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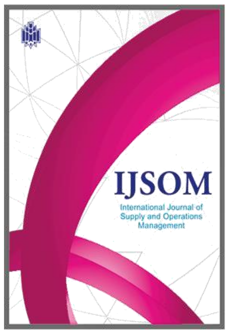

Integration of P-Hub Location Problem and 3M Supply Chain

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Abstract
The present study proposes an integrated model for hub location problem in a Multi-location, Multi-period, Multi-commodity (3M) three-echelon supply chain. The problem is formulated as a mixed integer programming model that could be solved using GAMS software. As the developed model is a mixed integer non-linear programming and NP-hard, a new algorithm for re-formulation is proposed to change it to a mixed integer leaner programming. Also, a new heuristic algorithm is proposed to solve it in a reasonable time. To prove the applicability of the model, the well-known real CAB data set is used. Numerical examples show the benefit of the proposed model in both solution time and result quality.

Keywords: Reliability; Ready-to-use systems; Markov chain; Fuzzy theory; Designed lifetime; Designing phase.

1. Introduction

The present study addresses two major issues, i.e. hub location and supply chain management (SCM). Both of these topics are particularly well-researched, but despite the obvious relationships between the two, very few studies have followed an integrated approach to address both concepts simultaneously. Simultaneous solution of Hub Location Problem (HLP) and SCM problem can lead to clear integration of the effective factors of a supply chain. Although such integration has been extensively investigated, according to many articles, there are still many gaps to be filled in the literature. This paper is concentrated on the integration of the aforementioned concepts into a model. To describe the problem, the following example is given:
One of the main contributions of the research is described in Figure 1. Figure 1a illustrates a simple network example with 4 nodes. In this network, hubs can be located at any of the nodes. The Demands ($\mathcal{D}_4$) and the transportation cost per unit ($\mathcal{CT}d_{ij}$) are shown on the links. Due to this problem, a 2-hub location could be considered on nodes (1, 4) or nodes (3, 4) (Figure 1b) which was described and proved by Campbell (1994).

This solution is theoretically correct, but it could be incorrect for a real supply chain, considering supplier, manufacturer, and consumer because each of these elements potentially can be located on each nodes. Suppose, two suppliers are located at nodes 2 and 4, and two manufacturers are located at nodes 1 and 3. In this case, the nodes pair (1, 2) should be chosen for 2- hub locations which are presented in Figure 1c.

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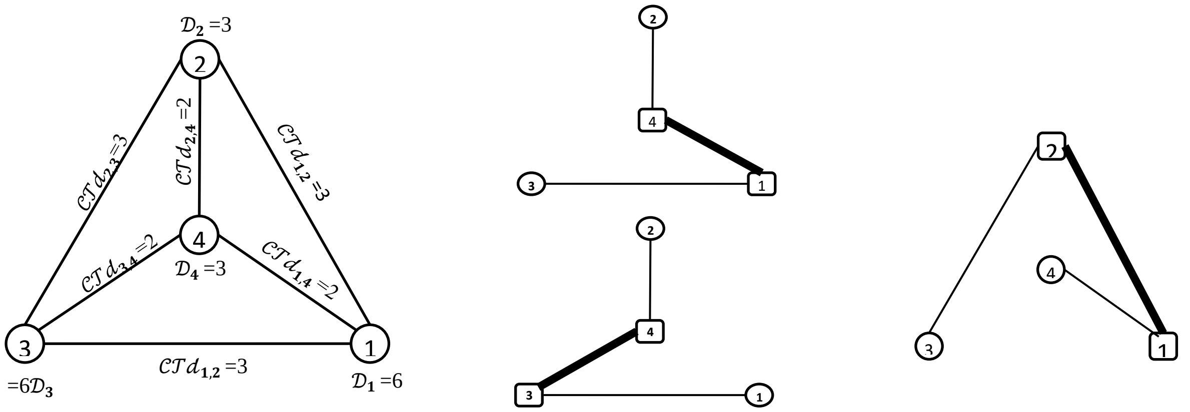

a. Network Structure

b. 2-hub Median in simple problem

c. 2-hub Median in considering the SCM elements

Figure 1. Network example

In other words, the solution to the problem in the first simple example is different from the second one, which includes the elements and conditions of a real supply chain, with its interactions and internal connections. For better clarity, assume a supply chain in which a number of manufacturers fulfill the customer demand for a final product, but depending on the chain conditions, a specific customer may have a unique link with a manufacturer (a customer may have active relationship with some manufacturers and inactive relationship with others), and the same also goes for the manufacturers. This type of relationship can be extended to more than two echelons (Figure 2).

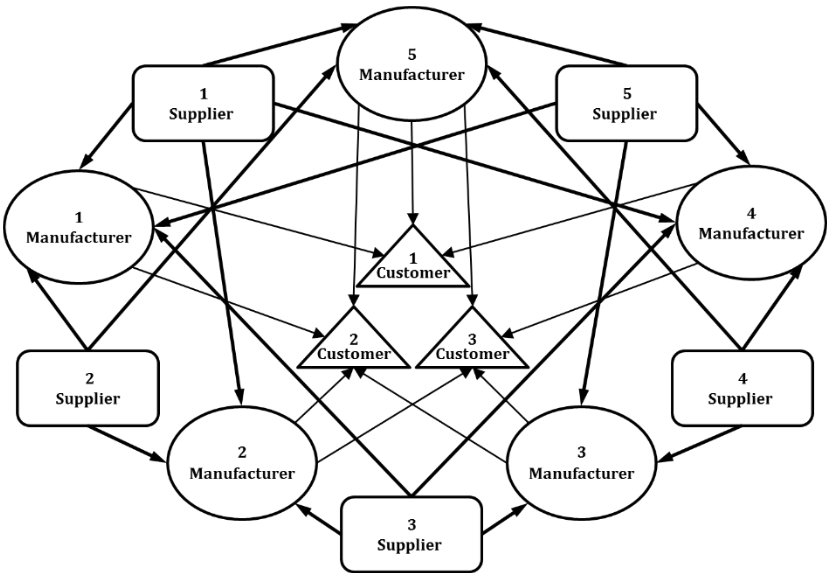

Figure 2. Relationships of a hypothetical supply chain elements without geographic location approach

In the supply chain literature, supply chain networks are typically modeled based on “the existence of entities” (Chopra & Meindl, 2016; Hugos, 2011; Webster, 2008), but in the hub location literature, the nature of relationships and constraints calls for emphasis on “geographical location” of the elements constituting the chain (Campbell, 1991; Talbi & Todosijević, 2017). So the question is whether it is possible to consider each element of the supply chain as a symbol of a geographical location and simultaneously as a symbol of entities in a multi-level chain. In Figure 3, this issue is shown in a supply chain with three levels consisting of the customer, manufacturer, and supplier. Here, an element may have 7 different statuses: three statuses in relation to a single entity, three statuses in relation to two entities, and one status in relation to all three entities. But if we combine this concept of a supply chain with the concept provided in the hub location literature, the status of each element in the supply chain can be shown with a 2×2 matrix consisting of zero and non-zero entries determined according to supply chain features and condition (see Figure 3).

