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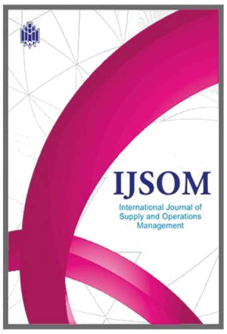

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Pricing, Service and Discount Policies for Substitutable Products in a Supply Chain with the Game Theoretical Approach

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Abstract
This study examines the impacts of three factors including service, price, and discount on the supply chain's profit. We consider a supply chain, including one traditional retailer and two manufacturers. By using the game-theoretic approach, we derived optimal solutions and analyzed competition between members under two scenarios: (1) the retailer buys the products from two manufacturers without price discount contracts, i.e. no products are sold with price discount contract. (2) The retailer buys the product from one of the manufacturers with price discount contract. We find that the price discount rate and the service level are very effective on the demand and profit of supply chain members and determining the appropriate discount rate is very important. The results show that increasing the service level provided by the retailer does not necessarily increase the profit of the manufacturer and he should set an appropriate discount rate to increase his profit. Our work contributes to three aspects: (1) joint and simultaneous examination of competition in the supply chain under the three factors of price discount, price, and service level; (2) examination of competition in the supply chain where the retailer and a manufacturer provide free service to consumers; and (3) analysis and comparison of the numerical example results in the two above scenarios according to the sensitivity analysis of various parameters.

Keywords: Supply chain; Discount; Service level; Competition; Price; Game theory.

1. Introduction

Competition under one of the discount types in the supply chain has been reviewed in many papers, but we have not found any studies that simultaneously examine the competitive factors of price, service, and price discount when the manufacturers are the market leader. Thus, we decided to study this matter in the supply chain. We are looking to answer the following questions: (1) what is the simultaneous effect of price, price discount and service on demand and profit? (2) Is the simple price discount contract a good mechanism to improve coordination and the performance of the supply chain? Many researchers such as Zhao et al. (2013), Lu et al. (2011) and Goffin (1999) studied providing service for the customer by manufacturers in the supply chains. By using a game-theoretic model, McGahan and Ghemawat (1994) found that the service can be used to retain old customers. Zhang et al. (2015) showed that the member's bargaining power influences the retail service level. Chen et al. (2016) analyzed the effects of member's bargaining power at the service level. Littler and Melanthiou (2006) investigated the impacts of retail service on consumer behavior. Yan et al. (2007) showed that the existence of an online channel will increase the level of retail service in the supply chain. Chen et al. (2008) investigated the impacts of the retail service level on demand and consumers' selection channel. In a system distribution with an electronic channel and a common channel, Xiao et al. (2010) analyzed the pricing decisions under service cooperation. By using the service cost-sharing, Luo et al. (2011) coordinated the suppliers and the retailers. Chen et al. (2010) showed that the performance of a supply chain with two-channel and different services is better than a supply chain with one channel and retail service. Li and Bo Li (2016) investigated a game theory model where the retailer has fairness concerns and retail service. Bucklin and Lattin (1991), and Rohm and Swaminathan (2004) showed that service value is effective for consumer's purchasing treatment. Kumar et al. (2017), Murali et al. (2016) and Sarkar et-

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al. (2016) stated that non-price factors such as service affect business performance and demand for products. Yuen and Chan (2010) showed that retail services are effective in customer loyalty. Ba et al. (2008), Bernstein and Federgruen (2007) and Koulamas (2006) considered the service to coordinate the manufacturers and the retailers. Kumar and Ruan (2006) showed that service value is influenced by the degree of brand and channel loyalty. Jena and Sarmah (2016) considered two systems, including an indirect system and a direct system and examined pricing policies and service co-opetition decisions. Borger and Dender (2006) investigated a distribution system with two enterprises which make decisions on service and price. Darian et al. (2005) showed that the service can be used for attracting the consumer. In a distributed system with two members and uncertainty demand, Xiao and Yang (2008) studied the competition under service. When two retailers compete under price and service, Iyer (1998) investigated how manufacturer coordinates the channel distribution. Mukhopadhyay et al. (2008) examined a game theory model where the retailer provides service for customers. Jeuland and Shugan (1983) presented a model with a non-price variable such as quality and services.

Tao et al. (2019) considered a perishable product supply chain consisting of one supplier and one retailer. They formulated a Stackelberg game and analyzed the joint advertising and preservation service decisions of the members. In a supply chain, including a manufacturer and an independent retailer, Zhang et al. (2019) explored after-sale service and information sharing decisions. They showed that if the manufacturer provides service for customers, the retailer prefers information sharing on the condition of higher cost efficiency for the manufacturer's service. Zhao et al. (2019) analyzed the decision models of the pricing, service, recycling and the profit of closed- loop supply chains members in different remanufacturing modes. Guan et al. (2019) considered two supply chains in each of which manufacturer provides free after-sales service for consumers and one retailer who has private information about uncertain demand. They studied the impacts of information sharing on price and service decisions. Many researchers explored the quantity discounts, but a few of their works are similar to our study. In a dual-fairness supply chain, Nie and Du (2017) used the quantity discount contract. Li et al. (2016) used the profit sharing method and price discount scheme. Zissis et al. (2015) analyzed a Stackelberg game which the manufacturer applies the quantity discounts. In a decentralized supply chain consisting of a supplier and a buyer, by using quantity discounts and game theoretic approach, Venegas and Ventura (2017) examined the coordination between the members. Our model includes the price discount contract, such as Cai et al. (2009) and Bernstein and Federgruen (2005). Cai et al. (2009) showed that the simple price discount contract can improve supply chain performance. Bernstein and Federgruen (2009) considered the price discount scheme as a coordination tool in the supply chain. In Table 1, the comparison of our study to other relevant researches shows the similarities and differences of the proposed model with other relevant works. However, in none of the studies mentioned in table 1, pricing, service and price discount decisions were considered, simultaneously. Also, in this supply chain, the retailer and a manufacturer simultaneously provide free service to consumers.

Table 1. Summary table of the existing literature

Service provider		**Competition factors**			**Reference**
Manufacturer	**Retailer**	**Price**	**Service**	**Discount**	
	*		*		Littler and Melanthiou (2006)
	*	*	*		Zhang et al. (2015)
*		*	*		Zhao et al. (2013)
*		*	*		Lu et al. (2011)
	*	*	*		Chen et al. (2016)
*			*		Goffin (1999)
	*		*		McGahan and Ghemawat (1994)
	*	*	*		Yan et al. (2007)
	*		*		Chen et al. (2008)
	*	*	*		Xiao et al. (2010)
	*		*		Luo et al. (2011)
	*	*	*		Chen et al. (2010)
	*		*		Li and Bo Li (2016)
	*		*		Rohm and Swaminathan (2004)
	*		*		Kumar et al. (2017)
	*		*		Murali et al. (2016)
	*		*		Sarkar et al. (2016)
	*		*		Yuen and Chan (2010)
	*	*	*		Ba et al. (2008)
	*	*	*		Bernstein and Federgruen (2007),

Table 1. Continued

Service provider		Competition factors			Reference
Manufacturer	**Retailer**	**Price**	**Service**	**Discount**	
	*	*	*		Koulamas (2006)
	*	*	*		Borger and Dender (2006)
	*	*	*		Darian et al. (2005)
*		*	*		Kumar and Ruan (2006)
*		*	*		Jena and Sarmah (2016)
	*	*	*		Xiao and Yang (2008)
	*	*	*		Mukhopadhyay et al. (2008)
	*	*	*		Iyer (1998)
*	*		*		Tao et al. (2019)
*			*		Zhang et al. (2019)
	*	*	*		Zhao et al. (2019)
*			*		Guan et al. (2019)
				*	Nie and Du (2017)
		*		*	Li et al. (2016)
				*	Zissis et al. (2015)
				*	Venegas and Ventura (2017)
		*		*	Bernstein and Federgruen (2005)
		*		*	Cai et al. (2009)
*	*	*	*	*	*Our paper

The remainder of this paper is organized as follows. The next section describes the main model and assumptions. In Section 3, the numerical examples and analysis are presented. The conclusion is given in the last section. The detailed proofs of the propositions presented are included in Appendix A.

