (i,j)\epsilon R, k\epsilon K, p\epsilon P \qquad\qquad (23)$$

$$q_{kp}^1, r_{ijkp}^1 \geq 0 \qquad\qquad\qquad (24)$$

Similarly, by introducing variables q_{kp}^2 and r_{ijkp}^2 and Γ_{kp}^2, Constraint (11) will be replaced with the these constraints:

$$d_{ijp} w_{ijj'p} + q_{kp}^2 \Gamma_{kp}^2 + \sum_{(i,j)\epsilon R} r_{ijkp}^2 \leq b_{j'p} \qquad\qquad \forall\, j'\epsilon J, k\epsilon K, p\epsilon P \qquad\qquad (25)$$

$$q_{kp}^2 + r_{ijkp}^2 \geq \hat{d}_{ijp} w_{ijj'p} \qquad\qquad \forall\, (i,j)\epsilon R, j'\epsilon J, k\epsilon K, p\epsilon P \qquad\qquad (26)$$

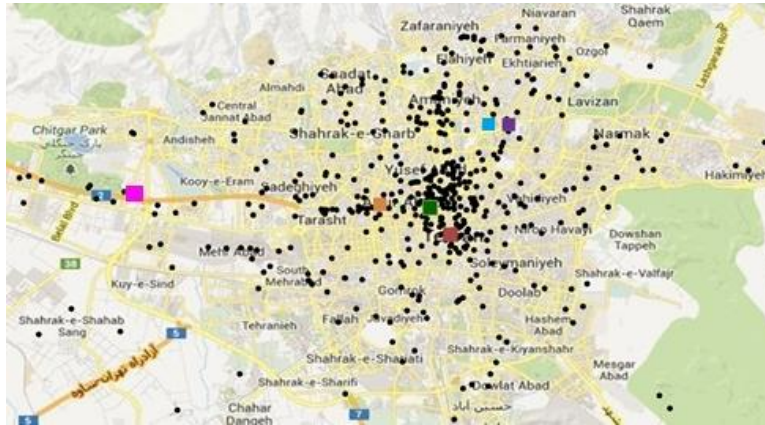$$q_{kp}^2, r_{ijkp}^2 \geq 0 \qquad\qquad\qquad (27)$$

Therefore, the proposed bi-objective robust model for the LARP is mixed-integer linear programming with objective functions (1) and (2) and Constraints (3) – (9) and (12) – (27). This robust proposed model is used for solving the location and routing problem of the bank as case study with uncertain demand for cash collection and delivery.

4. Bank case study

Processing the cash supply should be on time in the bank branches network. In this paper, the bank case study has two challenges in the cash supply. Challenge 1: according to the instructions of the Central Bank of Iran, cash collection and delivery should be done by the cash supply center of the bank or the National Bank by fee-paying. The cost-benefit of these solutions should be calculated with the number and location of the required cash supply center. Kahfi et al. (2018) solved this challenge with an LARP model. Challenge 2: the cash supply center of the bank in Tehran has several problems. In this case study, four cash supply centers (i.e., red, green, purple, and pink rectangles in Figure 1a) collect and deliver the cash for 517 banking units (250 branches, 202 counters, and 65 fuel stations) in Tehran. Also, a special

cash supply center (i.e.blue rectangles in Figure 1a) is communicated with the Central Bank of Iran and other cash supply. The problems of the cash supply of the Tehran center are as follows:

- Two cash supply centers were included in the urban development planning (blue and purple in Figure 1a) the bank is forced to move them to another cash supply center (i.e., green rectangle in Figure 1a).

- One of the cash supply centers (red rectangle in Figure 1a) had security problems. Therefore, it moved to a temporary cash supply center (orange rectangle in Figure 1a).

a) Current cash supply centers location

b) Potential cash supply centers location

● Branches   ■ Current cash supply centers   ▲ Potential cash supply centers

Figure 1. Current and potential cash supply centers location

Therefore, there are three cash supply centers (i.e., green, orange, and pink rectangles in Figure 1a) that service all branches in Tehran. These problems increase servicing time and transportation costs and decrease the security of cash supply centers and cash transfers. Determining the optimum number of the cash supply center and its location in Tehran was the main problem in this case study. Also, two cash supply centers are fixed. The potential locations are shown in Figure (1b). This problem is a node-basing problem with the closest node that could be converted arc-routing problem with smaller dimensions. For this purpose, branches with 2 km distance are replaced by the required arc. Therefore, 517 nodes in this case study problem are converted to 214 nodes that decrease problem dimensions.

