On Single Machine Scheduling Problem with Distinct due Dates under Fuzzy Environment

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Abstract

This research article proposes a single machine scheduling problem subject to distinct due dates in fuzzy environment. The proposed problem is considered in fuzzy environment with an attempt to sequence of the $n-$jobs. The total penalty cost is considered as the composition of all the total earliness and tardiness cost. The aim of this research work is to minimize the total penalty cost. A method to minimize the total penalty cost due to earliness or lateness of job in fuzzy environment is proposed. A numerical example is illustrated to support the proposed method in this study.

Keywords: Single machine; Distinct due dates; Early/ lately machine problem; $(\alpha, \beta)-$ interval- valued fuzzy number; Signed distance ranking.

1. Introduction

Single machine, distinct due dates, early (lately) machine problem and fuzzy environment closely to the situation faced by "just in time" manufacture (Gupta and Rambha, 2011). Scheduling is defined as prescribing of when and where each operation necessary to manufacture the product to be performed. It aims to plane a sequence of work so as to the production can be systematically arranged towards the end of completion of all products by due dates. Senthilkumar and Narayanan (2010) introduced a survey on the classification of the literatures of single machine scheduling problem in three major categories, viz. Offline scheduling, online scheduling and miscellaneous scheduling. Baker and Scudder (1990) developed sequences with earliness and tardiness penalties under Just in Time production. Recently, the of earliness and tardiness penalties in scheduling models is necessary and inquiry (Liaw, 1999; Biskup and Feldmann, 2001; Belkaid et al. 2013; Belkaid et al. 2016; Che et al. 2017; Arlk, 2019; and Gerstl and Mosheiov, 2020).

Single machine scheduling problems bear complex computations and such problem analyzed important for a better understanding problem. Among those, related to earliness and tardiness. Single machine scheduling with single machine consists of single machine to process n jobs; this problem aims to schedule these n jobs on the single machine to minimize the given measure of performance. These jobs may be independent or dependent. Mohamad and Said in (2011) studied n-job single machine scheduling problem with common due date so as to minimize the sum of the total inventory and penalty costs. Belkaid et al. (2012) presented a branch and bound algorithm to minimize the makespan. They considered the identical parallel machines for consumable resources. Zhao et al. (2014) considered a model with job- dependent position effects for a single machine scheduling and due date assignment with set up time, which is proportional to the length of the processed jobs. Zhao et al. (2014) aims to minimize the sum of the scheduling criterion of the accepted jobs and total penalty of the rejected jobs for the single- machine scheduling and due date assignment problem. Ben-Yehoshua and Mosheiov (2016) proposed a single machine-scheduling problem with the objective to minimize total early work. Wang and Xu (2016) considered a single machine scheduling problem with a mandatory maintenance whose duration is workload- dependent aims to determine the start time of the maintenance and schedule all the jobs to the machine such that the maximum lateness is minimized through a proposed an approximation algorithm based on the classical earliest due date.

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Processing time of a job varies in different ways, may be due to factor environmental or the different work places. When a contractor takes the work form his/her department, he (she) calculates expenditure at the time of allotment. But due to enormous factors as non-available of labor, weather not favorable or sometimes abnormal conditions. In many scientific areas, such as system analysis and operators research, a model has to be setup-using data, which is only approximately known. Fuzzy sets theory, introduced by Zadeh (1965) makes this possible. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Decision making in a fuzzy environment, developed by Bellman and Zadeh (1970) has an improvement and a great help in the management decision problems. Kaufmann and Gupta (1988) studied several fuzzy mathematical models with their applications to engineering and management sciences. Dubois and Prade (1980) extended the use of algebraic operations on real numbers to fuzzy numbers by use of a fuzzification principle. Fuzzy linear constraints with fuzzy numbers have been studied by Dubois and Prade (1980). Recently, the study of earliness and tardiness penalties in scheduling is relatively of inquiry. Yazdani et al. (2017) studied a single- machine scheduling problem subject to several unavailability constraints. To solve the model, they considered a mathematical model and an enhanced variable neighborhood search approach. An enormous number of authors studied machine scheduling in uncertainty environment (Arlk and Toksarl, 2019; Ertem et al. 2019; Wu and Wang, 2020; and Yue et al. 2020). The job sequencing scheduling problem when processing time represented as fuzzy numbers is studied by Cahon and Lee (1992). Chong (1995), Ishibuchi et al. (1996) and Ishii and Tada (1995) fuzzified the scheduling problems by using a fixed due date. Jadhav and Bajaj (2012) proposed n - jobs to processed on single machine scheduling problem with fuzzy processing and fuzzy due dates so as to minimize the total penalty cost in the schedule. Dueness and Petrivic (1995) developed an approach under fuzzy flow as fuzzy in nature. They used the concepts of Average High Ranking Method. Toksari and Arık (2017) studied the single machine scheduling problem involving the position dependent fuzzy learning effect as well as fuzzy processing times. Sotskov and Egorova (2018) applied the stability approach to the single machine problem with uncertain durations of the given jobs to minimize the sum of the job completion times. Ponnalagu and Mounika (2018) considered the sequence performance measurements and job mean Koulamas and Kyparisis (2019) shown that the single machine scheduling problem with past- sequence- dependent setup times and either the minimum maximum lateness/ tardiness objective was solvable by an index priority rule followed by backward intentions of certain qualifying jobs.