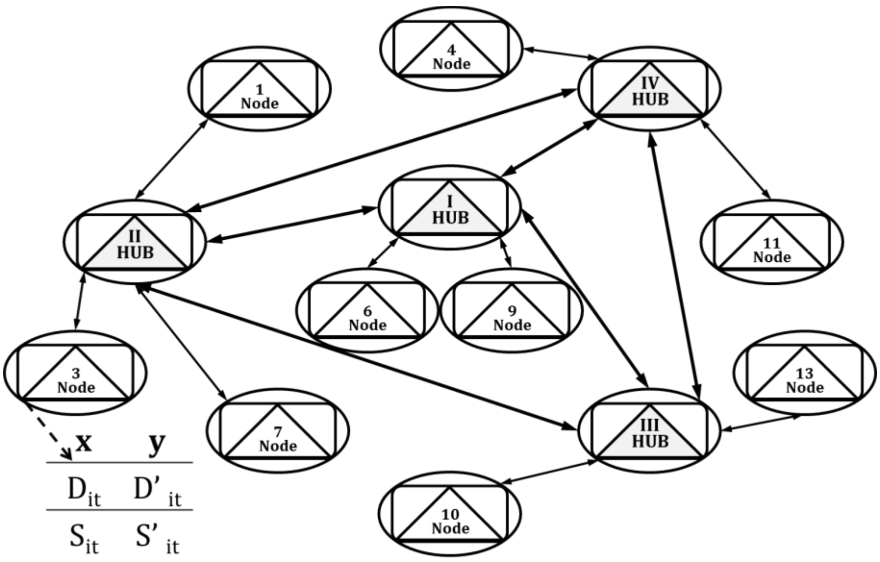

Figure 3. Applying the concept of hub location in a hypothetical three-level supply chain

The strategic goal pursued in this study is to make a significant reduction in the number and complexity of relations between different locations in a multilevel supply chain. In other words, this study seeks to improve the performance of multilevel supply chain by incorporating the concept of the hub into the supply chain to reduce the cost and complexity of relations and uncertainty in decision-making.

Chopra and Meindl (2016) categorized the elements of supply chain performance into five areas (production, inventory, facility location, transportation, and information) and stated that with proper management of these elements, one can achieve the capabilities required for the success of a supply chain. They added that proper combination of responsiveness and efficiency for each of these elements may increase productivity and reduce the costs of inventory and operations (Aykin, 1994; Bruce, Daly, & Towers, 2004; Hines, 2004; Hugos, 2011; Webster, 2008). In another study, Fakhrzad and Moobed (2010) presented a genetic algorithm for decision making in reverse supply chains and solving the corresponding NP-Hard problem to obtain the optimal solution of chain facilities. Fakhrzad and Esfahanib (2013) presented two heuristic algorithms for solving the transport routing problem in the supply chain.

Researchers believe that the competition between supply chains is growing day by day and therefore strategic and macro-planning for the entire supply chain has emerged as an important challenge and a prerequisite for any effort to reduce the cost of individual elements in the supply chain.

The importance of integration of these elements has been emphasized by many researchers (Al-Qahtani & Elkamel, 2008; Guajardo, Kylinger, & Rönnqvist, 2013). But despite the emphasis given to this integration, the majority of studies conducted in the field of supply chain have concentrated on only one or two of these five elements and the research studies that have considered 3 elements at the same time are significantly rare(Hosseini-Nasab, Fereidouni, Ghomi, and Fakhrzad, 2018; Fakhrzad, Heydari, 2008).

Some researchers have also presented heuristic methods for solving the model. de Camargo, de Miranda, O'Kelly, and Campbell (2017), formulated a decomposition method. Some researchers proposed a linear mixed integer programming problems and a Taboo Search (TS) based met heuristic. (Ghaffarinasab, 2018; Fakhrzad, M.B., Sadeghieh, A., Emami, L., 2012.), Ghaffarinasab, Motallebzadeh, Jabarzadeh, and Kara (2018) , Ghaffarinasab and Motallebzadeh (2018), and also Duan, Tavasszy, and Peng (2017) presented a Simulated Annealing (SA) based solution algorithm. Hoff, Peiró, Corberán, and Martí (2017), implemented memory structures to create advanced local search algorithm. Martins de Sá, Morabito, and de Camargo (2018a) present Benders decomposition frameworks and ILS-VND stochastic local search procedure. Mokhtar, Krishnamoorthy, and Ernst (2018) and Martins de Sá, Morabito, and de Camargo (2018b) implemented a modified Benders decomposition method. Silva and Cunha (2017), proposed a taboo search (TS) heuristic. Quadros, Costa Roboredo, and Alves Pessoa (2018) developed a branch-and-cut algorithm. da Costa Fontes and Goncalves (2018) developed A Variable Neighborhood Decomposition Search (VNDS).

Based on the results of the research, the researchers found that out of the 54 articles examined, 7 did not focused on either of these two concepts. 46 cases focused on one of the two concepts (SCM and Hub-Location concepts), and only one case partially addressed both concepts. Another study results in the first phase showed that among the papers examined, 8 cases focused on only one of the elements. The greatest attention is given to two of five elements, about 39 cases. 5 cases focused on 3 elements. Finally, the contribution of cases focused on 4 and 5 elements was only 1 item.

As some of the researchers in this field acknowledge, the majority of models used in the SCM domain have focused on topics, such as warehousing, production, and order management, and have chosen to avoid the role of factors, such as transportation, site selection, and information in the supply chain coordination, or their relationships with the previously said topics (Shah & Goh, 2006; Silver, 1992). This is why the authors working on this venue of research believe that an integrated optimization approach can result in great improvements in the supply chain (Lam, 2011; Fakhrzad, M.B., Heydari, M., 2008).

Since optimality can be guaranteed only with integration, the integration of hub location problem (which emphasizes location and transportation) with the existing supply chain models in the literature (which emphasize other elements), considering the exchange of information in the supply chain in both approaches, could be considered an attractive research field of research. This study intends to pursue this goal by combining the topics as shown in Figure 4. This issue is considered one of the contributions of this paper.