2. Main model and assumptions

The supply chain consists of two manufacturers and one independent retailer. The manufacturers' substitutable products are sold to the retailer, which are then sold to the consumers. Our model is a single period and perfect information, and there is not any cooperation between members. The demands for the products of two manufacturers depend on the retail prices and the service levels and they are considered to be deterministic. The bargaining power of two manufacturers is equal and their power is more than the retailer. So, two manufacturers are the leader. Manufacturer 2 provides service for the second product's customers and the customers of the first product benefit from the retailer's services. In this study, similar to the researches in the literature review, we assume that the services include all activities that the manufacturer or the retailer performs to attract customers and demand enhancing. By using the game-theoretic approach, we investigate the two Manufacturer Stackelberg games under the following two scenarios:

1-Without price discount contracts: the retailer buys the products from two manufacturers without price discount contracts.

2-With price discount contracts: the retailer buys the product from the first manufacturer with price discount contracts. (Figure 1).

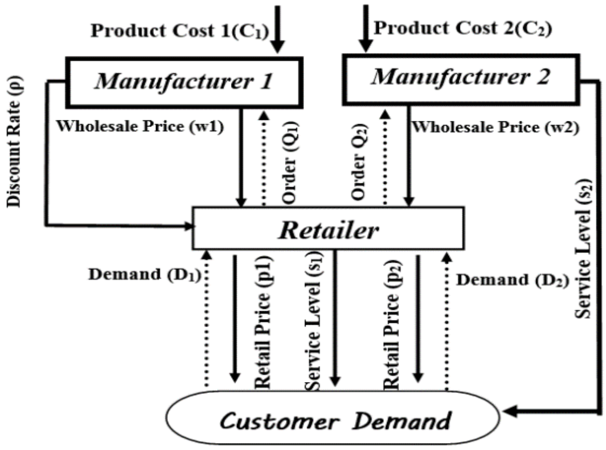

Figure 1. Supply chain structure in the second scenario

2.1. Without discount contracts

The demand Q_i is a linear function of service levels and prices. That is similar to the function used in Xiao et al. (2010). The following notations in **Table 2** are used for the formulation of mathematical models.

Table 2. Notations and Symbols

Symbol	Description
D_i	Demand of product i $i\epsilon\{1,2\}$
Q_i	Quantity ordered by the retailer from manufacturer *i*
a_i	The market base of product *i*
b_p	Price elasticity on market demand
b_s	Service elasticity on market demand
θ_p	Intensity of price competition
θ_s	Intensity of service competition
η_1	Service cost factor of the retailer
η_2	Service cost factor of manufacturer 2
c_i	Manufacturer i's product cost
w_i	Wholesale price of product *i*
p_i	Retailer price of product *i*
s_1	Service level provided by the retailer
s_2	Service level provided by manufacturer 2
π_{Mi}	Profit function of manufacturer *i*
π_R	Profit function of retailer

In this study, the demand function for product *i* is:

$$Q_i(p_1, p_2, s_1, s_2) = a_i - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i) \quad (1)$$

where $a_i > 0$, $b_p > 0$, $b_s > 0$, $\theta_p > 0$, $\theta_s > 0$, and $i, j\epsilon\{1,2\}$ $i \neq j$.

The costs of the second manufacturer include the production and the service, and manufacturer 1 carries the production cost. The function of service cost is similar to Lu et al. (2011). Thus, we obtain the profit functions for two manufacturers as follows:

$$\pi_{M1} = (w_1 - c_1)Q_1 \quad (2)$$

$$\pi_{M2} = (w_2 - c_2)Q_2 - \frac{\eta_2 s_2^2}{2} \quad (3)$$

The costs of the retailer include the retail service and wholesale prices. Then, the retailer's profit is:

$$\pi_R = (p_1 - w_1)Q_1 - \frac{\eta_1 s_1^2}{2} + (p_2 - w_2)Q_2 \quad (4)$$

2.1.1. Manufacturer Stackelberg

According to the retailer's response function, two manufacturers make the decision. So, by assuming retailers' observation and notification of the manufacturers' decisions, the retailer's response function is achieved.

2.1.1.1. Retailer's best response

In this stage, the retailer with the observation of manufacturers' decisions chooses the retail service s_1^* and retail prices p_1^* , p_2^* to maximize his profit.

Proposition 1. In this Manufacturer Stackelberg game, giving the wholesale prices $w_1,\ w_2$ and s_2 , the retailer's best reaction functions are obtained as:

$$p_1^* = Uw_1 + Yw_2 + Vs_2 - I \tag{5}$$
$$p_2^* = Zw_1 + Xw_2 + Ts_2 - J \tag{6}$$
$$s_1^* = \alpha w_1 + \beta w_2 + \gamma s_2 - \lambda \tag{7}$$

where $U,\ Y,\ V,\ I,\ Z,\ X,\ T,\ J,\ \alpha,\ \beta,\ \gamma\ and\ \lambda$ are described in Appendix A. Appendix A shows the proof of Proposition 1.

2.1.1.2. Manufacturers decisions

By applying the retailer's best response functions, we can obtain the two manufacturer's equilibrium wholesale prices and the second manufacture's equilibrium service level. The best response function of manufacturer 1 for wholesale price w_1 , and the best response functions of the second manufacturer for wholesale price w_2 ,and service level s_2 are obtained by maximizing the profit of two manufacturers, given $p_i^*$and s_1^* in Eqs. (5), (6) and (7), respectively. In this stage, the two manufacturers move simultaneously. Therefore, there is a Nash equilibrium between them.

Proposition 2. In the Manufacturer Stackelberg game, the two manufacturer's optimal wholesale prices and service level of the manufacturer 2, denoted as $w_1^*,\ w_2^*$ and s_2^* are given as follows:

$$w_1^* = \frac{\hat{\beta}\hat{\varphi} - \hat{\lambda}\hat{\gamma}}{\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha}} \tag{8}$$
$$w_2^* = \frac{\hat{\gamma}\hat{\mu} - \hat{\alpha}\hat{\varphi}}{\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha}} \tag{9}$$
$$s_2^* = \frac{\varphi(\hat{\gamma}\hat{\mu} - \hat{\alpha}\hat{\varphi}) - \Omega(\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha})}{\eta_2(\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha})} \tag{10}$$

where $\hat{\beta},\ \hat{\varphi},\ \hat{\lambda},\ \hat{\gamma},\ \hat{\mu},\ \hat{\alpha},\ \Omega$ and φ are described in Appendix A. Appendix A shows the proof of Proposition 2. With substituting (8), (9) and (10) into (5), (6) and (7), the equilibrium retail prices and service level are obtained.

2.2. With discount contracts

In this scenario, the simple price discount contracts are used and the first manufacturer's wholesale price is a discounted rate ρ of the retail price i.e. $w_1 = \rho p_1$. The demand and profit functions of the supply chain are similar to the one without discount scenario. Since the two manufacturers are the leaders, by solving the Manufacturer Stackelberg model, we can derive the following results.

2.2.1. Manufacturer Stackelberg

In this game, conditions, variables and parameters are similar to the first scenario, and the wholesale price of manufacturer 1 is a discounted rate of the retail price.

2.2.1.1. Retailer's best response

In this stage, the retail services s_1^* and the retail prices p_1^* and p_2^* are chosen by the retailer to maximize the optimal profit.

Proposition 3. In this Manufacturer Stackelberg game, given the wholesale prices $w_1 = \rho p_1,\ w_2$ and s_2 , the retailer's best response functions are obtained as:

$$p_1^* = Y_1 w_2 + V_1 s_2 - I_1 \tag{11}$$
$$p_2^* = X_1 w_2 + T_1 s_2 - J_1 \tag{12}$$
$$s_1^* = \beta_1 w_2 + \gamma_1 s_2 - \lambda_1 \tag{13}$$

where $Y_1,\ V_1,\ I_1,\ X_1,\ T_1,\ J_1,\ \beta_1,\ \gamma_1\ and\ \lambda_1$ are described in Appendix A. Appendix A shows the proof of Proposition 3.