In the case study, we use Tehran map with the required information, such as the branches location, the type of streets, the type of traffic allowed, the width of the street and the length of the street. For this purpose, the GIS Bank team used ArcMap 10 software. Figure 2 shows a small part of the map after identifying the permitted routes (bold green lines), the selected routes (blue lines) and the branches (green circular).
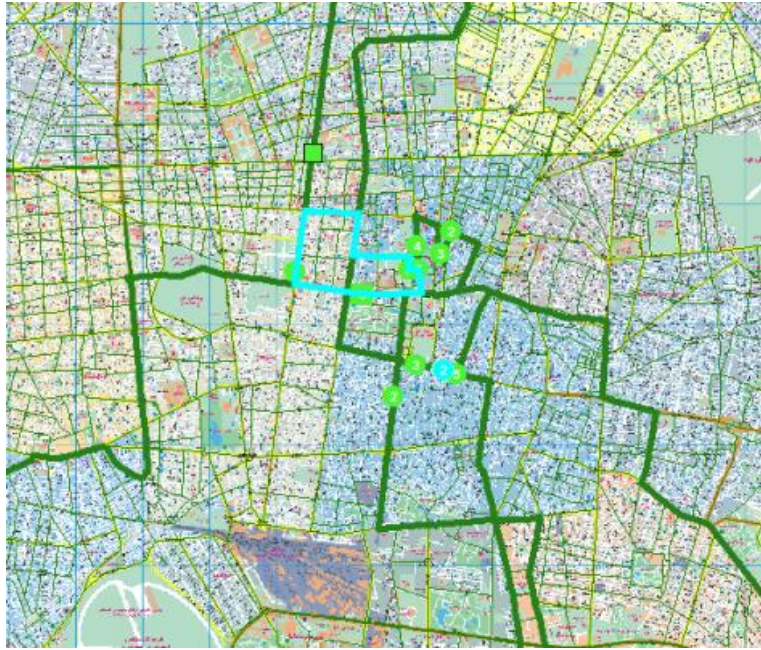
Figure 2. Part of the Tehran map with bank information

5. Solution approach

The proposed model for the LARP is NP-hard and bi-objective. Therefore, we use the MODA (Mirjalili, 2016) and NSGA-II (Deb et al. 2002). To solve this model, first, we present a solution representation of the problem as a necessary step in designing the algorithm. Then, we describe the MODA, a generation of the initial solution and a verified algorithm.

5.1. Solution representation

The solution representation in routing problems is based on a sequence of nodes number. Different solution representation is introduced in the research paper (Armas et al., 2018). In Lacomme et al. (2004), the solution representation is as a vector. The initial value of this vector is generated randomly that the vector is divided into several parts, and each part is assigned to a vehicle. The infeasible vector will be removed. In Kirlik and Sipahioglu (2012), the solution representation is defined as a matrix with a variable number of rows and columns, in which the number of rows is the number of the vehicle. In this representation type, selecting the vehicles are always in order. In this research, a solution representation is as a matrix with unknown dimensions (Figure 3a). The number of rows and columns is variable. The numbers of rows and columns are dependent on routes and nodes that are serviced by the vehicle, respectively.

For example, in a problem with three depots and four vehicles, if the set of required arcs is {1-4, 4-5, 5-7, 5-8, 7-8, 8-6, 2-6, 2-4, 8-10, 2-10, 9-10, 3-7}, the solution representation is shown in Figure 3b. The first row shows that vehicle 2 is assigned to depot 1 and services to the required arcs {1-4, 4-5, 5-8, 8-7, 7-5}. In route 2, the required arc of {8-5} is as a deadheading arc that is serviced in route 1. This solution is presented in Figure 3c.

