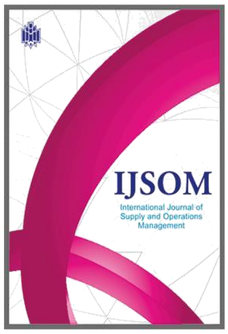
International Journal of Supply and Operations Management

IJSOM



August 2019, Volume 6, Issue 3, pp. 188-199
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com

Optimizing the Safety Stock with Guaranteed Service Model in Reverse Logistics Considering Internal and External Returns

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Abstract
In this paper, a guaranteed service model (GSM) is considered in a general supply chain with the releasing of the service time constraint. For this purpose, a bi-objective model is developed containing minimization of both service time and the holding cost of safety stock. In addition to considering the return of items from inside of the chain, the returns from outside of the chain are also considered to carry out remanufacturing, refurbishing, and repairing processes. The model is solved in a real-world case of the electronic product supply chain and the Pareto solution set is obtained to show the changes in the holding cost of safety stock based on the maximum time for the different services offered to customers.

Keywords: Guaranteed Service Model; Safety Stock; Inventory Control; Reverse logistics; Lead Time.

1. Introduction

The safety stock is used as a buffer to reduce the risk of inventory shortages and the risk of unnecessary inventory overage in both single and multiple echelon systems (Eruguz et al. 2014). In multi-echelon inventory-control systems, the safety stock bears extraordinary significance because it can adjust and optimize the lead times and consequently the inventory decisions at different levels, from the external sources to the final customer (Klosterhalfen and Minner 2010). The guaranteed service model (GSM) is one of approaches which is used for the optimization of the safety stock (Sitompul et al. 2008). In this approach, each stage meant for meeting the demand carries out the orders into its upstream stages (i.e. suppliers) after receiving the demand from downstream stages at a certain time. The topic is used in combination with other supply chain issues such as supply chain configuration, production scheduling, reverse logistics, and green supply chain with the aim of reaching into optimized inventory. Based on our review, it seems that there is only one study about the use of guaranteed service model with reverse logistics simultaneously. In that research, Minner (2001) considered the returned items from inside of the chain for recovery processes. Almost all previous studies of GSM have considered the customer's service time as a constraint. Therefore, it is essential that a study is conducted considering the return of items from both inside and outside of the chain by including more reverse logistics processes as well as considering the service time constraint as an objective.

2. Review of the Literature

As early works, Inderfurth (1991) provided a dynamic programming algorithm for distribution systems, Graves and Willems (2000) developed a guaranteed service model for general systems, and Minner (2000) considered the relaxation based on the assumption of similar service time for the downstream stages of distribution systems and proposed two different models. Later, Grahl et al. (2016) extended the Minner's (2000) model for general acyclic networks and showed that the computational complexity of the network increases significantly for different service periods. Eruguz et al. (2016) conducted a comprehensive review of the literature on this topic and provided a classification of optimizing methods for safety stock using a guaranteed service model.

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In another research, Minner (2001) analyzed the effect of byproduct items (which are returned due to defects in the production process) on the amount of the safety stock to ensure GSM and then developed the general cyclic network model in reverse logistics. In the model, the byproducts can be returned from one stage to the previous stages. Then, the gross and net demands at each stage are adjusted with the return quantities. The demands are characterized over different intervals depending on the values of service time, regular lead time, and the recovered lead time.

Graves and Willems (2003) integrated the GSM with the supply chain configuration problem. In the proposed model, the best option for each stage and the amount of safety stock is determined in the supply chain at the same time. Later, Graves and Willems (2008) considered the optimization of the safety stock in a supply chain with non-stationary demand bound. They indicated that if the non-stationary demand bound is a concave function, the optimization problem is equivalent to the case that the demand is stationary. Nepal et al. (2011) provided a multi-objective optimization model for new products involving the maximization of the total compatibility index for the selected options and the minimization of the total supply chain cost.

For the GSM problem, the effect of using extraordinary measures when the demand exceeds the pre-specified limit has been discussed only in a few cases. In this regard, Klosterhalfen and Minner (2010) assumed that the external demand would be indefinitely propagated within the system. They used accelerated tasks, such as increasing the speed of transport between the stages, as extraordinary measures.

Li and Jiang (2012) suggested a heuristic approach to the issue of safety stock placement which integrates the genetic algorithm with ε-constraint method by considering the balancing between solution speed and quality. Li et al. (2013) decomposed the optimization problem into two independent sub-problems consisting of decision on the order size and reorder point and then offered two dynamic programming algorithms to solve the sub-problems. Ni and Shu (2015) combined the safety stock placement problem with the environmental impact of carbon dioxide emissions and developed two models by taking into account the carbon tax and the carbon cap in trade-off between the service time and carbon emissions. Recently, Hong et al. (2018) considered a supply chain configuration problem for a green product family using a guaranteed service model and emission constraints. They decomposed the original problem into two sub-problems and suggested a hybrid algorithm (STA+PSO) to solve the original model. Magnanti et al. (2006) developed a GSM model with the maximum service time for which safety stock can be considered at each station by adding a constraint to the problem. Then the model is solved using an exact solution method without referring to the optimal solution properties. Graves and Schoenmeyr (2016) examined the capacity constraints in the GSM model with a modified single-stage base-stock policy and then extended it into a multi-stage policy. They showed that under the base-stock policy, adding capacity constraints leads to increases in inventory costs. Hua and Willems (2016) developed a two-stage serial line supply chain and indicated that the safety stock cost of downstream stage can be reduced by cutting the cost or increasing the lead time of the upstream stage. Martínez and Mastrocinque (2016) considered service time as the second objective of the model and proposed a bi-objective water drop algorithm for solving the model. Then they examined the algorithm for seven examples in the literature. The results showed that their proposed algorithm outperforms many other meta-heuristics such as ant colony optimization algorithm.

There are many other studies on the GSM but the most important research is reviewed here. To get a better perception of the researches done, they are summarized in Table 6 in Appendix B.