2.2.1.2. Manufacturers' decisions

Similar to the first scenario, we can obtain the two manufacturer's equilibrium wholesale prices and service level of the manufacturer 2, by using the retailer's best response function. The best response functions of manufacturer 2 for wholesale price w_2 and service level s_2 are derived from maximizing his profit, given $p_i^*$and s_1^* in Eqs. (11), (12) and (13), respectively. As regards$w_1 = \rho p_1$, the manufacturers do not move simultaneously and there is not a Nash game between them.

Proposition 4. In the MS game, the two manufacturers' optimal wholesale prices and service level of the manufacturer 2, defined as w_1^*, w_2^*, and s_2^* are given as follows:

$$w_1^* = \rho p_1^* \tag{14}$$
$$w_2^* = \frac{\eta_2 \varepsilon_1 + \varphi_1 \Omega_1}{\eta_2 \delta_1 + \varphi_1^2} \tag{15}$$
$$s_2^* = \frac{\varphi_1 \varepsilon_1 - \delta_1 \Omega_1}{\eta_2 \delta_1 + \varphi_1^2} \tag{16}$$

where ε_1, φ_1, Ω_1 *and* δ_1 are described in Appendix A. Appendix A shows the proof of Proposition 4.

3. Numerical examples

Due to the complexity of the formula, similar to Cai et al. (2009), we use a numerical approach to compare the performance of the two scenarios mentioned.

3.1. Without discount scenario

The base values of key parameters used in this numerical experiment are given by**:**

$a_i = 40, b_p = 0.3,\ b_s = 0.3, \theta_p = 0.3, \theta_s = 0.3, \eta_i = 2, c_i = 2$ and $i = 1,2$ **.**

The results of the numerical example are shown in Table 3.

Table 3. Results of the "without discount contract scenario"

Variable	value
P1	27.53919643
P2	65.90603623
W1	9.390817928
W2	37.48927248
S1	1.243130487
S2	4.618824433
Q1	2.608523974
Q2	11.1164927
πM1	19.27912576
πM2	373.1826994
πR	361.6898539

In this case, we analyze the effect of changes of parameters on the profit of the supply chain's members (See Figures 2 to 11). We study the behavior change of the profits in the supply chain when the value of a parameter is changing and the values of other parameters are constant.

Effect of market base a_1 and a_2: By increasing a_1 and a_2 ,the profit of the supply chain members increases. Figure 2 shows that the trend of increasing the first manufacturer's profit is greater than the second manufacturer with the reverse case in Figure 3.

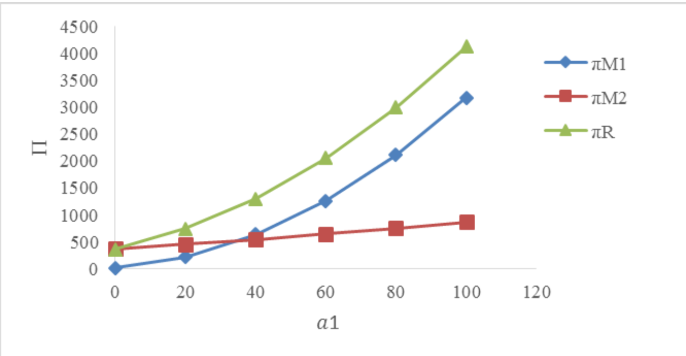

Figure 2. Effect of $a1$ on the profits

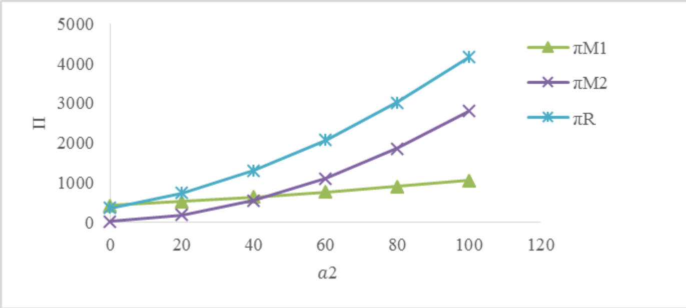

Figure 3. Effect of $a2$ on the profits

Effect of b_p: By increasing b_p, the profit of the supply chain members decreases in Figure 4.

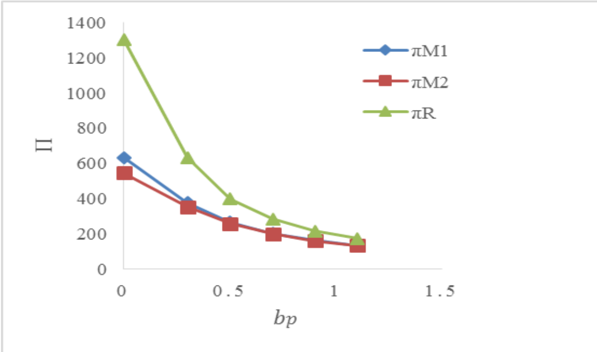

Figure 4. Effect of bp on the profits

Effect of b_s: An increase in b_s leads to an increase in the profit of the first manufacturer and the retailer and a decrease in the profit of the second manufacturer (Figure 5).

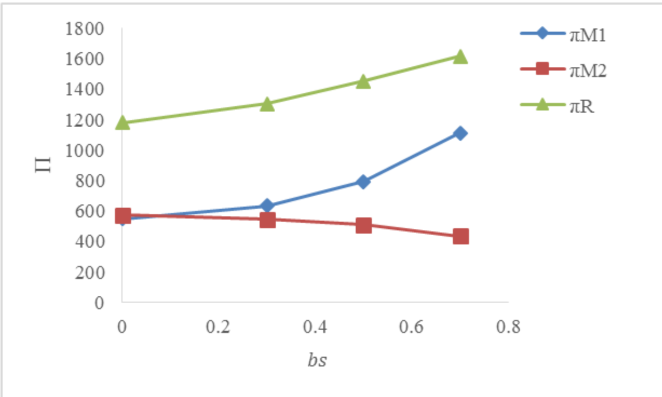

Figure 5. Effect of bs on the profits

Effect of θ_p: An increase in θ_p leads to a decrease in the profit of the two manufacturers, and an increase in the profit of the retailer (Figure 6).

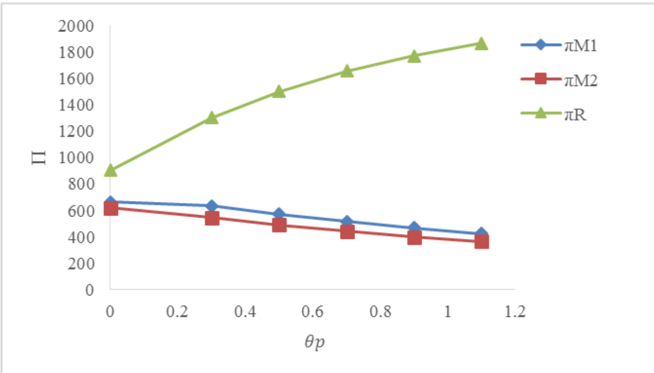

Figure 6. Effect of θp on the profits

Effect of $\boldsymbol{\theta_s}$: An increase in θ_s leads to a decrease in the profit of the two manufacturers and an increase in the profit of the retailer (Figure 7).

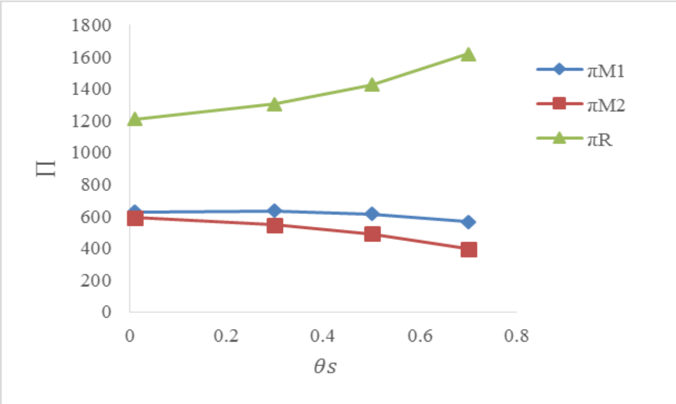

Figure 7. Effect of θs on the profits

Effect of $\boldsymbol{\eta_1}$: An increase in η_1 leads to a decrease in the profit of the first manufacturer and the retailer and an increase in the profit of the second manufacturer (Figure 8).