5.2. Multi-Objective Dragonfly Algorithm

The Dragonfly Algorithm (DA) is introduced by Mirjalili (2016). Dragonflies have special swarming behaviors. The aims of dragonflies swarm are hunting (i.e., static swarm) and migration (i.e., dynamic swarm). Dragonflies in a static swarm create small group that has abrupt changes and local movement in the flying path. The huge number of dragonflies in dynamic swarms create swarms to migrate over long rout in one way. The DA is originated from static and dynamic dragonflies swarming behavior. These two behavior are identical to the two main stages of optimization: exploration and exploitation in meta-heuristic algorithms. Dragonflies make sub-swarms and fly in disparate locations in a static swarm that is the aim of the exploration stage. In a dynamic swarm, dragonflies fly in larger swarms along one path that is the main objective in exploitation. The multi-objective DA (MODA) is used to solve the presented multi-objective model. The MODA is described in Figure 4.
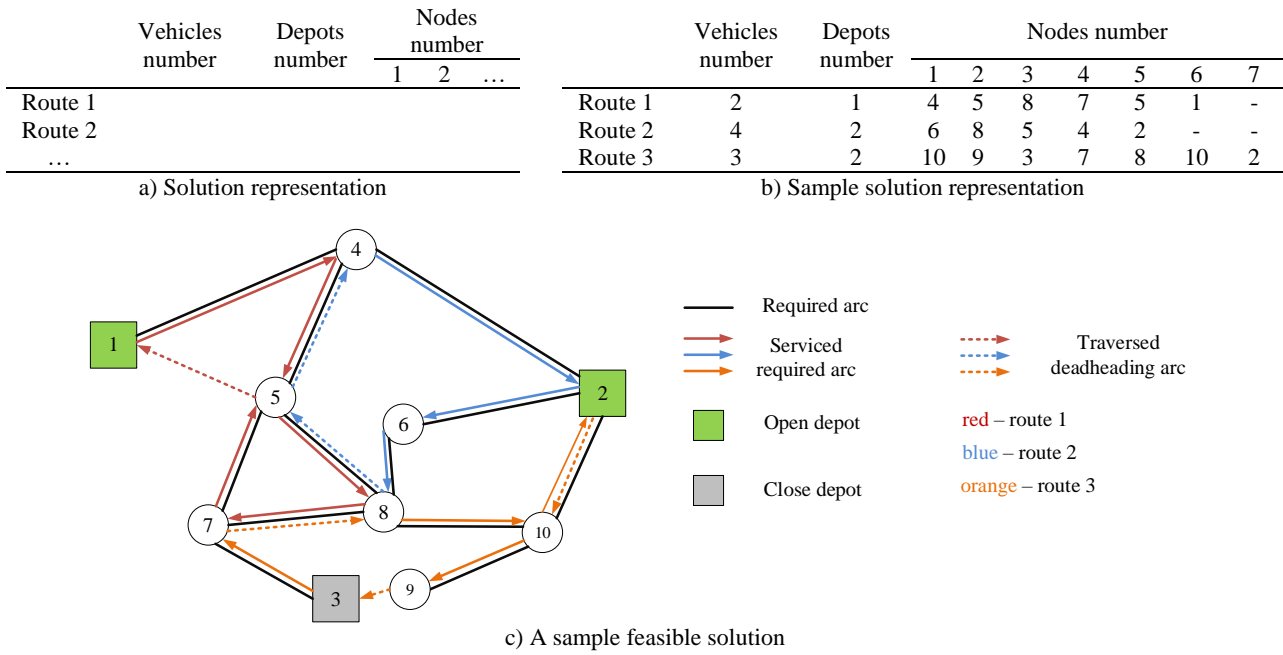
	Vehicles number	Depots number	Nodes number		
			1	2	...
Route 1					
Route 2					
...					

