

## An Integrated Production-Inventory Model for a Dual-Channel Supply Chain

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### Abstract

In this paper a dual-channel supply chain with one retailer and one manufacturer is considered. The manufacturer produces in a lot to supply an integer number of the retailer's order and ships his product to the retailer in equal-sized batches. The manufacturer has also an online channel to meet the demand directly. It is assumed that two members have integrated production-inventory system to minimize the total cost. A procedure is developed to find the optimal solution based on the convexity of the total cost. A numerical example is provided to clarify the characteristics of the model. The results show that the optimal number of the shipments from the manufacturer to the retailer is very sensitive to the holding cost of the manufacturer. When the manufacturer's holding cost is less than that of the retailer, most of the inventory is held in the manufacturer and is delivered to the retailer in low quantities with more shipments. It was also obtained that if the dual-channel supply chain reshapes to single one, the total cost of the supply chain would be minimized and the maximum value of total cost of the supply chain would occur when the demand is distributed equally between the two channels.

**Keywords:** Integrated production-inventory model; Joint Economic lot size; Dual-channel supply chain; E-commerce.

### 1. Introduction

Supply chain management (SCM) helps members of the chain to handle the flow of goods, reduce the costs and synchronize supply with demand. In traditional supply chain, each member orders from immediate upstream member and inbound orders have valuable information for downstream member. In this setting, this information may mislead the other members and impose additional costs to the chain. Modern competitive world enforces firms to integrate with each other for delivering high quality products with competitive prices to the end customers. Sahin and Robinson Jr (2005) showed that a fully integrated supply chain can reduce costs by 47.58%. Cárdenas-Barrón and Treviño-Garza (2014) developed a general mathematical model for an integrated three echelon supply chain network with multi-product and multi-period. Cárdenas-Barrón et al. (2014) proposed an alternative heuristic algorithm for a multi-product Economic Productin Quantity (EPQ) vendor-buyer integrated model with Just in Time (JIT) philosophy and a budget constraint. Khan et al. (2017) modelled the vendor-buyer supply chain exploring the effect of variable lead time learning in production and screening errors for defective items. Najafi et al. (2018) developed an economic productin quantity (EPQ) inventory model with scrap and rework for multiple products which are produced in a single machine. Taleizadeh et al. (2019) considered coordination of a supply chain comprising of one supplier and one retailer in which two complementary fashion products are manufactured and sold as a bundle and the demand rates are uncertain and dependent on selling price and a random noise effect on the market. Inventory control issue is one of the most important aspects of the supply chain management which can be handled in centralized or decentralized manner. Cárdenas-Barrón and Sana (2015) studied a multi-item EOQ inventory model in a two-layer supply chain where demand is sensitive to promotional effort. Mashud and Hasan (2019) proposed an inventory model with consideration of price, frequency of advertisement, continuous time and partially backlogged shortages for instantaneous decaying products.

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Joint economic lot sizing (JELS) policy is an effective centralized planning mechanism for integration of the members, which has received wide attention from both academics and practitioners. Goyal (1977) firstly introduced JELS for minimizing the total system cost. Banerjee (1986) developed Goyal's model based on lot-for-lot shipment policy with a finite production rate. Goyal (1988) extended his earlier work and assumed that the manufacturer produces for multiple retailer orders in a single lot and then delivers them to retailer in equal sized lot. Hill (1997) then developed more general policy which allows delivering shipments to the buyer during the production period. JELS models have been extended in different directions and more detailed review can be found in Glock (2012). The literature discussed a plethora of integrated production inventory models that may be implemented in supply chains including Ben-Daya et al. (2013), Sarkar (2013), Hoque (2013), Ouyang et al. (2015) and Aldurgam et al. (2017).

