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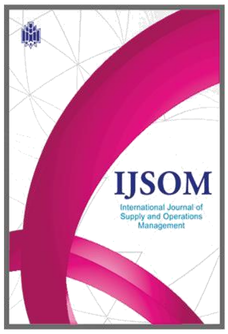

A General Methodology for Reducing Computing Times of Road Network Design Algorithms

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Abstract
In this paper, a general methodology for reducing computing times in procedures for solving road network design problems is proposed. Such problems which are studied extensively in the related literature concern the design of road networks, in terms of flow directions, capacity expansion and signal settings in urban contexts, and in terms of link addition and capacity expansion in rural contexts. The solution to them is almost always formulated as a bi-level model, where the upper level operates on the network design decision variables while the lower level estimates the equilibrium traffic flows, which must be known in order to determine objective function values. Computing times required for calculating equilibrium traffic flows at each iteration of the network design procedure significantly affect the total solution time. Hence, any reduction in computing times of the lower level, which has to be implemented numerous times at any step of the upper-level algorithm, allows the global computing time to be considerably reduced. In this context, the methodology proposed herein seeks to reduce computing times of the traffic assignment problem and in turn of the whole network design procedure, acting on the traffic flows adopted in the initialisation phase of the assignment algorithm. Obviously, this approach is feasible only if the topology of the network configuration remains unchanged and therefore only if the network design decision variables are limited to capacity expansion in rural contexts, or signal settings and capacity expansion in urban contexts. The proposed approach is tested on a real-scale case study: the rural road network of Vilnius County (Lithuania). Preliminary results underline the feasibility of the proposal and a significant reduction in computing times -- up to 80% -- compared to traditional approaches.

Keywords: Road network design problem; Bi-level optimisation model; Rural road network analysis; Road capacity expansion; Computing times reduction.

1. Introduction

Road network design both in rural and urban contexts is important for achieving several transport planning objectives such as travel time reduction, accessibility improvement, as well as noise and air pollution reduction. In the urban context, the problem often concerns only the management of existing roads (directions of links, signal setting optimisation, parking lot allocation, pricing). Less commonly, there is the possibility of adding new roads to the network or improving existing ones (capacity expansion). By contrast, in the rural context, the problem usually lies in the possibility of adding new roads to the network and improving the performance of existing roads. At times, only improvement interventions are considered (capacity expansion).

These problems are known in the literature as road network design problems (RNDPs) and are usually schematised with discrete, continuous or mixed optimisation models, depending on the kind of decision variables. Such optimisation problems, even in the simplest cases, are very difficult to solve: the variables are numerous and sometimes heterogeneous, some constraints are non-linear and others inexpressible in a closed form, the problem is often combinatorial, and the feasible solutions are so numerous that it is impossible to adopt, except for toy networks, exact -

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solution methods or exhaustive approaches. Indeed, almost all RNDPs are NP-hard and it is possible to find only local optimal solutions with heuristic or meta-heuristic algorithms, usually with high computing times.

A crucial point in these problems is assignment constraint, which is the main reason behind the high computing times. This constraint links traffic flows, which are considered 'descriptive' variables since it is impracticable to directly operate upon them, to decision variables: a configuration of equilibrium traffic flows corresponds to each configuration of the network. To calculate the performance of each network configuration it is necessary to compute the equilibrium traffic flows by means of equilibrium traffic assignment algorithms. Such procedures are well consolidated in the literature, but may prove extremely time-consuming since they have to be repeated several times as subroutines of the network design solution algorithms.

This paper contributes to the current literature by proposing a general method for reducing computing times of network design solution procedures. Indeed, since RNDPs may be formulated as bi-level optimisation problems where the lower level represents the assignment problem which has to be implemented at any objective function evaluation, any reduction in computing times of the single lower level phase may allow a reduction in global times of the overall procedure. In the traditional approach, assignment algorithms generally start from an initial solution where the traffic flows are all equal to zero. This paper proposes to use a different starting point: the equilibrium traffic flows estimated in the previous iteration of the network design procedure. Obviously, this innovative methodology is applicable only if the network topology remains unchanged throughout the iterations of the design algorithm and hence when the decision variables are limited to road improvements in rural contexts (no changes in road directions and no new links are feasible). This network design problem is common when the decision maker has to choose where to invest public money for improving the performance of a rural network. In this case, the decision variables are usually binary and refer to each road; the value 0 indicates that the road is unimproved, while the value 1 that the road is improved.

However, in order to show the main advantages in terms of computing time reduction of the proposed methodology, it was implemented and tested on a real-scale network, and some theoretical aspects were studied and explored.

The paper is structured as follows: Section 2 explores the literature on road network design problems; Section 3 provides the analytical formulation of the proposed approach; an application to a real dimension network is described in Section 4; finally, conclusions and research prospects are summarised in Section 5.

2. Background

Road network design problems (RNDPs) belong to the class of transportation network design problems (TNDPs) which include all problems where decisions have to be taken to change and improve transportation systems. These problems can be classified in several ways, according to the transportation system to design (road network, mass-transit network, multimodal network), the decision variables (topology, performance, pricing), the kind of variables (continuous, discrete, mixed) and the assumption on travel demand (rigid, elastic). The literature on TNDPs is so extensive that a review would merit a paper on its own. What follows is limited to the analysis of some significant contributions, focusing mainly on RNDPs.

Literature reviews on TNDPs can be found in Magnanti and Wong (1984), while Feremans et al. (2003) focused on the formal generalisation of network design, including transportation-related problems. Chen et al. (2011) explored TNDPs under uncertainty. Yang and Bell (1998) reviewed models and algorithms for RNDPs, and Farahani et al. (2013) focused on urban road networks. Guihaire and Hao (2008), instead, reviewed transit network design problems. In D'Acierno et al. (2013), some applications of metaheuristic algorithms to large-scale transportation network design problems are summarised. Xu et al. (2016) reviewed the sustainable road network design problem, considering three dimensions of sustainable development (economic, environmental and social). Interaction with land use and the sustainability aspects of the problem were also considered by Szeto et al. (2015), while Fontaine and Minner (2018) studied the TNDP for minimising the risk of shipment of hazardous goods on roads.

Focusing on RNDPs, it is possible to identify two main classes: urban (URNDPs) and rural (RRNDPs). In the former case, the decision variables are usually (i) the directions of links and, in some cases, also the insertion of new roads or capacity expansion and (ii) signal settings (Cantarella et al. 2006; Gallo et al., 2010; Khooban et al., 2015). Some problems also consider parking variables (Cantarella and Vitetta, 2006) or road tolls (Yang, 1997; Dimitriou et al., 2008). The decision variables are at times limited only to signal settings (Cascetta et al., 2006; Cantarella et al., 2015a; Memoli et al., 2017). In URNDPs, the objective function is often given only by total user costs; where applicable, construction costs are also considered. In some cases, other objective functions have been proposed, such as maximisation of consumer surplus, maximisation of reserve capacity, and minimisation of distances travelled.

By contrast, RRNDPs usually consider the insertion (i.e. the construction) of new roads or the capacity expansion of some roads in an existing network as decision variables. In this case, besides user costs, construction/maintenance costs always have to be considered in the objective function. Examples of these problems can be found in Billheimer and Gray (1973), Abdulaal and Le Blanc (1979), Solanki et al. (1998), Drezner and Wesolowsky (2003), Chiou (2005), Gao et al.

(2005), Poorzahedy and Abulghasemi (2005), Poorzahedy and Rouhani (2007), Lo and Szeto (2009), Obreque et al. (2010), Wang and Lo (2010), Babazadeh et al. (2011), Ukkusuri and Patil (2011), Gallo et al. (2012), Cao et al. (2013), Wang et al. (2013), Wang et al. (2015), Liu and Wang (2015, 2016), Bagloee and Sarvi (2018), and Di et al. (2018). The main features of these papers are summarised in Table 1. Most consider discrete or binary decision variables (insertion of new roads or capacity expansion), while others assume continuous variables (capacity expansion). As for solution methods, few papers attempt exact approaches (branch-and-bound), with almost all proposing heuristic or metaheuristic algorithms.

Other aspects of the problem are treated elsewhere. A time-dependent discrete network design problem was proposed by Hosseininasab and Shetab-Boushehri (2015). Tan et al. (2016) examined the problem in terms of different ownership regimes (free, public toll and private toll roads) and also investment returns. Bagloee et al. (2016) studied the problem *vis-à-vis* multimodal and multiclass traffic flows. Haas and Bekhor (2017) formulated a bi-level multi-objective optimisation model for also considering the maximisation of road safety within the problem. Xu et al. (2017) formulated a joint road toll pricing and capacity expansion network design problem. Di et al. (2018) optimised the network so as to maximise flow-based accessibility.

