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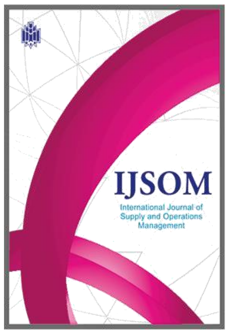

Efficiency Evaluation in Hybrid Three-Stage network Data envelopment analysis from the Double-Frontier Standpoint

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Abstract

The efficiency evaluation of a network opens the “black box” and focuses on the internal structures and innermost interactions of the system. In this paper, we have made efforts to consider a three-stage system, comprising of six sub-DMUs, in combination with additional inputs and undesirable outputs. The proposed models are simulations of a factory, with a production area and three warehouses for goods and two delivery points. Hence, an authentic example of production planning and inventory control in a factory for a year and within duration of 24 periods was taken under consideration as a dynamic structure. In this simulation, all costs were considered, including, production costs, setup cost, maintenance costs of the products, warehouse reservation costs, transportation costs, delay penalty costs and the profit obtained from the sale of products. We utilized the multiplicative DEA with a double-frontier approach to measure the efficiency of a general system and improve the accuracy of efficiencies. Moreover, a heuristic technique was used to convert non-linear models into linear models. The ranking results of the 24 time periods indicate that the time periods 24 and 1 respectively are the best and poorest periods in terms of efficiency. Finally, we suggest using a k-means method to cluster DMUs into several groups with similar characteristics based on double-frontier Standpoint.

Keywords: Network DEA; Three-stage; Double-frontier; Additional inputs; Undesirable outputs; k-means technique.

1. Introduction

It is evident that a lack of resources, the utilization of undesirable and limited resources available, and an increment in costs make it essential to take advantage of managerial techniques in more prominent organizations. Thereby, in order to be successful, organizations must improve their processes and produces consistently. Today, all organizations have somehow depicted the importance of having a measurement system for performance. As a principle, every organization should, wherever feasible, measure its performance capacities. The absence of an effective assessment or evaluation system is directly related to the disintegration of an organization and this shortcoming is considered an organizational disease; for without measuring, there shall be no basis for judgments, opinions and evaluations. Whatever cannot be evaluated cannot be even fittingly managed. So as to ensure an effective management, every organization must use scientific models for the evaluation of performance, so that its efforts and the results achieved from its performance can be appraised. Several factors have an impact on the growth and development of countries. Researches executed in this arena indicate that efficiency impacts enhance the speed of economic development. These surveys have revealed that in the past years, there was a difference in the economic growth and development of countries due to modification in the level of efficiency and productivity of factors relative to production. Thence, an increment in performance and efficiency

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of organizations is an inevitable necessity for survival in global markets today. This issue is not confined to a particular sector or industry and in a limited period of time shall encompass all the sectors of economy. A performance assessment is a process which appraises measures, evaluates and judges the performance of an organization during a given period. It is carried out by comparing the present circumstances with desirable or ideal conditions, which are based on pre-determined indexes. In general, the objectives of assessing the performance are a response to the results in specifying quality improvement measures and to reduce costs, as well as comprehend what is being evaluated.

Currently, efficiency measurement is an extremely crucial issue towards a better understanding of problems in a system and planning for future improvement (Kao, 2014). Data Envelopment Analysis (DEA) is one of the most important and appropriate approaches for measuring the performance of decision-making units (DMUs). (Ma et al., 2017). In Farrell (1957)'s initial DEA task, which was later developed by Charnes et al. (1978), CCR or the Charnes-Cooper-Rhodes Model was reputedly known. Then, Banker et al. (1984) developed the DEA and proposed BCC or the Banker-Charnes-Cooper Model. DEA is a non-parametric linear programming model for measuring the performance for a set of homogeneous DMUs with multiple inputs and multiple outputs (Chen et al., 2009). In classical DEA models, such as the CCR and BCC models, we do not consider the intermediate measures of DMUs or internal operations of DMUs and measure the efficiencies of DMUs as a "black box" (Lee et al., 2016; Kritikos, 2017). In other words, ignoring the internal structure of systems leads to the classical DEA models where important information and segregation between the efficient units cannot be presented (Lewis and Sexton, 2004). To overcome the problem, Fare and Grosskopf (1996; 2000) suggested a network DEA model. In the network DEA models, the internal structure of systems and internal interactions of DMUs are taken into consideration until there is an increase in the accuracy of efficiencies. The network DEA models can simulate systems with complex internal structures by using stages and sub-DMUs and then evaluate the overall efficiencies of systems, stages and sub-DMUs, respectively, (Kou et al., 2016; Wanke and Barros, 2014). The internal structure of systems can simulate with the sub-DMUs either in series or in parallel. Thus, the systems with series and parallel structures are two very significant areas in network DEA (Kao, 2009a). For the parallel structure, the sum of the inputs or outputs of all stages are considered as the inputs or outputs of the whole structure, but in the series structure the inputs of first stage and the outputs of last stage are the inputs and outputs of the whole structure (Kao and Hwang, 2008). The general efficiency of the parallel and series structures are measured by the multiplicative and additive methods, respectively, (Cook and Zhu, 2014). In recent years, the efficiency evaluation of the multi-stage is one of the most important topics in DEA, and the parallel and series structures are used by many researchers. Kao (2009b) proposed a "closed system" with a series structure to be taken into consideration for intermediate measures, but without any additional input or output in each stage; whereas, Yu and Lin (2008) used the network DEA to measure service effectiveness and technical efficiency. Kao (2014) utilized the network DEA approach to estimate the overall efficiency of the system with multi-stage and additional inputs. Hua and Bian (2008), Cook et al. (2010) and Tone and Tsutsui (2009) used the network DEA for the evaluation of efficiency. In the network structure, the sub-DMUs have desirable or undesirable outputs. Fare et al. (1989) developed the DEA and utilized the undesirable outputs initially. Seiford and Zhu (2002) considered a network structure and proposed a model for efficiency evaluation that increased the desirable output and decreased the undesirable output. A non-radial network DEA model is suggested by Jahanshahloo et al. (2005) for considering the undesirable outputs. Badiezadeh and Farzipoor (2014) reflected on a production line, as a system with undesirable outputs, and measured the overall efficiency of the system under consideration and the internal interactions of DMUs. Lu and Lo (2007) categorized the methods for working with undesirable outputs in DEA in the following three modes: 1-The first method is to ignore the undesirable outputs, which are done, in order to simplify the models, 2-the second method is to measure distances in such a manner, so as to limit the expansion of the undesired output or that the undesired output is modeled as a nonlinear DEA model, 3-the third method is to consider the undesired output as a desired input, or to employ the negative sign as a desirable output, or that a decrease in conversion is applied to them.

The Data Envelopment Analysis (DEA) with a double-frontier considers two efficiencies for each DMU. One is called the optimistic efficiency or the best relative efficiency and the other efficiency is known as the pessimistic or the poorest efficiency (Amirteimoori, 2007). In the optimistic efficiency, each DMU is compared with a set of efficient DMUs that are located on the efficiency frontier; whereas, in the pessimistic efficiency, each DMU is compared with a set of inefficient DMUs that are located on the inefficiency frontier (Wang and Chin, 2009; Parkan and Wang, 2000). The value of the optimistic approach is less than or equates to (1); and from the pessimistic viewpoint is it more than (1) or equal to (1). The efficiency value of the optimistic approach is less than (1) when the DMU under evaluation is not on the efficiency frontier; whereas, it equates to (1) when the DMU under assessment or evaluation is on the efficiency frontier. The pessimistic value approach is more than (1) when the DMU under evaluation is not on the inefficiency frontier; but is equivalent to (1) when the DMU under evaluation is on the efficiency frontier (Azizi and Ajirlu, 2011; Azizi and Wang, 2013; Jahanshahloo and Afzalinejad, 2006). In fact, the double-frontier views each DMU from two perspectives and any conclusion which implies to only one of the two viewpoints shall result in a one-sided and an incomplete perspective (Azizi and Ajirlu, 2011). The measurement of efficiency, based on the optimistic and pessimistic views in a mutual fashion, can lead to an increment in accuracy for the purpose of ranking the DMUs (Badiezadeh et al., 2018). Doyle et al.

for the first time in 1955 obtained the efficiency of DMUs from the two optimistic and pessimistic viewpoints. Entami et al. (2002) attained the double-frontier in order to measure the efficiency for each lower bound and upper bound DMU as optimistic and pessimistic efficiencies, respectively. So as to combine the results of the optimistic and pessimistic approaches, which would usher a general or overall efficiency, several other researchers suggested mathematical combinations (*i.e.* averaging out the optimistic and pessimistic values) (Azizi, 2014). Wang and Chen (2009) used a geometric mean to combine the results of an optimistic and pessimistic viewpoint for ranking the DMUs. In recent years, numerous other researchers have utilized the double-frontier to measure efficiency and in this regard Jiang et al. 2012; Wang and Lan 2013; Yang and Morita 2013; Azizi et al. 2015; Jahed et al. 2015 and Badiezadeh et al. 2018 can be helpful.

Throughout the past years, an increment in the importance of the production sector and anxiety over the development and efficiency growth of this segment is directly correlated with that of the economic system. A rise in costs has led to haul the production units towards incrementing their organizational performance. The optimal mode which would increase efficiency is to logically utilize, adopt and modify the available resources. This could only be achieved by ensuring a correct managerial performance, including a rational evaluation of the returns attained (Swell, 1997). In this paper, we simulate a factory in a factual world that produces three perishable goods and each of the goods is placed in a warehouse. The factory has a production area and three warehouses for goods and two delivery points. Therefore, we are faced with a hybrid system having a complex internal structure with three stages, six sub-DMUs, additional inputs and undesirable outputs in the second and third stages. We have a sub-DMU in the first stage and three sub-DMUs in the second stage that are parallel and two sub-DMUs in the third stage that are parallel and the stages are linked in series. The purpose of this paper is to estimate the overall efficiency of the system on the basis of a cooperative approach. We use the optimistic and pessimistic views to increase the accuracy of the measurement for ranking the DMUs. The cooperative models cannot be turned into linear models, from the optimistic and pessimistic views because of the additional inputs and outputs in the second stage. Therefore, we use a heuristic technique to convert the nonlinear models into linear models. This factory is considered as a dynamic network and measures the overall efficiency and the efficiencies of the DMUs. The paper is organized as follows: Section (2) issues the model description and presents a three-stage network DEA model with additional inputs and undesirable outputs. Section (3) renders the model solution and presents the solution of the models according to the heuristic method. Section (4) offers a case study description and a factory is described as a factual example, and finally, Section (5) concludes the paper.