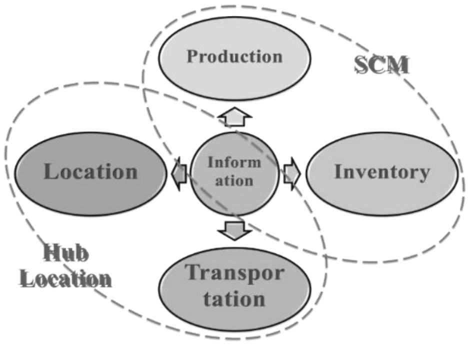

Figure 4. Integrating Supply Chain Elements

2. Problem statement

Assume a supply chain network with n geographical positions (sites), each having the potential to demand, produce, store, and supply products, raw material and finished product (See Figure 5). For site i, the raw materials produced in period t is $\mathcal{PR}_i(t)$, the amount of raw material carried over from the inventory of previous period to period t is $I\mathcal{R}_i(t-1)$, and the shortage left from the previous period is $\mathcal{S}l\mathcal{R}_i(t-1)$. Since different parts of the network are connected, a fraction of raw materials at site i is transferred to other sites ($\mathcal{R}_{ij}^{m_\mathcal{H}}(t)$), and it is assumed that this transfer is done via the hub network.

The subscript $m_\mathcal{H}$ is given to the hub site through which raw material is transferred from site i to site j. Likewise, site i receives a fraction of raw materials produced at other sites (like j), which has been sent to the previous period, but arrive at the present period ($\mathcal{R}_{ji}^{m_\mathcal{H}}(t-1)$). Site i can simultaneously produce the final product, so a portion of raw material available at site i in period t will be consumed to produce the final product $\mathcal{PP}_i(t)$. At the end of period t, site i will have $I\mathcal{R}_i(t)$ amount of raw in its inventory or will accrue $\mathcal{S}l\mathcal{R}_i(t)$ amount of shortage.

As mentioned, in period t, site i produces $\mathcal{PP}_i(t)$ amount of product, and may have $I\mathcal{P}_i(t-1)$ amount of product carried over from the inventory of previous period, or $\mathcal{S}l\mathcal{P}_i(t-1)$ amount of product shortage left from the previous period. Again, since different parts of the network are connected, $\mathcal{P}_{ij}^{m_\mathcal{H}}(t)$ amount of product at site i is transferred to other sites via the hub network ($m_\mathcal{H}$). The subscript m denotes the site through which raw material is transferred from site i to site j. Like before, site i receives $\mathcal{P}_{ji}^{m_\mathcal{H}}(t-1)$ amount of product from other sites, such as j, which has been sent in the previous period but arrive at the present period. On the other hand, site i also has demand for final product $\mathcal{DP}_i(t)$, which must be deducted from its inventory. At the end of period t, site i will have $I\mathcal{P}_i(t)$ amount of final product in its inventory or will accrue $\mathcal{S}l\mathcal{P}_i(t)$ amount of shortage.

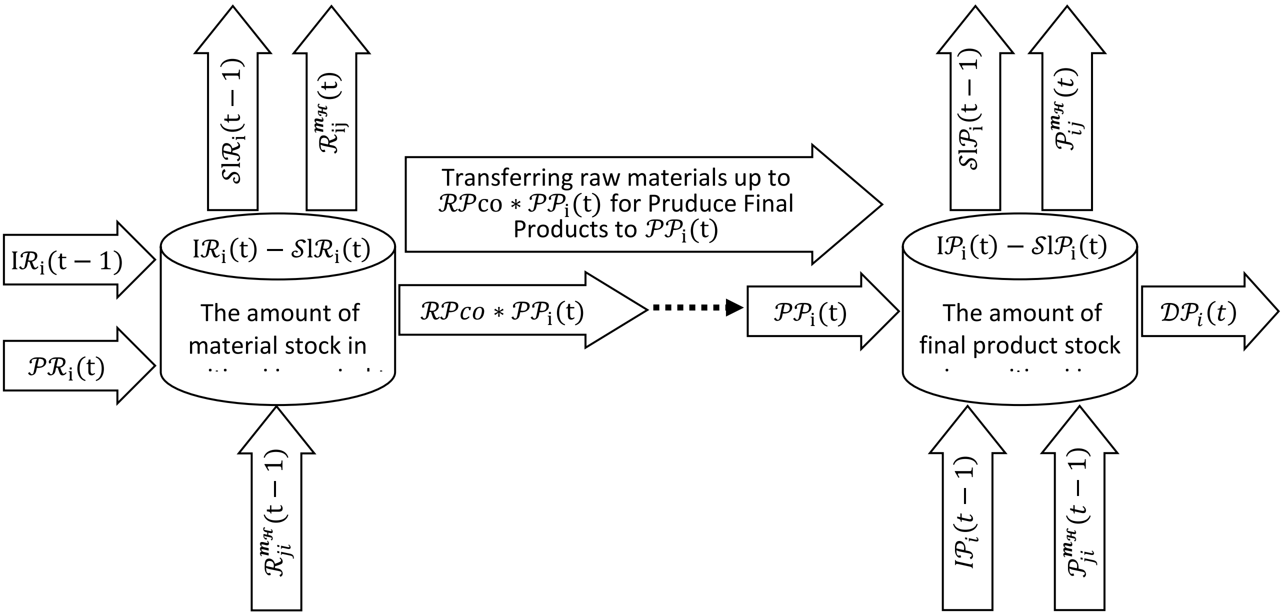

Figure 5. Demand, production, inventory and supply at Position i

Considering the focus on hub location concept, the routes of the supply chain network are limited to certain paths with rules specific to hub location literature. Figure 6 illustrates the route between points i and j on the network. Assume a solution where demand for product in period t at site j ($\mathcal{DP}_j(\mathrm{t})$) must be fulfilled partly by the inventory carried over from the previous period at site i ($\mathrm{I}\mathcal{P}_{\mathrm{i}}(\mathrm{t}-1)$). In that case, $\mathcal{P}_{\mathrm{ij}}^{\boldsymbol{m_{\mathcal{H}}}}(\mathrm{t}-1)$ amount of product must be transferred from site i to site j.

