

## Marine Inventory-Routing Problem for Liquefied Natural Gas under Travel Time Uncertainty

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### Abstract

In this paper we developed an uncertain LNG marine inventory-routing problem. For this purpose, we set a scenario for a hypothetical LNG manufacturer in Iran selling products in long-term and spot contracts. The purpose of the study was to compare shipping expenses of split and non-split delivery strategies in deterministic and uncertain situations. The objective function was to minimize total costs consisting operational costs, contract penalties, and spot fees regarding liquefaction port operational constraints, ship flows, customer and contractual constraints. Considering uncertainty in the problem is one of this paper's contributions which is modeled by assuming vessels speed a fuzzy parameter. As a solution method, we propose a metaheuristic that combines a heuristic with GA. According to the computational results, split delivery policy is only cost effective in the deterministic problem, hence split delivery is not recommended in maritime transportation with uncertain nature.

**Keywords:** Marine Inventory; Routing; Uncertainty; LNG.

### 1. Introduction

Inventory-routing problem (IRP), starting point of Bell and others (1983), is an integrated problem, covering two important sections of logistics value chain, "inventory management" and "vehicle routing." In the IRP, the most important issue is to optimize sending one or more types of products to several customers at different times, taking into account service level, the storage restrictions on the supply and the demand side. One of the strategic IRP application fields which are in maritime transportation sector is called marine inventory-routing problem (MIRP). In 2011, 80% of the world's exchanges took place through water transportation UNCTAD (2012) which shows its importance. Presenting a holistic view and modeling approaches, we have categorized MIRP components in Table 1.

Among marine cargos, liquefied natural gas (LNG) is one the most critical and strategic products. LNG is an odorless, colorless, non-destructive and non-toxic natural gas that is produced in a particular process by cooling the natural gas to the minus 161 ° C at atmospheric pressure; it is one of the main ways of exchanging gas in pipelines (Hashemi and Javan, 2005). Natural gas is one of the most important energy carriers, accounting for about 24 percent of the world's energy consumption portfolio. According to BP forecasts, this fuel will have the highest consumption growth rate among other fossil fuels by 2035, most of which will be due to increased gas reserves, including US shale gas and LNG. The exchange of gas as liquefied natural gas has had a growing trend in the last two decades, and by 2035 the volume of trade will surpass the pipelines. Increased demand and exchanges of liquefied natural gas have made the supply chain more complicated, and given the high costs of investment in the manufacturing and distribution sector in this industry, as well as price fluctuations it requires detailed planning and comprehensive management in this field.

In this research, a hypothetical LNG producer in Iran produces one kind of liquefied natural gas. The product is temporarily stored and then shipped to destination ports at a specified time window. In addition to long-term contracts, the producer sells parts of its products to the Spot Market. This paper develops a model for LNG inventory-routing problem in the context of tactical planning under travel time uncertainty and with the approach of reducing operational costs and contractual penalties.

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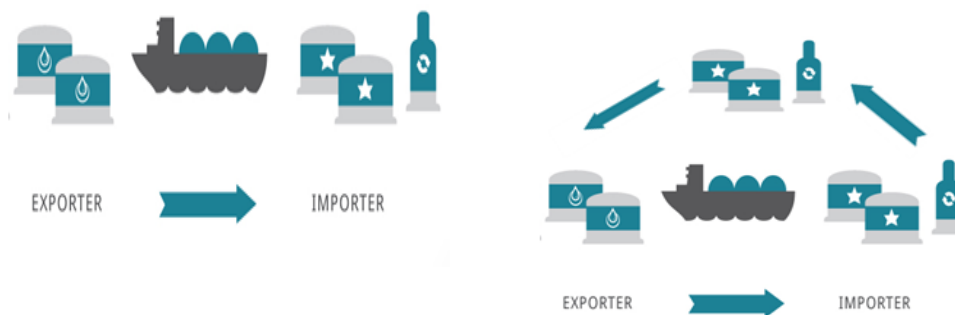
**Table1.** MIRP modelling components

Marine Supply chain	Modeling components
supply	Product type (L-bulk, crude oil, LNG,cement,Amonia,etc) Product Number (Single, Multiple) Production rate (Fix, Variable)
Storage and Loading	Inventory management (Lot sizing, not lot sizing) Inventory type (Deteriorating,non deteriorating)
Transportation & Maintenance	Shipping (Liner, tramp, industrial) Fleet composition (Hetrogenous, Homogenous) Transportation structure (1-1,1-m,m-m-m) Transportation type (Successive, not successive) Maintenance
Unloading & Storage	Storage in unloading port(storage considerations in only demand side/on both sides) Port type(offshore)
Consumption	Consumption rate(fix,variable) Contract type(long-term,spot,futures) Delivery(FOB.CIF.DES)



**Figure1.** Hypothetical LNG producer and destination ports

The purpose of the study is to compare the cost of one of the important policies in the field of maritime transportation, namely, split delivery (serving one destination on each trip from the origin) and non-split delivery (serving multiple destinations on each trip from the origin). (See Figure2).



**Figure2.** Split versus non-split delivery

The rest of this paper is organized as follows: first previous studies in this field are reviewed in section two and then in the third section, the problem and the main assumptions are described. In the fourth part the mathematical model is presented and in section 5 the algorithm is described. The results of the model implementation are presented in section 6 and finally in section 7 we present some concluding remarks.

## 2. Literature Review

The studies in the field of maritime routing can be categorized and reviewed from the perspective of solving methods. The basis for categorizing in this research is derived from the paper by SteadieSeifi et al. (2014). Accordingly, the solving methods are grouped into exact, approximation, heuristic, metaheuristic, and hybrid heuristic methods.

### 2.1. Exact methods

#### 2.1.1. Arc-flow models

Ethan et al. (2018) developed a large-scale, discrete time, path-based integer-programming for liquid helium global supply chain planning. Gustavo et al. (2016) improved the decision-making process in a Brazilian petroleum company significantly using a mixed-integer linear programming (MILP) model to represent crude oil offloading and supply problem. Nguyen et al. (2016) modeled and solved container shipping route design incorporating the costs of shipping, inland/feeder transport, and inventory and CO<sub>2</sub> emission. Agra et al. (2013a) defined the MIRPs in a discrete-time mode and used branching and constraint methods to solve them. Agra et al. (2012), looking at strategies to strengthen the proposed models, provided a new model that combines two discrete and continuous modes. Hewitt et al. (2013) used the guided search method of the branch and the price in order to provide a more effective approach to solve the real MIRP. Using the knapsack inequality, Rocha et al. (2013) remodeled the problem of crude-oil distribution. Rakke et al. (2014) who aimed at preparing a long-term LNG program took into account both spot and long-term contracts, as well as discussing penalties, to provide a model with minimal costs. Al-Khayyal and Hwang (2007) studied and modeled the marine routing problem with inventory constraints for multi-commodity shipments of liquid products. Halverson and Fagerholt (2013a) considered the real LNG maritime inventory-routing at the tactical level and in the form of a simple network structure. Fodstad et al. (2010) presented an optimization model as a decision-making tool for the LNG supply chain planning.