2. Model description

We consider a set of n homogeneous DMUs that are denoted by $\mathrm{DMU_j}$ (j=1,..., n), and each $\mathrm{DMU_j}$ (j=1,…,n) has three-stages with a complex internal structure, as shown in Fig. 1. where there is one sub-DMU is in the first stage (sub-$\mathrm{DMU_{1j}}$) and three sub-DMUs form a parallel structure in the second stage (sub-$\mathrm{DMU_{2j}}$, sub-$\mathrm{DMU_{3j}}$, sub-$\mathrm{DMU_{4j}}$) as well as two sub-DMUs form a parallel structure in the third stage (sub-$\mathrm{DMU_{5j}}$, sub-$\mathrm{DMU_{6j}}$) and all the stages are connected together in series. We denote the inputs to sub-$\mathrm{DMU_{1j}}$, sub-$\mathrm{DMU_{2j}}$, sub-$\mathrm{DMU_{3j}}$ and sub-$\mathrm{DMU_{4j}}$ by $x^1_{i_1j}$ (i_1=1,...,I_1), $x^2_{i_2j}$ (i_2=1,...,I_2), $x^3_{i_3j}$ (i_3=1,...,I_3) and $x^4_{i_4j}$ (i_4=1,...,I_4), respectively. We denote the intermediate measures between stage 1 and 2 by $z^1_{d_1j}$ (d_1=1,...,D_1), $z^2_{d_2j}$(d_2=1,...,D_2) and $z^3_{d_3j}$ (d_3=1,...,D_3), and between stage 2 and 3 by $z^4_{d_4j}$ (d_4=1,...,D_4), $z^5_{d_5j}$ ($d_5 = 1, \dots, D_5$), $z^6_{d_6j}$ ($d_6 = 1, \dots, D_6$), $z^7_{d_7j}$ ($d_7 = 1, \dots, D_7$), $z^8_{d_8j}$ (d_8=1,...,D_8) and $z^9_{d_9j}$ (d_9=1,...,D_9). The outputs of sub-$\mathrm{DMU_{2j}}$, sub-$\mathrm{DMU_{3j}}$ and sub-$\mathrm{DMU_{4j}}$ are denoted by $y^1_{r_1j}$ (r_1=1,...,R_1), $y^2_{r_2j}$(r_2=1,...,R_2), $y^3_{r_3j}$ (r_3=1,...,R_3), $y^4_{r_4j}$ (r_4=1,...,R_4), $y^5_{r_5j}$ (r_5=1,...,R_5) and $y^6_{r_6j}$ (r_6=1,...,R_6), respectively, where, $y^2_{r_2j}$, $y^4_{r_4j}$ and $y^6_{r_6j}$ are undesirable outputs. Finally, the outputs of sub-$\mathrm{DMU_{5j}}$ and sub-$\mathrm{DMU_{6j}}$ are denoted by $y^7_{r_7j}$ (r_7=1,...,R_7), $y^8_{r_8j}$ (r_8=1,...,R_8), $y^9_{r_9j}$ (r_9=1,...,R_9) and $y^{10}_{r_{10}j}$ (r_{10}=1,...,R_{10}), respectively, where, $y^8_{r_8j}$ and $y^{10}_{r_{10}j}$ are undesirable outputs.

We adopt $v^1_{i_1}$, $v^2_{i_2}$, $v^3_{i_3}$ and $v^4_{i_4}$ as the weights of the inputs to sub-$\mathrm{DMU_{1j}}$, sub-$\mathrm{DMU_{2j}}$, sub-$\mathrm{DMU_{3j}}$ and sub-$\mathrm{DMU_{4j}}$, respectively. We adopt $u^1_{r_1}$, $u^2_{r_2}$, $u^3_{r_3}$,$u^4_{r_4}$, $u^5_{r_5}$ and $u^6_{r_6}$ as the weights of the outputs to sub-$\mathrm{DMU_{2j}}$, sub-$\mathrm{DMU_{3j}}$ and sub-$\mathrm{DMU_{4j}}$, respectively. Kao and Hwang (2008) used the same weights for the intermediate measures. In accordance with this, we value the intermediate measures in this research, irrespective of their dual role (as an input in one stage and as an output in the next stage). We assume that the weights relative to the intermediate measures between stages 1 and 2 and similarly, weights related to the intermediate measures between stages 2 and 3 are uniform. We adopt $w^1_{d_1}$, $w^2_{d_2}$ and $w^3_{d_3}$ as the weights of the intermediate measures between stage 1and stage 2. The weights of the intermediate measures between stage 2 and stage 3 are denoted by $w^4_{d_4}$, $w^5_{d_5}$, $w^6_{d_6}$, $w^7_{d_7}$, $w^8_{d_8}$ and $w^9_{d_9}$. Finally, we adopt $u^7_{r_7}$, $u^8_{r_8}$, $u^9_{r_9}$and $u^{10}_{r_{10}}$ as the weights of the outputs to sub-$\mathrm{DMU_{5j}}$ and sub-$\mathrm{DMU_{6j}}$, respectively.

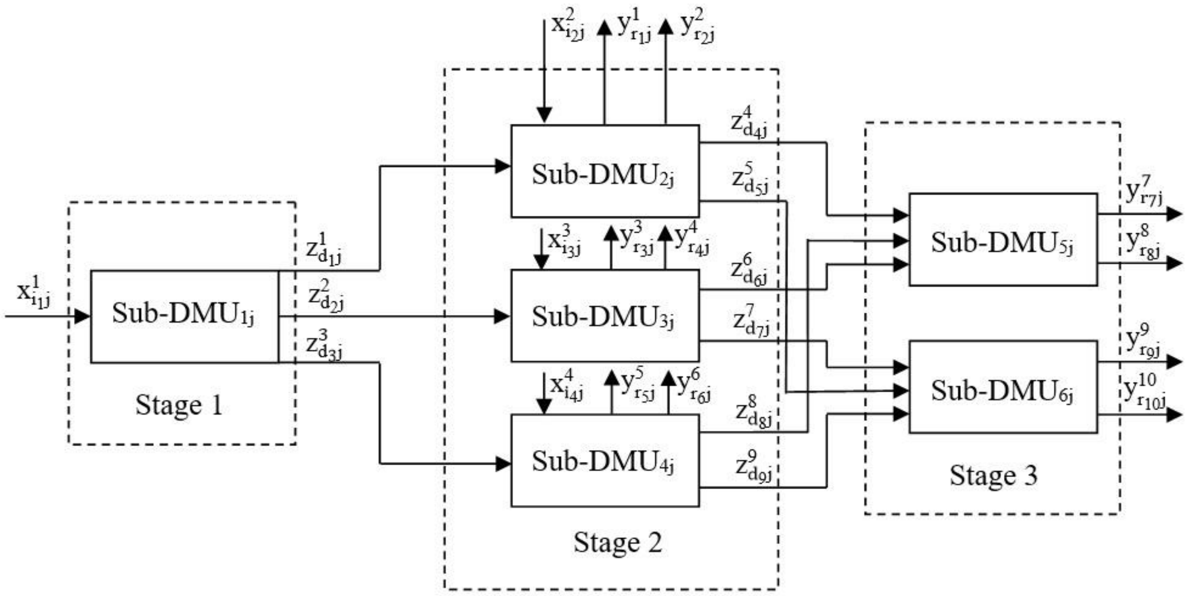

Figure 1. Structure of the three-stage system with additional inputs and undesirable outputs

Researchers doing efficiency analysis are likely to use input-oriented models, due to three major reasons. Firstly, demand is on the growth and estimating demand is an intricate matter. Secondly, managers have more control over inputs than outputs. Thirdly, these models reflect the primary goals of policymakers, based on being responsible in responding to the requirements of people and units must reduce costs, or else, limit the use of resources. Thereby, in this research we utilize the input-oriented model. In accordance with Korhonen and Luptacik (2004), we signify the undesirable outputs in the models with a negative mark. In the first stage, we have one DMU. The efficiency of sub-DMU_{1o} in the first stage is defined in model (1) as follows:

$$\theta_o^1 = \max \ \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1}$$

$$\text{s.t.} \quad \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1} \le 1, \quad j=1,\dots,n \tag{1}$$

$$v_{i_1}^1, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3 \ge \varepsilon;\ i_1=1,\dots,I_1;\ d_1=1,\dots,D_1;\ d_2=1,\dots,D_2;\ d_3=1,\dots,D_3.$$

The efficiencies of sub-DMU_{2o}, sub-DMU_{3o} and sub-DMU_{4o} in the second stage are defined, respectively, as follows:

$$\theta_o^2 = \max \ \frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4o}^4 + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5o}^5 + \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1 - \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1}$$

$$\text{s.t.} \quad \frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4 + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5 + \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1} \le 1, \quad j=1,\dots,n \tag{2}$$

$$u_{r_1}^1, u_{r_2}^2, v_{i_2}^2, w_{d_1}^1, w_{d_4}^4, w_{d_5}^5 \ge \varepsilon;\ r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ i_2=1,\dots,I_2;\ d_1=1,\dots,D_1;\ d_4=1,\dots,D_4;\ d_5=1,\dots,D_5.$$

$$\theta_o^3 = \max \ \frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6o}^6 + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7o}^7 + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3 - \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4o}^4}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2}$$

$$\text{s.t.} \quad \frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6 + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7 + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^4}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2} \le 1, \quad j=1,\dots,n \tag{3}$$

$u^3_{r_3}, u^4_{r_4}, v^3_{i_3}, w^2_{d_2}, w^6_{d_6}, w^7_{d_7} \geq \varepsilon$; $r_3=1,\ldots,R_3$; $r_4=1,\ldots,R_4$; $i_3=1,\ldots,I_3$; $d_2=1,\ldots,D_2$; $d_6=1,\ldots,D_6$; $d_7=1,\ldots,D_7$.

$$\theta^4_o = \max \ \frac{\sum_{d_8=1}^{D_8} w^8_{d_8} z^8_{d_8o} + \sum_{d_9=1}^{D_9} w^9_{d_9} z^9_{d_9o} + \sum_{r_5=1}^{R_5} u^5_{r_5} y^5_{r_5o} - \sum_{r_6=1}^{R_6} u^6_{r_6} y^6_{r_6o}}{\sum_{i_4=1}^{I_4} v^4_{i_4} x^4_{i_4o} + \sum_{d_2=1}^{D_2} w^3_{d_3} z^3_{d_3o}}$$

$$\text{s.t.} \quad \frac{\sum_{d_8=1}^{D_8} w^8_{d_8} z^8_{d_8j} + \sum_{d_9=1}^{D_9} w^9_{d_9} z^9_{d_9j} + \sum_{r_5=1}^{R_5} u^5_{r_5} y^5_{r_5j} - \sum_{r_6=1}^{R_6} u^6_{r_6} y^6_{r_6j}}{\sum_{i_4=1}^{I_4} v^4_{i_4} x^4_{i_4j} + \sum_{d_2=1}^{D_2} w^3_{d_3} z^3_{d_3j}} \leq 1, \quad j=1,\ldots,n \tag{4}$$

$u^5_{r_5}, u^6_{r_6}, v^4_{i_4}, w^3_{d_3}, w^8_{d_8}, w^9_{d_9} \geq \varepsilon$; $r_5=1,2,\ldots,R_5$; $r_6=1,2,\ldots,R_6$; $i_4=1,2,\ldots,I_4$; $d_3=1,2,\ldots,D_3$; $d_8=1,2,\ldots,D_8$;

$d_9=1,2,\ldots,D_9$.