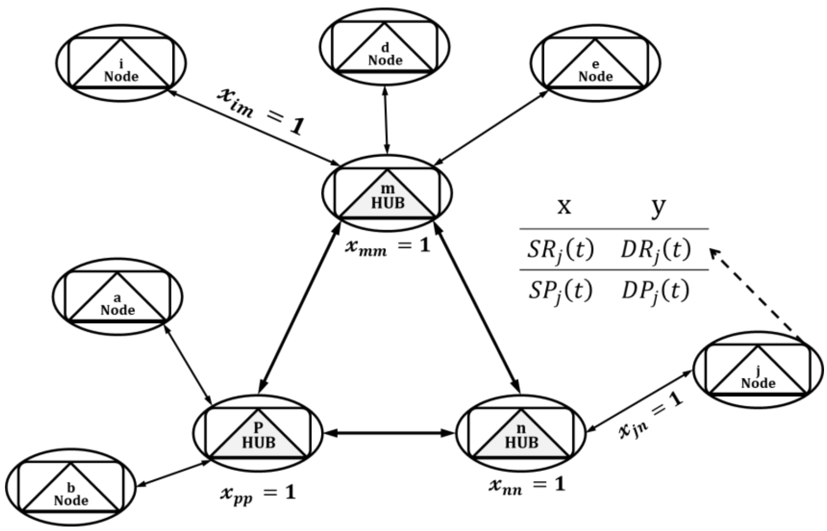

Figure 6. The supply chain network routes determined based on the hub location concept

According to hub location rules, transfer of product between the supply and demand points must be done through a hub network. More specifically, the product is sent from non-hub site i to hub site m ($\mathcal{X}_{\boldsymbol{m_{\mathcal{H}} m_{\mathcal{H}}}}$ =1), and then forwarded to another hub on the network, namely hub site n ($\mathcal{X}_{\boldsymbol{n_{\mathcal{H}} n_{\mathcal{H}}}} = 1$), which is geographically close to site j, and is finally forwarded to site j ($\mathcal{X}_{j\boldsymbol{n_{\mathcal{H}}}} = 1$) (Figure 8).

3. Problem Modeling

The incapacitated single allocation p-hub median problem (UCSApHMP) is one of the most widely used problems of hub location literature. In this problem, from n nodes existing in the network, p nodes must be selected as hubs and each of the remaining non-hub nodes must be allocated to at most one hub such that total network cost is minimized. The formulation proposed for combining the hub location and supply chain planning problems is provided below. Given the complexity of the model, to achieve a better solution speed, significant improvements are made in the definition of variables and constraints compared with similar work in the literature. These improvements are described below.

Table 1. Variable Construction

Variables	Definitions
Indices:	
$\boldsymbol{\Psi} = \{\mathbf{1}, \dots, \boldsymbol{N}\}$	Set of geographic locations that include at least one level of the supply chain
$\boldsymbol{\Phi} = \{\mathbf{1}, \dots, \boldsymbol{N}\}$	Geographic locations of supply chain hubs
$\boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{\Psi}$	Geographic location indexes
$\boldsymbol{m_{\mathcal{H}}}, \boldsymbol{n_{\mathcal{H}}} \in \boldsymbol{\Phi}$	Geographic location indexes of supply chain hubs
$\mathbf{t} \in \{\mathbf{1}, \dots, \boldsymbol{T}\}$	Time period index, t =1,2, ... , T (T denotes the length of the planning horizon)
Parameters:	
$DP_{\mathbf{i}}(\mathbf{t})$	Demand for finished product at location i in period t
$DR_{\mathbf{i}}(\mathbf{t})$	Demand for raw material at location i in period t
$SP_{\mathbf{i}}(\mathbf{t})$	Supply for finished product at location i in period t
$SR_{\mathbf{i}}(\mathbf{t})$	Supply for raw material at location i in period t
$CTd_{\mathbf{ij}}$	Raw material/ Finished Product direct transportation cost from location i to location j
$CH_{\boldsymbol{m_{\mathcal{H}}}}$	Fixed cost of establishing a hub at the location m
$CAPR_{\mathbf{i}}$	Production capacity of the finished product at the location i
$CAPP_{\mathbf{i}}$	Production capacity of raw material at the location I
CPR	Production cost of a raw material unit
CPP	Production cost of a finished product unit
CIR	Inventory holding cost of raw material in each period
$CSIR$	Shortage cost of raw material in each period
CIP	Inventory holding cost of finished products in each period
$CSIP$	Shortage cost of finished products in each period
PH	The number of required network hub
$RPco$	Conversion ratio of raw material required to produce a final product unit
T	Length of the planning horizon
N	Number of geographical locations (including supplier, manufacturer, customer or combination)
Decision variables:	
$\boldsymbol{\mathcal{X}}_{\mathbf{ij}}$, $\forall \mathbf{i} \neq \mathbf{j}$	Binary variable, assuming value 1 if there is a relationship between location i and location j
$\boldsymbol{\mathcal{X}}_{\boldsymbol{m_{\mathcal{H}}m_{\mathcal{H}}}}$	Binary variable, assuming value 1 if the location $\boldsymbol{m_{\mathcal{H}}}$ is a hub
$z_{\mathbf{i}\boldsymbol{m_{\mathcal{H}}n_{\mathcal{H}}}\mathbf{j}}$	Binary variable, assuming value 1 if the relationship between i and j is through m and n hubs respectively
$CTr_{\mathbf{ji}}$	Raw material/ Finished product transportation cost from location i to location j through hubs
$R_{\mathbf{ij}}^{\boldsymbol{m_{\mathcal{H}}}}(\mathbf{t})$	Amount of raw materials delivered from location i to location j through hub m
$IR_{\mathbf{i}}(\mathbf{t})$	Inventory level of raw materials at location i in period t

Table 1. Continued

Variables	Definitions
$SIR_i(t)$	Amount of raw material shortage in the location i in period t
$P_{ij}^{m_{\mathcal{H}}}(t)$	Amount of finished product delivered from location i to location j through hub $m_{\mathcal{H}}$
$IP_i(t)$	Inventory level of finished product at location i in period t
$SIP_i(t)$	Amount of finished product shortage in the location i in period t
$PR_i(t)$	Raw material production in location i in period t
$PP_i(t)$	Finished product production on location i in period t

Table 1 describes the variables and parameters used in the Model. We proceed to explain how we operationalize each notation. Assume n nodes (i.e. Sites or locations of entities) with known coordinates in the metric x-y plane. The Euclidean distance between sites i and j is denoted by D_{ij}. For each pair of sites i and j, the flow that must be transferred from site i to site j is denoted by $W_{ij}\geq 0$. It thus follows that $D_{ii}=0$, but W_{ii} is not necessarily zero. Thus, the matrix D is symmetrical, but W is not necessarily symmetrical. Matrix D can also be replaced by the time or cost of transport between the sites (t_{ij} and C_{ij}). Therefore, the statements that will be proved to matrix D_{ij} and its elements also hold true for matrices t and C and their elements t_{ij} and C_{ij}. The time, cost, or distance of transport on the edges between demand points and hubs and between demand points follow the triangle inequality rule i.e. ($t_{ij} \leq t_{im_{\mathcal{H}}} + t_{m_{\mathcal{H}}n_{\mathcal{H}}} + t_{jn_{\mathcal{H}}}$).

The model should select p hubs from n sites, (p is a known and fixed value). Assuming N as the set of all sites on the network and H as the set of hubs, N\H can be considered as the set of non-hub sites, where each non-hub site $\vartheta \in N\backslash H$ will be allocated to one hub h ∈ H. Direct flow between non-hub sites is not allowed. The objective of incapacitated single allocation p-hub median problem is the minimization of total network cost. In the network graph, transportation links are expressed as undirected edges connecting the supply points to hubs, hubs to hubs, and hubs to demand points.