#### 2.2.2. Path-flow models

Archetti et al. (2020) proposed and solved the Inventory Routing Problem with Pickups and Deliveries (IRP-PD) to solve the instances by a branch-and-cut algorithm and tested it on a large set of instances and showed the proper performance of the method. Fokkema et al. (2020) addressed supply driven IRP in continuous-time, inventory levels through a novel formulation model. Soroush and Al-Yakoob (2018) studied a stochastic maritime scheduling transportation-inventory problem to transport crude oil from a source using a set of fully loaded heterogeneous vessels to be fully unloaded at a destination over a finite time horizon. Gronhaug et al. (2010) examined the issue LNG-IRP at tactical level and in cases where fixed deterioration rates were considered. Engineer et al. (2012) solved a Marine Inventory using a price-cutting algorithm. Anderson et al. (2015a) proposed a decomposition algorithm for the LNG inventory-routing problem with regard to the rate of LNG boil-off. Anderson et al. (2015b) developed an annual delivery plan (ADP) for one of the large LNG manufacturers with two types of products using exact algorithms. Li et al. (2010) aimed at maintaining the flow between production and consumption and modeled shipping chemicals. Considering the preparation time for operation in ports, as well as the structure of fixed and variable costs, Yongheng and Grossmann (2015) addressed the issue. Nikhalat-Jahromi et al. (2016), by combining financial discussions and considering the arbiter discussion on the LNG spot sales, developed a model for extracting LNG short-term trade policies.

#### 2.2.3. Arc-flow & Path-flow models

Christensen (1999) used decomposition method to solve the combined problem of inventory management and routing for ammonia carriers. To solve the LNG IRP, Giami et al. (2016) used both models.

### 2.2. Approximation methods

To minimize logistic costs and satisfy service level, Yadollahi et al. (2019) solved a stochastic periodic inventory routing problem. Papageorgiou et al. (2014b) presented a definite marine inventory-routing problem in the long-term horizons with approximate dynamic programming.

### 2.3. Heuristic method

#### 2.3.1. Arc-flow models

Arijit et al. (2017) modeled and solved the sustainable maritime inventory routing problem with time window employing PSO-CP. Chengliang, et al. (2017) solved maritime Inventory Routing Problems with Delivery Time Windows through finding flexible solutions that can accommodate unplanned disruptions. Dung-Ying and Chang (2018) modeled and solved ship routing and freight assignment problem for liner shipping. They applied the model to the Northern Sea Route planning problem. Ronen et al. (2002) formulated the problem of scheduling ships in the form of arc-flow model based on mixed integer programming. Persson and Gothe-Lundgren (2005) investigated the issue of ship scheduling in oil refineries (Ninas Oil Refinery) suggesting the solutions based on column generation. Goel et al. (2012) presented a new version of LNG IRP for designing and analyzing supply chains, optimizing vessel shipments and inventory management on both sides of supply and demand. Hemmati et al. (2015) considered vendor managed inventory (VMI) service in tramp shipping. Papageorgio, et al. (2014a) presented a two-stage decomposition algorithm for the deep-seated and tactical level in the form of flow-path models. Goel et al. (2015) used a constraint programming method to solve the LNG inventory-routing problem. Song and Ferman (2013) presented a flexible modeling structure that incorporates many operational aspects on the basis of Fremen et al. (2011). Agra et al. (2016) studied the problem of MIRP for liquid petroleum, taking into account the stochastic travel time. With the goal of designing an optimal delivery plan for LNG, Al-Haidous (2016) designed a model to reduce the number of ships.

### 2.3.2. Path-flow models

Rakke et al. (2011) used a heuristic method of rolling the horizon based on the flow path model to prepare a LNG annual delivery plan for a large LNG manufacturer.

### 2.3.3. Arc-flow & Path-flow models

Urgent et al. (2011) proposed a new optimization method for marine-based navigation-based on fix and relax time decomposition. Stalhane, et al. (2012), in their research on creating and developing a heuristic method for the inventory-routing problem, solved the problem for the manufacturer and distributor of different products. Mutlu et al. (2015) modeled the problem and solved it using a heuristic algorithm called "vehicle routing".

## 2.4. Metaheuristic method

### 2.4.1. Arc-flow models

Siswanto et al. (2013) addressed a genetic algorithm to solve the marine-inventory-routing problem for bulk liquid products. Christiansen, et al. (2011) examined the same issue in the field of cement and multi-product. With a view to presenting a more efficient MIRP Shao, et al (2015) presented an innovative approach in this area which is a combination of rolling the horizon, grasp and neighborhood search methods.

## 2.5. Hybrid method

### 2.5.1. Arc-flow models

Alkaabneh et al. (2020) considered green inventory routing problem for perishable. They applied warm- a meta-heuristic (GRASP) and mathematical programming formulations. Mirzapour Al-e-hashem et al. (2019) solved a green inventory routing problem in a bi-objective model using a hybrid approach. Bertazzi et al. (2019), using a novel cluster base three-phase metaheuristic, solved a practical Multi-Depot Inventory Routing Problem (MDIRP). Giami et al. (2019) studied an inventory routing problem for inland distribution of LNG from a storage facility to several filling stations. They modeled the problem in an arc-flow model and solved it through a metaheuristic, a combination of a mixed integer programming formulation, and an adaptive large neighborhood algorithm. Hemmati et al. (2016) studied an iterative two-phase hybrid metaheuristic for a multi-product short sea inventory-routing problem.

### 2.5.2. Path-flow models

Shen et al. (2011) solved the problem of routing-crude oil inventory from a supplier to several customers through the Lagrangian relaxation approach.

Table 2 summarizes all of the studies mentioned above. Studies are categorized as 'discrete' and 'continuous' models. One of the factors influencing the structure of modeling is the nature of production and consumption rate. Generally, continuous models are used when the rate of production and consumption is constant and unchanged (Al-Khayyal and Hwang, 2007), and when these rates are variable or considered fixed but changing over the planning period, generally discrete-time models are used (Gronhaug et al., 2010; Ronen et al., 2002; Agra, Christiansen, 2013). Discrete-time models consider the time intervals discrete, and assume that events (such as loading, unloading, etc.) occur at specific points in time. In contrast, in the continuous-time models, there is no limit to the occurrence of events in terms of time.

Based on the modelling approaches, studies in the field are either "path-flow" or "arc-flow". Arc-flow models include decision variables for moving ships between ports and path-flow formulations include decision variables representing the entire sequence of ports visited by each vessel (Papageorgio et al., 2014).

The main contribution of this study in comparison with other previous studies is that we examine uncertain marine inventory problem as a less considered area in the field. To the best of our knowledge, comparison of split and non-split delivery strategies in uncertain conditions has never been studied before and it is a main contribution of this study. We consider a mixed-integer linear programming model for this problem and solve it for an LNG trading scenario for Iran through the proposed metaheuristic.