The rest of the paper is structured as follows: In section 3, notations and new equations regarding the return of items from outside of the chain are introduced. In section 4, a bi-objective mathematical model is developed. Section 5 is devoted to providing a numerical example and analytical results. In section 6, conclusions and future studies are presented.

3. Problem description

As mentioned above, to extend the Minner's model (2001), we consider more options of reverse logistics in the GSM model after the return of items from outside of the supply chain including remanufacturing, refurbishing, and repairing and then propose the developed model. Also, in this study, the Minner's model is extended by considering the replenishment lead time of the recycled items and by putting it along with the former items. Thus, the three permutation cases of lead times and service time in the Minner's model are extended into twelve permutation cases of lead times and service time in the model.

Meanwhile, we release the customer's service time constraint and consider it as a second objective function to reduce the service time. In addition, by regarding the total holding cost of the safety stock as another objective function, we develop a bi-objective model in which the trade-off results between service time and holding cost can be reported as a Pareto frontier.

The following notations are used to develop the proposed model:

3.1. Indices and Sets

I : Set of production and holding inventory stages, $i, j, k \in (1,..,i,...,I)$
$\hat{L}$: Index of initial permutation cases of lead times and service time developed by Minner $\ell \in (1,2,3)$
U : Index of extended permutation cases of lead times and service time developed in this study $u \in (1,..,12)$
T : Index of planning periods, $t \in T$
A, P, E : Sets of initial, in-process and final stages respectively
$v(i)$: Set of all direct predecessors of stage i
W : Set of direct connections in the network, example:$\{(1,2),(2,4),..,(n-1,n) \in W\}$
WW: Set of reverse connections in the network (i.e. backward flow), example:$\{(3,2),(12,8),... \in WW\}$
Wbp : Set of stages with the ability of receiving the by-products
rec: Set of stages with the ability of receiving the return items after recycling process
cap : stages with time limit
θ : The Overall set of reverse activities including remanufacturing, refurbishing, and repairing, $\alpha, \beta, \gamma \in \theta$

3.2. Scalars

b: Rate of return items from outside of the chain
$Esp.$, M : a very small & large number respectively

3.3. Parameters

$a_{i,j}$: Number of items required at stage j to produce one item at the successor stage i
$a_{j,i}$: The coefficient of by-products produced at stage i which is sent to stage j for corrective processes
C(b): The amount of return items
t_i: Processing time at stage i
t_i^{bp}: Processing time on by-product at stage i
t^{rec}: Recycling process time
t^{rco}: Recovery process time
D_i: Received demand by stage i $(i \in E)$
D_i^{gross}: Gross demand at stage i, $(i \in A \cup P \cup E)$
sdd_i: Standard deviation of demand at stage i, $(i \in E)$
$SL_i(k_i)$: Service level at stage i
Ca_i: The processing time limit at stage i
h_i: Holding cost at stage i
$\theta_\alpha, \theta_\beta, \theta_\gamma$: A Portion of the returned items for remanufacturing, refurbishing, and repairing respectively

3.4. Intermediate Decision Variables

$\delta_i, \eta_i, \tau_i, \omega_i, \Psi_i$: Binary variables of determining the active permutation cases of lead times and service time at stage i
$\delta\eta_i, \delta\eta_{i1}$: Continuously auxiliary variables for linearization of binary variables at stage i
$ab_{i,u}$: Continuously auxiliary variable for linearization of binary variables related to the permutation case u at stage i
OP_ℓ: Multiplication of binary variables at each of triple permutation cases of lead times and service time in the basic model
OC_u: Multiplication of binary variables in each of twelve permutation cases of lead times and service time in the developed model
D_i^{net}: Net demand at stage i, $(i \in A \cup P)$
V_i^{net}: The demand variance at initial and in-processing stages including returned items $i \in (A \cup P)$
V_i^{net*}: The demand variance at the final stages $i \in E$
τ_i: The net replenishment time
$\tilde{T}_i$: Total replenishment lead time for the gross requirement at stage i
L_i^r : Replenishment lead time of regular items at stage i
L_i^{bp} :Replenishment lead time of returned items at stage i
$L_{bp(i)}^r$: Regular replenishment lead time of the by-product items at stage i
L_i^{rec} : Replenishment lead time of recycled items at stage i

3.5. The main decision variables

SI_i, S_i: Inbound & outbound service time at stage i, respectively
Q_i: Regular replenishment quantity
sd_i^{net}: Net standard deviation of demand at stages i, $(i \in A \cup P)$
R_i: Number of returned items to stage i
RR_i: Number of recycled items to stage i
mx_i: The preparation time of recovered items at stages $i \in E$

Based on the GSM, at each stage i ∈ I, an outbound guaranteed service time S_i is imposed into downstream stages. Thus, any demand that occurs at time T would be satisfied at time $T+S_i$. Also, the time required to receive all inputs of stage i from stages j ((i, j) ∈ I) to start the process is defined as inbound service time (SI_i). At stage i, if demand is received at time T, the replenishment of items responding to this demand would be performed at time $T + SI_i + t_i$. In fact, demand received at time T is satisfied at time $T + S_i$. If $SI_i + t_i > S_i$, then demand during the lead time which is $\tau_i = SI_i + t_i - S_i$, (i.e. net replenishment time) should be covered.

Based on the Kimball control approach (1988), the internal returns can be considered as follows (Minner, 2000):

$$D_i^{gross} = D_i \qquad i \in E \qquad (1)$$

$$D_i^{gross} = \sum_{j \in n(i)} a_{i,j} Q_j \qquad i \in A \cup P \qquad (2)$$

$$L_i^r = max\{S_j\} + t_i \qquad i \in A \cup P \cup E, j \in v(i) \qquad (3)$$

$$R_i = a_{j,i} Q_j \qquad i \in A \cup P, (j,i) \in ww \qquad (4)$$

$$L_i^{bp} = max\{L_{bp(i)}^r\} + t_i^{bp} \qquad i \in A \cup P, j \,|(j,i) \in ww \qquad (5)$$

$$Q_i = D_{i,t}^{gross} - R_i \qquad i \in A \cup P \qquad (6)$$

In the case of external returns, according to the quality level of the returned items, remanufacturing, refurbishing, and repairing items are determined.