a) Solution representation

	Vehicles number	Depots number	Nodes number						
			1	2	3	4	5	6	7
Route 1	2	1	4	5	8	7	5	1	-
Route 2	4	2	6	8	5	4	2	-	-
Route 3	3	2	10	9	3	7	8	10	2

b) Sample solution representation

c) A sample feasible solution

Figure 3. Solution representation with a sample for the proposed LARP

Initialize the dragonflies population Xi ($i = 1, 2, ..., n$) and step vectors ΔXi ($i = 1, 2, ..., n$)

Define the maximum number of hyperspheres (segments) and the archive size

While the end condition is not satisfied

    Calculate the objective values of all dragonflies

    Find the non-dominated solutions

    Update the archive for the obtained non-dominated solutions

    If the archive is full

        Run the archive maintenance mechanism to omit one of the current archive members

        Add the new solution to the archive

    End if

    If any of the new added solutions to the archive is located outside the hyperspheres

        Update and re-position all of the hyperspheres to cover the new solution(s)

    End if

    Select a food source from archive

    Update w, s, a, c, f, and e

    Calculate S, A, C, F, and E using: $S_i = -\sum_{j=1}^{N} X - X_j$, $A_i = \frac{\sum_{j=1}^{N} V_j}{N}$, $C_i = \frac{\sum_{j=1}^{N} X_j}{N} - X$, $F_i = X^+ - X$, $E_i = X^- + X$

    Update neighboring radius

    Select an enemy from archive

    Update position vectors using

If a dragonfly has at least one neighboring dragonfly

    Update the velocity vector using $\Delta X_{t+1} = (sS_i + aA_i + cC_i + fF_i + eE_i) + w\Delta X_t$ and position vector using $X_{t+1} = X_t + \Delta X_t$

Else

    Update position vector using $X_{t+1} = X_t + Lévy(d) \times X_t$, $Lévy(x) = 0.01 \times \frac{r_1 \times \sigma}{|r_2|^{\frac{1}{\beta}}}$, $\sigma = \left(\frac{\Gamma(1+\beta) \times \sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1|\beta}{2}) \times \beta \times 2^{(\frac{\beta-1}{2})}} \right)^{1/\beta}$, $\Gamma(x) = (x - 1)!$

End if

    Check and correct the new positions based on the boundaries of variables
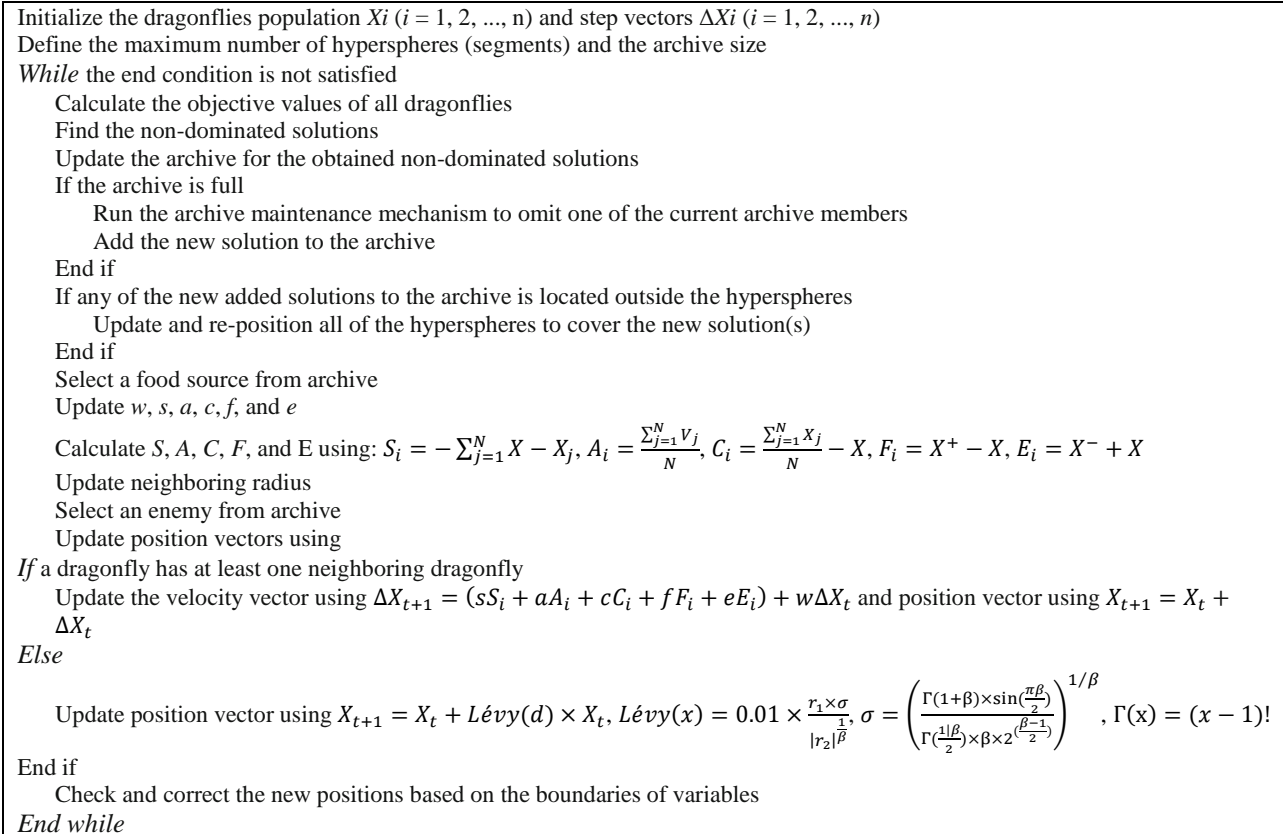
End while