**Table2.** Literature review summery

Author(s)	Product	level	*Time	*Model	Solving Method
				A	BC-local search heuristic
Agra, et al, 2013a	L-BULK	T	D	A	BC
Agra, et al, 2013b	FUEL OIL	O	D/C	A	BSSDP
Al-Khayyal and Hwang, 2007	LBULK	T	C	A	C-HUERISTIC
Al-Haidous,et al, 2016	LNG	T	D		ADP
Alkaabneh et al,2020	perishable products	T	D		GRASP
Andersson, et al, 2015a	LNG	T	C	P	BC
Andersson, et al, 2015b	LNG	T	D	P	BC
Anderson, et al, 2010	LNG	T	D	P	MODEL ONLY
Arijit, et al. 2017		T	D	A	PSO-CP
Archetti et al,2020	GENERAL	T	D	P	branch-and-cut algorithm
Bertazzi, et al. 2019	GENERAL	O	D	A	cluster-based matheuristic
Chengliang, et al. 2017		T		A	Heuristic method
Christiansen, 1999	AMMONIA	T	C	A/P	BPC
Christiansen& Fagerholt, 2009	GENERAL	T	C	A	construction-heuristic
Christiansen, et al. 2011	CEMENT	T			GENETIC
Drazen-Papvic,et al, 2011	GENERAL	O	D	P	HEURISTIC
Dung-Ying & Chang ,2018	GENERAL	O		A	Heuristic method
Engineer,et al, 2012	VGO	T	D	P	BPC
Ethan et al. (2018)	liquid helium	O	D	P	Integer programmin
Fokkema et al. 2020	biogas	O	C	P	Mixed-integer
Fodstad,et al, 2010	LNG	T	D		DEFALT SOLVER
Furman ,et al, 2011	VGO	T	D	A	DEFALT SOLVER
Giami, et al, 2019	LNG	O	D	A/P	Meta-Heuristic
Giami, et al, 2016	LNG	O	D	A/P	DEFALT SOLVER
Goel ,et al, 2015	LNG	S	D	A	MIP BASED LS
Goel ,et al, 2012	LNG	T	D	A	C&I- HEURISTIC
Gronhaug,et al, 2010	LNG	T	D	P	BPC
Halvorsen ,et al, 2013	LNG	T	D	A	DECOMPOSITION
Hemmati,et al, 2016	GENERAL	T	C	A	HYBRID-HUERISTIC
Hemmati,et al, 2015	GENERAL	T	C	A	2 PHASE HEURISTIC
Hewitt, et al, 2013	VGO	T	D	A	BPGS
Li ,et al, 2010	L-BULK	O	C	A	DEFALT SOLVER
Mirzapour, et al. 2019	GENERAL	O	D	P	Hybrid Algorithm
Mutlu, et al, 2015	LNG	T	D	P	VRH_HEURISTIC
Nikhalat-Jahromi, et al, 2016	LNG	O	C	P	MILP
Papageorgio,et al, 2014a	L-BULK	S	D	A	APPROXIMATE
Papageorgio, et al, 2014b	L-BULK	T	D	A	BENDERS LIKE
Person and Gothe, 2005	Oil Refinery	T	D	A/P	CG-HEURISTIC
Rakke, et al, 2014	LNG	T	D	A	BPC
Rakke, et al, 2011	LNG	T	D	P	ROLLING HORIZEN
Rocha, et al, 2013	CRUDE	T	D	A	BB
Ronen ,et al, 2002	L-BULK	T	D	A	GRASP
Shao, Furman, et al, 2015	LNG	S	D	A	LAGRANGIAN
Shen,et al, 2011	CRUDE	S	D	P	GENETC
Siswanto,et al, 2011	L-BULK			A	CIH
Stalhane, et al, 2012	LNG	T	D	P	MILP BASED LS
Song and Furman, 2013	VGO	T	D	A	FIX-RELAX
Uggen,Fodstad ,et al, 2011	LNG	T	D	P	FIX-RELAX
Yadollahia,et al.2019	GENERAL	O	D	P	Approximation
Yonneng ,et al, 2015	GENERAL	T	D	P	MODEL ONLY

### 3. Problem description

The overall problem in the form of the supply chain of liquid natural gas from the exploration to consumption is as shown in Figure3.

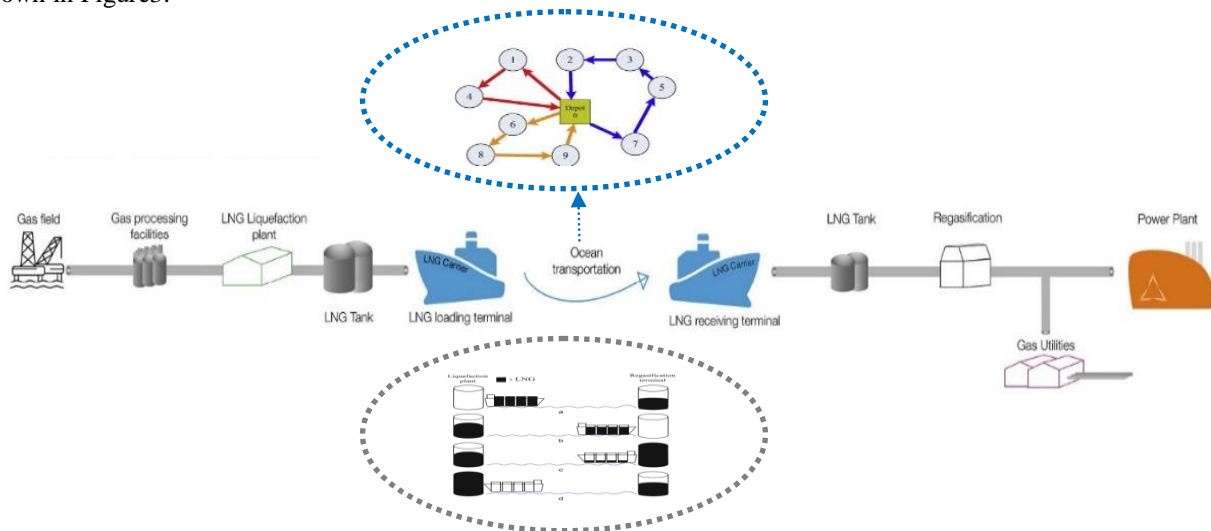


Figure3. LNG inventory routing problem

- Liquefaction: In this case, a producer is considered producing one kind of liquefied natural gas in a liquidation terminal. The product is temporarily stored and then loaded into the special vessels. Due to the technical specifications of the liquidation terminal, the production level is possible within a certain range and the daily production rate is considered as one of the parameters in the model.
- Storage and berth restrictions: Liquefied natural gas is stored in special tanks prior to being transferred to ships and that storage should be carried out at low temperatures, so the level of the LNG cannot be less than a certain amount. Therefore, minimum and maximum inventory levels should be included in the model. On the other hand, the number of ships that can be loaded in the port is also limited according to berth capacity.
- Transportation: One of the most important parts of the LNG scheduling model is shipping to meet long-term contracts demands. The producer has a fleet of specially transported liquefied natural gas vessels. On the other hand, each customer has a set of long-term contracts with main characteristics of the volume, acceptable interval of delivery and acceptable delivery period, which will be penalized outside the specified intervals. In this section we want to determine the route and time of ships according to the problem limits. Generally, in maritime routing successive travelling is assumed to be appropriate. That is, ships should only go from one source to one destination and not allowed to go to several destinations. In this research, this hypothesis which is examined in the uncertainty conditions is discussed later.
- Maintenance: ships have been scheduled for maintenance and repair program. The list of available ships is defined according to it.
- Unloading and regasification: For each customer, a port is considered and according to the port type, ships are allowed to enter.
- Spot sale: Depending on market conditions and probable imbalance between demand and supply, liquid natural-gas terminals sell part of the produced liquefied natural gas in the spot market.

The main objective of this problem is to determine logistics and inventory management policies to minimize costs. The optimization model is developed considering constraints of transportation, inventory management and contract obligations.

One of the important aspects of sea trips is climate change. In this study, the speed of ships is considered as a fuzzy parameter and the producer's goal is to minimize shipping costs which includes travel and operational costs, cost of penalties, and the cost of shipping spots.