Kao (2009) used the additive approach for the overall efficiency of a parallel structure where sub-DMUs are independent. In the second stage, we have three sub-DMUs forming a parallel structure and the sub-DMUs are independent. We then define the efficiency of the second stage as: $\theta^{234}_o=\max(w_1.\theta^2_o+w_2.\theta^3_o+w_3.\theta^4_o)$, where w_1, w_2 and w_3 are weights specified by experts such that, $w_1 + w_2 + w_3 = 1$. Chen et al. (2009) show that the relative size of the inputs of a stage expresses the importance of that stage. Hence, we compute the weights determined by the experts from the relative input value of each sub-DMU to obtain the value of inputs in the second stage. Thus, we define w_1, w_2 and w_3 as follows:

$$w_1 = \frac{\sum_{i_2=1}^{I_2} v^2_{i_2} x^2_{i_2o} + \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1o}}{\sum_{i_2=1}^{I_2} v^2_{i_2} x^2_{i_2o} + \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1o} + \sum_{i_3=1}^{I_3} v^3_{i_3} x^3_{i_3o} + \sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2o} + \sum_{i_4=1}^{I_4} v^4_{i_4} x^4_{i_4o} + \sum_{d_3=1}^{D_3} w^3_{d_3} z^3_{d_3o}},$$
$$w_2 = \frac{\sum_{i_3=1}^{I_3} v^3_{i_3} x^3_{i_3o} + \sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2o}}{\sum_{i_2=1}^{I_2} v^2_{i_2} x^2_{i_2o} + \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1o} + \sum_{i_3=1}^{I_3} v^3_{i_3} x^3_{i_3o} + \sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2o} + \sum_{i_4=1}^{I_4} v^4_{i_4} x^4_{i_4o} + \sum_{d_3=1}^{D_3} w^3_{d_3} z^3_{d_3o}}, \tag{5}$$
$$w_3 = \frac{\sum_{i_4=1}^{I_4} v^4_{i_4} x^4_{i_4o} + \sum_{d_3=1}^{D_3} w^3_{d_3} z^3_{d_3o}}{\sum_{i_2=1}^{I_2} v^2_{i_2} x^2_{i_2o} + \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1o} + \sum_{i_3=1}^{I_3} v^3_{i_3} x^3_{i_3o} + \sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2o} + \sum_{i_4=1}^{I_4} v^4_{i_4} x^4_{i_4o} + \sum_{d_3=1}^{D_3} w^3_{d_3} z^3_{d_3o}}$$

We defined w_1, w_2 and w_3 the parts of total input resources devoted to the sub-DMU_{2o}, sub-DMU_{3o} and sub-DMU_{4o}, respectively. In order to have more convenient models, we define I^{234}_o and O^{234}_o, the inputs and outputs to the second stage, respectively. Then, with models (2), (3) and (4) and formulas (5), the efficiency of the second stage is defined as follows:

$$\theta^{234}_o = \max \ \frac{O^{234}_o}{I^{234}_o}$$

$$\text{s.t.} \quad \frac{\sum_{d_4=1}^{D_4} w^4_{d_4} z^4_{d_4j} + \sum_{d_5=1}^{D_5} w^5_{d_5} z^5_{d_5j} + \sum_{r_1=1}^{R_1} u^1_{r_1} y^1_{r_1j} - \sum_{r_2=1}^{R_2} u^2_{r_2} y^2_{r_2j}}{\sum_{i_2=1}^{I_2} v^2_{i_2} x^2_{i_2j} + \sum_{d_1=1}^{D_1} w^1_{d_1} z^1_{d_1j}} \leq 1, \quad j=1,\ldots,n$$

$$\frac{\sum_{d_6=1}^{D_6} w^6_{d_6} z^6_{d_6j} + \sum_{d_7=1}^{D_7} w^7_{d_7} z^7_{d_7j} + \sum_{r_3=1}^{R_3} u^3_{r_3} y^3_{r_3j} - \sum_{r_4=1}^{R_4} u^4_{r_4} y^4_{r_4j}}{\sum_{i_3=1}^{I_3} v^3_{i_3} x^3_{i_3j} + \sum_{d_2=1}^{D_2} w^2_{d_2} z^2_{d_2j}} \leq 1, \quad j=1,\ldots,n \tag{6}$$

$$\frac{\sum_{d_8=1}^{D_8} w^8_{d_8} z^8_{d_8j} + \sum_{d_9=1}^{D_9} w^9_{d_9} z^9_{d_9j} + \sum_{r_5=1}^{R_5} u^5_{r_5} y^5_{r_5j} - \sum_{r_6=1}^{R_6} u^6_{r_6} y^6_{r_6j}}{\sum_{i_4=1}^{I_4} v^4_{i_4} x^4_{i_4j} + \sum_{d_3=1}^{D_3} w^3_{d_3} z^3_{d_3j}} \leq 1, \quad j=1,\ldots,n$$

$u^1_{r_1}, u^2_{r_2}, u^3_{r_3}, u^4_{r_4}, u^5_{r_5}, u^6_{r_6}, v^2_{i_2}, v^3_{i_3}, v^4_{i_4}, w^1_{d_1}, w^2_{d_2}, w^3_{d_3}, w^4_{d_4}, w^5_{d_5}, w^6_{d_6}, w^7_{d_7}, w^8_{d_8}, w^9_{d_9} \geq \varepsilon$;

$r_1=1,\ldots,R_1$; $r_2=1,\ldots,R_2$; $r_3=1,\ldots,R_3$; $r_4=1,\ldots,R_4$; $r_5=1,\ldots,R_5$; $r_6=1,\ldots,R_6$; $i_2=1,\ldots,I_2$; $i_3=1,\ldots,I_3$;

$i_4=1,\ldots,I_4$; $d_1=1,\ldots,D_1$; $d_2=1,\ldots,D_2$; $d_3=1,\ldots,D_3$; $d_4=1,\ldots,D_4$; $d_5=1,\ldots,D_5$; $d_6=1,\ldots,D_6$; $d_7=1,\ldots,D_7$;

$D_8=1,\ldots,D_8$; $d_9=1,\ldots,D_9$.

In the third stage, we have two sub-DMUs from a parallel structure. In the same way, we define the efficiency of the third stage, as we did in the second stage, as follows:

$$\theta_o^{56} = \max \ \frac{O_o^{56}}{I_o^{56}}$$

$$\text{s.t.} \ \frac{\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7j}^7 - \sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8j}^8}{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4 + \sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6 + \sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8} \le 1 \quad , \quad j=1,\ldots,n \tag{7}$$

$$\frac{\sum_{r_9=1}^{R_9} u_{r_9}^9 y_{r_9j}^9 - \sum_{r_{10}=1}^{R_{10}} u_{r_{10}}^{10} y_{r_{10}j}^{10}}{\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5 + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7 + \sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9} \le 1 \quad , \quad j=1,\ldots,n$$

$$u_{r_7}^7, u_{r_8}^8, u_{r_9}^9, u_{r_{10}}^{10}, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \ge \varepsilon;$$

$$r_7=1,\ldots,R_7;\ r_8=1,\ldots,R_8;\ r_9=1,\ldots,R_9;\ r_{10}=1,\ldots,R_{10};\ d_4=1,\ldots,D_4;\ d_5=1,\ldots,D_5;\ d_6=1,\ldots,D_6;$$

$$d_7=1,\ldots,D_7;\ d_8=1,\ldots,D_8;\ d_9=1,\ldots,D_9.$$

For the network structure as shown in Fig.1, the first, second and the third stages are linked in series. Kao and Hwang (2008) used the multiplicative approach to measure the overall efficiency of a series structure. With models (1), (6) and (7), we then define the overall efficiency of an integrated system shown in Fig.1 as $\theta_o^{overall}$=max θ_o^1. θ_o^{234}. θ_o^{56}. Thus,

$$\theta_o^{overall} = \max \ \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1} \cdot \frac{O_o^{234}}{I_o^{234}} \cdot \frac{O_o^{56}}{I_o^{56}}$$

$$\text{s.t.} \ \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1} \le 1, \quad j=1,\ldots,n$$

$$\frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4 + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5 + \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1} \le 1, \quad j=1,\ldots,n$$

$$\frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6 + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7 + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^4}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2} \le 1, \quad j=1,\ldots,n \tag{8}$$

$$\frac{\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8 + \sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9 + \sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5j}^5 - \sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6j}^6}{\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^4 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3} \le 1, \quad j=1,\ldots,n$$

$$\frac{\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7j}^7 - \sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8j}^8}{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4 + \sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6 + \sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8} \le 1 \quad , \quad j=1,\ldots,n$$

$$\frac{\sum_{r_9=1}^{R_9} u_{r_9}^9 y_{r_9j}^9 - \sum_{r_{10}=1}^{R_{10}} u_{r_{10}}^{10} y_{r_{10}j}^{10}}{\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5 + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7 + \sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9} \le 1 \quad , \quad j=1,\ldots,n$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3, u_{r_4}^4, u_{r_5}^5, u_{r_6}^6, u_{r_7}^7, u_{r_8}^8, u_{r_9}^9, u_{r_{10}}^{10}, v_{i_1}^1, v_{i_2}^2, v_{i_3}^3, v_{i_4}^4, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \ge \varepsilon;$$

$$r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;\ r_4=1,\ldots,R_4;\ r_5=1,\ldots,R_5;\ r_6=1,\ldots,R_6;\ r_7=1,\ldots,R_7;\ r_8=1,\ldots,R_8;$$

$$r_9=1,\ldots,R_9;\ r_{10}=1,\ldots,R_{10};\ i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3;\ i_4=1,\ldots,I_4;\ d_1=1,\ldots,D_1;\ d_2=1,\ldots,D_2;$$

$$d_3=1,\ldots,D_3;\ d_4=1,\ldots,D_4;\ d_5=1,\ldots,D_5;\ d_6=1,\ldots,D_6;\ d_7=1,\ldots,D_7;\ d_8=1,\ldots,D_8;\ d_9=1,\ldots,D_9.$$

In model (8), we measure the overall efficiency based on the efficiencies of the all sub-DMUs being less than one. The efficiency (Fig. 1) can be determined by the model (8) from the optimistic view. Wang et al (2005) measured the efficiencies of DMUs from both optimistic and pessimistic views (Azizi, 2014; Jahed et al, 2015). We then define the overall efficiency of the structure as shown in Fig. 1 from the pessimistic view based on the model (8) as follows:

$$\varphi_o^{overall} = \min \ \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1} \cdot \frac{O_o^{234}}{I_o^{234}} \cdot \frac{O_o^{56}}{I_o^{56}}$$

$$\text{s.t.}\quad \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1}\geq 1,\quad j=1,\dots,n$$

$$\frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5+\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1-\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1}\geq 1,\quad j=1,\dots,n$$

$$\frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7+\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3-\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^4}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2}\geq 1,\quad j=1,\dots,n \tag{9}$$

$$\frac{\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9+\sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5j}^5-\sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6j}^6}{\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3}\geq 1,\quad j=1,\dots,n$$

$$\frac{\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7j}^7-\sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8j}^8}{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4+\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6+\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8}\geq 1\ ,\quad j=1,\dots,n$$

$$\frac{\sum_{r_9=1}^{R_9} u_{r_9}^9 y_{r_9j}^9-\sum_{r_{10}=1}^{R_{10}} u_{r_{10}}^{10} y_{r_{10}j}^{10}}{\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9}\geq 1\ ,\quad j=1,\dots,n$$

$$u_{r_1}^1,u_{r_2}^2,u_{r_3}^3,u_{r_4}^4,u_{r_5}^5,u_{r_6}^6,u_{r_7}^7,u_{r_8}^8,u_{r_9}^9,u_{r_{10}}^{10},v_{i_1}^1,v_{i_2}^2,v_{i_3}^3,v_{i_4}^4,w_{d_1}^1,w_{d_2}^2,w_{d_3}^3,w_{d_4}^4,w_{d_5}^5,w_{d_6}^6,w_{d_7}^7,w_{d_8}^8,w_{d_9}^9 \geq \varepsilon;$$

$r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ r_3=1,\dots,R_3;\ r_4=1,\dots,R_4;\ r_5=1,\dots,R_5;\ r_6=1,\dots,R_6;\ r_7=1,\dots,R_7;\ r_8=1,\dots,R_8;$

$r_9=1,\dots,R_9;\ r_{10}=1,\dots,R_{10};\ i_1=1,\dots,I_1;\ i_2=1,\dots,I_2;\ i_3=1,\dots,I_3;\ i_4=1,\dots,I_4;\ d_1=1,\dots,D_1;\ d_2=1,\dots,D_2;$

$d_3=1,\dots,D_3;\ d_4=1,\dots,D_4;\ d_5=1,\dots,D_5;\ d_6=1,\dots,D_6;\ d_7=1,\dots,D_7;\ d_8=1,\dots,D_8;\ d_9=1,\dots,D_9.$

Models (8) and (9) are nonlinear models and in the third section of this paper an innovative approach is used to solve them. Wang and Chin (2009) used an approach for ranking DMUs from both optimistic and pessimistic views. We then define the overall efficiency according to the double-frontier from the solutions of models (8) and (9) as follows:

$$\emptyset_o=\sqrt{\theta_o^{overall}.\varphi_o^{overall}} \tag{10}$$

K-means clustering is a simple unsupervised learning algorithm that is used to solve clustering problems. It follows a simple procedure of classifying a given data set into a number of clusters, defined by the letter "k," which is fixed beforehand. The clusters are then positioned as points and all observations or data points are associated with the nearest cluster, computed, adjusted and then the process starts over using the new adjustments until a desired result is reached. Given a set of observations $(x_1, x_2, \dots, x_n)$, where each observation is a d-dimensional real vector, k-means clustering aims to partition the n observations into k ($\leq$ n) sets $S = \{s_1, s_2, \dots, s_k\}$ so as to minimize the within-cluster sum of squares (WCSS) (i.e. variance). Formally, the objective is to find $\arg\min_{s}\sum_{i=1}^{k}\sum_{x\in s_i}\|x-\mu_i\|^2 = \arg\min_{s}\sum_{i=1}^{k}|s_i|\mathrm{Var}\, s_i$, where μ_i is the mean of points in s_i. In this paper, we use the k-means technique to cluster the results of the described models (cluster based on the result of the formula (10)) and these results are shown in the case study section.