Figure 4. Pseudo-codes of the MODA

Where X, X_j, V_j, X^+, X^- and N are the position of the current dragonfly, the position of the *j*-th dragonfly in the neighborhood, speed of the *j*-th dragonfly in the neighborhood, position of the food source, position of the enemy, and the number of neighborhood dragonflies, respectively. Also, *s, a, c, f, e* are the coefficient of separation, alignment,

cohesion, attraction, the distraction of enemy, respectively. w is the weight of inertia and t is the counter of the algorithm. Also, d is the dimensions of the position vector, r_1 and r_2 are two random numbers in the interval [1-0] and β are assumed to be a constant value equal to 1.5. The position of dragonflies is updated using five factors as follows:

- *Separation*: Avoid colliding with other neighbors (S_i)
- *Alignment*: Adjust the speed with neighbors (A_i)
- *Cohesion*: Tendency to the center of gravity of neighbors (C_i)
- *Attraction*: Absorption to food sources (F_i)
- *Distraction*: Outward to enemy (E_i)

5.3. Initial solution Generation

To generate the initial solution, the following steps are performed:

1. Choose a random vehicle between the vehicles that have the volume and time capacity.

2. Choose a random depot between the depots that have the volume capacity.

3. Choose a random required arc with the following conditions:

   3.1. Start point is selected depot in Step 2.

   3.2. If the demand is less than the capacity of the depot, go to Step 3.3, otherwise, go back to Step 2. the number of this depot is added to the blacklist.

   3.3. If demand is smaller than the capacity of the vehicle and arc travel time is in time windows interval, the selected arc is removed from the required arc list and added to the deadheading arc list. Then, depot and vehicle capacity and vehicle time capacity are updated. Otherwise, go back to Step 1 and the vehicle number is added to the blacklist.

4. Choose a random required arc with the following conditions:

   4.1. If the ending node of selected arc is the starting node of another required arc goes to Step 4.2; otherwise, go to Step 6.

   4.2. If arc demand is less than the depot and vehicle capacity and arc travel time is in time windows interval, the selected arc is removed from required arc list and is added to deadheading arc list. The depot and vehicle capacity as well as vehicle time capacity are updated. Otherwise, return to Step 1 and add the vehicle number to the blacklist.

5. Choose a random deadheading arc with the following conditions and return to Step 4:

   5.1. Starting node arc is the ending node of selected arc in Step 4.

   5.2. Arc has smallest traveling time.

6. If all required arcs are serviced, go to the correction algorithm, otherwise, if the vehicle and depots are not on blacklist, go to step 4; otherwise, go back to Step 1 and the vehicle and depots add to the blacklist.