### 4. Mathematical model

In this study, the model proposed by Mutlu et al. (2015) is considered and is developed considering the fuzzy uncertainty for the ships' speed parameter.

#### Problem parameters

$T$  Planning horizon (Time in this model is discrete and day-to-day)

$r_t$	Daily Production Decisions in $[R^{\min}, R^{\max}]$ minimum and maximum production capacity
$s_t$	LNG inventory levels per day port in $[s^{\min}, s^{\max}]$ minimum and maximum storage levels
$\mathcal{V}$	LNG specialized fleet
$l$	Loading port
$B$	Berth number
$l_d$	Artificial Port
$G_v$	Isolated tanks
$C_v$	Tank's capacity
$g_{vt}$	Number of full tanks at the end of day t
$i$	Customers' actual port
$i_d$	Customers' artificial port
$C_i$	$\{C_{i1}, C_{i2}, \dots, C_{ini}\}$ Customers' contracts
$D_{in}$	Customers' demand in $[D_{in}^{\min}, D_{in}^{\max}]$ range in expected time $T_{in}^e$ and allowed time $T_{in}^a$ .
$o$	Artificial origin port
$d$	Artificial destination port
$\mathcal{V}^i$	A set of ships that are allowed to enter the port i
$P^v$	A set of ports that vessel $\mathcal{V}$ is allowed to enter
$P^{vi}$	A set of ports that vessel $\mathcal{V}$ can sail from port i
$P_0^{vi}$	A set of ports from where vessel $\mathcal{V}$ can reach to port i
$T_{vij}$	Travel time from i to j
$T_0$	$T \cup \{0\}$
$T_{ex}$	$T_0 \cup \{T+1, \dots, T+t_0\}$ , $t_0$ is max vessels' travel time
$\mathcal{V}^m$	Vessels that should go to maintenance
$T_v^m$	Maintenance time window
$m, m_d$	Actual and artificial maintenance ports
$C_{vij}$	Operational cost from i to j
$P_{in}^- / P_{in}^+$	Unit rate penalty for each cubic meter (over/under)
$d_{in}^- / d_{in}^+$	(over/under) volume deviation
$P_{int}$	Sending out of expected time penalty
$C_s$	Spot vessel capacity
$S$	Spot vessel operational cost

**Problem variables**

$x_{vijt}$	Binary variable which is 1 if vessel $\mathcal{V}$ arrives at port j from port i on day t and 0 else.
$y_t$	Binary variable which is 1 if spot vessel $\mathcal{V}$ is available on day t.
$q_{vit}$	Number of tanks loaded/unloaded on day t for vessel $\mathcal{V}$

Objective function and constraints:

$$\min \sum_{v \in \mathcal{V}} \sum_{i \in P^v} \sum_{j \in P^{vi}} \sum_{t \in T_{ex}} C_{vij} x_{vijt} + \sum_{i \in P^c} \sum_{n | C_{in} \in C_i} P_{in}^- d_{in}^- + P_{in}^+ d_{in}^+ + \tag{1}$$

$$\sum_{i \in P^c} \sum_{v \in \mathcal{V}} \sum_{n | C_{in} \in C_i} \sum_{t \in T_{in}^a} P_{int} x_{vi_d it} + St \sum_{t \in T} y_t$$

Subject to

$$x_{\omega o 0} = 1, \forall \omega \in \mathcal{U} \tag{2}$$

$$\sum_{t \in T_0} \sum_{j \in p^{\omega} \setminus \{o\}} x_{\omega j t} = 1, \forall \omega \in \mathcal{U} \tag{3}$$

$$x_{\omega j t} = 0, \forall \omega \in \mathcal{U}, j \in p^{\omega} \setminus \{0\}, t < T_{\omega j} \tag{4}$$

$$x_{\omega j t} = 0, \forall \omega \in \mathcal{U}, j \in p^{\omega} \setminus \{0\}, t < T_{\omega j}, j \in p^{vi}, t < T_{\omega j} + T_{vij} \tag{5}$$

$$x_{\omega d t} = 1, \forall \omega \in \mathcal{U}, t \in T_{\omega x} \setminus T_0 \tag{6}$$

$$\sum_{i \in p_o^{vi} \setminus \{d\}} \sum_{t \in T_{\omega x}} x_{vidt} = 1, \forall \omega \in \mathcal{U} \tag{7}$$

$$\sum_{k \in p_o^{vi}} x_{vkit} = \sum_{j \in p^{vi}} x_{vij}(t + T_{vij}), \forall \omega \in \mathcal{U}, i \in p^v, t \in T_0 \tag{8}$$

$$\sum_{i \in p^v} \sum_{j \in p^{vj}} x_{vijt} \leq 1, \forall \omega \in \mathcal{U}, t \in T_{\omega x} \tag{9}$$

$$g_{\omega t} = g_{\omega t-1} - \sum_{i \in p^{ic}} q_{vit} + q_{\omega t}, \forall \omega \in \mathcal{U}, t \in T \tag{10}$$

$$0 \leq g_{\omega t} \leq G_v, \forall \omega \in \mathcal{U}, t \in T \tag{11}$$

$$q_{vit} \leq G_v \sum_{j \in p_o^{vi}} x_{vjit}, \forall \omega \in \mathcal{U}, i \in p^{vc} \cup \{l\}, t \in T \tag{12}$$

$$s_t = s_{t-1} + r_t - \sum_{v \in \mathcal{V}} q_{\omega t} C_v - C_s y_t, t \in T \tag{13}$$

$$s^{\min} \leq s_t \leq s^{\max}, t \in T \tag{14}$$

$$\sum_{v \in \mathcal{V}} x_{\omega v_d t} + \sum_{v \in \mathcal{V}^m} (x_{\omega v_d t-1} + x_{\omega v_d t}) + y_t \leq B, t \in T \tag{15}$$

$$r_{30k+1} = r_{30k+\tau}, k = \{0, \dots, 11\}, \tau = \{2, \dots, 30\} \tag{16}$$

$$R^{\min} \leq r_t \leq R^{\max}, t \in \{30k+1, k = 0, 1, 2, \dots\} \tag{17}$$

$$\underline{D}_{in} \leq \sum_{v \in \mathcal{V}^j} \sum_{t \in T_{in}^a} q_{vit} C_v \leq \bar{D}_{in}, i \in p^c, n | C_{in} \in C_i \tag{18}$$

$$\sum_{v \in \mathcal{V}^j} \sum_{t \in T_{in}^a} q_{vit} C_v + d_{in}^- - d_{in}^+ = D_{in}, i \in p^c, n | C_{in} \in C_i \tag{19}$$

$$d_{in}^+, d_{in}^- \geq 0, i \in p^c, n | C_{in} \in C_i \tag{20}$$

$$\sum_{t=T_v^m}^{T_v^m - T_{vmm_d}} x_{\omega v_d mt} = 1, \forall \omega \in \mathcal{U}^m \tag{21}$$

$$\sum_{t \in T_v^m} x_{\omega v_d mt} = 0, \forall \omega \in \mathcal{U}^m \tag{22}$$

The first part in the objective function is the total operational cost of sending LNG ships. The second and third parts represent total contractual penalties due to the violations in the volume and time. The last part refers to the operating costs of sending and selling through spot vessels. Constraints (2) to (5) are initialization constraints and ensure that all vessels start from the origin and do not go to the same destination at the same time interval. Constraints (6) and (7) guarantee that all vessels will eventually reach the destination port. Constraint (8) controls the flow of vessels during the planning period. In this constraint, if a vessel arrives at port  $i$  at time  $t$ , it should go from that port to port  $j$  which can be reached from port  $i$ . Constraint (9) ensures that a vessel will not be present at two different locations at a same time. Along with the initialization constraints, flow constraints ensure a connected route for vessels' movement.