The previous p-hub median models are all based on two main assumptions. The first assumption is that the flow of material from site i to site j is a known value given as an input, and the second assumption is that the material transferred between sites is of the same kind or in other words the model is single-product (James F. Campbell, 1994; Miranda Junior, Camargo, Pinto, Conceição, & Ferreira, 2011). In the present study, however, the three-level supply chain is modeled without these two restrictive assumptions to achieve a higher degree of dynamism and interaction. In other words, the flow rate between the two sites (w_{ij}) in the graph G(H, N/H) is a factor of interaction and dynamics between demand($\mathcal{DR}_i(t)$ and $\mathcal{DP}_i(t)$), production ($\mathcal{PR}_i(t)$ and $\mathcal{PP}_i(t)$), inventory holding ($I\mathcal{R}_i(t)$,$\mathcal{S}I\mathcal{R}_i(t)$,$I\mathcal{P}_i(t)$ and $\mathcal{S}I\mathcal{P}_i(t)$), and supply ($\mathcal{SR}_i(t)$ and $\mathcal{SP}_i(t)$) of sites for multiple products ($\mathcal{R},\mathcal{P}$). This issue is considered one of the other contributions of the study. The proposed model is presented below (Eq. (1)-(13)).

$$\begin{aligned}\text{Min } \mathbb{Z} = &\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m_{\mathcal{H}}=1}^{N} CTr_{ij} * \left(R_{ij}^{m_{\mathcal{H}}}(t) + P_{ij}^{m_{\mathcal{H}}}(t)\right) + CIR\sum_{i=1}^{N}\sum_{t=1}^{T} IR_i(t) + CIP\sum_{i=1}^{N}\sum_{t=1}^{T} I\mathcal{P}_i(t) \\ &+ CSIP\sum_{i=1}^{N}\sum_{t=1}^{T} SIP_i(t) + CSIR\sum_{i=1}^{N}\sum_{t=1}^{T} SIR_i(t) + CPR\sum_{i=1}^{N}\sum_{t=1}^{T} PR_i(t) + CPP\sum_{i=1}^{N}\sum_{t=1}^{T} PP_i(t) \\ &+ \sum_{m_{\mathcal{H}}=1}^{N} CH_{m_{\mathcal{H}}}\mathcal{X}_{m_{\mathcal{H}}m_{\mathcal{H}}}\end{aligned} \quad (1)$$

S. t.

$$\begin{aligned}I\mathcal{R}_i(t) - \mathcal{S}I\mathcal{R}_i(t) = &\ I\mathcal{R}_i(t-1) - \mathcal{S}I\mathcal{R}_i(t-1) + \mathcal{PR}_i(t) + \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{R}_{ji}^{m_{\mathcal{H}}}(t-1) - \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{R}_{ij}^{m_{\mathcal{H}}}(t) - \mathcal{RP}co \\ &* \mathcal{PP}_i(t) \quad ,\forall t, i\end{aligned} \quad (2)$$

$$\mathcal{PR}_i(t) \leq \mathcal{CAPR}_i \quad ,\forall i = 1 \cdots N \quad (3)$$

$$\mathcal{X}_{im_{\mathcal{H}}} \leq \mathcal{X}_{m_{\mathcal{H}}m_{\mathcal{H}}} \quad ,\forall i, m_{\mathcal{H}} = 1 \cdots N \quad (4)$$

$$\sum_{j=1}^{N} \mathcal{X}_{ij} = 1 \quad ,\forall i = 1 \cdots N \quad (5)$$

$$\sum_{m=1}^{N} \mathcal{X}_{m_{\mathcal{H}}m_{\mathcal{H}}} = \wp \mathrm{H} \tag{6}$$

$$\mathcal{X}_{\mathrm{i}m_{\mathcal{H}}} + \mathcal{X}_{\mathrm{j}n_{\mathcal{H}}} - 1 \leq \mathcal{z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}} \quad ,\forall \mathrm{i}, m_{\mathcal{H}}, n_{\mathcal{H}}, (\mathrm{j} \neq \mathrm{i}) = 1 \cdots \mathrm{N} \tag{7}$$

$$\sum_{m=1}^{N}\sum_{n=1}^{N} \mathcal{Z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}} = 1 \quad ,\forall \mathrm{i} = 1 \cdots \mathrm{N} \ ,\forall \mathrm{j} = \mathrm{i} \cdots \mathrm{N} \tag{8}$$

$$\mathrm{C}\mathcal{T}\mathcal{r}_{\mathrm{ij}} = \sum_{m=1}^{N}\sum_{n=1}^{N} \mathcal{Z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}} \left(\mathcal{CT}d_{\mathrm{i}m_{\mathcal{H}}} + \mathcal{CT}d_{m_{\mathcal{H}}n_{\mathcal{H}}} + \mathcal{CT}d_{n_{\mathcal{H}}\mathrm{j}}\right) \quad ,\forall \mathrm{i} = 1 \cdots \mathrm{N} \ ,\forall \mathrm{j} = \mathrm{i} \cdots \mathrm{N} \tag{9}$$

$$\mathrm{I}\mathcal{P}_{\mathrm{i}}(\mathrm{t}) - \mathcal{S}\mathrm{l}\mathcal{P}_{\mathrm{i}}(\mathrm{t}) = \mathrm{I}\mathcal{P}_{\mathrm{i}}(\mathrm{t}-1) - \mathcal{S}\mathrm{l}\mathcal{P}_{\mathrm{i}}(\mathrm{t}-1) + \mathcal{PP}_{\mathrm{i}}(\mathrm{t}) + \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{P}_{\mathrm{ji}}^{m_{\mathcal{H}}}(\mathrm{t}-1) - \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{P}_{\mathrm{ij}}^{m_{\mathcal{H}}}(\mathrm{t}) - \mathcal{DP}_{\mathrm{i}}(\mathrm{t}) \ ,\forall \mathrm{t}, \mathrm{i} \tag{10}$$

$$\mathcal{PP}_{\mathrm{i}}(\mathrm{t}) \leq \mathcal{CAPP}_{\mathrm{i}} \quad ,\forall \mathrm{i} = 1 \cdots \mathrm{N} \tag{11}$$

$$\mathcal{PP}_{\mathrm{i}}(\mathrm{t}) \leq \frac{\mathrm{I}\mathcal{R}_{\mathrm{i}}(\mathrm{t}) - \mathcal{S}\mathrm{l}\mathcal{R}_{\mathrm{i}}(\mathrm{t})}{\mathcal{RP}\mathrm{co}} \tag{12}$$

$$\mathrm{C}\mathcal{T}\mathcal{r}_{\mathrm{ij}}, \mathcal{R}_{\mathrm{ij}}^{m_{\mathcal{H}}}(\mathrm{t}), \mathcal{P}_{\mathrm{ij}}^{m_{\mathcal{H}}}(\mathrm{t}), \mathrm{I}\mathcal{R}_{\mathrm{i}}(\mathrm{t}), \mathcal{S}\mathrm{l}\mathcal{R}_{\mathrm{i}}(\mathrm{t}), \mathrm{I}\mathcal{P}_{\mathrm{i}}(\mathrm{t}), \mathcal{S}\mathrm{l}\mathcal{P}_{\mathrm{i}}(\mathrm{t}), \mathcal{PR}_{\mathrm{i}}(\mathrm{t}), \mathcal{PP}_{\mathrm{i}}(\mathrm{t}) \geq 0 \quad , \quad \mathcal{X}_{\mathrm{ij}}, \mathcal{Z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}} = \{0,1\} \tag{13}$$