5.4. Correction algorithm

For an infeasible solution, the following corrective steps are required:

1. If the end of vehicle route is not the depot, remove it from the solutions.

2. If the route is not serviced by required arc, this route is deleted.

3. If there are consecutive deadheading arcs on the route and there is a shorter route between the starting and ending nodes of these arcs, consecutive deadheading is replaced by this arc.

6. Experimental results

The LARP is an NP-hard problem with a large number of binary variables (Khajepour et al. 2020). In this paper, we use the MODA to solve the proposed LARP for the case study problem. Also, the NSGA-II is used for the validation of the MODA. This model is tested on a PC with Intel Pentium 4, core i5, 2.4 GHz processor with 4 GB memory. Also, The Taguchi method is used for parameter tuning for the MODA and the NSGA-II (Table 2).

Table 2. Parameter tuning with the Taguchi method

	Population size	Separation	Alignment	Cohesion	Attraction	Distraction
MODA	400	0.1	0.1	0.7	1	1
NSGA-II	Population size	Cross over	Mutation			
	350	0.36	0.72			

To evaluate the performance of the proposed deterministic and robust models, the bank case study described in Section 4 is selected. The deterministic and robust model is solved for this case study under nominal data. In the proposed robust model of the LARP, the experiments are performed under four different uncertainty levels $\rho = 0.2, 0.4, 0.8, 1$. Then, for realization, ten problems are solved for this case study under uncertainty levels. The objective function value and computational time for solving each model are reported in Table 3. In the robust model, some demand values are in the worst case compared to the nominal value. Then, the robust objective value is larger than the deterministic objective value. A comparison results of the deterministic and robust model for both objectives shows that although the results of the robust model are worse than the deterministic model with nominal data, the mean and standard deviation present better solutions in the realization (Figure 5). Also, increasing the amount of uncertainty level ρ, increase the number of created depots, employed vehicles and costs. The solving time in the robust model compared to the deterministic model is increased, due to the increasing number of constraints.

Table 3. Result comparison of the proposed LARP model for the bank case study

Solving method	Uncertainty level (ρ)	Results with nominal data						Results under realizations							
		O_1*		O_2		T		M_1		M_2		S_1		S_2	
		D	R	D	R	D	R	D	R	D	R	D	R	D	R
MODA	0.2	15698	15949	6698	6864	1	1	15,346	15,000	6746	6632	743	59	81	53
	0.4		16106		7106		1.31	15,469	14,757	6382	6328	864	125	152	90
	0.8		16430		7430		1.67	15,326	14,648	6807	6173	978	267	207	136
	1		16957		7957		1.92	15,298	14,530	6820	6206	1602	324	236	210
NSGA-II	0.2	4478	19246	5478	5832	5720	7680	19951	18838	8783	8432	1367	170	204	85
	0.5		19884		6014		7825	19997	18542	9434	8246	2235	197	314	135
	0.8		19435		6273		8078	19423	18139	9950	8012	2518	415	567	162
	1		19103		6354		8234	19136	18678	10230	7754	2651	598	712	231