3. Model solution

Models (8) and (9) cannot be turned into linear models because of the additional inputs and outputs in the second stage in relative to DMU_{2o}, sub-DMU_{3o} and sub-DMU_{4o}, respectively. Thus, we present a heuristic technique to solve models (8) and (9), respectively.

3.1 A heuristic method from the optimistic view

First, we measure the maximum efficiencies of second and third stages provided that the efficiency of each sub-DMU in stages 1, 2 and 3 is less than one. Therefore, we define $\theta_o^{234\text{-}\max}$ and $\theta_o^{56\text{-}\max}$ maximum efficiencies from the optimistic view for stages 2 and 3, respectively, as follows:

$$\theta_o^{234\text{- max}}=\max\left\{\frac{O_o^{234}}{I_o^{234}}\,\middle|\,\theta_j^1\leq 1,\theta_j^2\leq 1,\theta_j^3\leq 1,\ \theta_j^4\leq 1,\theta_j^5\leq 1,\theta_j^6\leq 1,\ j=1,\ldots,n\right\} \tag{11}$$

$$\theta_o^{56\text{- max}}=\max\left\{\frac{O_o^{56}}{I_o^{56}}\,\middle|\,\theta_j^1\leq 1,\theta_j^2\leq 1,\theta_j^3\leq 1,\ \theta_j^4\leq 1,\theta_j^5\leq 1,\theta_j^6\leq 1,\ j=1,\ldots,n\right\}$$

In models (11), all variables are non-negative. The objective functions of models (11) are the same as those of models (6) and (7), respectively, but for the constraints, we shall consider the efficiency of each sub-DMU in stage 1, stage 2 and stage 3 as being less than one. With the Charnes–Cooper (1962) conversion, models (11) can be turned into linear models. We can solve models (11) and measure $\theta_o^{234\text{- max}}$ and $\theta_o^{56\text{- max}}$, respectively. Then, we convert model (8) to model (12) as follows:

$$\theta_o^{overall}=\max\left\{\theta_o^1.\,\theta_o^{234}.\,\theta_o^{56}\,\middle|\,\begin{array}{c}\theta_j^1\leq 1,\theta_j^2\leq 1,\theta_j^3\leq 1,\ \theta_j^4\leq 1,\theta_j^5\leq 1,\theta_j^6\leq 1,\theta_o^{234}=\frac{O_o^{234}}{I_o^{234}},\ \theta_o^{56}=\frac{O_o^{56}}{I_o^{56}},\\ \theta_o^{234}\in[0,\theta_o^{234\text{- max}}],\ \theta_o^{56}\in[0,\theta_o^{56\text{- max}}],\ j=1,\ldots,n\end{array}\right\} \tag{12}$$

In models (12), all variables are non-negative. It should be noted that we consider θ_o^{234} and θ_o^{56} as two variables in the objective functions of model (12) and add two constraints that specify these two variables, along with their range of changes. In fact, model (12) lets us consider stage 2 and stage 3 as two variables θ_o^{234} and θ_o^{56} that change between intervals $[0,\theta_o^{234\text{- max}}]$ and $[0,\theta_o^{56\text{- max}}]$, respectively. We should fix θ_o^{234} and θ_o^{56} until model (12) becomes a linear programing model that we can solve. For this purpose, we define θ_o^{234} and θ_o^{56} as follows:

$$\theta_o^{234}=\theta_o^{234\text{- max}}-k_2\Delta\varepsilon,\qquad k_2=0,1,\ldots,\left[\frac{\theta_o^{234\text{-max}}}{\Delta\varepsilon}\right]+1 \tag{13}$$

$$\theta_o^{56}=\theta_o^{56\text{- max}}-k_3\Delta\varepsilon,\qquad k_3=0,1,\ldots,\left[\frac{\theta_o^{56\text{-max}}}{\Delta\varepsilon}\right]+1$$

In formulas (13), $\Delta\varepsilon$ is a step size and we consider a smaller number as $\Delta\varepsilon$ =0.01. With the Charnes–Cooper (1962) conversion, model (13) can be turned into a linear model as follows:

$$\theta_o^{overall}=\max\ \theta_o^{234}.\,\theta_o^{56}.\left(\sum_{d_1=1}^{D_1}w_{d_1}^1 z_{d_1o}^1+\sum_{d_2=1}^{D_2}w_{d_2}^2 z_{d_2o}^2+\sum_{d_3=1}^{D_3}w_{d_3}^3 z_{d_3o}^3\right)$$

$$\text{s.t.}\ \sum_{d_1=1}^{D_1}w_{d_1}^1 z_{d_1j}^1+\sum_{d_2=1}^{D_2}w_{d_2}^2 z_{d_2j}^2+\sum_{d_3=1}^{D_3}w_{d_3}^3 z_{d_3j}^3-\sum_{i_1=1}^{I_1}v_{i_1}^1x_{i_1j}^1\leq 0 \tag{14}$$

$$\sum_{d_4=1}^{D_4}w_{d_4}^4 z_{d_4j}^4+\sum_{d_5=1}^{D_5}w_{d_5}^5 z_{d_5j}^5+\sum_{r_1=1}^{R_1}u_{r_1}^1y_{r_1j}^1-\sum_{r_2=1}^{R_2}u_{r_2}^2y_{r_2j}^2-\sum_{i_2=1}^{I_2}v_{i_2}^2x_{i_2j}^2-\sum_{d_1=1}^{D_1}w_{d_1}^1 z_{d_1j}^1\leq 0$$

$$\sum_{d_6=1}^{D_6}w_{d_6}^6 z_{d_6j}^6+\sum_{d_7=1}^{D_7}w_{d_7}^7 z_{d_7j}^7+\sum_{r_3=1}^{R_3}u_{r_3}^3y_{r_3j}^3-\sum_{r_4=1}^{R_4}u_{r_4}^4y_{r_4j}^4-\sum_{i_3=1}^{I_3}v_{i_3}^3x_{i_3j}^3-\sum_{d_2=1}^{D_2}w_{d_2}^2 z_{d_2j}^2\leq 0$$

$$\sum_{d_8=1}^{D_8}w_{d_8}^8 z_{d_8j}^8+\sum_{d_9=1}^{D_9}w_{d_9}^9 z_{d_9j}^9+\sum_{r_5=1}^{R_5}u_{r_5}^5y_{r_5j}^5-\sum_{r_6=1}^{R_6}u_{r_6}^6y_{r_6j}^6-\sum_{i_4=1}^{I_4}v_{i_4}^4x_{i_4j}^4-\sum_{d_3=1}^{D_3}w_{d_3}^3 z_{d_3j}^3\leq 0$$

$$\sum_{r_7=1}^{R_7}u_{r_7}^7\,y_{r_7j}^7-\sum_{r_8=1}^{R_8}u_{r_8}^8\,y_{r_8j}^8-\sum_{d_4=1}^{D_4}w_{d_4}^4 z_{d_4j}^4-\sum_{d_6=1}^{D_6}w_{d_6}^6 z_{d_6j}^6-\sum_{d_8=1}^{D_8}w_{d_8}^8 z_{d_8j}^8\leq 0$$

$$\sum_{r_9=1}^{R_9}u_{r_9}^9\,y_{r_9j}^9-\sum_{r_{10}=1}^{R_{10}}u_{r_{10}}^{10}\,y_{r_{10}j}^{10}-\sum_{d_5=1}^{D_5}w_{d_5}^5 z_{d_5j}^5-\sum_{d_7=1}^{D_7}w_{d_7}^7 z_{d_7j}^7-\sum_{d_9=1}^{D_9}w_{d_9}^9 z_{d_9j}^9\leq 0$$

$$\sum_{i_1=1}^{I_1}v_{i_1}^1x_{i_1o}^1=1$$

$$O_o^{234}=\theta_o^{234}.\,I_o^{234}$$

$$O_o^{56}=\theta_o^{56}.\,I_o^{56}$$

$$\theta_o^{234}\in[0,\theta_o^{234\text{- max}}]$$

$$\theta_o^{56}\in[0,\theta_o^{56\text{- max}}]$$

$$u_{r_1}^1,u_{r_2}^2,u_{r_3}^3,u_{r_4}^4,u_{r_5}^5,u_{r_6}^6,u_{r_7}^7,u_{r_8}^8,u_{r_9}^9,u_{r_{10}}^{10},v_{i_1}^1,v_{i_2}^2,v_{i_3}^3,v_{i_4}^4,w_{d_1}^1,w_{d_2}^2,w_{d_3}^3,w_{d_4}^4,w_{d_5}^5,w_{d_6}^6,w_{d_7}^7,w_{d_8}^8,w_{d_9}^9\geq\varepsilon;$$

$r_1=1,\ldots,R_1$; $r_2=1,\ldots,R_2$; $r_3=1,\ldots,R_3$; $r_4=1,\ldots,R_4$; $r_5=1,\ldots,R_5$; $r_6=1,\ldots,R_6$; $r_7=1,\ldots,R_7$; $r_8=1,\ldots,R_8$;

$r_9=1,\ldots,R_9$; $r_{10}=1,\ldots,R_{10}$; $i_1=1,\ldots,I_1$; $i_2=1,\ldots,I_2$; $i_3=1,\ldots,I_3$; $i_4=1,\ldots,I_4$; $d_1=1,\ldots,D_1$; $d_2=1,\ldots,D_2$;

$d_3=1,\ldots,D_3$; $d_4=1,\ldots,D_4$; $d_5=1,\ldots,D_5$; $d_6=1,\ldots,D_6$; $d_7=1,\ldots,D_7$; $d_8=1,\ldots,D_8$; $d_9=1,\ldots,D_9$.