The method described in this paper has not, to our knowledge, been proposed elsewhere. It can reduce the computing time for estimating the value of the objective function. All heuristic and meta-heuristic RNDP solution algorithms are based on the examination of solutions: the more the solutions examined, the better the final result. Therefore, by reducing the computing time of examining each solution, a better result can be obtained in the same total computing time.

Table 1 . Main literature on RRNDPs

Paper	Design subject	Decision variables	Algorithm
Billheimer and Gray (1973)	Link addition	Binary	Heuristic
Abdulaal and Le Blanc (1979)	Capacity expansion	Continuous	Heuristic
Solanki et al. (1998)	Link addition	Binary	Heuristic
Drezner and Wesolowsky (2003)	Link addition	Discrete	Heuristic and metaheuristic
Chiou (2005)	Capacity expansion	Continuous	Gradient-based
Gao et al. (2005)	Link addition	Binary	Heuristic
Poorzahedy and Abulghasemi (2005)	Link addition and capacity expansion	Binary	Metaheuristic
Poorzahedy and Rouhani (2007)	Link addition	Binary	Metaheuristic
Lo and Szeto (2009)	Capacity expansion	Continuous	Gradient-based
Obreque et al. (2010)	Link addition	Binary	Branch-and-Cut
Wang and Lo (2010)	Capacity expansion	Continuous	Mixed-integer linear program
Babazadeh et al. (2011)	Link addition and capacity expansion	Binary	Metaheuristic
Ukkusuri and Patil (2011)	Capacity expansion	Continuous	Interior point method
Gallo et al. (2012)	Capacity expansion	Binary	Metaheuristic
Cao et al. (2013)	Link addition	Binary	Metaheuristic
Wang et al. (2013)	Capacity expansion	Binary	Heuristic
Wang et al. (2015)	Link addition and optimal capacity	Mixed-integer	Heuristic
Liu and Wang (2015)	Capacity expansion	Continuous	Heuristic
Liu and Wang (2016)	Link addition	Binary	Branch-and-Bound and heuristic
Bagloee and Sarvi (2018)	Link addition	Binary	Hybrid exact-heuristic
Di et al. (2018)	Link addition	Binary	Heuristic

3. Optimisation model and the proposed approach

Any transportation network design problem (TNDP) can be formulated as a constrained optimisation problem. In this problem, the assignment constraint assumes an important role since it links user flows (descriptive variables) to decision variables (network configuration). In light of the above, the assignment constraint is almost always inexpressible in a closed form, requiring an algorithm for calculating user flows. The problem is formulated in bi-level terms, where the upper level concerns the decision variables and the lower level the solution of the assignment problem. This bi-level approach can be found in most papers dealing with TNDPs (see also Section 2). In the literature, the problem is usually analysed to refer to one or, less often, more simulation time windows. In this case, transportation demand changes with time (e.g. among hours of the day, days of the week) and the TNDP can be generalised as follows:

$$\hat{y} = \arg\min_{y \in S_y} \int_0^T Z\left(f(\tau, y), y\right) \cdot d\tau \tag{1}$$

subject to:

$$f(\tau, y) = \Lambda\left(f(\tau, y), f(\tau-1, y), \ldots, f(0, y)\right) \quad \forall \tau \in T \tag{2}$$

$$\Gamma\left(y, f(\tau, y)\right) \le B(\tau) \quad \forall \tau \in T \tag{3}$$

where $\hat{y}$ is the optimal value of vector y; y is the vector of decision variables to be optimised (i.e. designed); S_y is the feasibility set of y; T is the analysed time period (such as the life-cycle of the designed intervention or the plan duration); τ is the generic time period (or time interval); $f(\cdot)$ is the vector of passenger or vehicular flows; $Z(\cdot)$ is an integrand function whose primitive (i.e. antiderivative) function over the time interval T is the objective function to be minimised; $\Lambda(\cdot)$ is the simulation function providing (passenger or vehicular) flows associated with period τ; $\Gamma(\cdot)$ is a function expressing the budget consumed in the period τ; and $B(\cdot)$ is a function expressing the budget constraint in the period τ.

Constraint (2) imposes the coherence of transportation system performance and network flows. Indeed, network performance depends on the design solution considered (i.e. values of vector y), network flows in period τ and network flows in the previous time periods. Likewise, network flows in period τ depend on network performances since users make mobility choices based on them.

Constraint (3) represents the budget constraint expressed in monetary terms and/or resource terms (such as the number of facilities or vehicles to be used). Expenditure depends on the intervention solution y adopted. Moreover, in some cases (such as the fleet sizing of public transport services), the required budget depends on passenger flows in any period τ.

However, it is worth noting that in some conditions, resources which were left untapped during one period may be used in the next periods. In such contexts, equation (3) may be expressed as:

$$\Gamma\left(y, f(\tau, y)\right) \le \int_0^t B(x) \cdot dx - \int_0^{t-\tau} \Gamma\left(y, f(x, y)\right) \cdot dx \qquad \text{with } 0 \le \tau \le t \le T \tag{4}$$

Under the hypothesis that travel demand and network features are constant over a reference period of sufficient length concerning the journey times of the system, it is possible to analyse the transportation system by considering a set of stationary conditions. Hence the problem may be formulated as follows:

$$\hat{y} = \arg\min_{y \in S_y} \sum_0^T Z\left(f(\tau, y), y\right) \cdot \Delta\tau \tag{5}$$

subject to:

$$f(\tau, y) = \Lambda\left(y, f(\tau, y)\right) \quad \forall \tau \in T \tag{6}$$

$$\Gamma\left(y, f(\tau, y)\right) \le B(\tau) \quad \forall \tau \in T \tag{7}$$

where $\Delta\tau$ represents the duration of period τ and constraint (6) is called assignment constraint.

Obviously, in the case of resource re-use, equation (4) may be rewritten as follows:

$$\Gamma\left(y, f(\tau, y)\right) \le \sum_0^t B(x) \cdot dx - \sum_o^{t-\tau} \Gamma\left(y, f(x, y)\right) \cdot \Delta x \qquad \text{with } 0 \le \tau \le t \le T \tag{8}$$

Since it is generally impossible to assert the convexity of the objective function or that the objective function is manifestly non-convex, solution algorithms for the upper level (i.e. equation 1 or, equivalently, 5) are often based on heuristics or metaheuristic algorithms, which explore a large number of feasible (i.e. within the set S_y) solutions satisfying budget constraints (i.e. equations 3 and 7 or, equivalently, equations 4 and 8). These algorithms are based on exploratory rules which identify the subsequent solutions to be analysed or the termination condition depending on the value assumed by the objective function.

Calculation of the objective function requires the solution of the simulation problem (2) or, equivalently, (6) for any period τ. In the case of stationary conditions, the simulation problem (6), namely the assignment problem, may be formulated as a fixed-point problem whose theoretical property concerning the existence and uniqueness of the solution may be proved under generally verified conditions (Cantarella, 1997; Cascetta, 2009; D'Acierno et al., 2011).

Since the fixed-point problem is based on transcendental equations, it is impracticable to solve it in a closed form and it is therefore necessary to adopt recursive algorithms based on a sequence of network loading flows converging to the equilibrium solution (Sheffi and Powell, 1981; Daganzo, 1983; Cantarella, 1997; D'Acierno et al., 2006; Cantarella et al., 2015b). These algorithms terminate when two successive iterations provide similar results whose difference is lower than a prefixed threshold, ε, which means that the equilibrium condition is almost met.

For the above reasons, it is necessary to highlight the difference between the theoretical value of the fixed-point solution and its numerical value obtained by means of recursive algorithms. Indeed, although the fixed-point solution exists and is unique, its numerical value may be non-unique due to the recursive algorithm adopted (i.e. different algorithms or the same algorithm adopted with different initial conditions may yield different numerical values of the fixed-point problem solution).

Hence, theoretically, having fixed the value of the decisional variable $\boldsymbol{y}$, it is possible to univocally calculate the equilibrium flows for any period τ (i.e. lower level problem), that is:

$$\forall\, \boldsymbol{y} \in \boldsymbol{S}_y \;\; \exists!\; \boldsymbol{F}^*(\boldsymbol{y}) = \left[\boldsymbol{f}^*(\tau\,,\boldsymbol{y})\right]^{\mathrm{T}} \tag{9}$$

where $\boldsymbol{F}^*$ is a vector whose elements are equilibrium flow vectors, $\boldsymbol{f}^*$, for each period τ.