The inclusive growth of Internet and its great potential have persuaded manufacturers to use e-commerce as a sales channel. In real business world, many brick and mortar companies create e-commerce channels that deal with customers independently and indirectly (Yan and Wang (2009)). It was forecasted that the direct online sales in the United States and Canada will reach USD 500 billion by 2018 (Batarfi et al. (2017)). The widespread use of Internet has changed the shape of the supply chains and introduced the concept of the dual-channel supply chain. Moriarty and Moran (1990) pointed out that the dual or multiple channels would become the dominant design for computer industry in the 1990s. Many brand name manufacturers such as Hewlett-packard, IBM, Eastman Kodak, Nike, and Apple have dual-channel supply chains (Dumrongsiri et al. (2008)). The largest English-language publisher, Random House, has publicly said that it may sell books directly to the readers, putting them in direct competition with Barnes and Nobel and Amazon.com (Dumrongsiri et al. (2008)). Therefore in recent decade, the trends to use of the dual-channel supply chain have emerged in almost all of industries and many firms have to engage in an online channel for reaching the larger coverage of the market. Chiang et al. (2003) studied direct marketing and the dual channel in supply chain with lot-for-lot policy and without consideration of supply chain costs. Chiang and Monahan (2005) investigated a dual-channel supply chain with two-echelon including a manufacturer warehouse and a retail store, under stochastic demand from both channels. Takahashi et al. (2011) developed a contribution on production setup and delivery for a dual-channel supply chain, in which production and delivery are restarted according to an order point policy. Rasouli and Nakhai Kamalabadi (2014) studied a mathematical model for a joint pricing and inventory control problem by considering seasonal and substitutable goods in a competitive market over a finite time planning horizon. Panda et al. (2015) investigated pricing and replenishment decisions for a high-tech product in a dual-channel supply chain. Yazdi and Honarvar (2015) presented a model for designing integrated forward/reverse logistics based on pricing policy in direct and indirect sales channel. MohsenzadehLedari and ArshadiKhamseh (2018) considered a three-echelon dual-channel supply chain including two producers, distributor and retailer which both producers produce one kind of goods in different brands and quantities and exclusive and non-exclusive market. Fakhrzad et al. (2018) explored the influence channel synchronization on the supplier, the retailer, and the entire supply chain for dual-channel supply chains and presented a hybrid model and also surveyed the effects of free riding on sales effort.

To the best of our knowledge, most papers in literature have considered an integrated production-inventory model with a single retail channel and the papers in which the dual-channel supply chain was studied the costs of the supply chain and the integration of production and inventory were ignored. However, in today's modern competitive world and by changing the life style, it is necessary for manufacturers to add an online direct channel for selling their products. In this paper, we study an integrated dual-channel supply chain and develop a centralized structure for the production and shipment decisions in dual-channel supply chain and also explore the minimum relevant cost of the total supply chain based on Goyal (1988)'s model in which manufacturer produces in a single lot and then stops production and delivers orders in equal sized ships to the retailer. The demand of the customers in direct channel is responded during both production and shipment period. The convexity of the objective function would be proved by using algebraic method which has recently received an extraordinary attention from researchers for deriving optimal policy of inventory control in supply chain management. Seliaman et al. (2018) used an algebraic solution for the  $k$ -multiplier cycle time mechanism without the use of differential calculus.

Generally, the contributions of the paper are as follow:

- A two-level supply chain with a single manufacturer and a single buyer is considered in which the manufacturer has an online direct channel for selling his product as well as the retail channel.
- Production, shipment and inventory policies are jointly optimized for centralized dual-channel supply chain.

In the following, the paper is structured as follows. Section 2 includes the assumptions and definitions. Mathematical models are provided in section 3. Section 4 introduces a solution procedure for obtaining optimal values. A numerical example is elaborated in section 5 and the final section presents the concluding remarks.

## 2. Model description

Consider a dual-channel supply chain including one manufacturer and one retailer as illustrated in Figure 1.

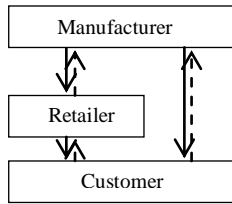


Figure 1. The dual-channel supply chain

The manufacturer produces a product and sells it to the customer through the both direct online channel and the retailer.