The penalizing concept in both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. The using of earliness and tardiness in fuzzy environment give rise to a non-regular performance measures, because this is led to a new methodological in the design of the procedures of solutions.

In this article, the research objective is to consider the sequence of the jobs on a single machine with fuzzy due dates so as to minimize the total penalty cost. A method to minimize the total penalty fuzzy cost due to earliness or lateness of job in fuzzy environment is proposed.

The outlay of the paper is organized as follows: In the next Section, the basic concepts and results related to fuzzy numbers, and (α, β) interval-valued fuzzy numbers are introduced. Section 3 presents assumptions and notation needed in the paper. Section 4 formulate single machine scheduling problem with processing time represented as (α, β) interval valued fuzzy bi-objective criteria. Section 5 proposes solution procedure for solving the problem. Section 6 provides a numerical example to illustrate the efficiency of the solution procedure. Finally, some concluding remarks are reported in Section 7.

2. Preliminaries

This Section introduces some of basic concepts and results related to fuzzy numbers, and (α, β) interval valued fuzzy numbers.

Definition 1. A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be fuzzy numbers if its membership function

$\mu_{\tilde{A}}(x): \mathbb{R} \rightarrow [0,1]$, have the following properties:

1. $\mu_{\tilde{A}}(x)$ is an upper semi- continuous membership function;

2. $\tilde{A}$ is convex fuzzy set, i.e., $\mu_{\tilde{A}}(\delta x + (1 - \delta) y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$ for all $x, y \in \mathbb{R}; 0 \leq \delta \leq 1$;

3. $\tilde{A}$ is normal, i.e., $\exists x_0 \in \mathbb{R}$, for which $\mu_{\tilde{A}}(x_0) = 1$;

4. $\text{Supp}(\tilde{A}) = \{x \in \mathbb{R}: \mu_{\tilde{A}}(x) > 0\}$ is the support of $\tilde{A}$, and the closure $\text{cl}(\text{Supp}(\tilde{A}))$ is compact set.

Definition 2. (Chiang, 2001). If the membership function of the fuzzy set $\tilde{A}$ on $\mathbb{R}$ is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{\alpha (x-r)}{(s-r)}, & r < x \leq s, \\ \frac{\alpha (t-x)}{(t-s)}, & s \leq x < t, \\ 0, & otherwise, \end{cases}$$

Where $0 < \alpha \leq \alpha$ then $\tilde{A}$ is called a level α fuzzy number and it is denoted as $\tilde{A} = (r, s, t;\ \alpha)$.

Definition 3. (Chiang, 2001).   An interval- valued fuzzy set $\tilde{A}$ on $\mathbb{R}$ is given by

$\tilde{A} \triangleq \{(x,\ [\ \mu_{\tilde{A}^-}(x), \mu_{\tilde{A}^+}(x)\ ]) : x \in \mathbb{R}\}$, where $\mu_{\tilde{A}^-}(x), \mu_{\tilde{A}^+}(x) \in [0,\ 1]$, and

$\mu_{\tilde{A}^-}(x) \leq \mu_{\tilde{A}^+}(x)$; for all $x \in \mathbb{R}$ and is denoted as $\tilde{A} = [\tilde{A}^-, \tilde{A}^+]$. Let

$$\mu_{\tilde{A}^-}(x) = \begin{cases} \dfrac{\alpha\,(x-r)}{(s-r)}, & r < x \leq s, \\ \dfrac{\alpha\,(t-x)}{(t-s)}, & s \leq x < t, \\ \quad 0, & \text{otherwise} \end{cases}$$

Then $\tilde{A}^- = (r, s, t; \alpha)$.