Likewise, since the objective function is a single-value function, for each value of vector $\boldsymbol{F}^*$, it is possible to univocally determine its value, that is:

$$\forall\, \boldsymbol{F}^*(\boldsymbol{y}) \;\exists!\; \Omega\left(\boldsymbol{F}^*(\boldsymbol{y})\right) = \sum_o^T Z\left(\boldsymbol{f}^*(\tau\,,\boldsymbol{y}),\,\boldsymbol{y}\right)\cdot \Delta\tau \tag{10}$$

where $\Omega(\cdot)$ is the objective function to be minimised.

Thus, by combining equations (9) and (10), the following is obtained:

$$\forall\, \boldsymbol{y} \in \boldsymbol{S}_y \;\; \exists!\; \Omega\left(\boldsymbol{F}^*(\boldsymbol{y})\right) = \sum_o^T Z\left(\boldsymbol{f}^*(\tau\,,\boldsymbol{y}),\,\boldsymbol{y}\right)\cdot \Delta\tau \tag{11}$$

The use of assignment recursive algorithms, which are based on termination thresholds, means that the computational value of vector $\boldsymbol{F}^*$ is affected by the initialisation value $\boldsymbol{F}^0$ and the termination threshold ε. So, equations (9) and, consequently, (11) have to be changed to:

$$\forall\, \boldsymbol{y} \in \boldsymbol{S}_y \;\; \exists!\; \boldsymbol{F}^*\left(\boldsymbol{y}\,,\boldsymbol{F}^0\,,\varepsilon\right) = \left[\boldsymbol{f}^*\left(\tau\,,\boldsymbol{y}\,,\,\boldsymbol{f}^0\,,\varepsilon\right)\right]^{\mathrm{T}} \tag{12}$$

$$\forall\, \boldsymbol{y} \in \boldsymbol{S}_y \;\; \exists!\; \Omega\left(\boldsymbol{F}^*\left(\boldsymbol{y}\,,\boldsymbol{F}^0\,,\varepsilon\right)\right) = \sum_o^T Z\left(\boldsymbol{f}^*\left(\tau\,,\boldsymbol{y}\,,\,\boldsymbol{f}^0\,,\varepsilon\right),\,\boldsymbol{y}\right)\cdot \Delta\tau \tag{13}$$

where $\boldsymbol{F}^0 = \left[\boldsymbol{f}^0\right]^{\mathrm{T}}$.

One of the most commonly used algorithms for solving the assignment problem is the Method of Successive Averages (MSA); this algorithm starts from an initial solution where all flows equal 0 (null vector), $\boldsymbol{f}^0 = \boldsymbol{0}$, for each period τ. This assumption means that once the termination threshold is fixed, for any feasible solution the computational value of the objective function $\Omega(\cdot)$ is univocally determined.

Although the use of this initial (null) flow vector allows the value of the objective function in an NDP to be determined univocally, the corresponding computing times could be high in real-scale networks since this assignment procedure has to be repeated at each iteration of the network design algorithm, often thousands of times.

Focusing on the convergence of the MSA assignment algorithm (the case of a two-link network is reported in Figure 1), two phases may be identified:

- A first phase, where initial flows approach the equilibrium solution (*approach phase*);
- A second phase, where flows are slightly modified to reach the equilibrium condition (*tuning phase*).

If the network design does not affect the topology but only some performance variables of roads (e.g. free-flow speed, capacity), the number of links is constant and the dimension of vector $\boldsymbol{f}$ is independent of decisional variable $\boldsymbol{y}$; all RRNDPs where the subject of design is road capacity expansion fall within this case.

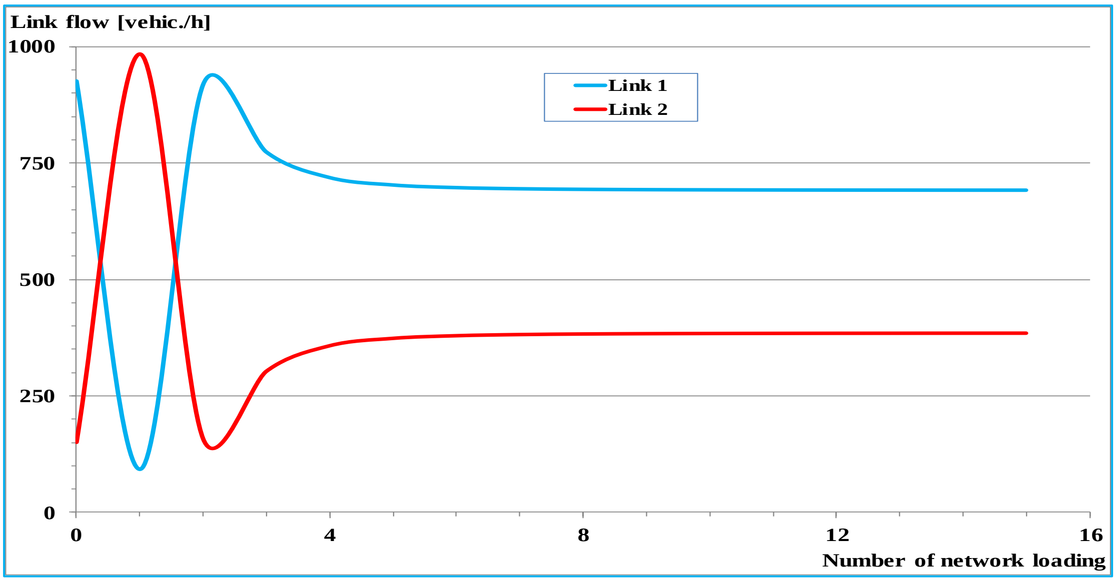

Figure 1. Convergence of network flows in a two-link network

The proposed approach is intended to exploit this feature of the problem in order to reduce the computing times of network design solution algorithms. For this purpose, at each iteration of the network design solution algorithm, except the first, the equilibrium traffic flows obtained at the end of the previous iteration are used as initial flows in the next assignment procedure. Operating in this way, a reduction in MSA iterations and in turn a significant benefit for global computing times in the network design procedure are expected. Naturally, this approach is applicable with regard to the same time interval τ. The actual benefits of this approach may be evaluated with experimental tests which will be performed in the next section.

A technical limit of the proposed approach is the evaluation of the network design objective function. Indeed, the objective function strictly depends on the equilibrium traffic flows which can assume different values (albeit very close) if the MSA algorithm starts from different initial values. Therefore, the same value of decisional variable $\boldsymbol{y}$, considered in different iterations of the solution algorithm, may yield different, albeit probably close, values of the objective function (because of the difference between the theoretical and numerical value of the fixed-point solution), that is:

$$\boldsymbol{y}_k = \boldsymbol{y}_h \quad \Rightarrow \quad \Omega\left(\boldsymbol{F}^*\left(\boldsymbol{y}_k\right)\right) \neq \Omega\left(\boldsymbol{F}^*\left(\boldsymbol{y}_h\right)\right) \tag{14}$$

This is, however, due only to the convergence threshold used for ending the algorithm: the assignment algorithm ends when the differences between flows of two successive iterations are lower than the fixed threshold. Theoretically speaking, instead, since it may be stated that the equilibrium solution exists and is unique (Cantarella, 1997) under some quite mild assumptions, the theoretical solution should be the same whatever the starting link flow vector.

Based on the above considerations, the authors proposed to solve the network design problem in two stages: (a) adopting the proposed method for calculating the equilibrium traffic flows at each network design algorithm iteration; (b) restarting the network design algorithm from the solution generated in the previous stage using the classical approach, which assumes that the initial flows inside the assignment algorithm equal zero.

4. Numerical results

The effectiveness of the proposed approach in reducing network design computing times was tested in a real-scale case study. The study area is Vilnius County (Lithuania), which consists of eight municipalities (see Figure 2).

The area was partitioned into 235 traffic zones (see Figure 3), using the grid-like division provided by the Lithuanian National Institute of Statistics. Zoning was obtained by using the 2.5 km grid in the Vilnius (city) area and the 10.0 km grid in the rest of the study area. Note that, unlike other countries usually providing non-regular territorial portions (census zones), in this case the territory is divided into squares and the socio-economic data are attributed to such elements.