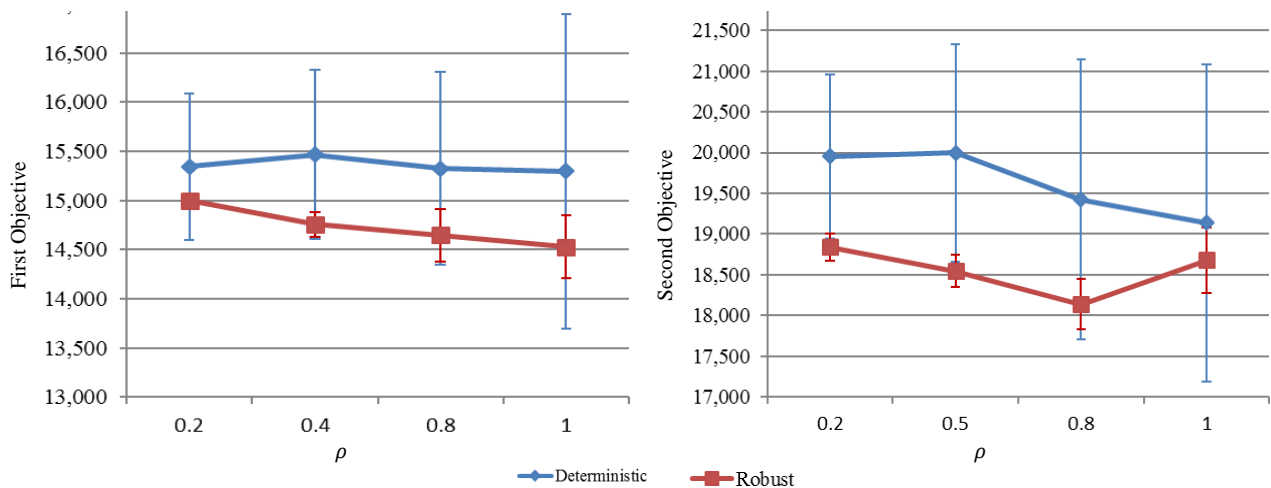* O: objective, T: Time (Sec), M: Mean, S: Standard deviation

Figure 5. Mean and standard deviation of the first and second objectives under uncertainty level ρ

To validate the MODA, the proposed model is also solved by the NSGA-II (Table 3). As usual, metrics (e.g., the number of non-dominated solutions, spacing, and quality solutions) are used for the comparison of the results in multi-objective algorithms (Collette and Siarry, 2003). Comparison of the MODA and NSGA-II results based on the number of non-dominated solutions (Figure 6), the spacing, and the quality of solutions (Table 3) shows that the proposed MODA for solving the model of the LARP has better performance. Also, the solving time of the MODA is better than NSGA-II.
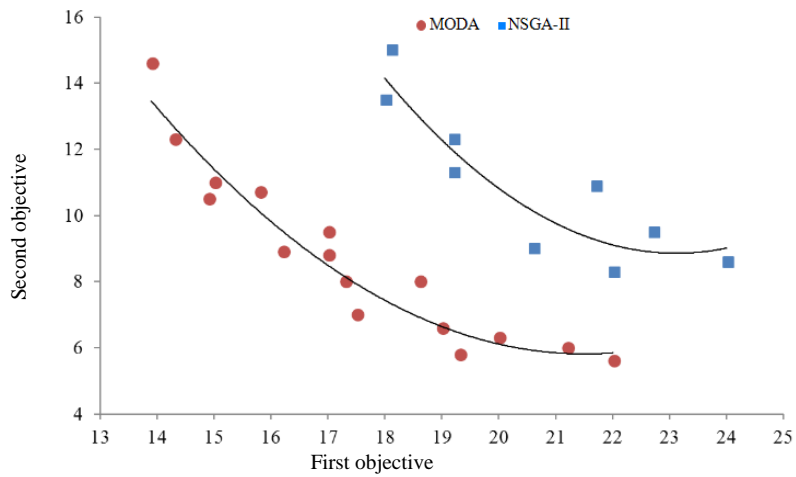
Figure 6. Pareto fronts of two meta-heiristics

The MODA Pareto front is demonstrated in Figure 6. However, the MODA Pareto front is not sufficient for the determination of cash supply centers in the bank case study. Selecting the best and most efficient final solution depends on the priorities of the decision-makers. Therefore, the bank experts have surveyed the Pareto front of the case study. They have selected a solution based on bank rules. The most important of these rules are as follows:

1) Availability of the estate, security issues and traffic in the proposed area of the solution.
2) Minimum waiting time as second objective in the solution.
3) Existence of at least four supply centers in the solution based on the bank experts experiences.