In model (14), we increase k_2 and k_3 from zero to the upper bound of each one, independently, and solve each linear model with each k_2 and k_3 and show the value of the objective function with $\theta_o^{overall}(k_2, k_3)$. Comparing all the values of the objective function in model (14), we define the maximal overall efficiency from the optimistic view as $\theta_o^{overall*}=\max \theta_o^{overall}(k_2, k_3)$. At the same time and with the same k_2 and k_3, the efficiencies of the second stage and third stage from the optimistic view based on formulas (13) are defined as $\theta_o^{234*}=\theta_o^{234}(k_2)$ and $\theta_o^{56*}=\theta_o^{56}(k_3)$, respectively. The efficiency of the first stage can be measured by $\theta_o^{1*}=\frac{\theta_o^{overall*}}{(\theta_o^{234*}.\theta_o^{56*})}$. We used a heuristic technique and estimated the overall efficiency and the efficiencies of the stages from the optimistic view. For the sub-DMUs in the first stage, we have sub-DMU_1, thus $\theta_o^1=\theta_o^{1*}$. For other sub-DMUs, we can calculate θ_o^{2*}, θ_o^{3*} and θ_o^{4*} for sub-DMU_2, sub-DMU_3 and sub-DMU_4 in the second stage and θ_o^{5*} and θ_o^{6*} for sub-DMU_5, sub-DMU_6 in the third stage respectively, and by the weights obtained from models (22), which are shown (with asterisks) * in formulas (15) as follows:

$$\theta_o^{2*}=\frac{\sum_{d_4=1}^{D_4} w_{d_4}^{4*} z_{d_4o}^4+\sum_{d_5=1}^{D_5} w_{d_5}^{5*} z_{d_5o}^5+\sum_{r_1=1}^{R_1} u_{r_1}^{1*} y_{r_1o}^1-\sum_{r_2=1}^{R_2} u_{r_2}^{2*} y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^{2*} x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^{1*} z_{d_1o}^1}$$

$$\theta_o^{3*}=\frac{\sum_{d_6=1}^{D_6} w_{d_6}^{6*} z_{d_6o}^6+\sum_{d_7=1}^{D_7} w_{d_7}^{7*} z_{d_7o}^7+\sum_{r_3=1}^{R_3} u_{r_3}^{3*} y_{r_3o}^3-\sum_{r_4=1}^{R_4} u_{r_4}^{4*} y_{r_4o}^4}{\sum_{i_3=1}^{I_3} v_{i_3}^{3*} x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^{2*} z_{d_2o}^2}$$

$$\theta_o^{4*}=\frac{\sum_{d_8=1}^{D_8} w_{d_8}^{8*} z_{d_8o}^8+\sum_{d_9=1}^{D_9} w_{d_9}^{9*} z_{d_9o}^9+\sum_{r_5=1}^{R_5} u_{r_5}^{5*} y_{r_5o}^5-\sum_{r_6=1}^{R_6} u_{r_6}^{6*} y_{r_6o}^6}{\sum_{i_4=1}^{I_4} v_{i_4}^{4*} x_{i_4o}^4-\sum_{d_3=1}^{D_{23}} w_{d_3}^{3*} z_{d_3o}^3} \quad (15)$$

$$\theta_o^{5*}=\frac{\sum_{r_7=1}^{R_7} u_{r_7}^{7*} y_{r_7o}^7-\sum_{r_8=1}^{R_8} u_{r_8}^{8*} y_{r_8o}^8}{\sum_{d_4=1}^{D_4} w_{d_4}^{4*} z_{d_4o}^4+\sum_{d_6=1}^{D_6} w_{d_6}^{6*} z_{d_6o}^6+\sum_{d_8=1}^{D_8} w_{d_8}^{8*} z_{d_8o}^8}$$

$$\theta_o^{6*}=\frac{\sum_{r_9=1}^{R_9} u_{r_9}^{9*} y_{r_9o}^9-\sum_{r_{10}=1}^{R_{10}} u_{r_{10}}^{10*} y_{r_{10}o}^{10}}{\sum_{d_5=1}^{D_5} w_{d_5}^{5*} z_{d_5o}^5+\sum_{d_7=1}^{D_7} w_{d_7}^{7*} z_{d_7o}^7+\sum_{d_9=1}^{D_9} w_{d_9}^{9*} z_{d_9o}^9}$$

We tested our proposed approach in three modes, and each time we considered two stages as variables. It should be observed that the optimal efficiency of the network structure, as shown in Fig. 1, shall be unique. Therefore, the results of these three approaches are similar to each other and we considered one of these three modes to describe our approach.

3.2 A heuristic method from pessimistic view

Primarily, we measure the minimum efficiencies of the second and third stages provided that the efficiency of each sub-DMU in stage 1, stage 2 and stage 3 is more than one. In Section (2), we explained Wang's approach to measure the overall efficiency from the pessimistic view. Therefore, we define $\varphi_o^{234\text{-}\min}$ and $\varphi_o^{56\text{-}\min}$ as minimum efficiencies from the pessimistic view for the stages 2 and 3, respectively, as follows:

$$\varphi_o^{234\text{-}\min}=\min\left\{\frac{O_o^{234}}{I_o^{234}} \,\middle|\, \varphi_j^1 \geq 1, \varphi_j^2 \geq 1, \varphi_j^3 \geq 1, \varphi_j^4 \geq 1, \varphi_j^5 \geq 1, \varphi_j^6 \geq 1, j=1,\ldots,n\right\} \quad (16)$$

$$\varphi_o^{56\text{-}\min}=\min\left\{\frac{O_o^{56}}{I_o^{56}} \,\middle|\, \varphi_j^1 \geq 1, \varphi_j^2 \geq 1, \varphi_j^3 \geq 1, \varphi_j^4 \geq 1, \varphi_j^5 \geq 1, \varphi_j^6 \geq 1, j=1,\ldots,n\right\}$$

With the Charnes–Cooper (1962) conversion, we can solve models (16) and measure $\varphi_o^{234\text{-}\min}$ and $\varphi_o^{56\text{-}\min}$, respectively. Then, we convert model (9) to model (17) as follows:

$$\varphi_o^{overall}=\min\left\{\varphi_o^1.\varphi_o^{234}.\varphi_o^{56} \,\middle|\, \begin{array}{c} \varphi_j^1 \geq 1, \varphi_j^2 \geq 1, \varphi_j^3 \geq 1, \varphi_j^4 \geq 1, \varphi_j^5 \geq 1, \varphi_j^6 \geq 1, \varphi_o^{234}=\frac{O_o^{234}}{I_o^{234}}, \varphi_o^{56}=\frac{O_o^{56}}{I_o^{56}}, \\ \varphi_o^{234} \in [\varphi_o^{234\text{-}\min}, M], \varphi_o^{56} \in [\varphi_o^{56\text{-}\min}, M], j=1,\ldots,n \end{array}\right\} \quad (17)$$

In models (17) all variables are non-negative. “M” means the upper bound for the interval efficiency from the pessimistic view, and based on the experts opinion, we consider a large number as M=3. It should be noted that like the optimistic approach, we consider φ_o^{234} and φ_o^{56} as two variables in the objective functions of model (17) and add two constraints that specify these two variables, along with their range of changes. We can fix φ_o^{234} and φ_o^{56} with formulas (18) as follows:

$$\varphi_o^{234} = \varphi_o^{234\text{-min}} + k_2\Delta\varepsilon, \qquad k_2=0,1,\ldots,\left[\frac{M\text{-}\varphi_o^{234\text{-min}}}{\Delta\varepsilon}\right]+1 \tag{18}$$
$$\varphi_o^{56} = \varphi_o^{56\text{-min}} + k_3\Delta\varepsilon, \qquad k_3=0,1,\ldots,\left[\frac{M\text{-}\varphi_o^{56\text{-min}}}{\Delta\varepsilon}\right]+1$$

With the Charnes–Cooper (1962) conversion, model (17) can be turned into a linear model as follows:

$$\varphi_o^{overall} = \min\ \varphi_o^{234}\,.\,\varphi_o^{56}.\left(\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3\right)$$

$$\text{s.t.}\ \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \geq 0 \tag{19}$$

$$\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4 + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5 + \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \geq 0$$

$$\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6 + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7 + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^4 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \geq 0$$

$$\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8 + \sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9 + \sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5j}^5 - \sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6j}^6 - \sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^4 - \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3 \geq 0$$

$$\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7j}^7 - \sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8j}^8 - \sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4 - \sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6 - \sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8 \geq 0$$

$$\sum_{r_9=1}^{R_9} u_{r_9}^9 y_{r_9j}^9 - \sum_{r_{10}=1}^{R_{10}} u_{r_{10}}^{10} y_{r_{10}j}^{10} - \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5 - \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7 - \sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9 \geq 0$$

$$\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1 = 1$$

$$O_o^{234} = \varphi_o^{234}.\, I_o^{234}$$

$$O_o^{56} = \varphi_o^{56}.\, I_o^{56}$$

$$\varphi_o^{234} \in [\varphi_o^{234\text{- min}}, M]$$

$$\varphi_o^{56} \in [\varphi_o^{56\text{- min}}, M]$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3, u_{r_4}^4, u_{r_5}^5, u_{r_6}^6, u_{r_7}^7, u_{r_8}^8, u_{r_9}^9, u_{r_{10}}^{10}, v_{i_1}^1, v_{i_2}^2, v_{i_3}^3, v_{i_4}^4, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \geq \varepsilon;$$

$r_1=1,\ldots,R_1$; $r_2=1,\ldots,R_2$; $r_3=1,\ldots,R_3$; $r_4=1,\ldots,R_4$; $r_5=1,\ldots,R_5$; $r_6=1,\ldots,R_6$; $r_7=1,\ldots,R_7$; $r_8=1,\ldots,R_8$;

$r_9=1,\ldots,R_9$; $r_{10}=1,\ldots,R_{10}$; $i_1=1,\ldots,I_1$; $i_2=1,\ldots,I_2$; $i_3=1,\ldots,I_3$; $i_4=1,\ldots,I_4$; $d_1=1,\ldots,D_1$; $d_2=1,\ldots,D_2$;

$d_3=1,\ldots,D_3$; $d_4=1,\ldots,D_4$; $d_5=1,\ldots,D_5$; $d_6=1,\ldots,D_6$; $d_7=1,\ldots,D_7$; $d_8=1,\ldots,D_8$; $d_9=1,\ldots,D_9$.