Our analysis specifically focused on the rural road network in the study area. The road network model was implemented using graph theory starting from the database of the road features: it represents 5,871 km of rural roads using 590 nodes (blue points), 250 centroids (red points), and 1,980 road links (see Figure 4).

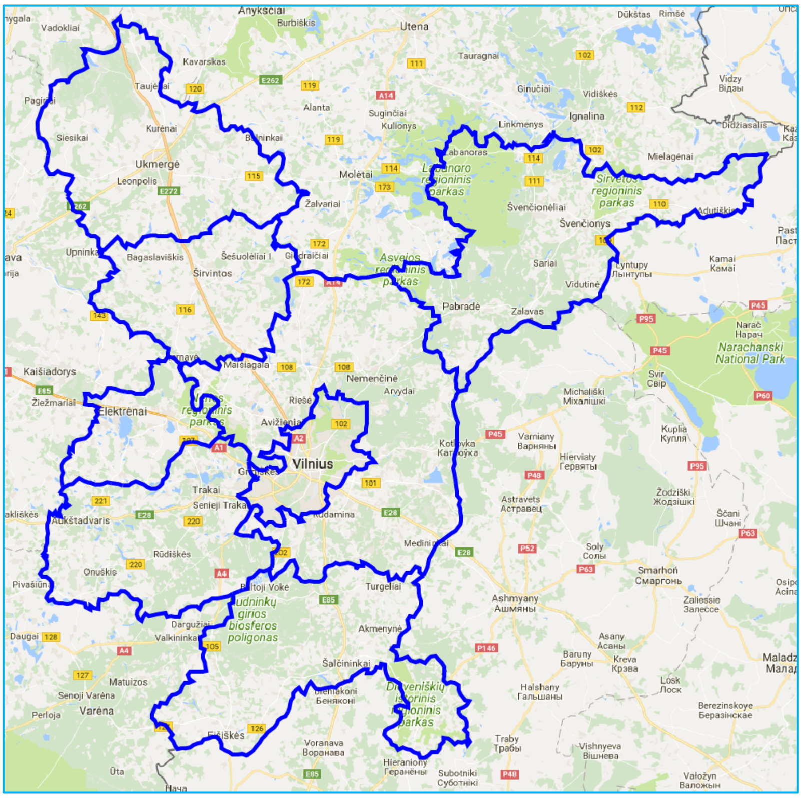

Figure 2. Vilnius County (Lithuania)

In accordance with Lithuanian road classification, all rural roads were attributed to three sets depending on their performance, namely:

- 1st type: main roads;
- 2nd type: national roads;
- 3rd type: regional roads.

In the test in question, only the roads belonging to the first two sets were considered: 865 km of main roads and 1,713 km of national roads were modelled (represented in Figure 5 with green and blue lines, respectively).

In terms of demand models, the peak-hour OD matrix was estimated using a system of mathematical models (Cascetta, 2009) which was calibrated in other case studies. The matrix generated was adjusted using traffic flows obtained from *Google APIs*, according to the procedures proposed in the literature based on traffic counts (Cascetta, 2009). The OD matrices corresponding to other periods were obtained starting from the peak-hour matrix according to the known data on the road traffic time variations. The demand thereby estimated can be assumed acceptable in a real-scale test. However, if the proposed procedure is adopted for actually designing a road network, a more accurate estimation of the demand has to be obtained with appropriate and specific surveys. Overall, eight hourly origin-destination matrices were generated, corresponding to eight different periods, as reported in Table 2. Considering several OD matrices is useful for evaluating the actual benefits of road improvements which could be overestimated if only the peak-hour period were to be considered in the network design problem.

Table 2. Details of the time periods

Day	Time period	Hours per year
Working day	Morning peak hour	486
	Afternoon peak hour	729
	Day-time off-peak hour	2,187
	Night-time hour	2,430
Pre-holiday	Day-time hour	720
	Night-time hour	720
Holiday	Day-time hour	744
	Night-time hour	744