Let

$$\mu_{\tilde{A}^+}(x) = \begin{cases} \dfrac{\beta\,(x-a)}{(s-a)}, & a < x \leq s, \\ \dfrac{\beta\,(b-x)}{(b-s)}, & s \leq x < c, \\ \quad 0, & \text{otherwise}. \end{cases}$$

Then $\tilde{A}^+ = (a, s, b;\ \beta)$.

It is clear that $0 < \alpha \leq \beta \leq 1$, and $a < r < s < t < b$. Then the interval-valued fuzzy set is defined as:

$\tilde{A} \triangleq \{(x,\ [\ \mu_{\tilde{A}^-}(x), \mu_{\tilde{A}^+}(x)\ ]) : x \in \mathbb{R}\}$, is denoted as

$\tilde{A} = [(r, s, t; \alpha), (a, s, b; \beta)] = [\tilde{A}^-, \tilde{A}^+]$.

$\tilde{A}$ is called a level (α, β) interval-valued fuzzy number as shown in the following Fig 1.

Figure 1. Level (α, β) interval- valued fuzzy number
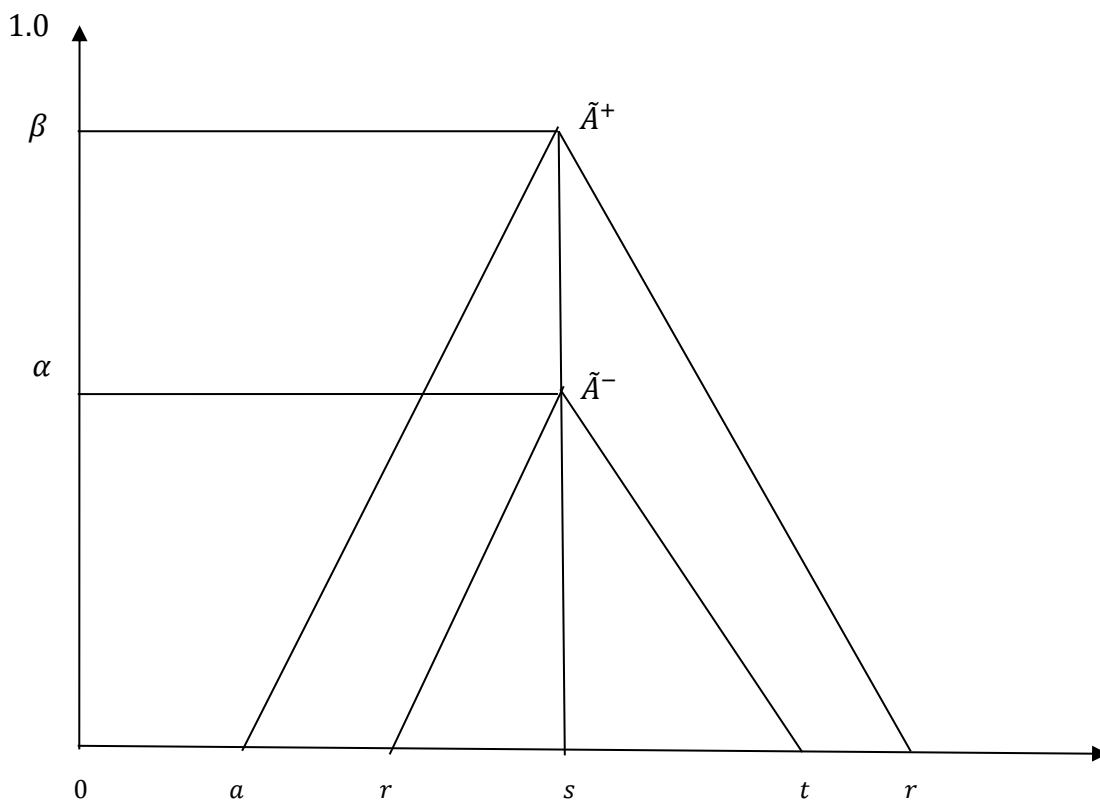
Property 1. (Chiang, 2001).