The following notations have been used to formulate the models:

- $S$ : Set-up cost for the manufacturer
- $c$ : Production cost per item for the manufacturer
- $c_d$ : Shipment cost per item for the manufacturer in direct channel
- $H$ : Holding cost per item, per time unit for the manufacturer
- $A$ : Order cost for retailer
- $h$ : Holding cost per item, per time unit for the retailer
- $Q$ : Order quantity for retailer
- $n$ : Number of shipments
- $P$ : Manufacturer's production rate
- $D$ : Total demand rate of the customer
- $\rho$ : The share of the demand goes to the direct channel
- $d_d$ : Demand rate in direct channel ( $= \rho D$ )
- $d_r$ : Demand rate in retail channel ( $=(1 - \rho)D$ )
- $T_m$ : Manufacturer's cycle ( $= \frac{nQ}{d_r}$ )
- $T_r$ : Retailer's cycle ( $= \frac{Q}{d_r}$ )
- $TPC_m$ : Total production cost of the manufacturer per time unit
- $THC_m$ : Total inventory holding cost of the manufacturer per time unit
- $TSC_m$ : Total setup cost of the manufacturer per time unit
- $C_m$ : Total cost of the manufacturer per time unit
- $C_r$ : Total cost of the retailer per time unit
- $C_S$ : Total cost of the supply chain per time unit

The assumptions of this paper are as follow:

- A single product is considered.
- Shipment lots are equal.
- Total demand is deterministic and constant over time.
- Shortage is not allowed.
- The production rate is larger than the total demand rate.

### 3. Mathematical model

In this section, we first illustrate the inventory behavior of the manufacturer and the retailer and then formulate the cost function. The manufacturer starts to produce a quantity equal to an integer ( $n$ ) multiplier of the retailer's order quantity ( $Q$ ), then he/she delivers them in equal-sized lots to the retailer. The inventory behavior for the manufacturer and the retailer is illustrated in Figure 2.

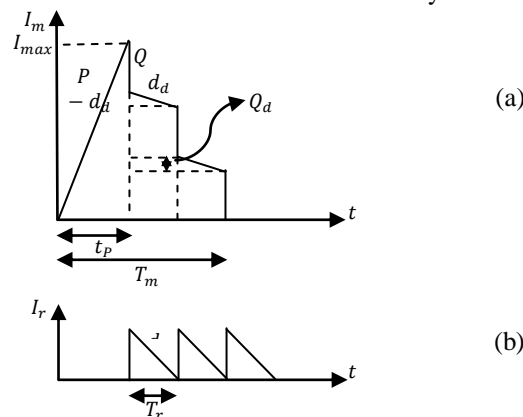


Figure 2. Inventory behavior for the manufacturer (a) and the retailer (b) ( $n = 3$ )

The different components of the total cost function are developed step-by-step in the following.

### 3.1. The total cost of the manufacturer

The total cost of the manufacturer has three components: the setup cost, the holding cost and the production cost.

#### 3.1.1. Setup cost per time unit at the manufacturer

The manufacturer incurs a setup cost  $S$  for each production cycle. So the average setup cost per time unit for the manufacturer is:

$$TSC_m = \frac{S}{T_m} = \frac{Sd_r}{nQ} \quad (1)$$

#### 3.1.2. Inventory holding cost per time unit at the manufacturer

The inventory held at the manufacturer per cycle can be gained as follows:

$$THC_m = H \times \frac{1}{T_m} \times \left( \frac{I_{max} \times t_p}{2} + (n-1) \times \frac{Q_d \times T_r}{2} + ((n-1) + \dots + 1) \times Q \times T_r + ((n-2) + \dots + 1) \times Q_d \times T_r \right) \quad (2)$$

In which,

$$T_m = nT_r = n \frac{Q}{d_r} \quad (3)$$

$$I_{max} = nQ + (n-1)Q_d \quad (4)$$

$$Q_d = d_d T_r = d_d \frac{Q}{d_r} \quad (5)$$

$$t_p = \frac{I_{max}}{P - d_d} = \frac{nQ + (n-1)Q_d}{P - d_d} \quad (6)$$

Therefore, by substituting  $T_m$ ,  $I_{max}$ ,  $Q_d$  and  $t_p$  in Equation (2), inventory holding cost per time unit at the manufacturer is:

$$THC_m = \frac{1}{2} \frac{Q}{nd_r} H(n(d_r + d_d) - d_d) \left( \frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1) \right) \quad (7)$$