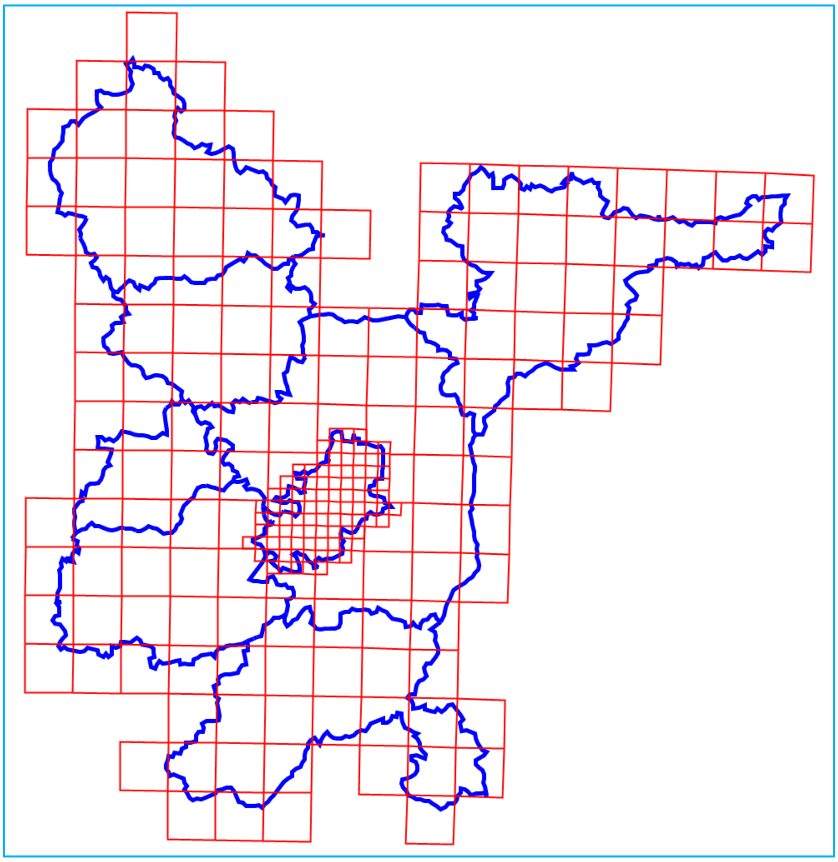

Figure 3. Zoning of Vilnius County

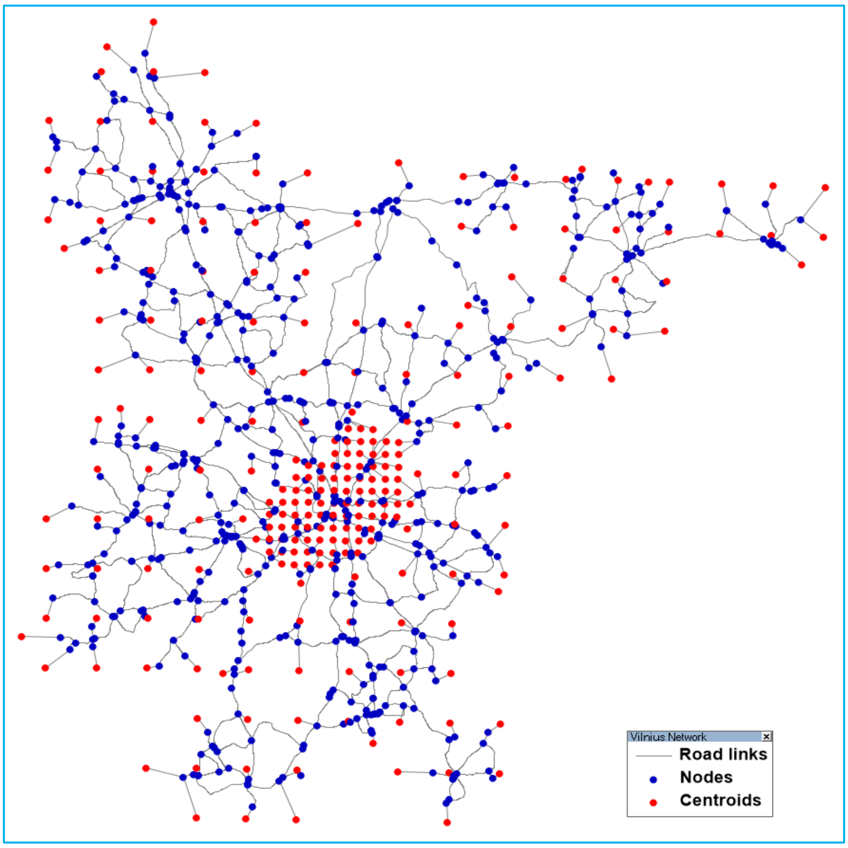

Figure 4. Network model of Vilnius County

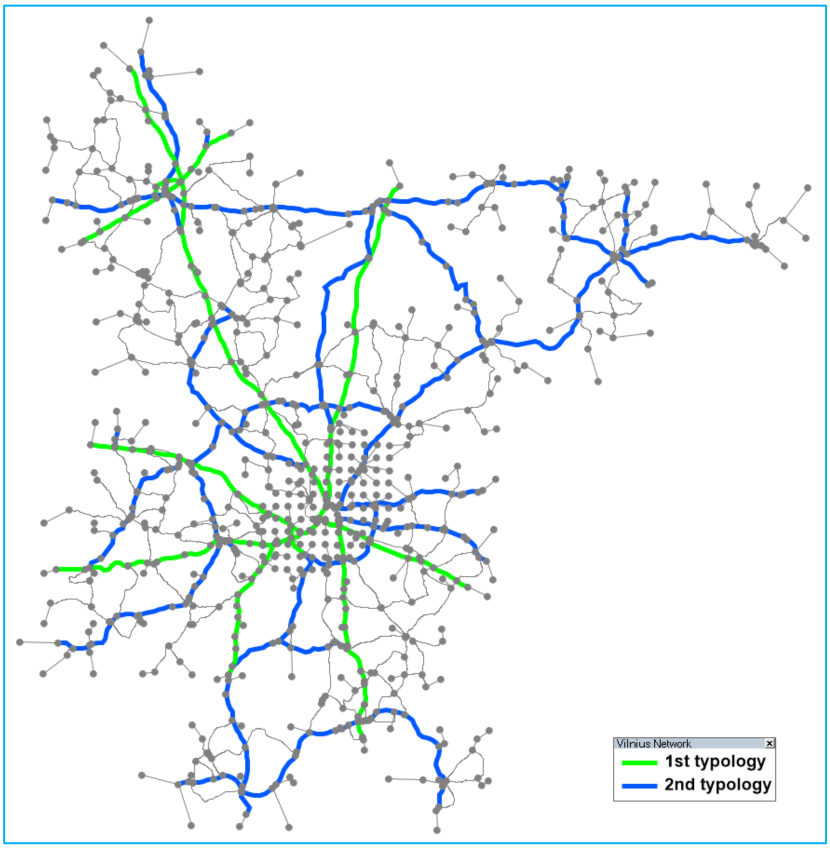

Figure 5. Representation of design variables (improvable road segments)

In this context, an RRNDP was formulated as follows:

$$\hat{\boldsymbol{y}} = \arg\min_{\boldsymbol{y}} Z\left(\boldsymbol{y}, \boldsymbol{F}^{*}(\boldsymbol{y})\right) \tag{15}$$

subject to:

$$y_h = 0/1 \quad \forall h \tag{16}$$

$$\sum_h y_h \cdot L_h \cdot ic_h \leq B \tag{17}$$

$$\boldsymbol{F}^{*}(\boldsymbol{y}) = \boldsymbol{\Lambda}\left(\boldsymbol{y}, \boldsymbol{F}^{*}(\boldsymbol{y})\right) \tag{18}$$

where $\hat{y}$ is the optimal solution, $\boldsymbol{y}$ is the vector of decision variables y_h; y_h is the binary variable corresponding to the improvement (value 1) or not (value 0) of the h-th road segment; $Z(.)$ is the objective function to be minimised; $\boldsymbol{F}^{*}$ represents the equilibrium flow vectors in different simulation periods; L_h is the length of road segment h; ic_h is the improvement cost of the road segment h; B is the available budget; $\boldsymbol{\Lambda}(.)$ is the assignment function linking the descriptive variables, $\boldsymbol{F}^{*}$, to the decision variables, $\boldsymbol{y}$. In particular, equation (16) is the constraint on the binary nature of the decision variables, equation (17) is the budget constraint, and equation (18) is the assignment constraint.

Table 3. Road segments

No.	Road name	Type	Current condition			Improved condition			Improvement costs per lane [€ / km / h]
			Number of lanes per direction	Capacity [vehic/h]	Free-flow speed [km/h]	Number of lanes per direction	Capacity [vehic/h]	Free-flow speed [km/h]	
1	A1	1st	2	3600.00	100.00	3	5400.00	110.00	5.71
2	A2	1st	2	3600.00	100.00	3	5400.00	110.00	5.71
3	A3	1st	2	3600.00	90.00	3	5400.00	100.00	5.71
4	A3	1st	1	1800.00	80.00	2	3600.00	90.00	5.71
5	A4	1st	1	1800.00	100.00	2	3600.00	110.00	5.71
6	A6	1st	1	1800.00	100.00	2	3600.00	110.00	5.71
7	A14	1st	2	3600.00	90.00	3	5400.00	100.00	5.71
8	A14	1st	1	1800.00	80.00	2	3600.00	90.00	5.71
9	A15	1st	1	1800.00	90.00	2	3600.00	100.00	5.71
10	A16	1st	1	1800.00	90.00	2	3600.00	100.00	5.71
11	A19	1st	2	3600.00	80.00	3	5400.00	90.00	5.71
12	A20	1st	1	1800.00	70.00	2	3600.00	80.00	5.71
13	A3 (urban segment)	1st	2	3600.00	60.00	3	5400.00	70.00	5.71
14	A2 (urban segment)	1st	2	3600.00	70.00	3	5400.00	80.00	5.71
15	101	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
16	102	2nd	1	1800.00	90.00	2	3600.00	100.00	3.81
17	103	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
18	104	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
19	105	2nd	1	1800.00	90.00	2	3600.00	100.00	3.81
20	106	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
21	107	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
22	108	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
23	109	2nd	1	1800.00	60.00	2	3600.00	70.00	3.81
24	110	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
25	111	2nd	1	1800.00	90.00	2	3600.00	100.00	3.81
26	114	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
27	115	2nd	1	1800.00	90.00	2	3600.00	100.00	3.81
28	116	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
29	120	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
30	126	2nd	1	1800.00	90.00	2	3600.00	100.00	3.81
31	127	2nd	1	1800.00	90.00	2	3600.00	100.00	3.81
32	143	2nd	1	1800.00	90.00	2	3600.00	100.00	3.81
33	145	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
34	171	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
35	172	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
36	173	2nd	1	1800.00	70.00	2	3600.00	80.00	3.81
37	174	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
38	176	2nd	1	1800.00	70.00	2	3600.00	80.00	3.81
39	202	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
40	214	2nd	1	1800.00	60.00	2	3600.00	70.00	3.81
41	220	2nd	1	1800.00	80.00	2	3600.00	90.00	3.81
42	221	2nd	1	1800.00	70.00	2	3600.00	80.00	3.81
43	231	2nd	1	1800.00	70.00	2	3600.00	80.00	3.81

Specifically, the decision variables, $\boldsymbol{y}$, identify some rural roads which could be improved with lane addition so as to increase their capacity and free-flow speed. Therefore, only the addition of a lane, with consequent effects on free-flow speed and capacity, is assumed as a possible road intervention. Hence, in order to identify decision variables in the examined context, the roads belonging to the analysed network were partitioned according to their main features (widths, lanes, free-flow speeds) so as to identify 43 segments.

Table 3 reports for all segments their main features in the current configuration, in the improved configuration and the equivalent hourly costs for the improvement. Indeed, although the improvement costs are generally expressed in terms of €/km, in this application we divided them by the number of life-cycle hours of the infrastructures considered in order to perform a comparison between costs (construction costs) and benefits (reduction in user travel times) on an hourly basis.

The objective function is given by:

$$Z\left(\boldsymbol{y}, \boldsymbol{F}^{*}(\boldsymbol{y})\right) = \sum_{\tau}\left(UTT_{\tau}\left(\boldsymbol{y}, \boldsymbol{F}^{*}(\boldsymbol{y})\right) + UVC_{\tau}\left(\boldsymbol{y}, \boldsymbol{F}^{*}(\boldsymbol{y})\right) + IC(\boldsymbol{y})\right) \cdot \Delta\tau \quad (19)$$

with:

$$UTT_{\tau}\left(\boldsymbol{y}, \boldsymbol{F}^{*}(\boldsymbol{y})\right) = VOT \cdot \sum_{l} tt_{l}^{\tau} \cdot f_{l}^{\tau}$$

$$UVC_{\tau}\left(\boldsymbol{y}, \boldsymbol{F}^{*}(\boldsymbol{y})\right) = \sum_{l} vmc_{l}^{\tau} \cdot f_{l}^{\tau}$$

$$IC(\boldsymbol{y}) = \sum_{h} y_{h} \cdot L_{h} \cdot ic_{h}$$

where $UTT_{\tau}(\cdot)$ is the *User Travel Time* in the period τ; $UVC_{\tau}(\cdot)$ is the *User Vehicular Cost* in the period τ; *IC* is the *Improvement Cost*; *VOT* is the value of time; tt_{l}^{τ} is the travel time of link *l* in period τ; f_{l}^{τ} is the vehicular flow of link *l* in period τ; vmc_{l}^{τ} is the vehicular monetary cost (such as fuel consumption) on link *l* in period τ; $\Delta\tau$ is period τ in hours.