Let, $F_{IVF}(\alpha, \beta) = \{[(r, s, t; \alpha), (a, s, b; \beta)]: for\ all\ a < r < s < t < b\}$,

$0 < \alpha \leq \beta \leq 1$ be the family of (α, β) interval- valued fuzzy numbers.

Let, $\widetilde{P} = [(r, s, t; \alpha), (a, s, b; \beta)] \in F_{IVF}(\alpha, \beta)$, and $\widetilde{Q} = [(r_1, s_1, t_1; \alpha), (a_1, s_1, b_1; \beta)] \in F_{IVF}(\alpha, \beta)$. Then

1. $\widetilde{P}(+)\widetilde{Q} = [(r + r_1, s + s_1, t + t_1; \alpha), (a + a_1, s + s_1, b + b; \beta)]$,

2. $k\widetilde{P} = \begin{cases} [(kr, ks, kt; \alpha), (ka, ks, k\ b; \beta)], & k > 0, \\ [(kt, ks, kr; \alpha), (kb, ks, k\ a; \beta)], & k < 0, \\ [(0,0,0; \alpha), (0, 0,0\ ; \beta)], k = 0. \end{cases}$

Definition 4. (Chiang, 2001). Let $\widetilde{P} = [(r, s, t; \alpha), (a, s, b; \beta)] \in F_{IVF}(\alpha, \beta), 0 < \alpha \leq \beta \leq 1$. The signed distance of $\widetilde{P}$ from $\widetilde{0}$ is given as

$$D_0(\widetilde{P}, \widetilde{0}) = \frac{1}{8}\left[6s + r + t + 4a + 4b + 3(2s - a - b)\frac{\alpha}{\beta}\right].$$

Remark 1. $\widetilde{P} = [(a, a, a; \alpha), (a, a, a; \beta)]$, then $D_0(\widetilde{P}, \widetilde{0}) = 2a$.

Definition 5. (Chiang, 2001). Let $\widetilde{P}, \widetilde{Q} \in F_{IVF}(\alpha, \beta)$, the ranking of level (α, β) interval- valued fuzzy numbers in $F_{IVF}(\alpha, \beta)$ using the distance function D_0 is defined as:

$\widetilde{Q} \prec \widetilde{P} \Leftrightarrow D_0(\widetilde{Q}, \widetilde{0}) < D_0(\widetilde{P}, 0)$,

$\widetilde{Q} \approx \widetilde{P} \Leftrightarrow D_0(\widetilde{Q}, \widetilde{0}) = D_0(\widetilde{P}, 0)$.

Property 2. (Chiang, 2001). Let $\widetilde{P} = \left[[(r, s, t; \alpha), (a, s, b; \beta)]\right]$ and

$\widetilde{Q} = [(r_1, s_1, t_1; \alpha), (a_1, s_1, b_1; \beta)]$ be (α, β) interval- valued fuzzy numbers in $F_{IVF}(\alpha, \beta)$. Then

1. $D_0(\widetilde{P} \oplus \widetilde{Q}, \widetilde{0}) = D_0(\widetilde{P}, \widetilde{0}) + D_0(\widetilde{Q}, \widetilde{0})$,

2. $D_0(k\ \widetilde{P}, \widetilde{0}) = k\ D_0(\widetilde{P}, \widetilde{0}), k > 0$.

3. Assumptions and Notation

3.1. Assumptions

The machine-scheduling problem introduced in this paper requires the following assumptions:

1. All jobs are available for processing at time zero
2. The single machine can process at most one job at a time
3. No pre- emption is allowed

3.2. Notation

In the single machine scheduling, the following notation may be used

S: Schedule for the i job.

$\quad P_i = [(r_i, s_i, t_i; \alpha), (a_i, s_i, b_i; \beta)]$: Processing time of job i on the machine under fuzzy environment.