#### 3.1.3. Production cost per time unit at the manufacturer

The production cost at the manufacturer per time unit is:

$$TPC_m = c(d_r + d_d) \quad (8)$$

Finally, the total cost of the manufacturer per time unit is:

$$C_m(n, Q) = c(d_r + d_d) + \frac{Sd_r}{nQ} + \frac{H}{2n} \frac{Q}{d_r} (n(d_r + d_d) - d_d) \left( \frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1) \right) + c_d d_d \quad (9)$$

### 3.2. The total cost of the retailer

The total cost of the retailer per time unit can be obtained in similar way as follows:

$$C_r(Q) = \frac{Ad_r}{Q} + \frac{hQ}{2} \quad (10)$$

### 3.3. The total cost of the supply chain

Based on the above discussions, the total cost of the supply chain per time unit is:

$$\begin{aligned} C_s(n, Q) &= C_m + C_r \\ &= c(d_r + d_d) + \frac{d_r}{Q} \left( A + \frac{S}{n} \right) + c_d d_d \\ &\quad + \frac{1}{2} Q \left( \frac{H}{nd_r} (n(d_r + d_d) - d_d) \left( \frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1) \right) + h \right) \end{aligned} \quad (11)$$

## 4. Solution method

For a certain value of  $n$ , by setting the derivative of Equation (11) with respect to  $Q$  equal to zero, the optimal value of  $Q$  is obtained as:

$$Q^*(n) = \sqrt{\frac{2d_r \left( A + \frac{S}{n} \right)}{\frac{H}{nd_r} (n(d_r + d_d) - d_d) \left( \frac{n(d_r + d_d) - d_d}{P - d_d} + (n-1) \right) + h}} \quad (12)$$

Then, by substituting Equation (12) in Equation (11), we have:

$$C_s(n) = c(d_r + d_d) + c_d d_d + \sqrt{2d_r \left(A + \frac{S}{n}\right) \left[\frac{H}{nd_r} (n(d_r + d_d) - d_d) \left(\frac{n(d_r + d_d) - d_d}{P - d_d} + (n - 1)\right) + h\right]} \quad (13)$$

**Proposition 1.** The total cost of the supply chain in Equation (13) is convex or increasing in  $n$ .

**Proof.** Minimizing the total cost of the supply chain is equivalent to minimizing:

$$z(n) = 2d_r \left(A + \frac{S}{n}\right) \left(\frac{H}{nd_r} (n(d_r + d_d) - d_d) \left(\frac{n(d_r + d_d) - d_d}{P - d_d} + (n - 1)\right) + h\right) \quad (14)$$

By taking the first and the second order derivatives of  $z(n)$  in  $n$  and simplifying them we would have:

$$\frac{\partial z(n)}{\partial n} = \frac{2 \left(d_r n (-hPS + AHd_r n^2 + HP(S + An^2)) + d_d (hd_r Sn + Hd_r n(S + An^2) + HP(n - 1)(2S + An(1 + n)))\right)}{(P - d_d)n^3} \quad (15)$$

$$\frac{\partial^2 z(n)}{\partial n^2} = -\frac{4(d_d hd_r Sn + (H - h)Pd_r Sn + d_d H(-APn + d_r Sn + PS(-3 + 2n)))}{(P - d_d)n^4} \quad (16)$$

From Equation (16), two cases can be considered:

- The numerator is negative, the second derivative would be positive and the convexity of  $z(n)$  with respect to  $n$  is proved.
- The numerator is positive, by simplifying the numerator of the second order derivative, we have:

$$d_d hd_r Sn + HPd_r Sn - hPd_r Sn - d_d HAPn + d_d Hd_r Sn - 3d_d HPS + 2d_d HPSn \geq 0$$

and by simplifying the numerator of the first order derivative, we have:

$$-hPSd_r n + AHd_r^2 n^3 + HPSd_r n + HPAd_r n^3 + hd_d d_r Sn + HSd_d d_r n + HAd_d d_r n^3 + 2SHPd_d n - 2SHPd_d + AHPd_d n^3 - AHPd_d n$$

which is equals to:

$$AHd_r^2 n^3 + HPAd_r n^3 + HAd_d d_r n^3 + AHPd_d n^3 + d_d HPS + (d_d hd_r Sn + HPd_r Sn - hPd_r Sn - d_d HAPn + d_d Hd_r Sn - 3d_d HPS + 2d_d HPSn)$$