We consider that the amount of returned items is determined based on the return rate of the items as well as the portion of remanufacturing, refurbishing, and repairing activities defined as follows:

$$C(b) = D_t . b \qquad (7)$$

$$\theta_\alpha = 3\frac{b}{4} \qquad (8.a)$$

$$\theta_\beta = \theta_\gamma = \frac{b}{8} \qquad (8.b)$$

The items are sent to recycling centers and after passing the recovery processes, they will be sent back to one of stages of the chain. The recycling time of the items is equal to $\mathrm{t^{rec}}$ and so the products or returned items can be used in the supply chain after:

$$L_i^{rec} = t^{rec} \qquad (9)$$

Now, let's use both receiving internal and external returns to determine demand at stage *i*; such that total replenishment time is equal to the maximum time of all three lead times (i.e. max [L_i^r, L_i^{bp}, Li^{rec}]). As shown in equation (10), the time duration which is needed to overcome the uncertainties of demand by safety stock would be:

$$\tilde{T}_i = max\{ L_i^r, L_i^{bp}, L_i^{rec}\} \; - S_i \qquad i \in A \cup P \qquad (10)$$

So, if demand is occurred at time t, the first period that can be fully affected would be $t+\tilde{T}_i + S_i$.

Let us suppose $D_i^{gross}[A,B,C,D\,]$ and $D_i^{net}[A,B,C,D\,]$ as gross and net demand values respectively over the interval $[\mathrm{t{+}S_i; t + \tilde{T}_i + S_i}]$. *A*, *B*, *C*, and *D* are defined respectively as the service time, the regular replenishment time, the byproduct replenishment time, and the recycling replenishment time at each stage. In the following, we will determine the required safety stock for dealing with the uncertainty of the demand during $\tilde{T}_i$. Depending on the values of L_i^{bp}, L_i^{rec}, S_i, and L_i^r, and by considering all possible permutations of these values and also considering that the service time is limited to the regular replenishing time (L_i^r) at each stage, four new permutation cases can be generated at each of the three previous permutation cases as shown in Appendix A.

But first, let's consider Minner's model and develop the mathematical relations for it. In Minner's model, for the stages that include regular replenishment as well as byproduct replenishment, three following cases are considered:

1) $S_i \le L_i^r \le L_i^{bp}$, 2) $S_i \le L_i^{bp} \le L_i^r$, 3) $L_i^{bp} \le S_i \le L_i^r$

In which, by defining binary variables and adding the following constraints, the cases are distinguished from each other.

$$L_i^r - L_i^{bp} \le \; M\delta_i \qquad (11.a)$$

$$L_i^{bp} - L_i^r \le \; M(1-\delta_i) \qquad (11.b)$$

$$L_i^{bp} - S_i \le \; M\eta_i \qquad (12.a)$$

$$S_i - L_i^{bp} \le \; M(1-\eta_i) \qquad (12.b)$$

Meanwhile, the following demand terms should be considered corresponding to the three cases:

1)$D_i^{net}\left[S_i \le L_i^r \le L_i^{bp}\right](1-\delta_i)$

2)$D_i^{net}[S_i \le L_i^{bp} \le L_i^r]\;\delta_i\eta_i$

3) $D_i^{net}[L_i^{bp} \le S_i \le L_i^r]\;\delta_i(1-\eta_i)$

Then, the net demand is obtained by integration of above terms into Equation 13.a:

$$D_i^{net} = \{D_i^{gross}[t+S_i; t+L_i^{bp}] - Q_i[t+L_i^r; t+L_i^{bp}]\}(1-\delta_i) + \{D_i^{gross}[t+S_i; t+L_i^r] - R_i[t+L_i^{bp}; t+L_i^r]\}(\delta_i\eta_i) + \{D_i^{gross}[t+S_i; t+L_i^r] - R_i[t+L_i^{bp}; t+L_i^r]\}\delta_i(1-\eta_i) \quad (13.a)$$

For simplification, the binary multiplications at each term are shown by OP_ℓ. Equivalently, by dividing the intervals at each term into sub-intervals and replacing OP_ℓ, Equation 13.b is obtained:

$$D_i^{net} = \{D_i^{gross}[t+S_i; t+L_i^r] + R_i[t+L_i^r; t+L_i^{bp}]\}OP_1 + \{D_i^{gross}[t+S_i; t+L_i^{bp}] + Q_i[t+L_i^{bp}; t+L_i^r]\}OP_2 + \{Q_i[t+S_i; t+L_i^r] - R_i[t+L_i^{bp}; t+S_i]\}OP_3 \quad (13.b)$$

Return to equations 3 and 5, the demand variance at each of three cases is determined based on the service time and replenishment time as follows.

$$V_{j,\ell}^{net} = \sigma^2.(SI_j + t_j - S_j) + a_{i,j}^2\sigma^2\left(S_i + t_j^{bp} - (SI_j + t_j)\right) \qquad \ell=1, j \in wbp, i\,|(i,j) \in ww \quad (14.a)$$

$$V_{j,\ell}^{net} = \sigma^2\left(S_i + t_j^{bp} - S_j\right) + (1-a_{i,j})^2\sigma^2\left(SI_j + t_j - \left(S_i + t_j^{bp}\right)\right) \qquad \ell=2, j \in wbp, i\,|(i,j) \in ww \quad (14.b)$$

$$V_{j,\ell}^{net} = (1-a_{i,j})^2\sigma^2(SI_j + t_j - S_j) + a_{i,j}^2\sigma^2\left(S_i - (S_i + t_j^{bp})\right) \qquad \ell=3, j \in wbp, i\,|(i,j) \in ww \quad (14.c)$$

It should be noted that in these equations, V_j^{net} is used instead of the classical sd_j^{net} (i.e. $sd_j^{net}(S_j, L_j^r, L_j^{bp}, L_j^{rec}) = \sqrt{V[D_j^{net}\left(S_j, L_j^r, L_j^{bp}, L_j^{rec}\right)]}$) and the term $\sum_{k\in E} sdd_k^2$ is replaced by σ^2.