Eq. **(1)** is the objective function of the proposed model, which minimizes the cost of the three-level supply chain. According to this equation, supply chain costs over the planning horizon consist of 8 terms. The first term represents the total cost of transport of goods (including raw materials and finished product).

The second and third terms of the objective function calculate the total cost due to inventory of raw materials and final product in the entire supply chain.

The fourth and fifth terms of the objective function calculate the total raw material and final product shortage costs.

The sixth and seventh term of the objective function calculate the total cost of production of raw materials and final product in the entire supply chain.

The eighth term of the objective function calculates the cost of hub construction. The sum of these costs.

Eq. (2) is the inventory balance constraint for raw materials and calculates the net inventory of raw materials at site i in period t by $\mathrm{I}\mathcal{R}_{\mathrm{i}}(\mathrm{t}) - \mathcal{S}\mathrm{l}\mathcal{R}_{\mathrm{i}}(\mathrm{t})$. Thus, the net inventory of raw materials in period t equals the net inventory in the previous period ($\mathrm{I}\mathcal{R}_{\mathrm{i}}(\mathrm{t}-1) - \mathcal{S}\mathrm{l}\mathcal{R}_{\mathrm{i}}(\mathrm{t}-1)$) plus the production in the present period ($\mathcal{PR}_{\mathrm{i}}(\mathrm{t})$) plus the goods received from other sites ($\mathcal{R}_{\mathrm{ji}}^{m_{\mathcal{H}}}(\mathrm{t}-1)$) minus the amount sent to other sites ($\mathcal{R}_{\mathrm{ij}}^{m_{\mathcal{H}}}(\mathrm{t})$) and the amount used for production of final product ($\mathcal{RP}\mathrm{co} * \mathcal{PP}_{\mathrm{i}}(\mathrm{t})$).

The constraint expressed as Eq. (3) guarantees that the amount of raw material produced at each site in each period ($\mathcal{PR}_{\mathrm{i}}(\mathrm{t})$) does not exceed the corresponding maximum production capacity ($\mathcal{CAPR}_{\mathrm{i}}$). Equations (2) and (3) are the constraints guaranteeing the propriety of producing, sending, receiving, and storage of raw materials in supply chain network.

Constraint (4) guarantees that node i can be linked to node m ($\mathcal{X}_{\mathrm{i}m_{\mathcal{H}}} = 1$) only if node m is designated as a hub ($\mathcal{X}_{m_{\mathcal{H}}m_{\mathcal{H}}} = 1$).

Constraint (5) guarantees that in any acceptable solution, site i can be linked to only one other site (i.e. a hub).

Constraint (6) controls the number of designated hubs in a supply chain network to make sure that their number ($\mathcal{X}_{m_{\mathcal{H}}m_{\mathcal{H}}}$) does not exceed a predetermined maximum ($\wp$H).

Equations (4), (5), and (6) form the supply chain network based on the number of required hubs determined according to hub location rules. In fact, these three constraints check the solutions to make sure that hub location rules are properly respected.

Constraint (7) oversees the status of the designated route between sites i and j. According to this equation, a route can be defined between sites i and j ($\mathcal{Z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}}$) only when site i is linked to hub m ($\mathcal{X}_{\mathrm{i}m_{\mathcal{H}}} = 1$) and site j is linked to hub n ($\mathcal{X}_{\mathrm{j}n_{\mathcal{H}}} = 1$). In that case, the left side of the equation will be equal to 1 ($x_{\mathrm{i}m_{\mathcal{H}}} + x_{\mathrm{j}n_{\mathcal{H}}} - 1 = 1$) and the route from i to j via hubs m and n will be activated ($\mathcal{Z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}} = 1$).

Constraint (8) states that at most one route can be defined between sites i and j. Thus, in an acceptable solution, the route that starts from site i and ends at site j is ($\mathcal{Z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}}$) unique.

Constraints (7 and (8) are defined to restrict the search for the best route, among different geographical locations (sites) in the supply chain network; a route which would be able to maintain the rules of not only hub location but also SCM. Naturally, these equations are associated on the one hand with constraints (4), (5), and (6) to maintain the hub rules, and on the other hand, with other constraints to maintain the supply chain rules.

Constraint (9) determines the cost of transport between sites according to the optimal route determined by constraints (2) to (8) based on the geographical location of sites and network specifications. In fact, assuming that the above equations have correctly determined the route between sites i and j via hubs m and n ($\mathcal{Z}_{\mathrm{i}m_{\mathcal{H}}n_{\mathcal{H}}\mathrm{j}} = 1$), this equation sums the costs of direct transport from i to m ($\mathcal{CT}d_{\mathrm{i}m_{\mathcal{H}}}$), from m to n ($\mathcal{CT}d_{m_{\mathcal{H}}n_{\mathcal{H}}}$), and from n to j ($\mathcal{CT}d_{n_{\mathcal{H}}\mathrm{j}}$).

Constraint (10) has a function similar to Constraint (2), but for the final product instead of raw material. Likewise, Constraint (11) functions similar to Constraint (3), but for the final product instead of raw material. Constraints (10) and (11) ensure the propriety of producing, sending, receiving, and storage of final product in supply chain network. Constraint (12) completes the relationship between production, sending, receiving and storage of raw materials and final product. In fact, this constraint serves as the link between constraints (2) and (3) dedicated to raw materials and constraint (10) and (11) dedicated to the final product. With this constraint, it is ensured that since the production of the final product depends on availability of raw materials at the right time, and location and acquisition of materials from other sites is not instantaneous, the product will be produced only if the site has enough materials at hand ($\frac{\mathrm{I}\mathcal{R}_{\mathrm{i}}(\mathrm{t})-\mathcal{S}\mathrm{I}\mathcal{R}_{\mathrm{i}}(\mathrm{t})}{\mathcal{RP}\mathrm{co}}$) , and otherwise, there will be no production. Constraint (13) expresses the sign restriction in model parameters.