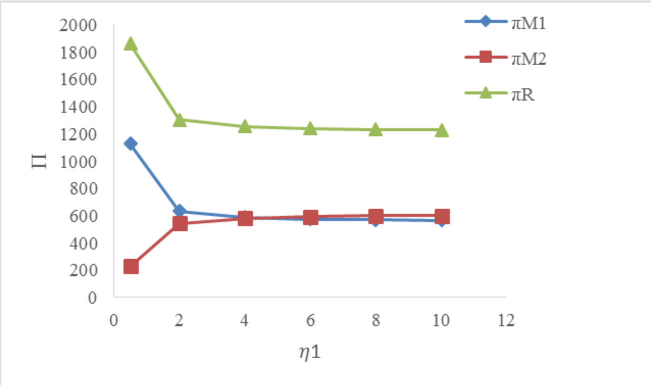

Figure 8. Effect of η_1 on the profits

Effect of $\boldsymbol{\eta_2}$: An increase in η_2 leads to an increase in the profit of the first manufacturers and a decrease in the profit of the second manufacturer and the retailer (Figure 9).

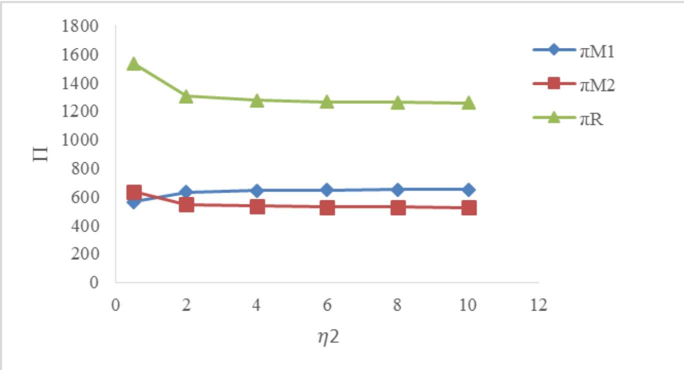

Figure 9. Effect of η_2 on the profits

Effect of $\boldsymbol{C_1}$: An increase in C_1 leads to a decrease in the profit of the first manufacturer and the retailer and an increase in the profit of the second manufacturer (Figure 10).

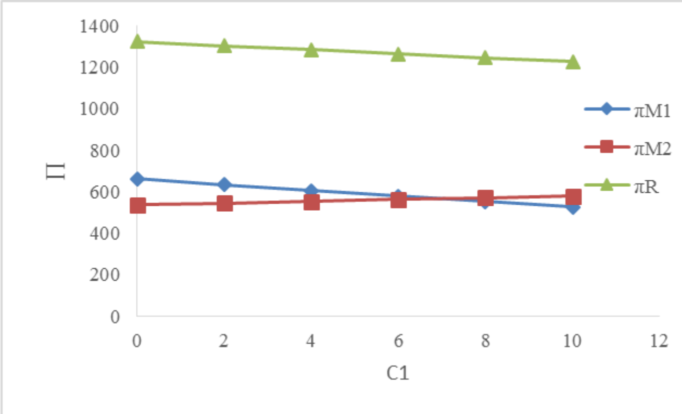

Figure 10. Effect of C_1 on the profits

Effect of C_2 : An increase in η_2 leads to an increase in the profit of the first manufacturers and an increase in the profit of the second manufacturer and the retailer (Figure 11).

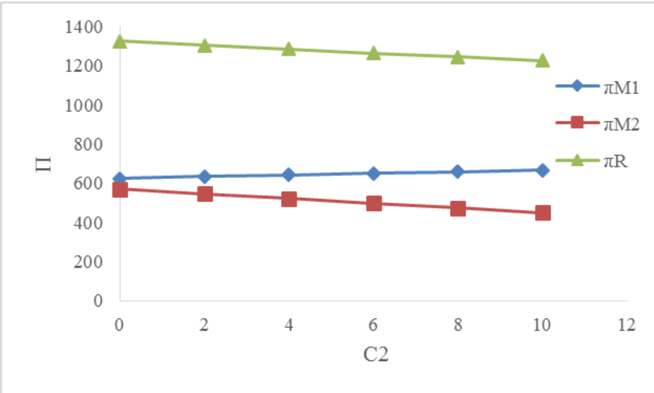

Figure 11.Effect of C_2 on the profits

3.2. With discount scenario

In this scenario, we analyzed the impacts of the change of discount rate ρ on the demand and profit of the supply chain members (Figures 12 to 16). The ranges of key parameters used in this numerical experiment are given in **Table4**.

Table 4. Values of parameters in the numerical example, in the "with discount contract scenario"

Parameters	a_i	b_p	b_s	θ_p	θ_s	η_i	c_i
Range1	40	0.3	0.3	0.3	0.3	2	2
Range2	60	0.5	0.5	0.5	0.5	4	4
Range3	80	0.7	0.7	0.7	0.7	6	6
Range4	100	0.9	0.9	0.9	0.9	8	8
Range5	120	1.1	1.1	1.1	1.1	10	10

All the following analyses were performed in a situation where all parameters and variables are greater than zero and optimality conditions exist. Figures 12 to 17 show that if the discount rate ρ increases, the second scenario includes the following points and results: 1) the demand for the first product has a descending trend. 2) The demand for the second product will have an ascending trend. 3) The profit of the retailer has a descending trend. 4) The profit of the second manufacturer has an ascending trend. 5) If the discount rate ρ increases, the profit of manufacturer 1 π_{M1} increases up to ρ_m and then falls. For example, the value of ρ_m in Figure 12 is $\rho_m \cong 0.6127$. 6) The service level provided by the retailer decreases. In Figures 12 to 16, $\boldsymbol{\pi_{M1}}$, $\boldsymbol{\pi_{M2}}$ and $\boldsymbol{\pi_R}$ are displayed on the scale of $\left(\frac{1}{10}\right)$.

Comparison of Figure 17 with other figures (Figures 12 to 16) shows that if the first manufacturer offers the retailer more discounts, the retailer will provide more service for customers, which does not necessarily increase the profit of the first manufacturer. He should then increase his profit by choosing an appropriate discount rate.

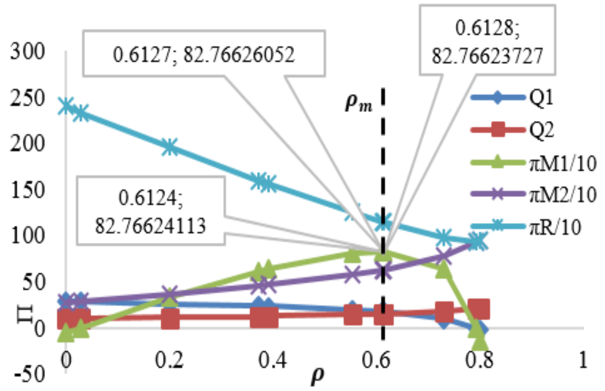

Figure 12. Effect of the change of discount rate ρ on profits and demands in Range1

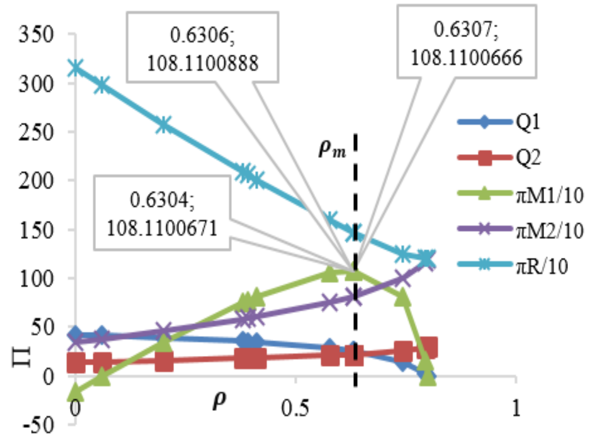

Figure 13. Effect of the change of discount rate ρ on profits and demands in Range 2