Moreover, by exerting the following replacements, the previous equations can be simply used for final stages.

$$mx_i \rightarrow max_{j\,|(j,i)\in ww}\{L_{bp(i)}^r\} + t_i^{bp} \quad (15)$$

$$\theta_\alpha \rightarrow a_{j,i}$$

$$sd_i^{net} \rightarrow \sum_{k\in E} sdd_k^2$$

Now, let's return into the model which is developed in this study. In our development, as mentioned before, there are twelve permutation cases. So, to make a distinction between the outlined cases, equations 16 to 18 are introduced in a similar way as used in equations 11 and 12.

$$L_i^{rec} - L_i^{bp} \leq M\tau_i \quad (16.a)$$

$$L_i^{bp} - L_i^{rec} \leq M(1-\tau_i) \quad (16.b)$$

$$L_i^{rec} - L_i^r \leq M\omega_i \quad (17.a)$$

$$L_i^r - L_i^{rec} \leq M(1-\omega_i) \quad (17.b)$$

$$L_i^{rec} - S_i \leq M\Psi_i \quad (18.a)$$

$$S_i - L_i^{rec} \leq M(1-\Psi_i) \quad (18.b)$$

As a result, the following binary multiplication terms are developed at each of twelve permutation cases.

$$\begin{array}{llll}
1)if & S_i \leq L_i^r \leq L_i^{bp} \leq L_i^{rec} & then & (1-\delta_i)\eta_i\tau_i \\
2)if & S_i \leq L_i^r \leq L_i^{rec} \leq L_i^{bp} & then & (1-\delta_i)\eta_i(1-\tau_i)\omega_i \\
3)if & S_i \leq L_i^{rec} \leq L_i^r \leq L_i^{bp} & then & (1-\delta_i)\eta_i(1-\tau_i)(1-\omega_i)\Psi_i \\
4)if & L_i^{rec} \leq S_i \leq L_i^r \leq L_i^{bp} & then & (1-\delta_i)\eta_i(1-\tau_i)(1-\omega_i)(1-\Psi_i) \\
5)if & S_i \leq L_i^{bp} \leq L_i^r \leq L_i^{rec} & then & \delta_i\eta_i\omega_i \\
6)if & S_i \leq L_i^{bp} \leq L_i^{rec} \leq L_i^r & then & \delta_i\eta_i(1-\omega_i)\tau_i \\
7)if & S_i \leq L_i^{rec} \leq L_i^{bp} \leq L_i^r & then & \delta_i\eta_i(1-\omega_i)(1-\tau_i)\Psi_i \\
8)if & L_i^{rec} \leq S_i \leq L_i^{bp} \leq L_i^r & then & \delta_i\eta_i(1-\omega_i)(1-\tau_i)(1-\Psi_i \\
9)if & L_i^{bp} \leq S_i \leq L_i^r \leq L_i^{rec} & then & \delta_i(1-\eta_i)\omega_i \\
10)if & L_i^{bp} \leq S_i \leq L_i^{rec} \leq L_i^r & then & \delta_i(1-\eta_i)(1-\omega_i)\Psi_i \\
11)if & L_i^{bp} \leq L_i^{rec} \leq S_i \leq L_i^r & then & \delta_i(1-\eta_i)(1-\omega_i)(1-\Psi_i)\tau_i \\
12)if & L_i^{rec} \leq L_i^{bp} \leq S_i \leq L_i^r & then & \delta_i(1-\eta_i)(1-\omega_i)(1-\Psi_i)(1-\tau_i)
\end{array} \quad (19)$$

Again for simplicity, the binary multiplication terms will be shown by OC_u ($OC_1, OC_2, \ldots, OC_{12}$).

Now, by considering the net demand terms introduced in Appendix A, the demand variance at each case ($V_{j,u}^{net}$) can be obtained similarly based on the service time. For instance, the equations of the first three cases are shown as follows:

$$V_{j,u}^{net} = \sigma^2(SI_j + t_j - S_j) + (a_{i,j} + \theta_\gamma)^2\sigma^2\left(S_i + t_j^{bp} - (SI_j + t_j)\right) + \theta_\gamma^2\sigma^2(t^{rec} - (S_i + t_j^{bp}) \quad u=1,\ j \in rec \cap wbp, i \mid (i,j) \in ww \tag{20.a}$$

$$V_{j,u}^{net} = \sigma^2(SI_j + t_j - S_j) + (a_{i,j} + \theta_\gamma)^2\sigma^2\left(t^{rec} - (SI_j + t_j)\right) + a_{i,j}^2\sigma^2(S_i + t_j^{bp} - t^{rec}) \quad u=2,\ j \in rec \cap wbp, i \mid (i,j) \in ww \tag{20.b}$$

$$V_{j,u}^{net} = \sigma^2(t^{rec} - S_j) + (1 - \theta_\gamma)^2\sigma^2(SI_j + t_j - t^{rec}) + a_{i,j}^2\sigma^2(S_i + t_j^{bp} - (SI_j + t_j)) \quad u=3,\ j \in rec \cap wbp, i \mid (i,j) \in ww \tag{20.c}$$

The equations for other cases can be developed in a similar way. Now, based on the above equations, the mathematical model can be developed.