3.1. The proposed solution method

As the developed model is a mixed integer nonlinear programming and NP-hard, a new algorithm for re-formulation is proposed to change it to a mixed integer leaner programming and also a new heuristic algorithms is proposed to solve it in a reasonable time. The exact solution of the model with mathematical modeling software is very time-consuming and sometimes impossible, especially in the case of realistically large instances (because of NP-Hard complexity), so the model needs to be solved by heuristic algorithms. This section presents a heuristic method capable of making a significant reduction in solution time and guaranteeing the acquisition of optimal solutions. Before presenting this heuristic method, the following Proposition needs to be proven:

Proposition 1 For each arbitrarily selected combination of hub sites on a network, if $\mathcal{CT}d_{m_{\mathcal{H}}n_{\mathcal{H}}}$ in Constraint (9) approaches zero, the lowest value of the objective function occurs when all non-hub points (i) are allocated to their closest hub (k) with$min\left(\mathcal{CT}d_{im_{\mathcal{H}}}\right)$.

Proof Assuming that in Constraint (9) $\mathcal{CT}d_{m_{\mathcal{H}}n_{\mathcal{H}}}$ =0, the cost of transporting raw materials and finished product from a site, such as i to another site such as j is$\mathcal{CTr}_{ij} = \mathcal{CT}d_{im_{\mathcal{H}}} + \mathcal{CT}d_{n_{\mathcal{H}}j}$. According to the premise, m and n are selected such that $\mathcal{CT}d_{im_{\mathcal{H}}}$and $\mathcal{CT}d_{n_{\mathcal{H}}j}$reach their minimum values:

$$\mathbf{CTd}_{\mathbf{m}_{\mathcal{H}}\mathbf{j}} = \min_{\mathbf{k} \in \boldsymbol{\Phi}}(\mathbf{CTd}_{\mathbf{ki}}) \tag{14}$$

$$\mathbf{CTd}_{\mathbf{n}_{\mathcal{H}}\mathbf{j}} = \min_{\mathbf{k}' \in \boldsymbol{\Phi}}\left(\mathbf{CTd}_{\mathbf{k}'\mathbf{j}}\right) \tag{15}$$

Therefore, since $\mathcal{CT}d_{ki} \geq 0$ and $\mathcal{CT}d_{k'j} \geq 0$, Eq. (36) holds:

$$\mathbf{CTr}_{\mathbf{ij}} = \min_{\mathbf{k} \in \boldsymbol{\Phi}}(\mathbf{CTd}_{\mathbf{ki}}) + \min_{\mathbf{k}' \in \boldsymbol{\Phi}}\left(\mathbf{CTd}_{\mathbf{k}'\mathbf{j}}\right) = \min_{\mathbf{k},\mathbf{k}' \in \boldsymbol{\Phi}}\left(\mathbf{CTd}_{\mathbf{kj}} + \mathbf{CTd}_{\mathbf{k}'\mathbf{i}}\right) \tag{16}$$

This equation means that the cost of transporting one unit of raw material or finished product from site i to site j equals the sum of the lowest cost of transport between each site and it nearest hub. In the present problem, this cost is known as the lower bound of transportation cost. Since $\mathcal{CTr}_{ij}$is minimized, it can be shown that the sum of transportation costs obtained from expression (17) will also be minimized.

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m_{\mathcal{H}}=1}^{N} \mathcal{CTr}_{ij} * \left(\mathcal{R}_{ij}^{m_{\mathcal{H}}}(t) + \mathcal{P}_{ij}^{m_{\mathcal{H}}}(t)\right) \tag{17}$$

Meanwhile, it can be shown that optimal inventory levels and shortages in a network with predetermined hubs are independent of the allocation of non-hub points to hubs. Therefore, other costs present in the objective function, including the initial inventory and shortage costs are also minimized. Thus, the sum of all cost terms in the objective function will be minimized, and this concludes the proof.

Based on the subject mentioned in the appendix, since $\mathcal{CT}d_{m_{\mathcal{H}}n_{\mathcal{H}}}$ is within the interval (0,1), a heuristic algorithm can be developed for a solution. In that case, the hubs nearest to non-hub points can be used to obtain a good estimation of the optimal solution for all possible hub combinations in the network. Then, for each hub combination, the model of production and inventory planning and control for raw materials and the finished product can be determined. This operation will be repeated for all possible hub combinations and the corresponding optimal solution will be determined. At the end, the final optimal solution of the model will be obtained by minimizing the optimal solution obtained for all combinations.

Therefore, one of the contributions of this paper is to convert a Mixed Integer non-linear Programming to several MIPs with much fewer variables and constraints, so that the resolution time is considerably less consumed. The following MIP model has been developed to solve each of the above combinations.

$$\begin{aligned}\text{Min } \mathbb{Z} = &\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m_{\mathcal{H}}=1}^{N} \mathcal{CTr}_{ij}^{fx} * \left(\mathcal{R}_{ij}^{m_{\mathcal{H}}}(t) + \mathcal{P}_{ij}^{m_{\mathcal{H}}}(t)\right) + \mathcal{CIR}\sum_{i=1}^{N}\sum_{t=1}^{T} \mathrm{I}\mathcal{R}_i(t) + \mathcal{CIP}\sum_{i=1}^{N}\sum_{t=1}^{T} \mathrm{I}\mathcal{P}_i(t) \\ &+ \mathcal{CS}\mathrm{l}\mathcal{P}\sum_{i=1}^{N}\sum_{t=1}^{T} \mathcal{S}\mathrm{l}\mathcal{P}_i(t) + \mathcal{CS}\mathrm{l}\mathcal{R}\sum_{i=1}^{N}\sum_{t=1}^{T} \mathcal{S}\mathrm{l}\mathcal{R}_i(t) + \mathcal{CPR}\sum_{i=1}^{N}\sum_{t=1}^{T} \mathcal{PR}_i(t) \\ &+ \mathcal{CPP}\sum_{i=1}^{N}\sum_{t=1}^{T} \mathcal{PP}_i(t) + \sum_{m_{\mathcal{H}}=1}^{N} \mathcal{CH}_{m_{\mathcal{H}}} \mathcal{X}_{m_{\mathcal{H}}m_{\mathcal{H}}}^{fx}\end{aligned} \tag{18}$$