D_i: Signed distance ranking of the processing time $[(r_i, s_i, t_i; \alpha), (a_i, s_i, b_i; \beta)]$,

$\quad i = 1, 2, \ldots, n$

d_i: Due date for the job i

c_i : Completion time for job i

T_i : $Max. (0, c_i - d_i)$

E_i : $Max. (0, d_i - dc_i)$

V_i : Slack time of job i

e_i: Penalty per unit time for the earliness of job i

l_i: Penalty per unit time for the tardiness of job i

4. Fuzzy processing time statement

Consider the following single machine scheduling problem. Assume there are a set of n jobs $(J_1, J_2, \ldots, J_n)$ to be scheduled for processing on a single machine. Continuously, the machine is available from $t = 0$ and process only one job at a time. Also, the jobs are continuously available from $t = 0$ and requiring processing times $(P_1, P_2, \ldots, P_n)$ and due date $d_i(i = 1, 2, \ldots, n)$ associated with each job are positive integers. The customer satisfaction decreases the completion time of job passes in between due date d_i and late date d_i° until vanishes in the delay case.

Mathematically, the problem can be formulated as follows

$$f(s) = \min \sum |c_i - d_i| = \sum |E_i + T_i|.$$

5. Solution approach

In this section, the steps of the approach to determine the total penalty cost are:

Step 1: Calculate the signed distance ranking for the fuzzy processing time of all the jobs.

Step 2: Find the slack time of all the jobs $V_i = |D_i - d_i|$

Step 3: Arrange the jobs in increasing order of its slack time. If two jobs have the same slack time, then take the one with the lowest processing at the earlier position.

Step 4: Determine the total penalty cost of all jobs using the sequence in step3.

6. Numerical Example

Consider 7- jobs having fuzzy processing time, single machine and distinct due dates. Penalty cost for earliness is shown in the following Table 1:

Table 1. Penalty cost for earliness

Job	P_i	D_i	d_i	V_i	e_i	l_i
1	$[(0.5, 2, 5; 0.6), (0.25, 2, 7; 0.9)]$	5	8	3	2	3
2	$[(2, 3, 4; 0.6), (1, 3, 13; 0.9)]$	8	9	1	2	3
3	$[(4, 6, 12; 0.6), (2, 6, 16; 0.9)]$	14	9	6	2	3
4	$[(4, 5, 6; 0.6), (3, 5, 9; 0.9)]$	11	13	4	2	3
5	$[(2, 4, 64; 0.6), (1, 4, 11; 0.9)]$	9	11	2	2	3
6	$[(3, 5, 7; 0.6), (2, 5, 8; 0.9)]$	10	17	7	2	3
7	$[(5, 6, 13; 0.6), (4, 6, 17; 0.9)]$	15	11	5	2	3

7 jobs having (α, β) interval- valued fuzzy numbers due dates $[(r, s, t; \alpha), (a, s, b; \beta)]$ which are converted into its average high ranking using the signed distance as in Definition 4 and referring to the solution approach in Section 5, the near optimal sequence is

$$S = 2 > 5 > 1 > 4 > 7 > 3 > 6.$$

The total flow time of the system and the total optimized penalty cost are displayed in the following Table 2:

Table 2. Total flow time of the system and the total optimized penalty cost

Job	Processing Time	d_i	V_i	Cost
2	0-8	8	1	3*2
5	8- 17	9	2	1*3
1	17-22	9	3	6*3
4	22-33	13	4	4*3
7	33-48	11	5	2*3
3	48-82	17	6	7*3
6	82- 92	11	7	5*3

It is clear from table2 that the flow time of the system and the total optimized penalty cost due date to earliness / tardiness of the job is equal to $81

7. Concluding Remarks

In this paper, a single machine scheduling problem with distinct due dates has studied. We determined the optimal scheduling which attempted to minimize a cost function consisting of the earliness and tardiness costs with penalties under fuzzy environment. The criteria to calculate the optimal schedule according to the signed distance of fuzzy numbers has investigated. Moreover, it is determined that the proposed technique facilitates to determine the optimal solution for small systems, satisfactory to the decision maker (DM), and help the DM to determine the best schedule for a given set of jobs that is to control penalty cost.

In real life situations, the uncertainty may be handled by considering the intuitionistic fuzzy set. While the proposed work did not consider the intuitionistic fuzzy set. This may be a limitation of the present study. For future research scope, there are several research directions to work. The better one is to introduce the concept of intuitionistic fuzzy set to the present work.

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Conflicts and Interest

The author declares no conflict of interest.

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