4. Mathematical Model

In this section, a bi-objective model for considering safety stocks in reverse logistics based on guaranteed service model will be presented.

$$\boldsymbol{Min}\, Z_1 = \sum_{j \in A \cup P \backslash wbp \cup rec} h_j k_j sd_j^{net\,2}\sqrt{SI_j + t_j - S_j} + \sum_{j \in wbp}\sum_{i \mid (i,j) \in ww} h_j k_j \left(\sum_{\ell=1}^{3} V_{j,\ell}^{net}\, OP_\ell\right)^{0.5} + \sum_{j \in wbp \cap rec}\sum_{i \mid (i,j) \in ww} h_j k_j \left(\sum_{u=1}^{12} V_{j,u}^{net} OC_u\right)^{0.5} + \sum_{j \in E} h_j k_j \left(\sum_{\ell=1}^{3} V_{j,\ell}^{net*}\, OP_\ell\right)^{0.5} \tag{21}$$

$$\boldsymbol{Min}\, Z_2 = max_{j \in E}\{S_j\} \tag{22}$$

$$SI_j + t_j \geq S_j \quad j \in A \cup P \cup E \tag{23}$$

$$SI_j \geq S_i \quad \forall (i,j) \in w \tag{24}$$

$$SI_j + t_j - S_j \leq Ca_j \quad j \in cap \tag{25}$$

$$Q_j = (1 - \theta_\alpha)\, D_j^{gross} \quad j \in E \tag{26}$$

$$D_j^{gross} = \sum_{i \mid (j,i) \in w} Q_i \quad j \in A \cup P \tag{27}$$

$$R_j = \sum_{i \mid (i,j) \in ww} a_{i,j} Q_i \quad j \in A \cup P \tag{28}$$

$$RR_j = \theta_\gamma \sum_{i \in E} D_i^{gross} \quad j \in rec \tag{29}$$

$$Q_j = D_j^{gross} - (R_j + RR_j) \quad j \in A \cup P \tag{30}$$

$$sd_j^{net} = \left(\frac{Q_j}{D_j^{gross}}\right) sdd_j \quad j \in E \tag{31}$$

$$sd_j^{net} = \left(\frac{Q_j}{\sum_{i \in E} Q_i}\right)\sigma^{0.5} \quad j \in A \cup P \tag{32}$$

$$mx_j = \theta_\gamma\, .\, t^{rco}.\sum_{i \in E} D_i \quad j \in E \tag{33}$$

$$SI_j + t_j - (S_i + t_j^{bp}) \leq M\, \delta_j \quad j \in (rec \cap wbp \& wbp \& E), i \mid (i,j) ww \tag{34.a}$$

$$S_i + t_j^{bp} - (SI_j + t_j) \leq M\,(1 - \delta_j) \quad j \in (rec \cap wbp \& wbp \& E), i \mid (i,j) ww \tag{34.b}$$

$$(S_i + t_j^{bp}) - S_j \leq M\eta_j \quad j \in (rec \cap wbp \& wbp \& E), i \mid (i,j) ww \tag{35.a}$$

$$S_j - (S_i + t_j^{bp}) \leq M(1 - \eta_j) \quad j \in (rec \cap wbp \& wbp \& E), i \mid (i,j) ww \tag{35.b}$$

$$t^{rec} - \left(S_i + t_j^{bp}\right) \leq M\tau_j \quad j \in rec \cap wbp, i \mid (i,j) \in ww \tag{36.a}$$

$$\left(S_i + t_j^{bp}\right) - t^{rec} \leq M(1 - \tau_j) \quad j \in rec \cap wbp, i \mid (i,j) \in ww \tag{36.b}$$

$$t^{rec} - (SI_j + t_j) \leq M\omega_j \quad j \in rec \cap wbp, i \mid (i,j) \in ww \tag{37.a}$$

$$(SI_j + t_j) - t^{rec} \leq M(1 - \omega_j) \qquad j \in rec \cap wbp, i \,|(i,j) \in ww \qquad (37.b)$$
$$t^{rec} - (S_j) \leq M\Psi_j \qquad j \in rec \cap wbp, i \,|(i,j) \in ww \qquad (38.a)$$
$$S_j - t^{rec} \leq M(1 - \Psi_j) \qquad j \in rec \cap wbp, i \,|(i,j) \in ww \qquad (38.b)$$
$$\delta_j, \eta_j, \tau_j, \omega_j, \Psi_j \in \{0,1\} \qquad j \in A \cup P \cup E \qquad (39)$$
$$SI_j, S_j, mx_j, Q_j, R_j, RR_j, sd_i^{net} \geq 0 \qquad j \in A \cup P \cup E \qquad (40)$$

The holding cost of the safety stock is minimized at the first objective function. This cost includes four terms that represent the holding cost of the safety stock with no return, with only internal returns, with both internal and external returns, and with only the final stages. The second objective function minimizes the service time. As mentioned before, in previous studies the service time is used in the model as a constraint. But in this study, we consider the service time as an objective function. In this way, we can generate a Pareto frontier solutions that shows the balance between service time and holding cost of the safety stock.

Equation 23 shows that the safety stock is needed only when the net replenishment time at each stage is positive. Equation 24 indicates that the inbound service time at each stage should be no shorter than the outbound service time of its upstream stages. Equation 25 guarantees that the processing time at each stage does not exceed the time limit of that stage. Equation 26 represents the amount of replenishment of new items at the final stages. Equation 27 denotes the gross demands at the mid and initial stages. Equations 28 and 29 show the amount of internal and external returns to the chain, respectively. Equation 30 represents the amount of replenishment at each of the initial and mid-level stages. Equations 31 and 32 show the net standard deviation of the final stages and the net standard deviation of the initial and mid-level stages, respectively. Equation 33 indicates the preparation time of recovered items at the final stages. Equations 34 to 38 determine which of the twelve permutation cases for the stages with both types of returns are active. In addition, these equations reveal which of the three basic permutation cases for the final stages and the stages with internal returns are active. Finally, equations 39 and 40 determine the state of the decision variables of the problem.