Constraint (10) is related to storage balance of vessels. Constraint (11) ensures storage capacity per vessel. Constraint (12) indicates that loading or unloading from the vessel to a port is possible when the vessel is present at the specified port at a certain time.

Constraints (13) and (14) respectively indicate the inventory balance and loading port restrictions, and Constraint (15) is related to the limitation of berths capacity in the port. Constraint (16) ensures that the rate of production can only be changed in time horizon. Constraint (18) ensures that the volume is within the scope of the contract. Constraint (19) shows the violation of the sent volume with the amount agreed in the contract, and Constraint (20) indicates their being non-negative. Ultimately, constraint (21) represents the timely start of repairs and maintenance, and constraint (22) prevents access to maintenance beyond the scheduled time. As discussed in the previous sections, the aim of this study is to examine the total cost in split and non-split delivery modes.

In this research, trapped membership function is considered for speed. When the vessel's speed is fuzzy due to climate changes, the travel time cannot be considered as a definite parameter. Travel time for the vessel from port  $i$  to  $j$  is calculated as follows:

$D_{ij}$  is the distance between port  $i$  and  $j$  and  $V_v$  is the speed of the vessel  $v$ .

$$T_{vij} = \frac{D_{ij}}{V_v} \tag{23}$$

Accordingly, constraint (4) is replaced with constraint (24), constraint (8) with constraint (25) and constraint (8) with constraint (26) in the fuzzy model.

$$x_{vojt} = 0, v \in \mathcal{V}, j \in p^{vo} \setminus \{0\}, t < \tilde{T}_{voj} \tag{24}$$

$$x_{vojt} = 0, v \in \mathcal{V}, j \in p^{vo} \setminus \{0\}, t < \tilde{T}_{voj}, j \in p^{vi}, t < T_{voj} + \tilde{T}_{vij} \tag{25}$$

$$\sum_{k \in p_o^{vi}} x_{okit} = \sum_{j \in p^{vi}} x_{vij}(t + \tilde{T}_{vij}), v \in \mathcal{V}, i \in p^v, t \in T_0 \tag{26}$$

Jemines approach is used to defuzzify the related parameters and constraints. Accordingly, constraints (24) to (26) are changed as follows:

$$x_{vojt} = 0, v \in \mathcal{V}, j \in p^{vo} \setminus \{0\}, t < (1-\alpha) \frac{T_{voj3} + T_{voj4}}{2} + \alpha \frac{T_{voj1} + T_{voj1}}{2} \tag{27}$$

$$x_{vojt} = 0, v \in \mathcal{V}, j \in p^{vo} \setminus \{0\}, t < \tilde{T}_{voj}, j \in p^{vi},$$

$$t < (1-\alpha) \frac{T_{voj3} + T_{voj4}}{2} + \alpha \frac{T_{voj1} + T_{voj1}}{2} + (1-\alpha) \frac{T_{vij3} + T_{vij4}}{2} + \alpha \frac{T_{vij1} + T_{vij1}}{2} \tag{28}$$

$$\sum_{k \in p_o^{vi}} x_{okit} \leq \sum_{j \in p^{vi}} x_{vij} (t + (1-\frac{\alpha}{2})(\frac{T_{vij1} + T_{vij2}}{2}) + (\frac{\alpha}{2})(\frac{T_{vij3} + T_{vij4}}{2})), v \in \mathcal{V}, i \in p^v, t \in T_0 \tag{29}$$

$$\sum_{k \in p_o^{vi}} x_{okit} \geq \sum_{j \in p^{vi}} x_{vij} (t + (1-\frac{\alpha}{2})(\frac{T_{vij3} + T_{vij4}}{2}) + (\frac{\alpha}{2})(\frac{T_{vij1} + T_{vij2}}{2})), v \in \mathcal{V}, i \in p^v, t \in T_0$$

### 5. Solution method

The basis of the metaheuristic algorithm used in this research is the Vehicle Routing Heuristic Algorithm (VRH), presented by Mutlu et al. (2015). Considering the delivery time window and volume required, the metaheuristic algorithm generates the annual delivery plans in a sequential manner. In order to simplify the general process, the algorithm is divided into 7 sub algorithms (See Table 4).

- "Contract ranking" algorithm
- "Available Vessels" algorithm
- "Picking Vessels & the optimal number of trips" algorithm

- "Preparing to send" algorithm
- "Sending" algorithm
- "Review undelivered contracts" algorithm.
- "Spot sale" algorithm

*Algorithms sequences*

1. Set all vessels at the origin port.
2. Initialize the inventory level of the ships and port of loading.
3. Ranking contracts (call "Contract ranking" algorithm)
4. For all contracts, ranked respectively
5. Call " Available Vessels" algorithm
6. Call "Picking Vessels & the optimal number of trips "algorithm
7. Call "Preparing to send" algorithm
8. Call "Sending "algorithm
9. Define total cost
10. Are all contracts covered?
11. If "Yes "go to step (13)
12. If "No" go to step (5)
13. Bring all ships to the destination.
14. Call "Review undelivered contracts" algorithm
15. Call" Spot sale" algorithm

**6. Computational study**

The schematic diagram of the algorithms used and the general process of work are described in Figure 4. After determining the list of available vessels, the optimal number of each trip must be determined satisfying goal of minimizing operational and penalty costs through covering contractual obligations. The input variable of the genetic algorithm is the number of trips for the selected vessels, taking into account the limit of the maximum number of possible trips per vessel. Based on this, a randomized initial solution for the selected vessels is firstly created by considering the ceiling for each trip, which is the initial population for the genetic algorithm. Though multiple iterations, the optimum number of trips for the list of available vessels will be determined and next steps will be run according to the sequence described above.

**6.1. Scenario setting**

The model presented in this study is run for a hypothetical LNG plant in Iran; we suppose the producer sells its LNG via long term and spot contracts to its customers. Vessel characteristics, distances between the ports and other inputs of the model are derived from "Argos Global LNG" and neighboring countries. We conduct the computational study to assess split and non split delivery costs under both deterministic and uncertain conditions. To this end, we conduct a three step process; first, we evaluate performance of the proposed algorithm comparing it with a commercial solver, and; second, we run a sensitivity analysis on the quality of results and ultimately we test the split and non-split policy under deterministic and uncertain situations.

All instances are implemented using MATLAB version 2016b and CPLEX version 12.6.1. on an Intel® Core™ 17-6500u CPU @ 2.50 GHZ processors and 8 GB of RAM.

**6.2. Definite model**

First, we set the metaheuristic algorithm parameters (genetic algorithm) through Taguchi method to gain qualified results in shorter calculation time; the results are provided in Table 5.

**Table3.** Parameter setting results

Parameters	Definition	Result
Npop	Population size	100
Pc	Cross-over percentage	0.9
Pm	Mutation percentage	0.1
Mu	Mutation rate	0.02

Table 6 presents critical parameters of 12 instances. We implement all these instances in 3 different levels (36 instances); according to contract volumes, level 1 is dedicated to 400,000 mcm LNG per year for each contact, second level is 600,000 mcm, and third level is 900,000 mcm.