In this case, such a model cannot be solved with an exhaustive approach since the number of feasible solutions is equal to $2^{43} = 8.80 \cdot 10^{12}$ and the evaluation of each solution requires the estimation of equilibrium traffic flows for each period τ. The problems are usually solved with heuristic or metaheuristic algorithms which, since the problem is usually not convex, lead to one or more local optima. In light of the above, this paper aimed to test the potential of the proposed approach to reduce computing times: any algorithm which explicitly requires equilibrium traffic flows to be computed at each iteration could be used. In the tests in question, the *Neighbourhood Search Algorithm* with a *Steepest Descent Method* (NSA-SDM) was used, whose details can be found in Gallo et al. (2010) and D'Acierno et al. (2013). The algorithm is often used inside multi-start procedures or scatter search algorithms for improving values of the objective function starting from more than one solution generated with various techniques. Conversely, here, the NSA-SDM was applied only once starting from the current solution, corresponding to all decision variables set equal to zero. The final solution obtained with the algorithm is therefore one of the possible local optima.

Moreover, the MSA algorithm was adopted for estimating the equilibrium traffic flows. In particular, the approach proposed in this paper is compared with the traditional approach. Basically, the difference lies in the fact that in the presented methodology, the MSA algorithm adopts as initial flows the equilibrium traffic flows obtained in the previous iteration of the network design solution algorithm, while in the traditional approach initial traffic flows are always assumed equal to 0.

Table 4 and Figures 6-8 summarise the main results of the test. The traditional approach required 345 iterations and a computational time of 7.83 h. However, although the proposed approach required 389 iterations (+12.75 %), because of the second stage, computational times are lower (6.31 h) corresponding to −19.34%. Both approaches led to the same solution (see Table 5) which is a local optimum. In this case, the second stage is actually useless since the final solution is the same. The differences in objective function values are only due to the termination criterion which was fixed at 0.50%. Neglecting this aspect of the problem, the computing time of the proposed approach decreases to 5.32 h, amounting to a reduction of –31.99%.

Table 4. Number of network loadings and computing times

Solution algorithm	**Analysed solutions**	**Network loadings**	**Analysed neighbourhoods**	**Computing times [h]**
Traditional approach	345	9,317	8	7.83
Proposed approach	389 (345+44)*	6,719 (5,531+1,188)*	9 (8+1)*	6.31 (5.32+0.99)*
**Performances subdivided between the first and second stage of the proposed approach*				

Table 5. Final solution

Variable	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	y_{19}	y_{20}	y_{21}	y_{22}
Value	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1
Variable	y_{23}	y_{24}	y_{25}	y_{26}	y_{27}	y_{28}	y_{29}	y_{30}	y_{31}	y_{32}	y_{33}	y_{34}	y_{35}	y_{36}	y_{37}	y_{38}	y_{39}	y_{40}	y_{41}	y_{42}	y_{43}	
Value	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	

Table 6. Number of network loadings for any time period

Day	Time period	Traditional approach	Proposed approach
Working day	Morning peak hour	11	2
	Afternoon peak hour	4	2
	Day-time off-peak hour	2	2
	Night-time hour	2	2
Pre-holiday	Day-time hour	2	2
	Night-time hour	2	2
Holiday	Day-time hour	2	2
	Night-time hour	2	2
TOTAL		**27**	**16**