In model (19) we increase k_2 and k_3 from zero to the upper bound for each one independently and solve each linear model with each k_2 and k_3 and show the value of the objective function with $\varphi_o^{overall}(k_2, k_3)$. Comparing all the values of the objective function in the model (19), we define the minimal overall efficiency from the pessimistic view as $\varphi_o^{overall*} = \min \varphi_o^{overall}(k_2, k_3)$. At the same time and with the same k_2 and k_3, the efficiencies of second stage and third stage from the pessimistic view based on formulas (18) are defined as $\varphi_o^{234*} = \varphi_o^{234}(k_2)$ and $\varphi_o^{56*} = \varphi_o^{56}(k_3)$, respectively. The efficiency of the first stage can be measured by $\varphi_o^{1*} = \frac{\varphi_o^{overall*}}{(\varphi_o^{234*}.\varphi_o^{56*})}$. For the sub-DMUs in the first stage we have sub-DMU$_1$, thus $\varphi_o^1 = \varphi_o^{1*}$. For other sub-DMUs, we can use the weights obtained from models (19) and calculate the efficiencies of sub-DMUs in formula (20) as follows:

$$\varphi_o^{2*} = \frac{\sum_{d_4=1}^{D_4} w_{d_4}^{4*} z_{d_4o}^4 + \sum_{d_5=1}^{D_5} w_{d_5}^{5*} z_{d_5o}^5 + \sum_{r_1=1}^{R_1} u_{r_1}^{1*} y_{r_1o}^1 - \sum_{r_2=1}^{R_2} u_{r_2}^{2*} y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^{2*} x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^{1*} z_{d_1o}^1}$$

$$\varphi_o^{3*} = \frac{\sum_{d_6=1}^{D_6} w_{d_6}^{6*} z_{d_6o}^6 + \sum_{d_7=1}^{D_7} w_{d_7}^{7*} z_{d_7o}^7 + \sum_{r_3=1}^{R_3} u_{r_3}^{3*} y_{r_3o}^3 - \sum_{r_4=1}^{R_4} u_{r_4}^{4*} y_{r_4o}^4}{\sum_{i_3=1}^{I_3} v_{i_3}^{3*} x_{i_3o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^{2*} z_{d_2o}^2}$$

$$\varphi_o^{4*} = \frac{\sum_{d_8=1}^{D_8} w_{d_8}^{8*} z_{d_8o}^8 + \sum_{d_9=1}^{D_9} w_{d_9}^{9*} z_{d_9o}^9 + \sum_{r_5=1}^{R_5} u_{r_5}^{5*} y_{r_5o}^5 - \sum_{r_6=1}^{R_6} u_{r_6}^{6*} y_{r_6o}^6}{\sum_{i_4=1}^{I_4} v_{i_4}^{4*} x_{i_4o}^4 - \sum_{d_3=1}^{D_3} w_{d_3}^{3*} z_{d_3o}^3} \tag{20}$$

$$\varphi_o^{5*}=\frac{\sum_{r_7=1}^{R_7} u_{r_7}^{7*} y_{r_7 o}^{7} - \sum_{r_8=1}^{R_8} u_{r_8}^{8*} y_{r_8 o}^{8}}{\sum_{d_4=1}^{D_4} w_{d_4}^{4*} z_{d_4 o}^{4} + \sum_{d_6=1}^{D_6} w_{d_6}^{6*} z_{d_6 o}^{6} + \sum_{d_8=1}^{D_8} w_{d_8}^{8*} z_{d_8 o}^{8}}$$

$$\varphi_o^{6*}=\frac{\sum_{r_9=1}^{R_9} u_{r_9}^{9*} y_{r_9 o}^{9} - \sum_{r_{10}=1}^{R_{10}} u_{r_{10}}^{10*} y_{r_{10} o}^{10}}{\sum_{d_5=1}^{D_5} w_{d_5}^{5*} z_{d_5 o}^{5} + \sum_{d_7=1}^{D_7} w_{d_7}^{7*} z_{d_7 o}^{7} + \sum_{d_9=1}^{D_9} w_{d_9}^{9*} z_{d_9 o}^{9}}$$

Like an optimistic approach, we tested our proposed approach in three modes, and each time we considered two stages as variables. The results of these three approaches are similar to each other and we considered one of these three modes to describe our approach.

4. Case study description

We consider a factual example of production planning and inventory control in a factory within duration of 24 intervals in a year. Therefore, we have a dynamic structure with 24 DMUs, where some of the outputs in the second stage within a period t convert to the inputs for the same stage at a period t + 1. The inputs-outputs of each DMU are defined as follows: the first stage is the production area and the inputs of the first stage are production cost (x_1^1, x_2^1, x_3^1) and setup cost (x_4^1, x_5^1, x_6^1) for three goods 1,2 and 3, separately. The intermediate measures between stage 1 and 2 are the quantity of each of the goods produced (z_1^1, z_1^2, z_1^3). In the second stage we have three warehouses and the additional inputs of each warehouse, that are costs for reserving storage location (x_1^2, x_1^3, x_1^4), cost of moving goods from the production area to the warehouse (x_2^2, x_2^3, x_2^4), cost of holding goods into warehouse (x_3^2, x_3^3, x_3^4) and the goods remaining in the warehouse from the last period (x_4^2, x_4^3, x_4^4). The desirable and undesirable outputs of each warehouse in the second stage are defined by the goods remaining in each warehouse for next period (y_1^1, y_1^3, y_1^5), in addition to the costs of moving the goods from the warehouse to each delivery point (y_1^2, y_1^4, y_1^6). The intermediate measures between stage 2 and 3 are the quantity of goods delivered from each warehouse to each delivery point (z_1^4, z_1^5, z_1^6, z_1^7, z_1^8, z_1^9). Finally, in the third stage the desirable and undesirable outputs are profits due to the sale of the goods (y_1^7,y_1^9) and the delay penalty (y_1^8,y_1^{10}). The inputs-outputs for each stage are summarized in Table 1.

Table 1. Variables of inputs and outputs

Stage-SubDMU	Input-Output	Variable	Symbol
Stage1- sub-DMU_1	Input	Production cost Setup cost	x_1^1, x_2^1, x_3^1 x_4^1, x_5^1, x_6^1
Stage1- sub-DMU_1	Output	Quantity of each of the goods produced	z_1^1, z_1^2, z_1^3
Stage2- sub-DMU_2	Input	Quantity of goods 1 produced Cost for reserving storage location 1 Cost of Transport goods 1 to warehouse 1 Cost of holding goods 1 Goods 1 remaining from the last period	z_1^1 x_1^2 x_2^2 x_3^2 x_4^2
Stage2- sub-DMU_2	Output	Quantity of goods 1 delivered Goods 1 remaining for next period Cost of Transport goods 1 to delivery points	z_1^4, z_1^5 y_1^1 y_1^2
Stage2- sub-DMU_3	Input	Quantity of goods 2 produced Cost of reserving storage location 2 Cost of Transport goods 2 to warehouse 2 Cost of holding goods 2 Goods 2 remaining from the last period	z_1^2 x_1^3 x_2^3 x_3^3 x_4^3
Stage2- sub-DMU_3	Output	Quantity of goods 2 delivered Goods 2 remaining for next period Cost of Transport goods 2 to delivery points	z_1^6, z_1^7 y_1^3 y_1^4
Stage2- sub-DMU_4	Input	Quantity of goods 3 produced Cost of reserving storage location 3 Cost of Transport goods 3 to warehouse 3 Cost of holding goods 3 Goods 3 remaining from the last period	z_1^3 x_1^4 x_2^4 x_3^4 x_4^4
Stage2- sub-DMU_4	Output	Quantity of goods 3 delivered Goods 3 remaining for next period Cost of Transport goods 3 to delivery points	z_1^8, z_1^9 y_1^5 y_1^6
Stage3- sub-DMU_5	Input	Quantity of each of the goods delivered	z_1^4, z_1^6, z_1^8
Stage3- sub-DMU_5	Output	Profit Delay Penalty	y_1^7 y_1^8
Stage3- sub-DMU_6	Input	Quantity of each of the goods delivered	z_1^5, z_1^7, z_1^9
Stage3- sub-DMU_6	Output	Profit Delay Penalty	y_1^9 y_1^{10}

We have many variables of inputs and outputs. Therefore, Tables 2 and 3 provide the data for the above-mentioned factory for duration of 24 intervals in 2016. The inputs of the factory are shown in Table 2 and the outputs and the intermediate measures of the factory are reported in Table 3. We consider each period as a DMU with three stages. Thereby, we measure the efficiency of the structure as shown in Fig. 1 as a dynamic network from both the optimistic and pessimistic views.

Table 2. The inputs of the factory for 24 periods in 2016

DMU	Production cost			Setup cost			Cost of reserving storage location			Cost of Transport goods to warehouse			Cost of holding goods			Goods remaining from last period		
	x_1^1	x_2^1	x_3^1	x_4^1	x_5^1	x_6^1	x_1^2	x_1^3	x_1^4	x_2^2	x_2^3	x_2^4	x_3^2	x_3^3	x_3^4	x_4^2	x_4^3	x_4^4
1	2480	3040	4480	432	576	672	65	60	45	64	84	48	50	48	45	0	0	0
2	4340	5320	6720	756	1008	1008	65	60	45	112	147	72	50	48	45	0	0	0
3	6820	8360	11200	1188	1584	1680	65	60	45	176	231	120	50	48	45	0	0	0
4	8680	10640	15680	1512	2016	2352	65	60	45	224	294	168	50	48	45	0	0	0
5	3720	4560	6720	648	864	1008	65	60	45	96	126	72	50	48	45	0	0	0
6	4340	5320	8960	756	1008	1344	65	60	45	112	147	96	50	48	45	0	0	0
7	8060	10640	13440	1404	2016	2016	65	60	45	208	294	144	50	72	45	0	0	0
8	12400	15200	22400	2160	2880	3360	65	60	45	320	420	240	113	144	105	0	2	0
9	12400	15200	22400	2160	2880	3360	65	60	45	320	420	240	92	120	105	6	8	4
10	12400	15200	22400	2160	2880	3360	65	60	45	320	420	240	50	48	75	4	6	4
11	12400	15200	22400	2160	2880	3360	65	60	45	320	420	240	50	48	45	0	0	2
12	12400	15200	22400	2160	2880	3360	65	60	45	320	420	240	50	48	45	0	0	0
13	7440	8360	11200	1296	1584	1680	65	60	45	192	231	120	50	48	45	0	0	0
14	4340	5320	6720	756	1008	1008	65	60	45	112	147	72	50	48	45	0	0	0
15	4340	5320	8960	756	1008	1344	65	60	45	112	147	96	50	48	45	0	0	0
16	3720	4560	6720	648	864	1008	65	60	45	96	126	72	50	48	45	0	0	0
17	6820	8360	11200	1188	1584	1680	65	60	45	176	231	120	50	48	45	0	0	0
18	8060	9880	13440	1404	1872	2016	65	60	45	208	273	144	50	48	45	0	0	0
19	6200	7600	11200	1080	1440	1680	65	60	45	160	210	120	50	48	45	0	0	0
20	7440	9120	13440	1296	1728	2016	65	60	45	192	252	144	50	48	45	0	0	0
21	7440	9120	11200	1296	1728	1680	65	60	45	192	252	120	50	48	45	0	0	0
22	9300	11400	15680	1620	2160	2352	65	60	45	240	315	168	134	144	105	0	0	0
23	12400	15200	22400	2160	2880	3360	65	60	45	320	420	240	50	48	45	8	8	4
24	12400	15200	22400	2160	2880	3360	65	60	45	320	420	240	50	48	45	0	0	0

In Table 2, the zero values for each period indicate that the goods from the previous period have not remained in the warehouse (the last three columns). The Table 3 also shows zero values indicating that there is no goods lingering in the warehouse for the next period (columns 11 to 13), or that, we do not have a delay penalty for delivery of goods at that period (the last two columns).