**Table4.** Problem instances (V, P, C, Pr, St denote the number of vessels, customer ports, number of contracts, production rate and storage in loading port respectively)

instance	V	P	C	C/Pr	S/pr
A1	15	2	40	0.8	6
A2	15	2	40	0.74	6
A3	15	2	40	0.67	6
A4	15	2	40	0.59	6
B1	30	4	80	0.8	6
B2	30	4	80	0.74	6
B3	30	4	80	0.67	6
B4	30	4	80	0.59	6
C1	40	8	120	0.8	6
C2	40	8	120	0.74	6
C3	40	8	120	0.67	6
C4	40	8	120	0.59	6

Feasible results in the exact method are achieved after average 22 hours of run time and in metaheuristic it takes 600 seconds on average. Based on the lower bound calculated in the exact method, gaps and improvement are defined and results of the implementation of the meta-heuristic and exact algorithm are illustrated in Table 7. The optimal gap changes versus C/Pr rates in 3 contract levels and on average are shown in Figure 5 – Figure 8.

**Table5.** Results of meta-heuristic and exact algorithm

Contract level	instance	Lower bound	Exact method		Metaheuristic		Metaheuristic Improvement
			Cost	Gap (%)	Cost	Gap (%)	
Level 1	A1	389,070,691.49	590,395,586.48	34.10	476,802,318.00	18.40	0.19
	A2	372,601,481.70	541,571,921.08	31.20	477,082,563.00	21.90	0.12
	A3	389,517,267.41	596,504,237.99	34.70	482,075,826.00	19.20	0.19
	A4	370,915,062.65	543,067,441.66	31.70	477,368,163.00	22.30	0.12
	B1	3,224,930,486.82	9,320,608,343.41	65.40	4,305,648,180.00	25.10	0.54
	B2	3,393,607,597.30	8,197,119,800.23	58.60	4,328,581,119.00	21.60	0.47
	B3	3,449,998,049.15	9,078,942,234.60	62.00	4,323,305,826.00	20.20	0.52
	B4	3,371,276,402.33	8,344,743,570.11	59.60	4,305,589,275.00	21.70	0.48
	C1	9,604,679,578.38	41,399,480,941.29	76.80	13,121,146,965.00	26.80	0.68
	C2	10,409,256,813.41	23,183,200,029.88	55.10	13,109,895,231.00	20.60	0.43
C3	10,405,836,629.50	26,819,166,570.87	61.20	13,138,682,613.00	20.80	0.51	
C4	10,302,872,745.25	36,150,430,685.09	71.50	13,175,029,086.00	21.80	0.64	
Level 2	A1	639,861,113.29	963,646,254.95	33.60	876,522,073.00	27.00	0.09
	A2	683,904,415.74	1,034,651,158.46	33.90	876,800,533.00	22.00	0.15
	A3	707,678,494.88	1,064,178,187.79	33.50	871,525,240.00	18.80	0.18
	A4	632,570,949.31	949,806,230.20	33.40	866,535,547.00	27.00	0.09
	B1	5,490,309,162.52	16,291,718,583.15	66.30	7,541,633,465.00	27.20	0.54
	B2	5,630,652,712.64	17,540,974,182.67	67.90	7,517,560,364.00	25.10	0.57
	B3	5,457,934,741.43	20,289,720,228.36	73.10	7,486,878,932.00	27.10	0.63
	B4	6,015,702,758.11	15,999,209,463.07	62.40	7,500,876,257.00	19.80	0.53
	C1	16,214,733,157.77	82,308,290,140.96	80.30	22,709,710,305.00	28.60	0.72
	C2	17,451,026,131.97	86,391,218,475.09	79.80	22,722,690,276.00	23.20	0.74
C3	16,451,227,759.82	103,466,841,256.76	84.10	22,722,690,276.00	27.60	0.78	
C4	16,244,091,064.91	96,118,882,040.88	83.10	22,750,827,822.00	28.60	0.76	
Level 3	A1	958,666,736.29	1,536,324,897.90	37.60	1,276,520,288.00	24.90	0.17
	A2	1,013,898,892.64	1,596,691,169.51	36.50	1,281,793,796.00	20.90	0.20
	A3	987,976,498.43	1,553,422,167.33	36.40	1,271,527,025.00	22.30	0.18
	A4	972,914,528.26	1,496,791,581.93	35.00	1,266,815,792.00	23.20	0.15
	B1	7,758,622,764.34	28,524,348,398.31	72.80	10,716,329,785.00	27.60	10.62
	B2	7,591,028,333.86	69,642,461,778.50	89.10	10,721,791,432.00	29.20	1
	B3	7,700,409,863.28	29,279,124,955.44	73.70	10,695,013,699.00	28.00	0.63
	B4	7,785,903,910.00	23,035,218,668.65	66.20	10,709,633,989.00	27.30	0.54
	C1	23,070,744,622.93	324,940,065,111.63	92.90	32,311,967,259.00	28.60	0.90
	C2	22,036,761,670.64	247,604,063,715.03	91.10	32,311,967,259.00	31.80	0.87
C3	22,151,764,472.76	177,214,115,782.08	87.50	32,338,342,296.00	31.50	0.82	
C4	22,720,948,150.94	324,584,973,584.79	93.00	32,319,983,145.00	29.70	0.90	