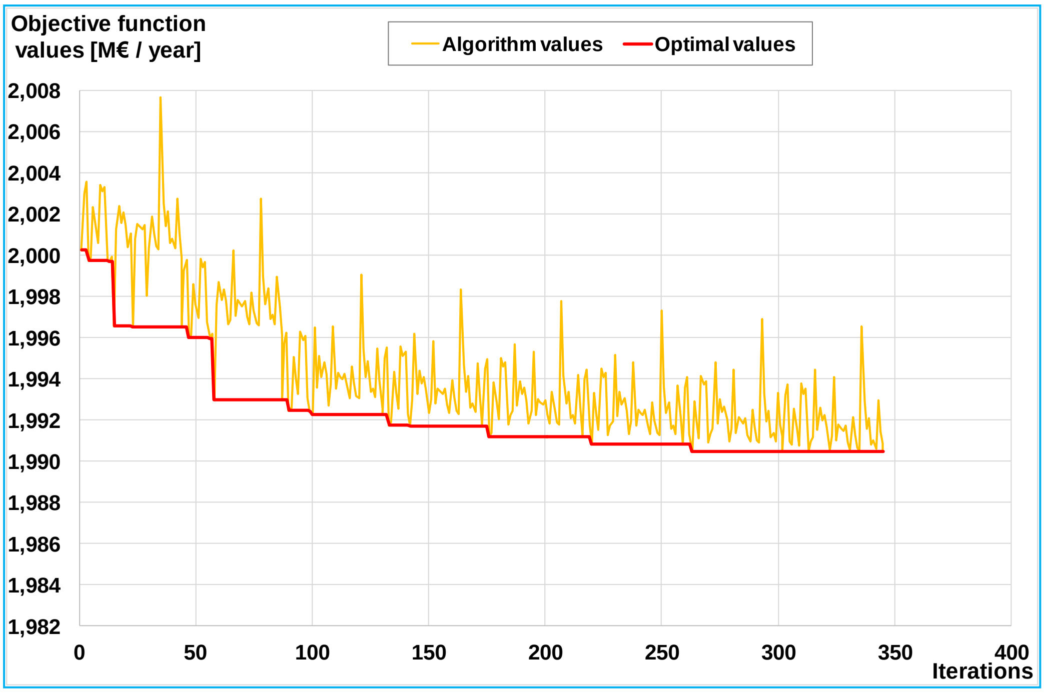

Figure 6. Objective function with the traditional approach

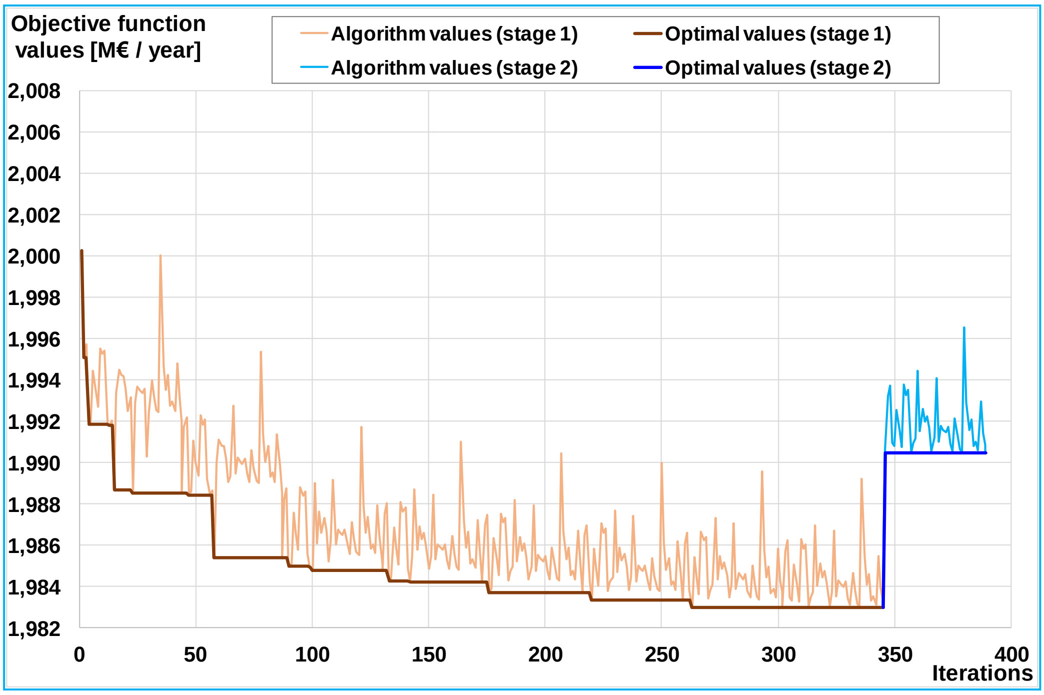

Figure 7. Objective function with the proposed approach

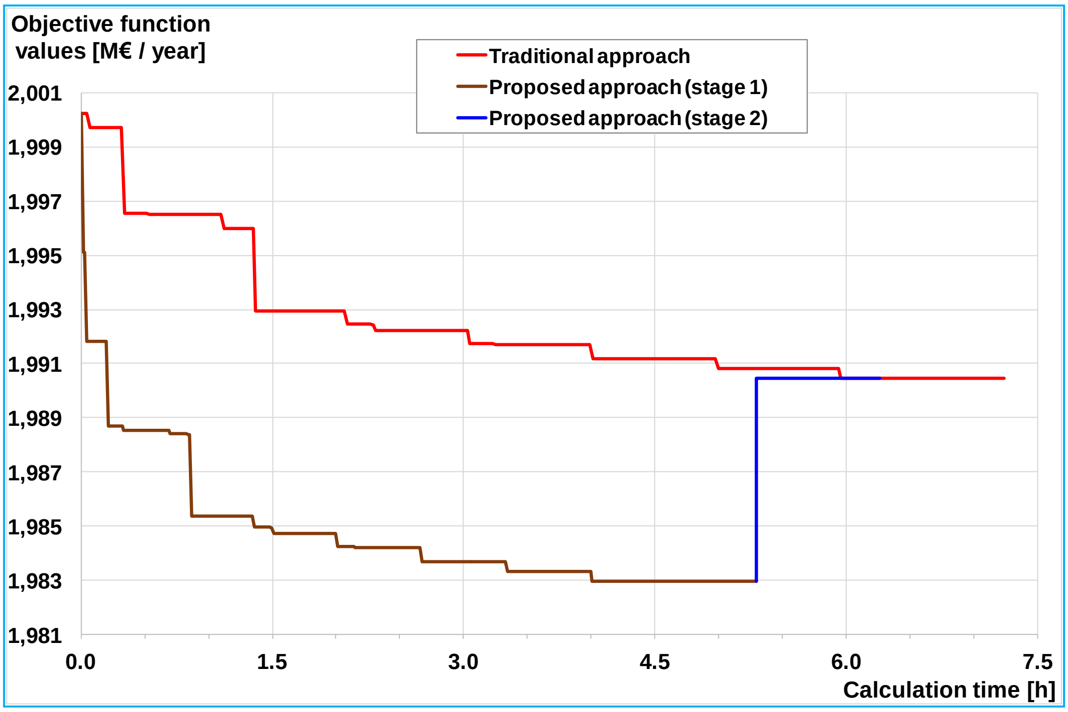

Figure 8. Comparison of analysed approaches

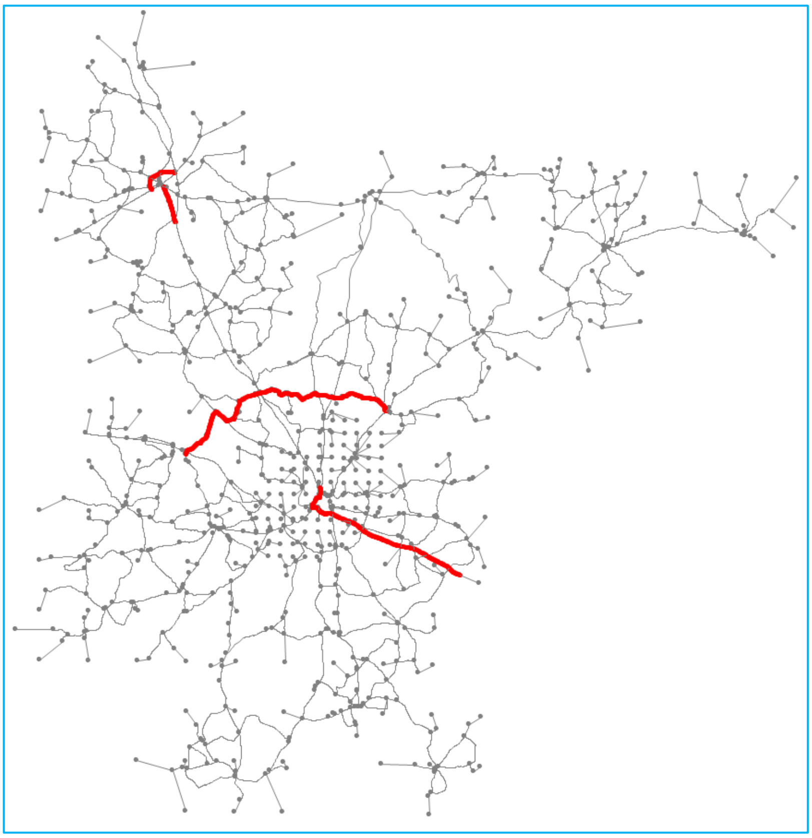

Figure 9. Roads to be improved

The solution obtained identified six roads to be improved (see Figure 9): A2 (only the two urban segments), A3, A20, 108 and 231.

Finally, Table 6 provides a comparison of the two approaches in terms of network loadings that is the number of iterations required to reach the equilibrium condition. In particular, although a full-year simulation yields a 40.74% reduction in computational effort, higher reductions occur in the simulation of peak hours of working days. Hence, if the RNDP is implemented in more critical conditions, the related reduction in terms of network loading number is 73.33% in the case of a working-day analysis and 81.82% in the case of a morning peak-hour analysis.

5. Conclusions

In this paper, a general method was proposed for reducing computing times in road network design problems (RNDPs). The method is applicable to all problems where the decision variables keep the topological configuration of the network unchanged, such as capacity expansion problems in rural contexts. A bi-level model has to be formulated since it is necessary to calculate equilibrium traffic flows for evaluating network performance. The proposed approach acts on the lower level of the problem, changing the initialisation phase of the assignment algorithm: it assumes initial traffic flows equal to the equilibrium values calculated at the previous iteration of the network design algorithm.

In order to show the main advantages of using the proposed approach, some tests were carried out on a real-scale case: the rural road network of Vilnius County (Lithuania). The main result of the application was that the approach yields the same final solution as the traditional approach. In terms of calculation times, the proposed approach leads to a 40.74% reduction in the case of a full-year analysis that can increase up to 81.82% in the case of a morning peak-hour analysis. Hence, the proposed methodology allows the same results to be obtained but in lower computation times compared to the traditional approach. Final results may thus be achieved in lower computation times or a large number of alternative solutions may be analysed (for instance by adopting meta-heuristic algorithms) at the same time as traditional approach procedures.

Future research may test the proposed methodology:

- on other real-scale cases in order to verify whether the reduction in computing times may be affected by the network dimension, as well as demand variability;
- with different traffic congestion levels since the main reductions in the case of peak-hour conditions seem to show that the higher the congestion level, the greater the time reduction upon adopting the proposed approach;
- by adopting meta-heuristic algorithms, such as Multi-Start techniques or Scatter Search which are able to generate more local optima since such algorithms are based on the recursive use of bi-level optimisation procedures. Therefore, any time reduction techniques including the proposed approach may allow their overall computing times to be considerably reduced.

References

Abdulaal M. and Le Blanc L.J. (1979). Continuous equilibrium network design models. *Transportation Research Part B*, Vol. 13, pp. 19-32.

Babazadeh A., Poorzahedy H. and Nikoosokhan S. (2011). Application of particle swarm optimization to transportation network design problem. *Journal of King Saud University – Science*, Vol. 23, pp. 293-300.

Bagloee S.A. and Sarvi M. (2018). An outer approximation method for the road network design problem. *PLoS ONE*, Vol. 13, art.no. e0192454. pp. 1-28.

Bagloee S.A., Sarvi M., Rajabifard A., Thompson R.S. and Saberi M. (2016). A solution to the road network design problem for multimodal flow. *Proceedings of the 19th IEEE International Conference on Intelligent Transportation Systems (IEEE ITSC 2016)*, pp. 235-240.

Billheimer J.W. and Gray P. (1973). Network design with fixed and variable cost elements. *Transportation Science*, Vol. 7, pp. 49-74.

Cantarella G.E. (1997). A general fixed-point approach to multimode multi-user equilibrium assignment with elastic demand. *Transportation Science*, Vol. 31, pp. 107-128.