Table 3. The outputs and the intermediate measures of the factory for 24 periods in 2016

DMU	Quantity of goods produced			Quantity of goods delivered						Goods remaining for next period			Cost of Transport goods to delivery points			Profit		Delay Penalty	
	z_1^1	z_1^2	z_1^3	z_1^4	z_1^5	z_1^6	z_1^7	z_1^8	z_1^9	y_1^1	y_1^3	y_1^5	y_1^{12}	y_1^4	y_1^6	y_1^7	y_1^9	y_1^8	y_1^{10}
1	8	8	4	4	4	4	4	2	2	0	0	0	132	154	82	1590	1590	0	0
2	14	14	6	7	7	7	7	3	3	0	0	0	231	269.5	123	2555.5	2555.5	0	0
3	22	22	10	11	11	11	11	5	5	0	0	0	363	423.5	205	4145.5	4145.5	0	0
4	28	28	14	14	14	14	14	7	7	0	0	0	462	539	287	5565	5565	0	0
5	12	12	6	6	6	6	6	3	3	0	0	0	198	231	123	2385	2385	0	0
6	14	14	8	7	7	7	7	4	4	0	0	0	231	269.5	164	3009.5	3009.5	0	0
7	26	28	12	13	13	13	13	6	6	0	2	0	429	500.5	246	4940.5	4940.5	0	0
8	40	40	20	17	17	17	17	8	8	6	8	4	561	654.5	328	6530.5	6530.5	0	0
9	40	40	20	21	21	21	21	10	10	4	6	4	693	808.5	410	8120.5	8120.5	0	0
10	40	40	20	23	21	23	23	11	11	0	0	2	719	885.5	451	8915.5	8842.5	0	20
11	40	40	20	16	24	18	22	11	11	0	0	0	688	783	451	7990	8818	120	0
12	40	40	20	17	17	27	13	12	8	0	0	0	561	724.5	396	9686.5	5994.5	60	270
13	24	22	10	18	6	6	16	3	7	0	0	0	354	456	219	2823	5541	0	0
14	14	14	6	7	7	7	7	3	3	0	0	0	231	269.5	123	2555.5	2555.5	0	0
15	14	14	8	7	7	7	7	4	4	0	0	0	231	269.5	164	3009.5	3009.5	0	0
16	12	12	6	6	6	6	6	3	3	0	0	0	198	231	123	2385	2385	0	0
17	22	22	10	11	11	11	11	5	5	0	0	0	363	423.5	205	4145.5	4145.5	0	0
18	26	26	12	13	13	13	13	6	6	0	0	0	429	500.5	246	4940.5	4940.5	0	0
19	20	20	10	10	10	10	10	5	5	0	0	0	330	385	205	3975	3975	0	0
20	24	24	12	12	12	12	12	6	6	0	0	0	396	462	246	4770	4770	0	0
21	24	24	10	12	12	12	12	5	5	0	0	0	396	462	205	4316	4316	0	0
22	30	30	14	11	11	11	11	5	5	8	8	4	363	423.5	205	4145.5	4145.5	0	0
23	40	40	20	24	24	24	24	12	12	0	0	0	792	924	492	9540	9540	0	0
24	40	40	20	20	20	20	20	10	10	0	0	0	660	770	410	7950	7950	0	0

An inattention or negligence in relative to the control of weights is one of the most crucial problems in DEA models. Each DMU can assign the weights to the factors in order to maximize its efficiency. In this case, very few weights may be allocated to important factors or high weights to the least important factors. To overcome the problem, we utilized a questionnaire that was completed by managers. Table 4 illustrates the importance of weights. The value of ε in models was also considered to be 0.001 according to the opinion of managers.

Table 4. Constraints to control weights

Inputs		Intermediate measures	Outputs
$\frac{v_3^1}{v_2^1} \geq 1.09$	$\frac{v_3^4}{v_3^3} \geq 1.20$	$\frac{w_1^8}{w_1^9} \geq 1.03$	$\frac{u_1^7}{u_1^9} \geq 1.03$
$\frac{v_2^1}{v_1^1} \geq 1.12$	$\frac{v_3^3}{v_3^2} \geq 1.08$	$\frac{w_1^9}{w_1^6} \geq 1.12$	$\frac{u_1^9}{u_1^8} \geq 1.03$
$\frac{v_1^1}{v_6^1} \geq 1.02$	$\frac{v_3^2}{v_2^4} \geq 1.01$	$\frac{w_1^6}{w_1^7} \geq 1.04$	$\frac{u_1^8}{u_1^{10}} \geq 1.04$
$\frac{v_6^1}{v_5^1} \geq 1.04$	$\frac{v_2^4}{v_2^3} \geq 1.08$	$\frac{w_1^7}{w_1^3} \geq 1.15$	$\frac{u_1^{10}}{u_1^3} \geq 1.17$
$\frac{v_5^1}{v_4^1} \geq 1.02$	$\frac{v_2^3}{v_2^2} \geq 1.30$	$\frac{w_1^3}{w_1^2} \geq 1.05$	$\frac{u_1^3}{u_1^2} \geq 1.10$
$\frac{v_4^1}{v_4^4} \geq 1.02$	$\frac{v_2^2}{v_1^4} \geq 1.06$	$\frac{w_1^2}{w_1^4} \geq 1.05$	$\frac{u_1^2}{u_1^1} \geq 1.16$
$\frac{v_4^4}{v_4^3} \geq 1.04$	$\frac{v_1^4}{v_1^3} \geq 1.07$	$\frac{w_1^4}{w_1^5} \geq 1.06$	$\frac{u_1^1}{u_1^6} \geq 1.38$
$\frac{v_4^3}{v_4^2} \geq 1.16$	$\frac{v_1^3}{v_1^2} \geq 1.16$	$\frac{w_1^5}{w_1^1} \geq 1.21$	$\frac{u_1^6}{u_1^5} \geq 1.08$
$\frac{v_4^2}{v_3^4} \geq 1.20$			$\frac{u_1^5}{u_1^4} \geq 1.33$

Model (11) measure the maximum optimistic performance and model (16) measure the minimum pessimistic performance of the second and third stages. These values are shown in Table 5 with the values of k_2 and k_3 obtained for the optimal overall optimistic and pessimistic performance.

Table 5. Results of the maximum and minimum overall efficiencies of the second and third stages

DMU	Optimistic View					Pessimistic View			
	$\theta_o^{234\text{-max}}$	$\theta_o^{56\text{-max}}$	k_2	k_3		$\varphi_o^{234\text{-min}}$	$\varphi_o^{56\text{-min}}$	k_2	k_3
1	0.58965	1.00000	7	6		1.00000	1.00070	0	3
2	0.73641	1.00000	7	8		1.11336	1.00020	10	1
3	0.86243	1.00000	8	7		1.19736	1.00099	8	12
4	0.92627	1.00000	7	6		1.23606	1.00244	17	2
5	0.70994	1.00000	7	6		1.09702	1.00104	5	7
6	0.77012	1.00000	7	4		1.14184	1.00096	16	7
7	0.91584	1.00000	7	7		1.32023	1.00134	4	5
8	0.94502	1.00000	2	7		1.15998	1.00204	5	1
9	0.80458	1.00000	2	7		1.16398	1.00273	8	3
10	0.81510	0.99978	3	7		1.27057	1.00279	9	6
11	0.94861	1.00000	5	20		1.23324	1.00207	19	6
12	0.99940	1.00000	5	13		1.28330	1.00000	3	12
13	0.89539	1.00000	10	11		1.24592	1.00000	9	2
14	0.73641	1.00000	7	8		1.11336	1.00020	10	1
15	0.77012	1.00000	7	4		1.14184	1.00096	16	7
16	0.70994	1.00000	7	6		1.09702	1.00104	5	7
17	0.86243	1.00000	8	7		1.19736	1.00099	8	12
18	0.90291	1.00000	8	7		1.22168	1.00134	1	9
19	0.84857	1.00000	8	6		1.19021	1.00174	15	7
20	0.89220	1.00000	8	6		1.21641	1.00209	5	6
21	0.87525	1.00000	8	9		1.20397	1.00000	2	17
22	0.95773	1.00000	6	7		1.00000	1.00099	0	1
23	0.80183	1.00000	3	6		1.17005	1.00417	17	11
24	0.99872	1.00000	6	6		1.27407	1.00348	10	5

Table 6 gives the overall efficiency and the efficiencies of stages and sub-DMUs from models (14) and (15) based on the optimistic view in relevance to considering constraints in order to control weights.

Table 6. Results based on the optimistic view

DMU	$\theta_o^{overall*}$	θ_o^{1*}	θ_o^{234*}	θ_o^{56*}	θ_o^{2*}	θ_o^{3*}	θ_o^{4*}	θ_o^{5*}	θ_o^{6*}
1	0.23663	0.48443	0.51965	0.94	0.512357	0.586409	0.47756	0.978973	0.900973
2	0.29337	0.478505	0.66641	0.92	0.635742	0.762186	0.608481	0.955317	0.884609
3	0.35627	0.489611	0.78243	0.93	0.690658	0.839468	0.782225	0.964254	0.895506
4	0.39901	0.49573	0.85627	0.94	0.72601	0.903115	0.889023	0.978973	0.900973
5	0.29104	0.483822	0.63994	0.94	0.601413	0.709667	0.60885	0.978973	0.900973
6	0.32794	0.487922	0.70012	0.96	0.628221	0.739518	0.706541	1	0.919883
7	0.39071	0.496688	0.84584	0.93	0.71458	0.925034	0.841362	0.966592	0.89339
8	0.43652	0.507423	0.92502	0.93	0.908467	0.980833	0.879161	0.962447	0.897531
9	0.37108	0.508566	0.78458	0.93	0.696431	0.847283	0.779707	0.960634	0.899341
10	0.36524	0.500349	0.7851	0.92978	0.658318	0.842691	0.804549	0.972073	0.886469
11	0.35395	0.492358	0.89861	0.8	0.74686	0.994601	0.904237	0.689819	0.893642
12	0.33808	0.409309	0.9494	0.87	0.821087	0.980487	1	0.971958	0.715375
13	0.3492	0.493292	0.79539	0.89	0.769192	0.843033	0.766934	0.739868	1
14	0.29337	0.478505	0.66641	0.92	0.635742	0.762186	0.608481	0.955317	0.884609
15	0.32794	0.487922	0.70012	0.96	0.628221	0.739518	0.706541	1	0.919883
16	0.29104	0.483822	0.63994	0.94	0.601413	0.709667	0.60885	0.978973	0.900973
17	0.35627	0.489611	0.78243	0.93	0.690658	0.839468	0.782225	0.964254	0.895506
18	0.38027	0.496886	0.82291	0.93	0.712499	0.873204	0.841473	0.966592	0.89339
19	0.35747	0.494798	0.76857	0.94	0.678504	0.808289	0.782341	0.978973	0.900973
20	0.38016	0.497938	0.8122	0.94	0.700729	0.849319	0.841497	0.978973	0.900973
21	0.35642	0.492512	0.79525	0.91	0.707493	0.863933	0.782271	0.951083	0.868944
22	0.42327	0.506978	0.89773	0.93	0.944822	1	0.770276	0.964816	0.895186
23	0.36081	0.497312	0.77183	0.94	0.588739	0.82617	0.844259	0.978973	0.900973
24	0.4372	0.495469	0.93872	0.94	0.768295	0.99556	0.990161	0.978973	0.900973

From the second column of Table 6, we note that the efficiency scores of period 24 is highest and the efficiency scores of period 1 is lowest from the optimistic view. Table 7 demonstrates the overall efficiency and the efficiencies of stages and sub-DMUs from models (19) and (20) based on the pessimistic view in relative to considering constraints on control weights.