4.1. Linearization of the model

Because of the similarities in the linearization process, this process is carried out only for a couple terms (i.e. $\delta_j.\eta_j$ and $\delta_j.(1-\eta_j)$) and the process for other binary terms are not discussed. For this purpose, new variables $\delta\eta_j$ and $\delta\eta_{i1}$ are defined and consequently new constraints (41, 42) are added to the model (Glover and Wolsey, 1974).

$$\delta\eta_j \leq \delta_j \qquad j \in wbp(i), j \in E \qquad (41.a)$$
$$\delta\eta_j \leq \eta_j \qquad j \in wbp(i), j \in E \qquad (41.b)$$
$$\delta\eta_j \geq \delta_j + \eta_j - 1 \qquad j \in wbp(i), j \in E \qquad (41.c)$$
$$\delta\eta_j \geq 0 \qquad j \in wbp(i), j \in E \qquad (41.d)$$
$$\delta\eta_{i1} \leq \delta_j \qquad j \in wbp(i), j \in E \qquad (42.a)$$
$$\delta\eta_{i1} \leq (1 - \eta_j) \qquad j \in wbp(i), j \in E \qquad (42.b)$$
$$\delta\eta_{i1} \geq \delta_j - \eta_j \qquad j \in wbp(i), j \in E \qquad (42.c)$$
$$\delta\eta_{i1} \geq 0 \qquad j \in wbp(i), j \in E \qquad (42.d)$$

5. Computational results

The model has been solved for an example of supply chain in an electronic product with 18 stages, as shown in Fig. 1. In this example, stages 15 to 18 are the final stages that are directly faced with customer demands. The service level at all stages is assumed to be equal to 0.99. The returns are shown with bidirectional arrows. The return rate from stage 3 to stage 2 and from stage 12 to stage 8 is equal to 0.1. The processing time on by-products at stages 2 and 8 is 2 and 4, respectively. Besides, only stage 14 has the time limit in the size of 30 units (i.e. Ca_{14}=30). The rest of the input data is shown in Tables 1 to 3. It should be noted that some parts of data are gathered from historical data records and the other parts, which are not accessible, are collected through interviews with experienced experts in the field.

Table 1. Process time and holding cost of the stages in the real example

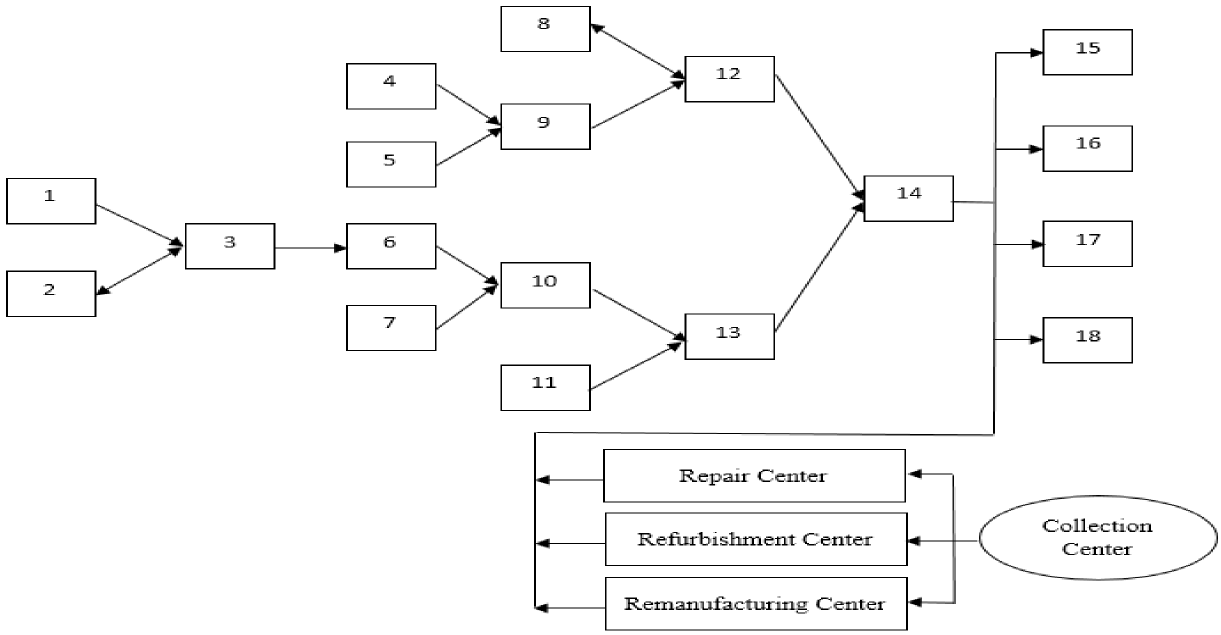
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
t_j	5	5	7	8	6	10	8	6	10	6	7	5	6	8	8	8	10	9
h_j	35	15	30	45	35	20	50	20	45	60	40	20	35	45	63	71	65	69

Table 2. Demand, Standard deviation of demand for the real example

stap	**15**	**16**	**17**	**18**
D_j^{gross}	250	400	500	**350**
sdd_j	50	80	100	**70**

Table 3 Other input information of the real example

M	b	θ_α	θ_β	θ_γ	t^{rec}	t^{rco}
10000	0.112	0.084	0.014	0.014	11	0.38



The model is coded in GAMS 24.1.2 on a PC with Intel Quad Core 2.2 GHz and 6 GB RAM. BARON (version 12.3.3) is used for solving the model and presenting the computational results. The computation time for each run in the developed cases took about four minutes on average.

Since the model is bi-objective, the ε-constraint method has been used, which generates efficient solutions under an appropriate runtime. Table 4 depicts the changes in the first objective function regarding the different ε values for two general situations. The first situation is a state in which both types of returns, i.e. internal and external, are considered. In the second situation, only internal returns are considered. The range of ε value is obtained by optimizing each objective function separately, that is $(\{(Z_1, Z_2) = (251826,0)\})$ and $(\{(Z_1, Z_2) = (29357,52)\})$ respectively. Since Z_2 changes in the interval (0, 52), the ε_2 values are defined by setting 26 steps for both situations. The results are reported in Table 4, in which for eachε_2, the first value corresponds to the first situation ($Z_{1,developed}$) and the second value corresponds to the second situation ($Z_{1,primary}$).