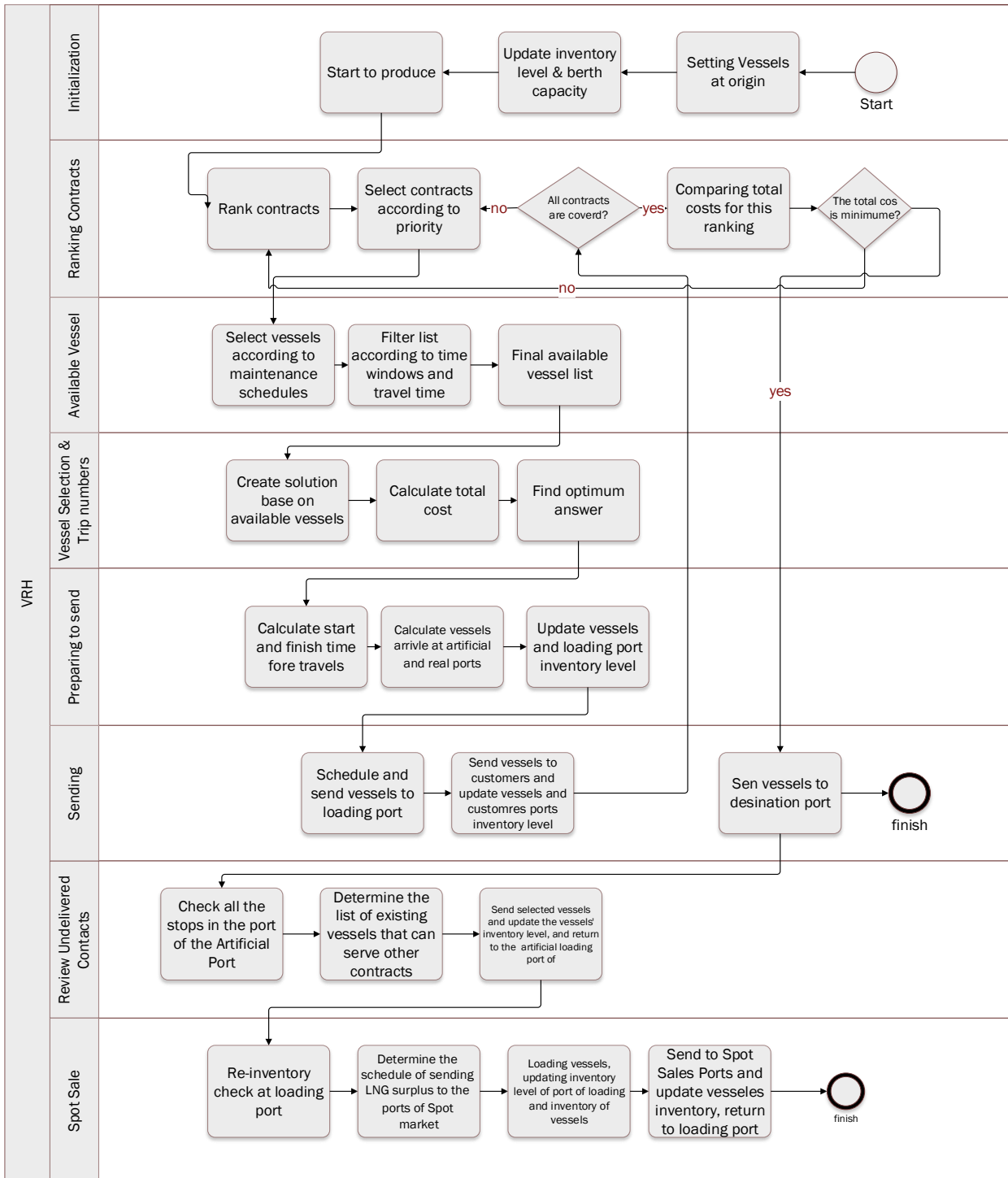


Figure 4. Metaheuristic process map

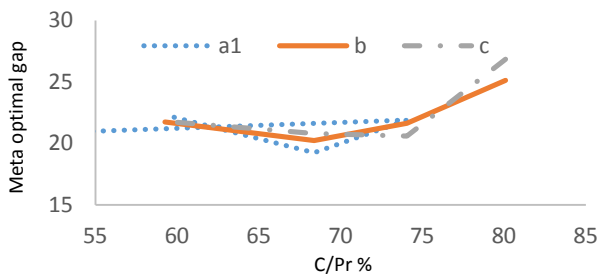


Figure 5. Optimaly gap vs. C/Pr % for metaheuristic in level 1

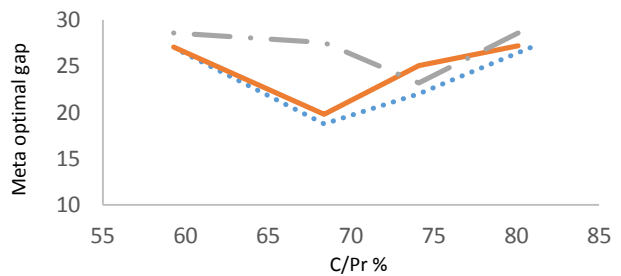


Figure 6. Optimaly gap vs. C/Pr % for metaheuristic in level 2

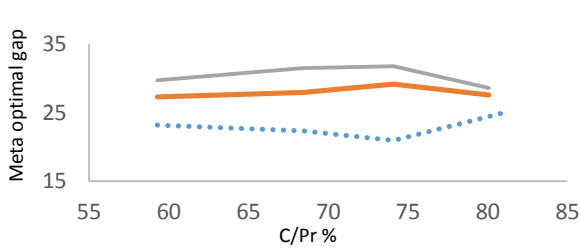


Figure 7. Optimality gap vs. C/Pr % for metaheuristic in level 3

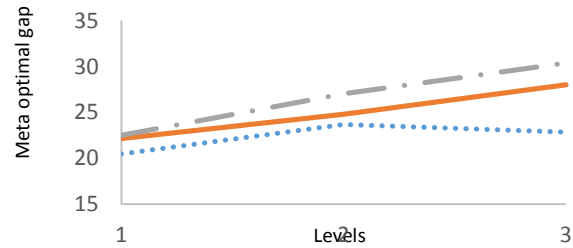


Figure 8. Average Optimal gap vs. C/Pr % for 3 levels

6.3. Fuzzy model

In order to investigate and compare the costs of shipping in uncertain conditions, in split and non-split deliveries, a fuzzy model is generated; details are provided in the previous section. The membership function for speed is assumed to be as follows:

$$f(V : 480, 520, 600, 700) = \left\{ \begin{array}{l} 0, V \leq 15 \\ \frac{V - V_1}{V_2 - V_1}, 15 \leq V \leq 20 \\ 20 \leq V \leq 25 \\ \frac{V_4 - V}{V_4 - V_3}, 25 \leq V \leq 30 \\ 0, 30 \leq V \end{array} \right\} \tag{31}$$

The de-fuzzified model was run for different feasibility degrees ( $\alpha$ ). In order to select the appropriate feasibility degrees, the satisfaction function for the manufacturer is defined as the weighted average of the two factors of total cost (total travel cost and penalty cost) and the ratio of the operational cost to the total cost with the following coefficients and calculations:

$$\Omega = \sum_i w_i \lambda_i = 0.9\lambda_1 + 0.1\lambda_2 \tag{32}$$

Since the lower total and the smaller proportion of operational cost to the total cost will increase the percentage of producer's satisfaction,  $\lambda_1$  and  $\lambda_2$  are defined as follows:

$$\lambda_1 = \frac{(total\ cost - \alpha\ operational\ cost)}{Max\ total\ cost} \tag{33}$$

$$\lambda_2 = \frac{\alpha\ operational\ cost}{\alpha\ total\ cost} \tag{34}$$

Table 6. Results of meta-heuristic and exact algorithm

instance	$\alpha$	Total cost	Operational cost	Satisfaction degree
1	0.1	478,415,347.00	403,655,347.00	21.08
2	0.2	532,308,958.00	397,208,958.00	30.30
3	0.3	457,244,960.00	409,084,960.00	17.09
4	0.4	419,150,309.00	332,350,309.00	22.60
5	0.5	446,703,401.00	388,883,401.00	18.48
6	0.6	527,974,304.00	392,874,304.00	30.28
7	0.7	458,279,546.00	410,119,546.00	17.09
8	0.8	477,257,890.00	392,837,890.00	22.50
9	0.9	476,290,546.00	411,190,546.00	19.64
10	1	476,774,218.00	402,014,218.00	21.08

According to the results presented in Table 8, the feasibility degrees of 0.4 with the highest satisfaction percentage was selected.

6.4. Sensivity analysis

Implementation results of both deterministic and fuzzy models on 36 instances are provided in Table 9. According to the results, Split delivery policy in the deterministic instances saved about 4.3 % while employing this policy under uncertainty imposed an average of 35% raise of total costs.