As an example, two solutions from the Pareto front (Figure 6) obtained from the MODA are shown in Figure 7. Although solution (a) has the minimum cost between Pareto front and there are at least four supply centers; however, solution (b) has all of conditions and is selected as the best solution by bank experts.
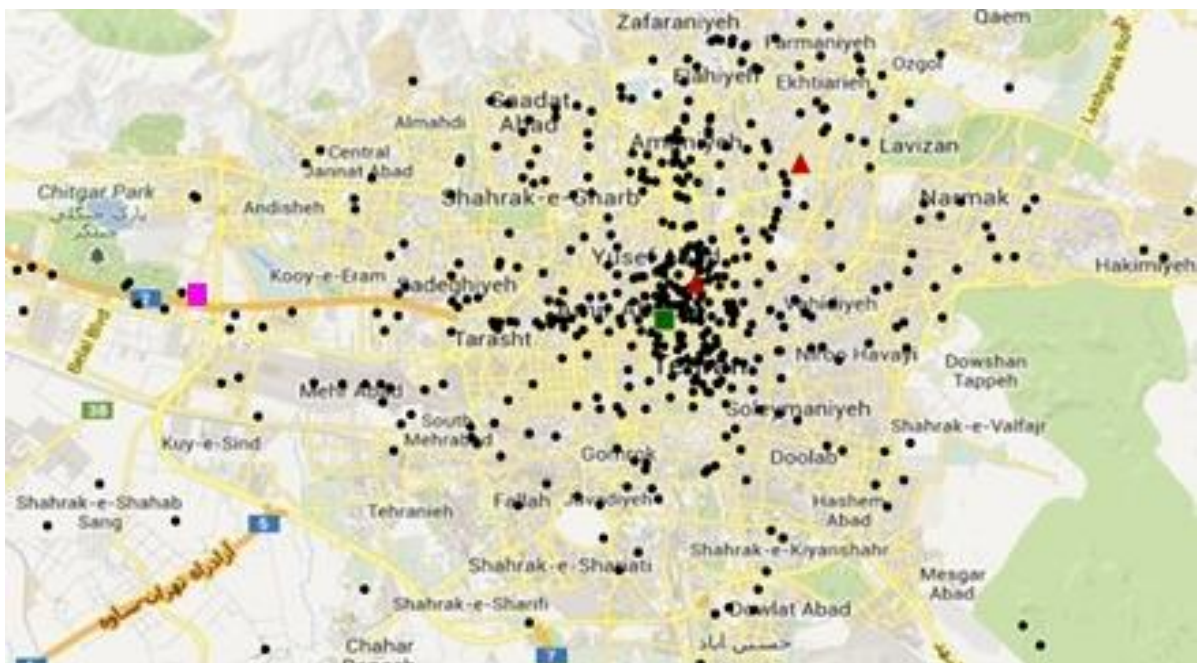

Figure 7. (a) Sample of Pareto front

Figure 7. (b) Sample of Pareto front

7. Conclusion and future research

To cope with the issue of the location of cash supply centers that servicing to bank branches under uncertain demand, this paper proposed a new mixed-integer linear programming model for a bi-objective and multi-period Location-Arc Routing Problems (LARP) with time windows. To the best of our knowledge, for the first time, the time windows were modeled in the LARP with a condition of converting the required arc with time windows to a deadheading arc without time windows after service. Also, the model forces the vehicle to service multiple required arc with the minimum deadheading arc in route. The case study was a node-routing problem with the closest nodes that were converted to the model of the LARP with a reduced dimension. The proposed model was solved by the Multi-Objective Dragonfly Algorithm (MODA) and Non-dominated Sorting Genetic Algorithm (NSGA-II), in which the MODA based on the multi-objective metrics has better performance. The approach proposed by Bertsimas and Sim was used due to the uncertainty in demand. The computational results showed that the robust model under the realization outperforms the deterministic model in terms of the mean and standard deviation. However, the deterministic model with nominal data had better objective function values. In this case study to determine the cash supply centers, bank experts surveyed the MODA Pareto front based on the availability of the estate, security issues, minimum waiting time, etc. Finally, five cash supply centers in Tehran were selected. Considering other features for developing the model of the LARP, investigating the uncertainty in other model parameters and solving the model with an exact method can be a good direction for future research. Another direction is to consider the risk of cash-in-transit for the bank case study.

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