Table 7. Results based on the pessimistic view

DMU	$\varphi_o^{overall*}$	φ_o^{1*}	φ_o^{234*}	φ_o^{56*}	φ_o^{2*}	φ_o^{3*}	φ_o^{4*}	φ_o^{5*}	φ_o^{6*}
1	1.09539	1.062763	1	1.0307	1	1	1	1.043384	1.018144
2	1.25799	1.026314	1.21336	1.0102	1.444636	1.255204	1.129217	1.02883	0.991978
3	1.43191	1.000002	1.27736	1.12099	1.068206	1.192991	1.459057	1.150346	1.091819
4	1.43761	1	1.40606	1.02244	1.125902	1.271352	1.658689	1.018829	1.026129
5	1.29446	1.053688	1.14702	1.07104	1.221698	1.144869	1.12809	1.105617	1.036718
6	1.43789	1.031323	1.30184	1.07096	1.262163	1.207759	1.3891	1.092297	1.049753
7	1.50029	1.049107	1.36023	1.05134	1.701783	1.239278	1.395674	1.084704	1.018288
8	1.2883	1.052062	1.20998	1.01204	1.396423	1.146618	1.23735	1.014243	1.009815
9	1.32689	1.032844	1.24398	1.03273	1.068958	1.240253	1.302052	1.05243	1.013236
10	1.49841	1.036245	1.36057	1.06279	1.220424	1.395808	1.37457	1.087052	1.038447
11	1.52662	1.00995	1.42324	1.06207	1.358902	1.395931	1.482932	1	1.114823
12	1.4709	1.000003	1.3133	1.12	1.172873	1.293982	1.465096	1.218566	1
13	1.36264	1.000001	1.33592	1.02	1.483196	1.146201	1.4485	1	1.030653
14	1.25799	1.026314	1.21336	1.0102	1.444636	1.255204	1.129217	1.02883	0.991978
15	1.43789	1.031323	1.30184	1.07096	1.150021	1.147987	1.304811	1.227224	1.114752
16	1.29446	1.053688	1.14702	1.07104	1.221698	1.144869	1.12809	1.105617	1.036718
17	1.43191	1.000002	1.27736	1.12099	1.068206	1.192991	1.459057	1.150346	1.091819
18	1.34418	0.999999	1.23168	1.09134	1.149171	1.13437	1.343746	1.167193	1.023843
19	1.51425	1.05423	1.34021	1.07174	1.486785	1.294474	1.338915	1.107239	1.0365
20	1.34504	1	1.26641	1.06209	0.770898	1.416272	1.466106	1.099244	1.026881
21	1.43561	1.00249	1.22397	1.17	1.142857	1.151533	1.401944	1.322582	1.01703
22	1.0483	1.036904	1	1.01099	1	1	1	1.017277	1.004691
23	1.55034	1.038376	1.34005	1.11417	1.035937	1.335427	1.503392	1.155136	1.073416
24	1.44756	1.000003	1.37407	1.05348	1.047982	1.415367	1.563872	1.074888	1.036668

From the second column of Table 6, we note that the efficiency scores of period 23 are the highest and the efficiency scores of period 22 are the lowest from the pessimistic view. By comparing the results of Tables 6 and 7, we observe the difference in optimistic and pessimistic views in some cases, for example, by looking at the second column of Table 6, we find that the efficiency scores of period 24 are higher than period 23 (0.36081 < 0.4372) from the optimistic view. But, from the second column of Table 7, it can be noted that period 23 is higher than period 24 (1.44756 < 1.55034) from the pessimistic view. Therefore, for the final ranking of DMUs, Tables 6 and 7 have different results on their own. Finally, Table 8 shows the overall efficiency as well as the efficiencies of stages and the sub-DMUs based on the double-frontier, or the optimistic and pessimistic views that we have explained in the Section (2) by formula (10).

Table 8. Results based on the double-frontier view

DMU	$\emptyset_o^{overall*}$	$\emptyset_o^{1*}$	$\emptyset_o^{234*}$	$\emptyset_o^{56*}$	$\emptyset_o^{2*}$	$\emptyset_o^{3*}$	$\emptyset_o^{4*}$	$\emptyset_o^{5*}$	$\emptyset_o^{6*}$
1	0.509119	0.71752	0.720868	0.984306	0.715791	0.765773	0.691057	1.010666	0.957768
2	0.6075	0.700783	0.899219	0.964046	0.95834	0.97811	0.828919	0.991392	0.936756
3	0.714245	0.699723	0.999722	1.021039	0.858932	1.000739	1.068322	1.053198	0.988803
4	0.757378	0.704081	1.097254	0.980354	0.904111	1.07153	1.214336	0.998702	0.961517
5	0.613791	0.714001	0.856752	1.003383	0.857173	0.901374	0.828757	1.04037	0.966465
6	0.686689	0.70937	0.954696	1.013963	0.890459	0.945071	0.990685	1.04513	0.982675
7	0.765623	0.721858	1.072631	0.98881	1.102751	1.070689	1.083636	1.023946	0.953797
8	0.749912	0.730644	1.057949	0.970153	1.126323	1.060491	1.042991	0.988006	0.952019
9	0.7017	0.724755	0.987928	0.98002	0.862818	1.025107	1.007581	1.005485	0.954591
10	0.739783	0.720058	1.03353	0.994063	0.896341	1.084544	1.051622	1.027956	0.959454
11	0.735083	0.705164	1.130901	0.921768	1.007427	1.178301	1.157982	0.830553	0.998125
12	0.705182	0.639774	1.116623	0.987117	0.981341	1.12638	1.210412	1.088299	0.845798
13	0.689807	0.702348	1.030814	0.952785	1.068112	0.982998	1.053994	0.860156	1.015211
14	0.6075	0.700783	0.899219	0.964046	0.95834	0.97811	0.828919	0.991392	0.936756
15	0.686689	0.70937	0.954696	1.013963	0.849981	0.921389	0.960158	1.107801	1.012641
16	0.613791	0.714001	0.856752	1.003383	0.857173	0.901374	0.828757	1.04037	0.966465
17	0.714245	0.699723	0.999722	1.021039	0.858932	1.000739	1.068322	1.053198	0.988803
18	0.714948	0.704901	1.006758	1.007445	0.904866	0.995257	1.063356	1.062167	0.956395
19	0.73573	0.72224	1.014911	1.003711	1.004385	1.022893	1.023469	1.041133	0.966364
20	0.715074	0.705647	1.014188	0.999182	0.734977	1.096753	1.110731	1.037367	0.961869
21	0.715318	0.702665	0.986591	1.031843	0.899201	0.99742	1.047235	1.121555	0.940076
22	0.666119	0.725043	0.947486	0.96965	0.97202	1	0.877654	0.990699	0.948359
23	0.747916	0.718608	1.017001	1.023386	0.780959	1.050376	1.126611	1.063413	0.983422
24	0.795533	0.703897	1.135723	0.995124	0.897307	1.187048	1.244381	1.02581	0.966442

For ranking the DMUs, we use the second column of Table 8. Therefore, the performance of 24 DMUs is rated as follows:

$$DMU_{24} > DMU_7 > DMU_4 > DMU_8 > DMU_{23} > DMU_{10} > DMU_{19} > DMU_{11} > DMU_{21} >$$

$$DMU_{20} > DMU_{18} > DMU_3 = DMU_{17} > DMU_{12} > DMU_9 > DMU_{13} > DMU_6 = DMU_{15} >$$

$$DMU_{22} > DMU_5 = DMU_{16} > DMU_2 = DMU_{14} > DMU_1,$$

Where symbol " $>$ " means that the performance is better than and symbol "$=$ " means that the performance is equal. It should be noted that in some cases, for example, inDMU_3 and DMU_{17}, the rank of the DMUs are equal. This is because of the demand, the amount of production of each good, the amount of delivery and maintenance of each good and other items during periods 3 and 17 were absolutely equivalent and the factory has the same performance. Table 9 demonstrates the clustering of the DMUs by k-means method that we have explained in the Section (2). In accordance with the opinions of managers, we suggest clustering the DMUs into three groups with similar characteristics as follows:

Table 9. Clustering results based on the double-frontier view

DMU	K-means label	DMU	K-means label
1	3	13	2
2	3	14	3
3	2	15	2
4	1	16	3
5	3	17	2
6	2	18	2
7	1	19	1
8	1	20	2
9	2	21	2
10	1	22	2
11	1	23	1
12	2	24	1

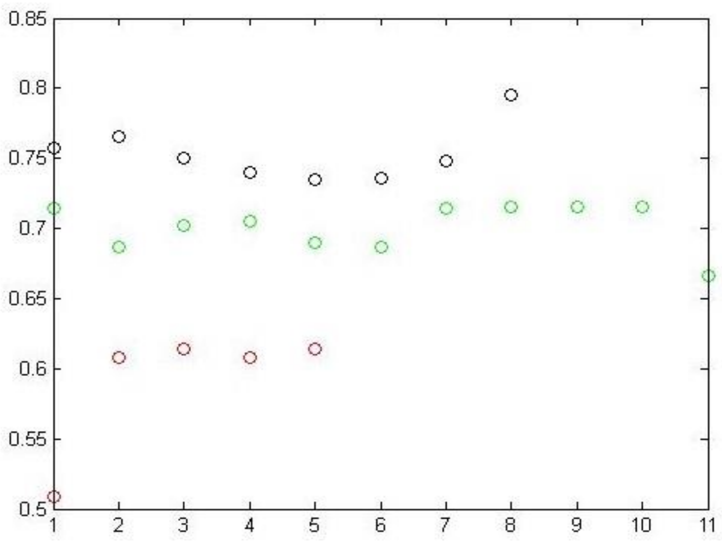

Figure 2. The three groups' classification of the DMUs

In Fig. 2, we identified three groups of DMUs based on the double-frontier view. Since there are no important differences in the inputs across the three groups, it implies that groups 2 and 3 have all abilities in place, but are poor in executing these capabilities and changing them into high level of performance. Thus, these groups must benchmark themselves against group 1 and identify ways to execute their abilities better. We have put this research at the managers' disposal, so that the best decisions can be adopted for the abovementioned factory.

5. Conclusions

Time and the resources involved in both the fundamental considerations, awareness of managers and owners of production and services, as well as the necessity of increasing the efficiency and effectiveness of activities related to the missions of organizations and the management suite have become extremely prominent and even more evident. The black box approach neglects the internal activities of systems and evaluates performance based on the final inputs and outputs. According to the belief of many researchers, this causes a lack of confidence in the evaluation results. In this research, we tried to pay attention to the intra-system activities using the proposed model, which was based on our knowledge. It has not been performed so far in the area of production planning and inventory control. A hybrid system with a complex internal structure is developed by a DEA approach. This system is comprised of three stages, six sub-DMUs, additional inputs and undesirable outputs. We utilize the cooperative approach multiplicative model to measure the efficiency of the overall system and the performances of DMUs from both optimistic and pessimistic views. The cooperative models from these views cannot be converted into linear models due to the additional inputs and outputs in the second stage. Therefore, a heuristic method is proposed to convert the nonlinear models into linear models.

We simulate a factory in a real world that has a production area and three warehouses for goods and two delivery points. The factory produces three goods and each good is placed in a warehouse. The factory is considered as a dynamic network and the efficiency of the network is measured from both optimistic and pessimistic views. We use a geometrical mean's approach for ranking the DMUs of the views. In the simulation, all costs are considered, including production costs, maintenance costs of the products, warehouse reservation costs, transportation costs from the production yard to the warehouses, transportation costs from the warehouses to the product delivery sites, product's delay penalty and the profit obtained from the sale of products.

The ranking results of the dynamic network under study indicated that the periods (24) and (1) were the best and the poorest periods, respectively, in terms of efficiency. The efficiency results of the other periods show a fluctuating situation. Based on the solar calendar in Iran, period 1 includes (the Iranian New Year) or Nowruz vacations and period 24 relates to the period prior to the holidays. According to the Iranian culture, demand and consumption achieve their maximum level before the vacations. Hence, experimental results confirm the results obtained from the model. In this paper, we suggest using a k-means technique to cluster the DMUs into three groups with similar characteristics based on the double-frontier view. Moreover, this paper presents the modeling method and solution for evaluating the efficiency of complex systems. The model allows us to open the structure of the "black box" by considering intermediate measures, and can help to obtain important information about efficient and inefficient points of the system. The results of this study show that the heuristic method can be considered as a beneficial technique to find the optimal efficiency in the complex internal structure. The heuristic approach proposed in this research can be used to solve a hybrid three-stage system. The model becomes complex for higher-stage systems, due to the presence of additional inputs and outputs, and increases the solution time significantly. To overcome the problem, we can change the movement step ($\Delta\varepsilon$). The findings of this study can assist managers to improve the performance of factories. We suggest developing models for imprecise data for tasks in the future.

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