Table 4. Values of the first objective function regarding the changes in ε values

ε_2	Z_1		ε_2	Z_1	ε_2	Z_1	ε_2	Z_1
0	$Z_{1,developed}$	251826	14	110819	28	93188	42	**46951**
	$Z_{1,primary}$	263754		111668		83018		**26508**
2	$Z_{1,developed}$	235678	16	109610	30	90991	44	**45434**
	$Z_{1,primary}$	247471		108495		79644		**24108**
4	$Z_{1,developed}$	215998	18	108031	32	88608	46	**43601**
	$Z_{1,primary}$	227874		105193		76113		**21384**
6	$Z_{1,developed}$	191287	20	106154	34	84202	48	**41286**
	$Z_{1,primary}$	203342		101743		70414		**18152**
8	$Z_{1,developed}$	144543	22	98734	36	78914	50	**38056**
	$Z_{1,primary}$	157136		92371		63787		**13941**
10	$Z_{1,developed}$	116458	24	97057	38	60379	52	**29357**
	$Z_{1,primary}$	124464		89367		43125		**3744**
12	$Z_{1,developed}$	111320	26	95208	40	54707		
	$Z_{1,primary}$	114725		86253		35867		

Meanwhile, Fig. 2 represents the optimal Pareto frontier with and without considering the external returns. As shown, for smaller values of ε_2 (i.e. 0–10), the first objective function is significantly reduced, but after $\varepsilon_2 = 12$, the changes are very slight. This behavior can be interpreted according to the less influential of ε_2 at larger values. In other words, by increasing ε_2, the effect of customer service time in the model is decreased. Also, the first objective function, which is the holding cost of safety stock, is less decreased. In fact, for service time values greater than 12, there is enough time to replenish the items through a regular source. Therefore, considering the external returns in the model would only lead to an increase in the amount of holding cost of the safety stock, without having any effect on the service time. Therefore, the holding cost of the safety stock would be increased.

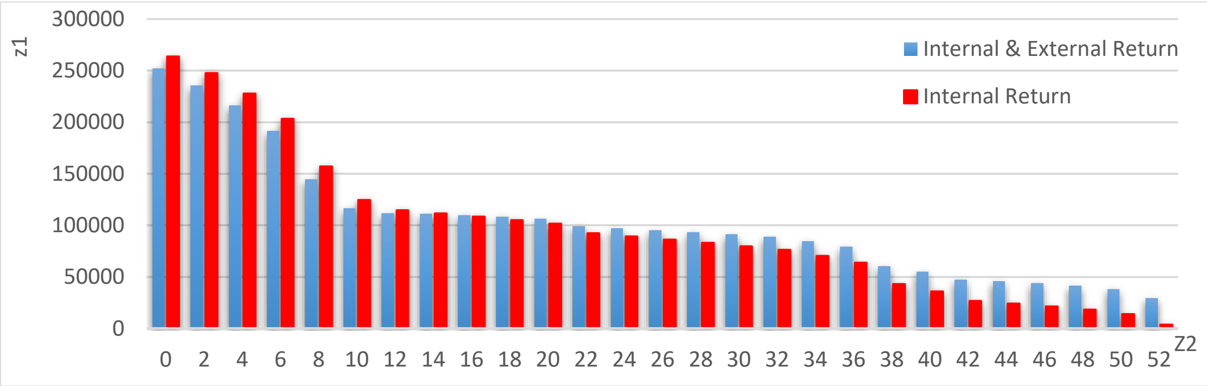


Figure 2. Pareto frontier solutions of the real example

6. Conclusions

This study aimed at developing a bi-objective optimization model for minimizing the holding cost of safety stock and minimizing the service time, using the guaranteed service model used for reverse logistics (RGSM) by considering both internal and external returns simultaneously. To this end, with extending the previous models, in which only the internal returns were investigated as recovery activity, the effect of remanufacturing, refurbishing, and repairing activities were also considered for the external returned items. Meanwhile, considering the time required for replenishment from outside of the chain, the new cases for replenishment time permutations were developed.

Typically in the literature, it is assumed that the delivery of products to the customers would happen immediately. In other words, the service time of the final stages in supply chain is equal to zero. But, consideration of such assumptions generally leads to a large inventory in the supply chain. In this paper, this problem was solved by releasing the service time constraint and adding the minimization of the service time as the second objective function to the model. For this purpose, a bi-objective model was developed and the ε-constraint method was used to solve it. Then, the model was examined in a real-world problem of an electronic product supply chain and the results are shown in the form of Pareto solution set. Since higher levels of customer satisfaction (i.e. lower levels of service times) obviously rises costs for decision-makers and stakeholders, the Pareto frontier solutions help managers and decision-makers to simultaneously manage their costs and customers satisfaction with the awareness of the holding cost of safety stock for different amounts of service time levels. This research can be extended by classifying the customers based on their information and requirements and solving the classified model by coordinating supply chain parties. Moreover, in the solution approach, heuristic and meta-heuristic algorithms can be used to decrease the runtime of the model in large scale problems.

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Appendix A
Extended Relationships.