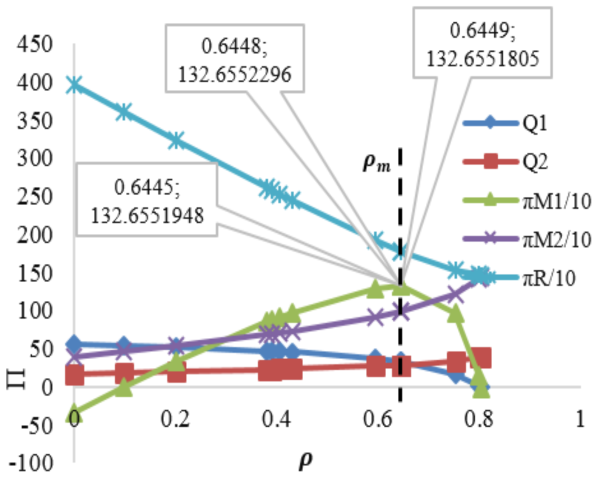

Figure 14. Effect of the change of discount rate ρ on profits and demands in Range 3

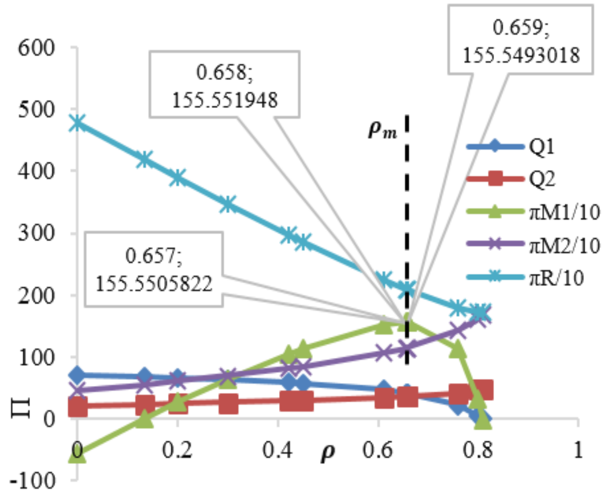

Figure 15. Effect of the change of discount rate ρ on profits and demands in Range 4

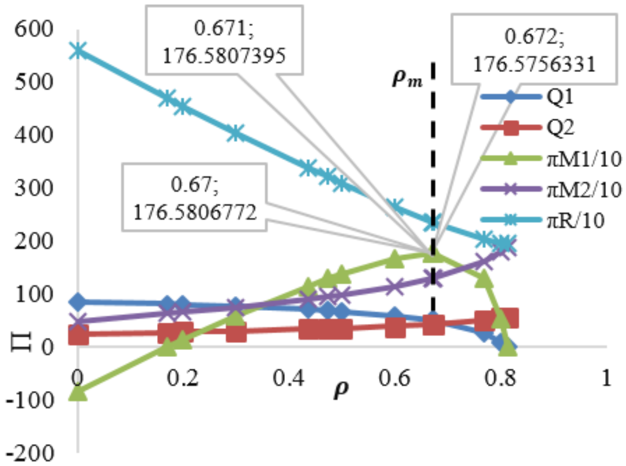

Figure 16. Effect of the change of discount rate ρ on profits and demands in Range 5

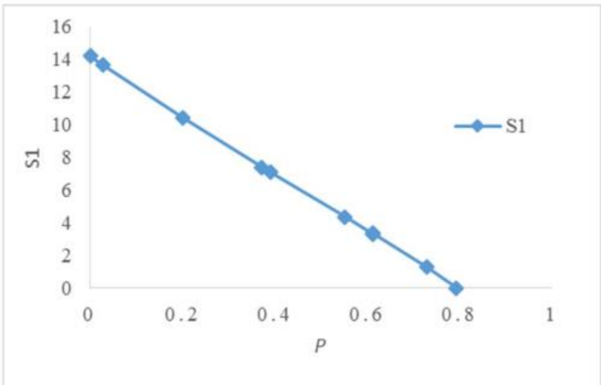

Figure 17. Effect of the change of discount rate ρ on the service provided by the retailer

3.3. Comparison and analysis of the results of the first scenario and the second scenario

To explore the effects of the discount rate ρ on the profit and demand of supply chain members, we compared and analyzed the numerical example between the two scenarios. The notations in **Table 2** and the following notations are used in this section:

$\boldsymbol{\pi_{M11}}$	Profit function of manufacturer 1 in scenario 1
$\boldsymbol{\pi_{M12}}$	Profit function of manufacturer 1 in scenario 2
$\boldsymbol{\pi_{M21}}$	Profit function of manufacturer 2 in scenario 1

$\boldsymbol{\pi_{M22}}$ Profit function of manufacturer 2 in scenario 2
$\boldsymbol{\pi_{R1}}$ Profit function of the retailer in scenario 1
$\boldsymbol{\pi_{R2}}$ Profit function of the retailer in scenario 2

Figures 18 to 22 illustrate that:

1) If the discount rate increases, the trend of the difference between the profit of the retailer in the second scenario and his profit in the first scenario ($\Delta\pi R = \pi_{R2} - \pi_{R1}$) will fall. In addition, this difference is positive up to ρ_1 and negative afterward. 2) If the discount rate increases, the trend of the difference between the profit of the manufacturer 2 in the second scenario and the first scenario ($\Delta\pi M2 = \pi_{M22} - \pi_{M21}$) is ascending. Also, this difference is negative up to ρ_2 and positive afterwards. 3) If the discount rate increases, the difference between the profit of the manufacturer 1 in the second scenario and the first scenario ($\Delta\pi M1 = \pi_{M12} - \pi_{M11}$) changes direction from the ascending mode to the descending mode, i.e. it shows an increasing trend up to ρ_m and afterward, it has a decreasing trend. For instance, the value of ρ_m in Figure 18 is $\rho_m \cong 0.6128$. 4) Regarding Figures 18 to 22, it can be observed that, in some intervals of the discount rate, the profit of the manufacturer 1 is greater than the profit of the manufacturer 2 in the second scenario and in some other intervals, this mode is reverse.

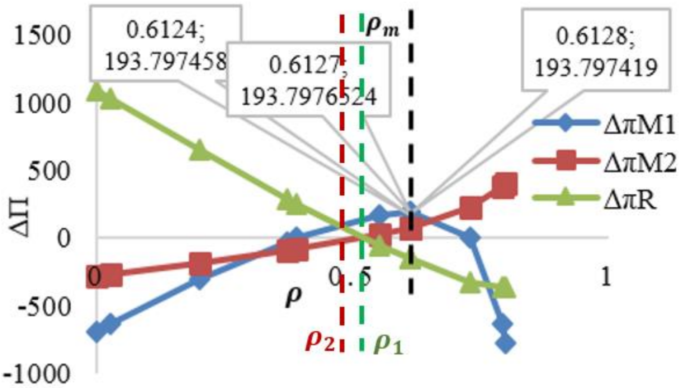

Figure 18. The difference between the profits in the second scenario and the first scenario in Range 1

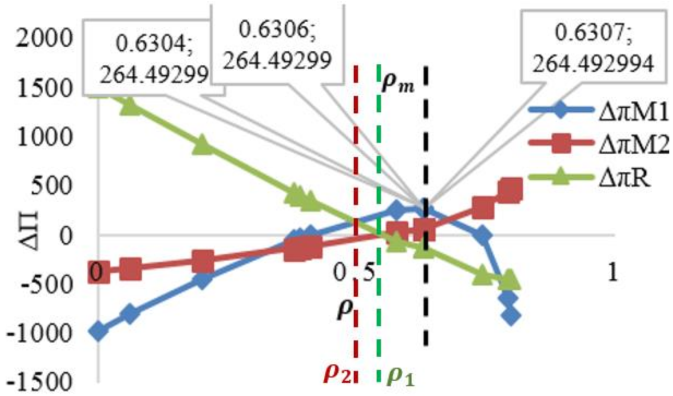

Figure 19. The difference between the profits in the second scenario and the first scenario in Range 2