Sentences in parentheses are positive based on assumption and the other phrases are also positive. As the first derivative is positive  $z(n)$  would be an increasing function of  $n$  and the optimal value of  $n$  is 1. □

The solution procedure for finding optimal values of  $Q$  and  $n$  is as follows:

- The economic value of  $n = n^*$  will be obtained when
 
$$z(n - 1) \geq z(n^*)$$

$$z(n + 1) \geq z(n^*)$$
- The optimal economic order quantity for the retailer, can be obtained by substituting  $n^*$  in Equation (12).

### 5. Numerical examples

In this section, a numerical example would be provided to explore the effect of different parameters on the total cost. Table 1 gives the values of the parameters.

**Table 1.** Input parameters

$D$ u/year	$P$ u/year	$S$ \$/order	$A$ \$/order	$H$ \$/u/year	$h$ \$/u/year	$c$ \$/u	$\rho$	$c_d$ \$/u
1,000	6,000	400	25	4	5	20	0.1	2

The optimal values of variables for this case are as follow:

**Table 2.** The optimal values

$n^*$	$Q^*$	$C^*(n)$
2	196	22,270.19

To analyse the effect of the parameter on optimal values, we consider different values for them and show results in Tables 3 and 4.

**Table 3.** Effects of parameters on optimal values

		$n^*$	$Q^*$	$C^*$
$\rho$	0.1	2	196	22,270.19
	0.3	1	330	22,407.56
	0.5	1	282	22,509.82
	0.7	1	221	22,554.44
	0.9	1	130	22,457.01
$c_d$	1	2	196	22,170.19
	1.5	2	196	22,220.19
	2	2	196	22,270.19
	2.5	2	196	22,320.19
	3	2	196	22,370.19
$A$	10	2	190	22,199.99
	25	2	196	22,270.19
	50	1	380	22,331.72
	100	1	401	22,447.03
	150	1	421	22,556.71
$h$	4	1	408	22,077.97
	5	2	196	22,270.19
	20	7	54	22,957.03
	50	12	32	23,557.67
	100	17	22	24,202.00
$H$	1	7	108	21,578.52
	2	4	132	21,905.80
	3	3	148	22,126.93
	4	2	196	22,270.19
	5	1	365	22,299.64

From Table 3, one can find that:

- As the direct channel demand increases the optimal number of shipments would be decreased. In fact, when direct channel share goes up, the retailer tends to the lot for lot policy for procurement.
- total supply chain cost would rise be increased by higher shipment cost in direct channel. The optimal number of shipments and order quantity for the retailer are not sensitive to this parameter.
- by increasing the ordering cost of the retailer, the total cost of the supply chain increases. Obviously, increasing the ordering cost lowers the number of shipments and increases the order quantity.
- by increasing the holding cost of the retailer, the total cost of the supply chain would rise . It can also be obtained that the optimal order quantity (number of shipments) is decreased (increased). The optimal number of shipments is very sensitive to the holding cost of the retailer.
- the optimal number of the shipments is very sensitive to the holding cost of the manufacturer. The optimal number of the shipments (the order quantity of the retailer) is reduced (increased) by increasing the holding cost of the manufacturer. When the manufacturer's holding cost is less than that of the retailer, most of the inventory is held in the manufacturer and is delivered to the retailer in low quantities with more shipments.