Comparing the total cost's rate of change in both situations shows that increasing the complexity of the problem in uncertain situations has more negative effects and in deterministic conditions it changes a lot. (See Fig.9)

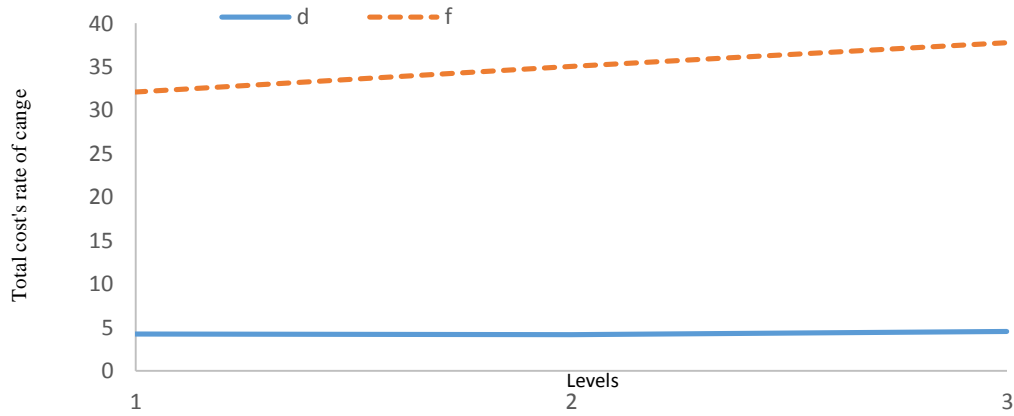


Figure9. Average absolute total cost's rate of change (d: deterministic, f: fuzzy)

Table7. Results of meta-heuristic and exact algorithm

level		VRHM			Fuzzy VRHM		
		Split	Non-Split	Cost Reduction	Split	Non-Split	Cost Reduction
Level 1	A1	465,835,864.69	476,802,318.00	2.30	613,889,883.78	483,131,338.53	-27.06
	A2	436,816,794.68	477,082,563.00	8.44	639,449,031.06	491,736,304.89	-30.04
	A3	472,723,554.98	482,075,826.00	1.94	786,083,081.46	482,655,012.01	-62.87
	A4	466,627,379.33	477,368,163.00	2.25	582,049,398.00	487,757,395.52	-19.33
	B1	4,232,452,160.94	4,305,648,180.00	1.70	7,206,466,365.43	4,374,325,083.82	-64.74
	B2	4,230,322,327.60	4,328,581,119.00	2.27	5,711,948,878.85	4,363,928,943.44	-30.89
	B3	4,135,242,022.57	4,323,305,826.00	4.35	4,836,550,938.49	4,362,568,946.52	-10.86
	B4	4,171,685,448.55	4,305,589,275.00	3.11	4,823,294,719.77	4,398,844,784.43	-9.65
	C1	11,910,065,100.13	13,121,146,965.00	9.23	20,632,914,303.21	13,184,432,239.75	-56.49
	C2	12,546,169,736.07	13,109,895,231.00	4.30	17,713,441,557.34	13,515,355,908.25	-31.06
	C3	12,896,930,852.92	13,138,682,613.00	1.84	16,350,243,153.99	13,456,250,115.73	-21.51
	C4	11,984,006,456.63	13,175,029,086.00	9.04	16,465,682,576.33	13,650,050,855.78	-20.63
Level 2	A1	790,622,909.85	876,522,073.00	0.10	1,004,561,583.80	891,046,124.83	-12.74
	A2	838,308,989.60	876,800,533.00	0.04	1,360,038,550.43	905,785,674.59	-50.15
	A3	861,764,157.31	871,525,240.00	0.01	995,449,664.66	873,009,355.91	-14.03
	A4	844,092,276.33	866,535,547.00	0.03	1,051,486,183.50	869,579,073.76	-20.92
	B1	7,233,180,656.28	7,541,633,465.00	0.04	12,584,994,501.52	7,664,261,651.42	-64.20
	B2	7,070,265,522.34	7,517,560,364.00	0.06	11,666,150,981.67	7,722,991,949.87	-51.06
	B3	7,289,974,016.09	7,486,878,932.00	0.03	8,639,259,372.60	7,740,776,397.85	-11.61
	B4	7,048,573,418.70	7,500,876,257.00	0.06	10,970,029,660.96	7,657,080,703.35	-43.27
	C1	21,092,778,931.28	22,709,710,305.00	0.07	29,380,416,305.50	23,680,615,542.23	-24.07
	C2	22,218,246,551.87	22,722,690,276.00	0.02	43,489,401,241.75	23,571,255,473.03	-84.50
	C3	22,454,562,530.74	22,722,690,276.00	0.01	27,261,318,077.44	23,880,914,635.84	-14.16
	C4	22,075,128,235.69	22,750,827,822.00	0.03	30,432,991,295.01	23,372,537,314.57	-30.21
Level 3	A1	1,235,799,290.81	1,276,520,288.00	0.03	1,998,215,938.51	1,322,818,951.30	-51.06
	A2	1,227,317,559.67	1,281,793,796.00	0.04	1,741,383,108.86	1,288,623,500.55	-35.14
	A3	1,206,933,452.13	1,271,527,025.00	0.05	2,088,636,849.70	1,311,663,941.61	-59.24
	A4	1,255,921,176.19	1,266,815,792.00	0.01	1,935,721,924.51	1,318,226,630.59	-46.84
	B1	10,434,490,311.65	10,716,329,785.00	0.03	21,014,273,784.66	10,948,436,641.81	-91.94
	B2	9,861,903,759.15	10,721,791,432.00	0.08	12,254,083,724.84	10,771,339,594.13	-13.77
	B3	10,662,928,657.90	10,695,013,699.00	0.00	12,590,329,108.07	10,840,273,362.05	-16.14
	B4	9,714,708,991.42	10,709,633,989.00	0.09	12,623,085,464.51	10,792,738,072.16	-16.96
	C1	29,949,962,452.37	32,311,967,259.00	0.07	49,358,556,946.62	32,774,081,812.56	-50.60
	C2	30,731,912,060.03	32,311,967,259.00	0.05	50,678,220,669.12	33,042,199,876.27	-53.37
	C3	30,465,952,277.06	32,338,342,296.00	0.06	34,384,290,077.93	33,215,224,215.28	-3.52
	C4	31,550,767,546.15	32,319,983,145.00	0.02	37,932,863,426.83	33,077,456,908.20	-14.68

7. Conclusion

We focus on the LNG inventory-routing Problem in this paper. The issue is the planning of shipping fleet and the level of inventory in ports for LNG manufacturers with a cost minimization approach. Mathematical modeling of the problem is presented in the form of two deterministic and fuzzy models. The uncertainty parameter in this study is the speed of vessels that varies due to climate changes during travel.

For solving the model, the fuzzy parameter and related fuzzy constraints were de-fuzzified with the Jemines method and the model was solved with both an exact method and the proposed meta-heuristic algorithm in the paper.

In order to investigate employing split and nonsplit delivery policy effect on total cost under both deterministic and uncertain conditions, 12 problems in 3 levels were designed based on the initial scenario and the model was implemented in all instances. Comparing the total costs shows that split policy in uncertain situations is not cost effective. One of the important factors in the price increase in the fuzzy conditions is the issue of contractual penalties. In other words, in conditions close to reality, the manufacturer acts more inappropriately to comply with the contractual obligations, which is multiplicative in split delivery due to increased travel time.

Despite conducting significant studies on the maritime inventory-routing problem, there are still many ways to develop it in the future. The sustainable development in the green maritime routing problem, considering hazards in marine travelling, combining geo-location discussions with the help of software such as Arc-GIS, considering pricing policies with inventory-routing using game theory with optimization issues in multi producer problems, use of neural network models to predict demand and price levels in the market are some suggested developing areas in marine inventory-routing problems.

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