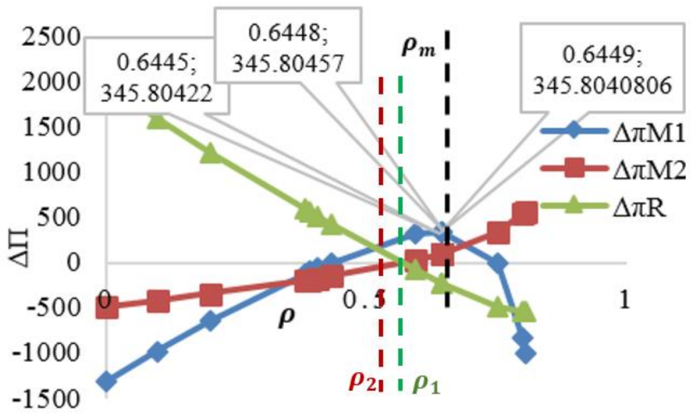

Figure 20. The difference between the profits in the second scenario and the first scenario in Range 3

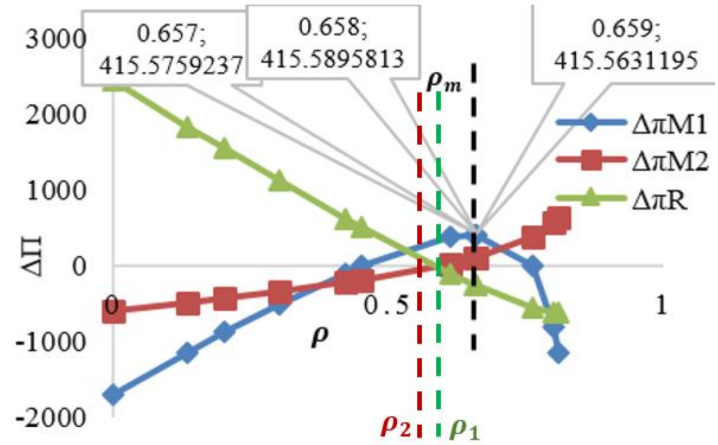

Figure 21. The difference between the profits in the second scenario and the first scenario in Range 4

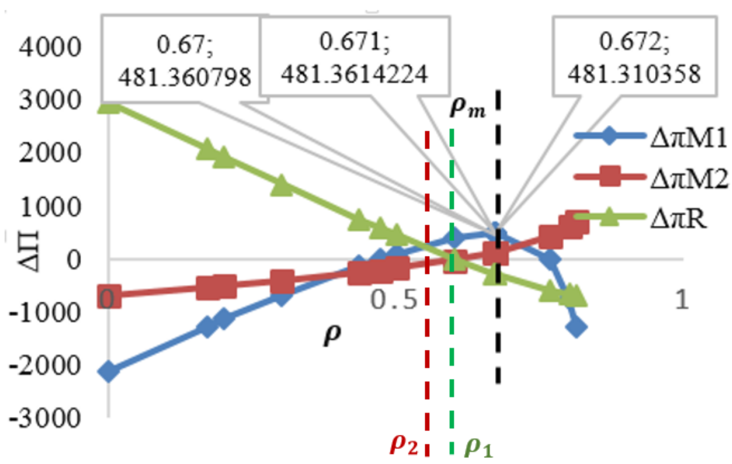

Figure 22. The difference between the profits in the second scenario and the first scenario in Range 5

The above results can be summarized in the following points:

1. If manufacturer 1 sells his product to the retailer by offering the appropriate discount rates, the profit and the demand for his product increases compared to the time when he offered his products without any price discounts.

2. Increased discount rate from the first manufacturer increases the presented service level by the retailer. Thus, the discount rate can be used as an incitement tool for providing retail services and acts as the coordinator of manufacturer and retailer.

4. Conclusion and Future Research

We studied the joint and simultaneous impacts of service, price and simple price discount contract on a supply chain's profit which consists of two manufacturers and one retailer. In this study, we assumed that the two manufacturers are the leader of the supply chain where a manufacturer undertakes the service for customers by herself and the retailer provides service for the customers of the other manufacturer. We obtained the optimal solutions of Manufacturer Stackelberg games and analyzed the following two scenarios: 1) the retailer buys the products from two manufacturers without price discount contracts. 2) The first product is sold to the retailer with price discount contracts. We compared and analyzed equilibrium solutions for the profit and demand of supply chain members under two decision scenarios. By using the sensitivity analysis of the numerical example, we showed that the price discount contract and service level have a huge impact on the profit of the supply chain members and the value of the discount rate is important. We concluded that increasing the service provided by the retailer does not necessarily increase the profit of the first manufacturer and he should set an appropriate discount rate to increase his profit. The results show that the price discount rate can be used as an incitement for providing retail services and coordination between the manufacturer and the retailer. There are several possible scenarios for future research. In our model, the demand is deterministic, sothis model can be extended with the demand uncertainty. In this study, the competitive factors include the service, price and the price discount, and in other studies factors such as advertising, quantity, production, customer supports, service quality, product quality and product delivery can be considered. Our model was applied in a single period, and another possibility is to consider the problem over multiple periods. Also, the supply chain structure and power structure can be changed.

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Appendix A

Proof of Proposition 1. By the first derivative of π_R with respect to p_i and s_1 ie. $\partial\pi_R/\partial p_i = 0$ and $\partial\pi_R/\partial s_1 = 0$, the retailer's reaction functions are obtained as:

$$Ap_1 + 2\theta_p p_2 + Bs_1 = D \quad (17)$$
$$2\theta_p p_1 + Ap_2 - \theta_s s_1 = E \quad (18)$$
$$Bp_1 - \theta_s p_2 - \eta_1 s_1 = F \quad (19)$$

By assuming $A = -2(b_p + \theta_p)$ (20), $B = (b_s + \theta_s)$ (21)

$$D = (\theta_s s_2 + \theta_p w_2 - (b_p + \theta_p)w_1 - a_1 = \theta_s s_2 + \theta_p w_2 + 0.5Aw_1 - a_1) \quad (22)$$
$$E = (\theta_p w_1 - (b_s + \theta_s)s_2 - (b_p + \theta_p)w_2 - a_2 = \theta_p w_1 - Bs_2 + 0.5Aw_2 - a_2), \quad (23)$$
$$F = (b_s + \theta_s)w_1 - \theta_s w_2 = Bw_1 - \theta_s w_2 \quad (24)$$

By solving (17), (18) and (19), simultaneously, we get

$$p_1 = \frac{MO - LR}{KO - LN} \quad (25)$$

$$p_2 = \frac{KR - MN}{KO - LN} \quad (26)$$
$$s_1 = \frac{BMO - BLR - \theta_s KR + \theta_s MN - FKO + FLN}{\eta_1(KO - LN)} \quad (27)$$

where $K = A\theta_s + 2\theta_p B$ (28), $L = 2\theta_s\theta_p + AB$ (29), $M = D\theta_s + BE$ (30)

$$N = 2\theta_p\eta_1 - \theta_s B \quad (31)$$
$$O = \eta_1 A + \theta_s^2 \quad (32)$$
$$R = \eta_1 E - \theta_s F \quad (33)$$

Taking the second order conditions, we have the Hessian Matrix:

$$H = \begin{bmatrix} A & 2\theta_p & B \\ 2\theta_p & A & -\theta_s \\ B & -\theta_s & -\eta_1 \end{bmatrix} \quad (34)$$

if $\left(H_{11} = \frac{\partial^2 \pi_R}{\partial {p_1}^2} = A < 0\right)$, $\det\left(\begin{bmatrix} A & 2\theta_p \\ 2\theta_p & A \end{bmatrix}\right) = A^2 - 4\theta_p^2 > 0$ and $\det(H) = [A(-A\eta_1 - \theta_s^2) - 2\theta_p(-2\theta_p\eta_1 + B\theta_s) + B(-2\theta_p\theta_s - AB)] < 0$, then π_R is concave. By substituting Eqs. (20), (21), (22), (23), (24), (28), (29), (30), (31), (32) and (33) into Eqs. (25), (26) and (27), the following expression provides the retailer's best reaction function:

$p_1^* = Uw_1 + Yw_2 + Vs_2 - I$ (35)

$p_2^* = Zw_1 + Xw_2 + Ts_2 - J$ (36)