1) $S_i \leq L_i^r \leq L_i^{bp} \leq L_i^{rec}$

$$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^{rec}] - Q_i[t+L_i^r, t+L_i^{rec}] - R_i[t+L_i^{bp}, t+L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^r] + (D_i^{gross} - Q_i)[t+L_i^r, t+L_i^{bp}] + (D_i^{gross} - Q_i - R_i)[t+L_i^{bp}, t+L_i^{rec}] \quad \text{(A.1)}$$

2) $S_i \leq L_i^r \leq L_i^{rec} \leq L_i^{bp}$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^{bp}] - Q_i[t+L_i^r, t+L_i^{bp}] - RR_i\,[t+L_i^{rec}, t+L_i^{bp}] \quad = D_i^{gross}[t+S_i, t+L_i^r] + (D_i^{gross} - Q_i)\,[t+L_i^r, t+L_i^{rec}] + (D_i^{gross} - Q_i - RR_i)\,[t+L_i^{rec}, t+L_i^{bp}]$ (A.2)

3) $S_i \le L_i^{rec} \le L_i^r \le L_i^{bp}$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^{bp}] - RR_i\,[t+L_i^{rec}, t+L_i^{bp}] - Q_i[t+L_i^r, t+L_i^{bp}] = D_i^{gross}[t+S_i, t+L_i^{rec}] + (D_i^{gross} - RR_i)\,[t+L_i^{rec}, t+L_i^r] + (D_i^{gross} - RR_i - Q_i)\,[t+L_i^r, t+L_i^{bp}]$ (A.3)

4) $L_i^{rec} \le S_i \le L_i^r \le L_i^{bp}$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^{bp}] - RR_i\,[t+L_i^{rec}, t+L_i^{bp}] - Q_i\,[t+L_i^r, t+L_i^{bp}] = -RR_i[t+L_i^{rec}, t+S_i] + (D_i^{gross} - RR_i)\,[t+S_i, t+L_i^r] + (D_i^{gross} - RR_i - Q_i)\,[t+L_i^r, t+L_i^{bp}]$ (A.4)

5) $S_i \le L_i^{bp} \le L_i^r \le L_i^{rec}$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^{bp}] + (D_i^{gross} - R_i)\,[t+L_i^{bp}, t+L_i^r] + (D_i^{gross} - R_i - Q_i)\,[t+L_i^r, t+L_i^{rec}]$ (A.5)

6) $S_i \le L_i^{bp} \le L_i^{rec} \le L_i^r$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^{bp}] + (D_i^{gross} - R_i)\,[t+L_i^{bp}, t+L_i^{rec}] + (D_i^{gross} - R_i - RR_i)\,[t+L_i^{rec}, t+L_i^r]$ (A.6)

7) $S_i \le L_i^{rec} \le L_i^{bp} \le L_i^r$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = D_i^{gross}[t+S_i, t+L_i^{rec}] + (D_i^{gross} - RR_i)\,[t+L_i^{rec}, t+L_i^{bp}] + (D_i^{gross} - RR_i - R_i)\,[t+L_i^{bp}, t+L_i^r]$ (A.7)

8) $L_i^{rec} \le S_i \le L_i^{bp} \le L_i^r$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = -RR_i[t+L_i^{rec}, t+S_i] + (D_i^{gross} - RR_i)[t+S_i, t+L_i^{bp}] + (D_i^{gross} - RR_i - R_i)\,[t+L_i^{bp}, t+L_i^r]$ (A.8)

9) $L_i^{bp} \le S_i \le L_i^r \le L_i^{rec}$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = -R_i[t+L_i^{bp}, t+S_i] + (D_i^{gross} - R_i)\,[t+S_i, t+L_i^r] + (D_i^{gross} - R_i - Q_i)\,[t+L_i^r, t+L_i^{rec}]$ (A.9)

10) $L_i^{bp} \le S_i \le L_i^{rec} \le L_i^r$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = -R_i[t+L_i^{bp}, t+S_i] + (D_i^{gross} - R_i)\,[t+S_i, t+L_i^{rec}] + (D_i^{gross} - R_i - RR_i)\,[t+L_i^{rec}, t+L_i^r]$ (A.10)

11) $L_i^{bp} \le L_i^{rec} \le S_i \le L_i^r$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = -R_i[t+L_i^{bp}, t+L_i^{rec}] + (-R_i - RR_i)\,[t+L_i^{rec}, t+S_i] + (D_i^{gross} - R_i - RR_i)\,[t+S_i, t+L_i^r]$ (A.11)

12) $L_i^{rec} \le L_i^{bp} \le S_i \le L_i^r$

$D_i^{net}[S_i, L_i^r, L_i^{bp}, L_i^{rec}] = -RR_i[t+L_i^{rec}, t+L_i^{bp}] + (-RR_i - R_i)\,[t+L_i^{bp}, t+S_i] + (D_i^{gross} - RR_i - R_i)\,[t+S_i, t+L_i^r]$ (A.12)

Appendix B

Table 6 Review of the Literature

Reference	Supply Chain Network						Modeling Assumptions							Solution Approach		Objective Function			
								Demand		Service Times		Lead Times				Number		Type	
	Serial	Assembly	Distribution	Spanning Tree	General Acyclic	General Cyclic	Capacity Constraints	Stationary	Non-Stationary	Constant	Customer-Specific	Constant	Stochastic	Exact Solution	Approximate Solution	Single-Objective	Bi-Objective	Concave	Arbitrary
Simpson (1958)	■							■		■		■		■		■		■	
Inderfurth (1991)			■					■		■		■		■		■		■	
Graves and Willems (2000)				■				■		■		■		■		■		■	
Minner (2000)	■	■	■		■	■		■			■		■	■	■	■		■	
Minner (2001)						■		■		■		■		■		■		■	
Magnanti (2006)					■		■	■		■		■		■		■		■	
Sitompul et al. (2008)	■						■	■		■		■		■		■		■	
Graves and Willems (2008)				■					■	■		■		■		■		■	
Nepal et al (2011)		■						■		■		■					■		■
Li and Jiang (2012)					■		■	■		■		■			■	■		■	
Li et al (2013)	■	■						■		■		■		■		■		■	
Eruguz(2014)					■			■		■		■			■	■			■
Ni and Shu (2015)					■		■	■		■		■		■	■	■		■	
Grahl et al (2016)					■			■			■	■			■	■			■
Graves and Schoenmeyr (2016)	■						■	■		■		■		■		■		■	
Hua and Willems (2016)	■							■		■		■		■		■		■	
This Study						■	■	■		■		■		■			■	■	