Table 4 shows that the minimum total supply chain cost occurred in 0.1 or 0.9 of  $\rho$ . In fact, the values of 0.1 and 0.9 for  $\rho$  are extreme states for this supply chain. When  $\rho$  goes to 0.1, the supply chain tends to use only one channel which is the retailer. Conversely, when  $\rho$  goes to 0.9, the supply chain tends towards the direct channel only. Therefore, one-channel supply chain is an optimum structure for this kind of supply chain. For example, when  $c_d$  is less than 1.7, the manager should move the supply chain structure towards the direct channel. Thus, if the policy is to move the supply chain towards the direct channel, the manger should plan to reduce  $c_d$  less than 1.7. Similar results can be obtained for the other parameters.

**Table 4.** The joint effects of  $\rho$  and effective parameters on total supply chain cost

		$\rho$				
		0.1	0.3	0.5	0.7	0.9
$c_d$	1.4	22,210.19	22,227.56	22,209.82	22,134.44	<b>21,917.01</b>
	1.6	22,230.19	22,287.56	22,309.82	22,274.44	<b>22,097.01</b>
	1.7	<b>22,246.84</b>	22,337.50	22,393.05	22,390.97	<b>22,246.84</b>
	1.8	<b>22,250.19</b>	22,347.56	22,409.82	22,414.44	22,277.01
	2	<b>22,270.19</b>	22,407.56	22,509.82	22,554.44	22,457.01
	2.2	<b>22,290.19</b>	22,467.56	22,609.82	22,694.44	22,637.01
	2.4	<b>22,310.19</b>	22,527.56	22,709.82	22,834.44	22,817.01
A	15	<b>22,223.66</b>	22,386.17	22,491.95	22,540.78	22,449.24
	25	<b>22,270.19</b>	22,407.56	22,509.82	22,554.44	22,457.01
	140	<b>22,535.18</b>	22,637.49	22,701.87	22,701.29	22,540.59
	143.7	<b>22,543.10</b>	22,644.40	22,707.64	22,705.70	<b>22,543.10</b>
	200	22,661.50	22,747.70	22,793.93	22,771.68	<b>22,580.65</b>
	400	23,042.30	23,079.95	23,071.45	22,983.88	<b>22,701.42</b>
H	1	<b>21,578.52</b>	21,898.72	22,191.83	22,419.43	22,453.20
	2	<b>21,905.80</b>	22,186.93	22,408.58	22,541.87	22,454.47
	3	<b>22,126.93</b>	22,350.64	22,496.97	22,548.17	22,455.74
	4	<b>22,270.19</b>	22,407.56	22,509.82	22,554.44	22,457.01
	5	<b>22,299.64</b>	22,427.66	22,522.56	22,560.68	22,458.28
h	4	<b>22,077.97</b>	22,234.71	22,361.82	22,438.14	22,388.78
	5	<b>22,270.19</b>	22,407.56	22,509.82	22,554.44	22,457.01
	77	<b>23,934.43</b>	24,094.76	24,210.03	24,236.65	23,939.34
	77.9	<b>23,946.53</b>	24,105.61	24,219.51	24,245.09	<b>23,946.53</b>
	100	24,202.00	24,334.82	24,418.75	24,411.28	<b>24,101.32</b>
	500	26,854.03	26,689.26	26,432.94	26,017.62	<b>25,194.29</b>

### 6. Concluding remarks

This paper extends the work of Goyal (1988) in which the manufacturer sells his product to the end customer directly through online channel. It is assumed that the supply chain is integrated and then production and ordering policy is investigated. Based on convexity of total supply chain cost, the global optimal values for variables are obtained by using a derivative method. Some sensitivity analyses are made on input parameters and their effects on optimal decisions have been investigated. In a numerical example, we showed that the total supply chain cost would decreased if the demand of the direct channel increased and the optimal number of shipment is very sensitive to the holding cost of the manufacturer and the retailer. It was also obtained that if the dual-channel supply chain reshapes to single one, the total cost of the supply chain would be minimized and the maximum value of total cost of the supply chain emerges when the demand is distributed equally between two channels.

There are some extensions that can be considered in future researches. The demand can be considered as a function of different parameters such as price, lead time, etc. A dual-channel supply chain with many manufacturers and many retailers can be investigated. There are many other production policies which were studied in the literature so it is interesting to assume other policies by considering dual-channel supply chain and compare them. This study can be expanded for substitutable products or complementary products.

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