Cantarella G.E., de Luca S., Di Pace R. and Memoli S. (2015a). Network Signal Setting Design: Meta-heuristic optimisation methods. *Transportation Research Part C*, Vol. 55, 24-45.

Cantarella G.E., de Luca S., Di Gangi M. and Di Pace R. (2015b). Approaches for solving the stochastic equilibrium assignment with variable demand: internal vs. external solution algorithms. *Optimization Methods & Software*, Vol. 30, pp. 338-364.

Cantarella G.E., Pavone G. and Vitetta A. (2006). Heuristics for urban road network design: lane layout and signal settings. *European Journal of Operational Research*, Vol. 175, pp. 1682-1695.

Cantarella G.E. and Vitetta A. (2006). The multi-criteria road network design problem in an urban area. *Transportation*, Vol. 33, pp. 567-588.

Cao J.X., Wang Y., Wei Z.M. and Wu J. (2013). Solve the discrete network design problem under construction cost uncertainties with the stochastic programming approach. *Procedia - Social and Behavioral Sciences*, Vol. 96, pp. 1039-1049.

Cascetta E. (2009). *Transportation systems analysis: Models and applications*. Springer, New York (NY), USA.

Cascetta E., Gallo M. and Montella B. (2006). Models and algorithms for the optimization of signal settings on urban networks with stochastic assignment. *Annals of Operations Research*, Vol. 144, pp. 301-328.

Chen A., Zhou Z., Chootinan P., Ryu S., Yang C. and Wong S.C. (2011). Transport Network Design Problem under Uncertainty: A Review and New Developments. *Transport Reviews*, Vol. 31, pp. 743-768.

Chiou S.W. (2005). Bilevel programming for the continuous transport network design problem. *Transportation Research Part B*, Vol. 39, pp. 361-383.

D'Acierno L., Gallo M. and Montella B. (2011). A fixed-point model and solution algorithms for simulating urban freight distribution in a multimodal context. *Journal of Applied Sciences*, Vol. 11, pp. 647-654.

D'Acierno L., Gallo M. and Montella B. (2013). Application of metaheuristics to large-scale transportation problems. *Lecture Notes in Computer Science*, Vol. 8353, pp. 215-222.

D'Acierno L., Montella B. and De Lucia F. (2006). A stochastic traffic assignment algorithm based on Ant Colony Optimisation. *Lecture Notes in Computer Science*, Vol. 4150, pp. 25-36.

Daganzo C.F. (1983). Stochastic network equilibrium with multiple vehicle types and asymmetric, indefinite link cost Jacobians. *Transportation Science*, Vol. 17, pp. 282-300.

Di Z., Yang L., Qi J. and Gao Z. (2018). Transportation network design for maximizing flow-based accessibility. *Transportation Research Part B*, Vol. 110, pp. 209-238.

Dimitriou L., Tsekeris T. and Stathopoulos A. (2008). Genetic computation of road network design and pricing Stackelberg games with multi-class users. In: Giacobini, M. et al. (Eds.), *Applications of Evolutionary Computing*. Springer, Berlin, Heidelberg, pp. 669-678.

Drezner Z. and Wesolowsky G.O. (2003). Network design: Selection and design of links and facility location. *Transportation Research Part A*, Vol. 37, pp. 241-256.

Farahani R.Z., Miandoabchi E., Szeto W.Y. and Rashidi H. (2013). A review of urban transportation network design problems. *European Journal of Operational Research*, Vol. 229, pp. 281-302.

Feremans C., Labbé M. and Laporte G. (2003). Generalized network design problems. *European Journal of Operational Research*, Vol. 148, pp. 1-13.

Fontaine P. and Minner S. (2018). Benders decomposition for the Hazmat Transport Network Design Problem. *European Journal of Operational Research*, Vo. 267, pp. 996-1002.

Gallo M., D'Acierno L. and Montella, B. (2010). A meta-heuristic approach for solving the urban network design problem. *European Journal of Operational Research*, Vol. 201, pp. 144-157.

Gallo M., D'Acierno L. and Montella B. (2012). A meta-heuristic algorithm for solving the road network design problem in regional contexts. *Procedia – Social and Behavioral Sciences*, Vol. 54, pp. 84-95.

Gao Z., Wu J. and Sun H. (2005). Solution algorithm for the bi-level discrete network design problem. *Transportation Research Part B*, Vol. 39, pp. 479-495.

Guihaire V. and Hao J. (2008). Transit network design and scheduling: a global review. *Transportation Research Part A*, Vol. 42, pp. 1251-1273.

Haas I. and Bekhor S. (2017). Network design problem considering system time minimization and road safety maximization: formulation and solution approaches. *Transportmetrica A*, Vol. 13, pp. 829-851.

Hosseininasab S.-M. and Shetab-Boushehri S.-N. (2015). Integration of selecting and scheduling urban road construction projects as a time-dependent discrete network design problem. *European Journal of Operational Research*, Vol. 246, pp. 762-771.

Khooban Z., Farahani R.Z., Miandoabchi E. and Szeto W.Y. (2015). Mixed network design using hybrid scatter search. *European Journal of Operational Research*, Vol. 247, pp. 699-710.

Memoli S., Cantarella G.E., de Luca S. and Di Pace R. (2017). Network signal setting design with stage sequence optimisation. *Transportation Research Part B*, Vol. 100, pp. 20-42.

Liu H. and Wang D.Z.W. (2015). Global optimization method for network design problem with stochastic user equilibrium. *Transportation Research Part B*, Vol. 72, pp. 20-39.

Liu H. and Wang D.Z.W. (2016). Modeling and solving discrete network design problem with stochastic user equilibrium. *Journal of Advanced Transportation*, Vol. 50, pp. 1295-1313.

Lo H.K. and Szeto W.Y. (2009). Time-dependent transport network design under cost-recovery. *Transportation Research Part B*, Vol. 43, pp. 142–158.

Magnanti T.L. and Wong R.T. (1984). Network design and transportation planning: models and algorithms. *Transportation Science*, Vol. 18, pp. 1-55.

Poorzahedy H. and Abulghasemi F. (2005). Application of ant system to network design problem. *Transportation*, Vol. 32, pp. 251-273.

Poorzahedy H. and Rouhani O.M. (2007). Hybrid meta-heuristic algorithms for solving network design problem. *European Journal of Operational Research*, Vol. 182, pp. 578-596.

Sheffi Y. and Powell W.B. (1981). A comparison of stochastic and deterministic traffic assignment over congested networks. *Transportation Research Part B*, Vol. 15, pp. 53-64.

Solanki R.S., Gorti J.K. and Southworth F. (1998). Using decomposition in large-scale highway network design with a quasi-optimization heuristic. *Transportation Research Part B*, Vol. 32, pp. 127-140.

Szeto W.Y., Jiang Y. and Wang D.Z.W. (2015). A sustainable road network design problem with land use transportation interaction over time. *Networks and Spatial Economics*, Vol. 15, pp. 791-822.

Tan Z., Yang H., Tan W. and Li Z. (2016). Pareto-improving transportation network design and ownership regimes. *Transportation Research Part B*, Vol. 91, pp. 292-309.

Ukkusuri S.V. and Patil G. (2009). Multi-period transportation network design under demand uncertainty. *Transportation Research Part B*, Vol. 43, pp. 625-642.

Wang D.Z.W., Liu H. and Szeto W.Y. (2015). A novel discrete network design problem formulation and its global optimization solution algorithm. *Transportation Research Part E*, Vol. 79, pp. 213-230.

Wang D.Z.W. and Lo H.K. (2010). Global optimum of the linearized network design problem with equilibrium flows. *Transportation Research Part B*, Vol. 44, pp. 482-492.

Wang S., Meng Q. and Yang H. (2013). Global optimization methods for the discrete network design problem. *Transportation Research Part B*, Vol. 50, pp. 42-60.

Xu M., Wang G., Grant-Muller S. and Gao Z. (2017). Joint road toll pricing and capacity development in discrete transport network design problem. *Transportation*, Vol. 44, pp. 731-752.

Xu X., Chen A. and Yang C. (2016). A review of sustainable network design for road networks. *KSCE Journal of Civil Engineering*, Vol. 20, pp. 1084-1098.

Yang H. (1997). Sensitivity analysis for the elastic-demand network equilibrium problem with applications. *Transportation Research Part B*, Vol. 31, pp. 55-70.

Yang H. and Bell M.G.H. (1998). Models and algorithms for road network design: a review and some new developments. *Transport Reviews*, Vol. 18, pp. 257-278.