$s_1^* = \alpha w_1 + \beta w_2 + \gamma s_2 - \lambda$ (37)

where $U = \left[\frac{(0.5\theta_s\eta_1 A^2 + 0.5\theta_s^3 A + 3\theta_s^2\theta_p B - 2\theta_s\eta_1\theta_p^2 + \theta_s AB^2)}{(KO - LN)}\right]$ (38)

$Y = \left[\frac{(-\theta_s^3\theta_p - 0.5\theta_s^2 BA)}{(KO - LN)}\right]$ (39)

$V = \left[\frac{(\theta_s^2\eta_1 A + \theta_s^4 - \theta_s^2 B^2 + 2\theta_s\eta_1\theta_p B)}{(KO - LN)}\right]$ (40)

$I = \left[\frac{(a_1\theta_s\eta_1 A + a_1\theta_s^3 + a_2\theta_s^2 B - 2a_2\theta_s\eta_1\theta_p)}{(KO - LN)}\right]$ (41)

$Z = \left[\frac{(-\theta_s\theta_p B^2 - 0.5A\theta_s^2 B)}{(KO - LN)}\right]$ (42)

$X = \left[\frac{(0.5\eta_1 A^2\theta_s + 0.5A\theta_s B^2 + A\theta_s^3 + 3\theta_p B\theta_s^2 - 2\theta_p^2\eta_1\theta_s)}{(KO - LN)}\right]$ (43)

$T = \left[\frac{(-\eta_1 A\theta_s B - 2\theta_p\eta_1\theta_s^2)}{(KO - LN)}\right]$ (44)

$J = \left[\frac{(a_2\eta_1 A\theta_s + a_2\theta_s B^2 - 2a_1\theta_p\eta_1\theta_s + a_1\theta_s^2 B)}{(KO - LN)}\right]$ (45)

$\alpha = \left[\frac{(-0.5B\theta_s\eta_1 A^2 + 2\theta_s\theta_p^2\eta_1 B)}{\eta_1(KO - LN)}\right]$ (46)

$\beta = \left[\frac{(-2\theta_s^2\theta_p^2\eta_1 + 0.5A^2\theta_s^2\eta_1)}{\eta_1(KO - LN)}\right]$ (47)

$\gamma = \left[\frac{(2B\theta_s^2\eta_1 A + 2\theta_s^3\theta_p\eta_1 + 2\theta_s\theta_p\eta_1 B^2)}{\eta_1(KO - LN)}\right]$ (48)

$\lambda = \left[\frac{(a_1 B\theta_s\eta_1 A + 2a_1\theta_s^2\theta_p\eta_1 - 2a_2\theta_s\theta_p\eta_1 B - a_2 A\theta_s^2\eta_1)}{\eta_1(KO - LN)}\right]$ (49)

Proof of Proposition 2. In this Nash Equilibrium, by substituting Eqs. (35), (36) and (37) into Eqs. (2) and (3), the profit functions of the two manufacturers can be expressed as:

$\pi_{M1} = (w_1 - c_1)[a_1 - b_p(Uw_1 + Yw_2 + Vs_2 - I) + \theta_p(Zw_1 + Xw_2 + Ts_2 - J - Uw_1 - Yw_2 - Vs_2 + I) + (b_s + \theta_s)(\alpha w_1 + \beta w_2 + \gamma s_2 - \lambda) - \theta_s s_2]$ (50)

$\pi_{M2} = (w_2 - c_2)[a_2 - b_p(Zw_1 + Xw_2 + Ts_2 - J) + \theta_p(Uw_1 + Yw_2 + Vs_2 - I - Zw_1 - Xw_2 - Ts_2 + J) + (b_s + \theta_s)s_2 - \theta_s(\alpha w_1 + \beta w_2 + \gamma s_2 - \lambda)] - \frac{\eta_2 s_2^2}{2}$ (51)

By solving $\partial(\pi_{M_i}|p_1, p_2, s_1)/\partial w_i = 0$ and $\partial(\pi_{M_2}|p_1, p_2, s_1)/\partial s_2 = 0$, the results in the following are obtained:

$\frac{\partial \pi_{M1}}{\partial w_1} = 0 \rightarrow (AU + 2\theta_p Z + 2\alpha B)w_1 + (\theta_p X + 0.5AY + \beta B)w_2 + (\theta_p T + 0.5AV + \gamma B - \theta_s)s_2 + a_1 - 0.5AI - \theta_p J - \lambda B - 0.5AUc_1 - \theta_p Zc_1 - \alpha Bc_1 = 0$ (52)

$\frac{\partial \pi_{M2}}{\partial w_2} = 0 \rightarrow (0.5AZ + \theta_p U - \alpha\theta_s)w_1 + (2\theta_p Y + AX - 2\beta\theta_s)w_2 + (0.5AT + \theta_p V + B - \gamma\theta_s)s_2 + a_2 - 0.5AJ - \theta_p I + \lambda\theta_s - 0.5AXc_2 - \theta_p Yc_2 + \beta\theta_s c_2 = 0$ (53)

$\frac{\partial \pi_{M2}}{\partial s_2} = 0 \rightarrow (0.5AT + \theta_p V + B - \gamma\theta_s)w_2 - \eta_2 s_2 - 0.5ATc_2 - \theta_p Vc_2 - Bc_2 + \theta_s\gamma c_2 = 0$ (54)

By assuming $\mu = AU + 2\theta_p Z + 2\alpha B$ (55)

$\sigma = \theta_p X + 0.5AY + \beta B$ (56)

$\xi = \theta_p T + 0.5AV + \gamma B - \theta_s$ (57)

$\tau = -(a_1 - 0.5AI - \theta_p J - \lambda B - 0.5AUc_1 - \theta_p Zc_1 - \alpha Bc_1)$ (58)

$\upsilon = 0.5AZ + \theta_p U - \alpha\theta_s$ (59)

$\delta = 2\theta_p Y + AX - 2\beta\theta_s$ (60)

$\varphi = 0.5AT + \theta_p V + B - \gamma\theta_s$ (61)

$\varepsilon = -(a_2 - 0.5AJ - \theta_p I + \lambda\theta_s - 0.5AXc_2 - \theta_p Yc_2 + \beta\theta_s c_2)$ (62)

and $\Omega = 0.5ATc_2 + \theta_p Vc_2 + Bc_2 - \theta_s\gamma c_2$ (63)

the response functions of manufacturers include:

$\mu w_1 + \sigma w_2 + \xi s_2 = \tau$ (64)

$\upsilon w_1 + \delta w_2 + \varphi s_2 = \varepsilon$ (65)

$\varphi w_2 - \eta_2 s_2 = \Omega$ (66)

We have optimal second condition and the Hessian Matrix, respectively, $\frac{\partial^2 \pi_{M1}}{\partial {w_1}^2} = \mu < 0$,$H = \begin{bmatrix} \delta & \varphi \\ \varphi & -\eta_2 \end{bmatrix}$, $(H_{11} = \delta < 0)$, $Det(H) = (-\delta\eta_2 - \varphi^2) > 0$
Given the above first and second order conditions and solving Eqs. (64), (65) and (66), simultaneously, we get:

$$w_1^* = \frac{\hat{\beta}\hat{\varphi} - \hat{\lambda}\hat{\gamma}}{\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha}} \quad (67)$$

$$w_2^* = \frac{\hat{\gamma}\hat{\mu} - \hat{\alpha}\hat{\varphi}}{\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha}} \quad (68)$$

$$s_2^* = \frac{\varphi(\hat{\gamma}\hat{\mu} - \hat{\alpha}\hat{\varphi}) - \Omega(\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha})}{\eta_2(\hat{\beta}\hat{\mu} - \hat{\lambda}\hat{\alpha})} \quad (69)$$

where $\hat{\alpha} = \eta_2\mu$ (70)

$$\hat{\beta} = \eta_2\sigma + \xi\varphi \quad (71)$$

$$\hat{\gamma} = \eta_2\tau + \Omega\xi \quad (72)$$

$$\hat{\mu} = \eta_2\upsilon \quad (73)$$

$$\hat{\lambda} = \eta_2\delta + \varphi^2 \quad (74)$$

$$\hat{\varphi} = \varepsilon\eta_2 - \varphi\Omega \quad (75)$$

Proof of Proposition 3. given the first manufacturer wholesale price $w_1 = \rho p_1$ and by obtaining the first order conditions $\partial\pi_R/\partial p_i = 0$ and $\partial\pi_R/\partial s_1 = 0$, the retailer's reaction functions are obtained as:

$$A_1p_1 + B_1p_2 + H_1s_1 = D_1 \quad (76)$$

$$B_1p_1 + Ap_2 - \theta_s s_1 = E_1 \quad (77)$$

$$H_1p_1 - \theta_s p_2 - \eta_1 s_1 = F_1 \quad (78)$$

By assuming $A_1 = -2(1-\rho)(b_p + \theta_p) = (1-\rho)A$ (79)

$$H_1 = (1-\rho)(b_s + \theta_s) = (1-\rho)B \quad (80)$$

$$B_1 = (2-\rho)\theta_p \quad (81)$$

$$D_1 = (1-\rho)\theta_s s_2 + \theta_p w_2 - a_1(1-\rho) \quad (82)$$

$$E_1 = -(b_s + \theta_s)s_2 - (b_p + \theta_p)w_2 - a_2 = -Bs_2 + 0.5Aw_2 - a_2 \quad (83)$$

$$F_1 = -\theta_s w_2 \quad (84)$$

By solving Eqs. (76), (77) and (78), simultaneously, response functions of retailer are obtained as:

$$p_1 = \frac{M_1O_1 - L_1R_1}{K_1O_1 - L_1N_1} \quad (85)$$

$$p_2 = \frac{K_1R_1 - M_1N_1}{K_1O_1 - L_1N_1} \quad (86)$$

$$S_1 = \frac{H_1M_1O_1 - H_1L_1R_1 - \theta_sK_1R_1 + \theta_sM_1N_1 - F_1K_1O_1 + F_1L_1N_1}{\eta_1(K_1O_1 - L_1N_1)} \quad (87)$$

where $K_1 = (A_1\theta_s + H_1B_1)$ (88)

$$L_1 = (B_1\theta_s + H_1A) \quad (89)$$

$$M_1 = \theta_sD_1 + H_1E_1 \quad (90)$$

$$N_1 = (B_1\eta_1 - \theta_sH_1) \quad (91)$$

$$O_1 = (\eta_1A + \theta_s^2) \quad (92)$$

$$R_1 = (\eta_1E_1 - \theta_sF_1) \quad (93)$$

Taking the second order conditions, we obtain the Hessian Matrix: $H = \begin{bmatrix} A_1 & B_1 & H_1 \\ B_1 & A & -\theta_s \\ H_1 & -\theta_s & -\eta_1 \end{bmatrix}$ (94)

If $(H_{11} = \frac{\partial^2\pi_R}{\partial {p_1}^2} = A_1 < 0), \det\left(\begin{bmatrix} A_1 & B_1 \\ B_1 & A \end{bmatrix}\right) = A_1A - B_1^2 > 0$ and $\det(H) = [A_1(-\eta_1A - \theta_s^2) - B_1(-\eta_1B_1 + H_1\theta_s) + H_1(-\theta_sB_1 - AH_1)] < 0$, then π_R is concave
Given the first and the second order optimality conditions and substituting Eqs. (79), (80), (81), (82), (83), (84), (88), (89), (90), (91), (92) and (93) into Eqs. (85), (86) and (87), the following expression provides the retailer's response function:

$$p_1^* = Y_1w_2 + V_1s_2 - I_1 \quad (95)$$

$$p_2^* = X_1w_2 + T_1s_2 - J_1 \quad (96)$$

$$s_1^* = \beta_1w_2 + \gamma_1s_2 - \lambda_1 \quad (97)$$

where $Y_1 = \frac{\theta_s\theta_p\eta_1A + \theta_p\theta_s^3 - 0.5B_1A\theta_s\eta_1 - B_1\theta_s^3 - 0.5(1-\rho)BA\theta_s^2}{K_1O_1 - L_1N_1}$ (98)

$$V_1 = \frac{(1-\rho)\theta_s^2\eta_1A + (1-\rho)\theta_s^4 - (1-\rho)B^2\theta_s^2 + B_1B\theta_s\eta_1}{K_1O_1 - L_1N_1} \quad (99)$$

$$I_1 = \frac{a_1(1-\rho)\theta_s\eta_1A + a_1(1-\rho)\theta_s^3 + a_2(1-\rho)B\eta_1A + a_2(1-\rho)B\theta_s^2 - a_2B_1\theta_s\eta_1 + a_2(1-\rho)BA\eta_1}{K_1O_1 - L_1N_1} \quad (100)$$

$$X_1 = \frac{0.5A_1\theta_s\eta_1 A + 0.5(1-\rho)^2B^2\theta_s A + A_1\theta_s^3 + (1-\rho)BB_1\theta_s^2 - \theta_s B_1\eta_1\theta_p + \theta_s^2(1-\rho)B\theta_p}{K_1O_1 - L_1N_1} \tag{101}$$

$$T_1 = \frac{-A_1\theta_s\eta_1 B - (1-\rho)^2B^3\theta_s - (1-\rho)\theta_s^2 B_1\eta_1 + (1-\rho)^2\theta_s^3 B}{K_1O_1 - L_1N_1} \tag{102}$$

$$J_1 = \frac{a_2A_1\theta_s\eta_1 + a_2(1-\rho)^2B^2\theta_s - a_1(1-\rho)\theta_s B_1\eta_1 + a_1(1-\rho)^2\theta_s^2 B}{K_1O_1 - L_1N_1} \tag{103}$$

$$\beta_1 = \frac{(1-\rho)B\theta_s\eta_1 A\theta_p + \theta_s^2 B_1\eta_1\theta_p + 0.5A_1\theta_s^2\eta_1 A - B_1^2\theta_s^2\eta_1 - 0.5A\theta_s(1-\rho)BB_1\eta_1}{\eta_1(K_1O_1 - L_1N_1)} \tag{104}$$

$$\gamma_1 = \frac{(1-\rho)^2\theta_s^2 B\,\eta_1 A + (1-\rho)\theta_s^3 B_1\eta_1 + A_1\theta_s^2\eta_1 B + \theta_s(1-\rho)B^2B_1\eta_1}{\eta_1(K_1O_1 - L_1N_1)} \tag{105}$$

$$\lambda_1 = \frac{a_1(1-\rho)^2B\theta_s\eta_1 A + a_1(1-\rho)\theta_s^2 B_1\eta_1 - a_2A_1\theta_s^2\eta_1 - a_2\theta_s(1-\rho)BB_1\eta_1}{K_1O_1 - L_1N_1} \tag{106}$$

Proof of Proposition 4. In this stage, given the first manufacturer wholesale price $w_1 = \rho p_1$, by solving $\partial(\pi_{M_2}|p_1, p_2, s_1)/\partial w_2 = 0$ and $\partial(\pi_{M_2}|p_1, p_2, s_1)/\partial s_2 = 0$, the second manufacturer's reaction functions are obtained as:

$$\delta_1 w_2 + \varphi_1 s_2 = \varepsilon_1 \tag{107}$$

$$w_2 - \eta_2 s_2 = \Omega_1 \tag{108}$$

By assuming $\delta_1 = AX_1 + 2\theta_p Y_1 - 2\beta_1\theta_s$ (109)

$$\varphi_1 = 0.5AT_1 + \theta_p V_1 + B - \gamma_1\theta_s \tag{110}$$

$$\varepsilon_1 = -(a_2 - 0.5AJ_1 - \theta_p I_1 + \lambda_1\theta_s - 0.5AX_1c_2 - c_2\theta_p Y_1 + c_2\theta_s\beta_1) \tag{111}$$

$$\Omega_1 = -(-0.5AT_1c_2 - \theta_p V_1c_2 - Bc_2 + \theta_s\gamma_1c_2) \tag{112}$$

By solving Eqs. (107) and (108) we obtain:

$$w_2^* = \frac{\eta_2\varepsilon_1 + \varphi_1\Omega_1}{\eta_2\delta_1 + \varphi_1^2} \tag{113}$$

$$s_2^* = \frac{\varphi_1\varepsilon_1 - \delta_1\Omega_1}{\eta_2\delta_1 + \varphi_1^2} \tag{114}$$