S. t.

$$\begin{aligned}\mathrm{I}\mathcal{R}_i(t) - \mathcal{S}\mathrm{l}\mathcal{R}_i(t) = &\ \mathrm{I}\mathcal{R}_i(t-1) - \mathcal{S}\mathrm{l}\mathcal{R}_i(t-1) + \mathcal{PR}_i(t) + \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{R}_{ji}^{m_{\mathcal{H}}}(t-1) - \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{R}_{ij}^{m_{\mathcal{H}}}(t) \\ &- \mathcal{RP}\mathrm{co} * \mathcal{PP}_i(t) \quad ,\forall t, i\end{aligned} \tag{19}$$

$$\mathcal{PR}_i(t) \le \mathcal{CAPR}_i \quad ,\forall i = 1 \cdots N \tag{20}$$

$$\begin{aligned}\mathrm{I}\mathcal{P}_i(t) - \mathcal{S}\mathrm{l}\mathcal{P}_i(t) = &\ \mathrm{I}\mathcal{P}_i(t-1) - \mathcal{S}\mathrm{l}\mathcal{P}_i(t-1) + \mathcal{PP}_i(t) + \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{P}_{ji}^{m_{\mathcal{H}}}(t-1) - \sum_{m_{\mathcal{H}}=1}^{N}\sum_{j=1}^{N} \mathcal{P}_{ij}^{m_{\mathcal{H}}}(t) \\ &- \mathcal{DP}_i(t) ,\forall t, i\end{aligned} \tag{21}$$

$$\mathcal{PP}_i(t) \le \mathcal{CAPP}_i \quad ,\forall i = 1 \cdots N \tag{22}$$

$$\mathcal{PP}_i(t) \le \frac{\mathrm{I}\mathcal{R}_i(t) - \mathcal{S}\mathrm{l}\mathcal{R}_i(t)}{\mathcal{RP}\mathrm{co}} \tag{23}$$

$$\mathcal{R}_{ij}^{m_{\mathcal{H}}}(t), \mathcal{P}_{ij}^{m_{\mathcal{H}}}(t), \mathrm{I}\mathcal{R}_i(t), \mathcal{S}\mathrm{l}\mathcal{R}_i(t), \mathrm{I}\mathcal{P}_i(t), \mathcal{S}\mathrm{l}\mathcal{P}_i(t), \mathcal{PR}_i(t), \mathcal{PP}_i(t) \ge 0 \quad , \tag{24}$$

In the above model, variables $\mathcal{CTr}_{ij}^{fx}$ represent the cost of transfer from position i to position j, which is specified for each determined combinations.

Proposition 2 For all specified Hub-network combinations, the above MIP model will findthe optimum solution for the proposed MINLP model.

Proof Suppose a combination of hubs and non-hubs with defined conditions is specified. Therefore, for each of the mentioned combinations, the $\mathcal{CTr}_{ij}$, $\mathcal{X}_{ij}$, $\mathcal{Z}_{im_{\mathcal{H}}n_{\mathcal{H}}j}$ variables are converted to the $\mathcal{CTr}_{ij}^{fx}$, $\mathcal{X}_{ij}^{fx}$, $\mathcal{Z}_{im_{\mathcal{H}}n_{\mathcal{H}}j}^{fx}$ constant parameters.

With this transformation, the constraints (4), (5), (6), (7), (8) and (9) of the proposed MINLP model change as follows.

$$\mathcal{X}_{i\boldsymbol{m}_{\mathcal{H}}}^{fx} \leq \mathcal{X}_{\boldsymbol{m}_{\mathcal{H}}\boldsymbol{m}_{\mathcal{H}}}^{fx} \quad ,\forall i, m_{\mathcal{H}} = 1 \cdots N \tag{25}$$

$$\sum_{j=1}^{N} \mathcal{X}_{ij}^{fx} = 1 \quad ,\forall i = 1 \cdots N \tag{26}$$

$$\sum_{m=1}^{N} \mathcal{X}_{\boldsymbol{m}_{\mathcal{H}}\boldsymbol{m}_{\mathcal{H}}}^{fx} = \wp H \tag{27}$$

$$\mathcal{X}_{i\boldsymbol{m}_{\mathcal{H}}}^{fx} + \mathcal{X}_{j\boldsymbol{n}_{\mathcal{H}}}^{fx} - 1 \leq z_{i\boldsymbol{m}_{\mathcal{H}}\boldsymbol{n}_{\mathcal{H}}j} \quad ,\forall i, m_{\mathcal{H}}, n_{\mathcal{H}}, (j \neq i) = 1 \cdots N \tag{28}$$

$$\sum_{m=1}^{N}\sum_{n=1}^{N} \mathcal{Z}_{i\boldsymbol{m}_{\mathcal{H}}\boldsymbol{n}_{\mathcal{H}}j}^{fx} = 1 \quad ,\forall i = 1 \cdots N \ ,\forall j = i \cdots N \tag{29}$$

$$\mathcal{CTr}_{ij}^{fx} = \sum_{m=1}^{N}\sum_{n=1}^{N} \mathcal{Z}_{i\boldsymbol{m}_{\mathcal{H}}\boldsymbol{n}_{\mathcal{H}}j}^{fx}\left(\mathcal{CTd}_{i\boldsymbol{m}_{\mathcal{H}}} + \mathcal{CTd}_{\boldsymbol{m}_{\mathcal{H}}\boldsymbol{n}_{\mathcal{H}}} + \mathcal{CTd}_{\boldsymbol{n}_{\mathcal{H}}j}\right) \quad ,\forall i = 1 \cdots N \ ,\forall j = i \cdots N \tag{30}$$

With a logical review of constraints (25) through (30), it is easy to conclude that all of the above constraints are obvious and always satisfied. For this reason, the above constraints can be eliminated from the model.

Also, considering that the variable $\mathcal{CTr}_{ij}$ is converted to constant parameter$\mathcal{CTr}_{ij}^{fx}$, expression (25) can also be reformulated as follows.

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{m_{\mathcal{H}}=1}^{N} \boldsymbol{\mathcal{CTr}_{ij}^{fx}} * \left(\boldsymbol{\mathcal{R}_{ij}^{m_{\mathcal{H}}}(t)} + \boldsymbol{\mathcal{P}_{ij}^{m_{\mathcal{H}}}(t)}\right) \tag{31}$$

Finally, if the constraints (25) through (30) are eliminated and equation (25) is replaced by expression and the variables, $\mathcal{CTr}_{ij}$, $\mathcal{X}_{ij}$, $\mathcal{Z}_{i\boldsymbol{m}_{\mathcal{H}}\boldsymbol{n}_{\mathcal{H}}j}$ are converted to constant parameters $\mathcal{CTr}_{ij}^{fx}$, $\mathcal{X}_{ij}^{fx}$, $\mathcal{Z}_{i\boldsymbol{m}_{\mathcal{H}}\boldsymbol{n}_{\mathcal{H}}j}^{fx}$, then the MINLP model is reformulated to the proposed MIP model. As a result, the MINLP model (Expressions (1) to (13)) solutions will be equal to the MIP (Expressions (18) to (24)). And obviously, the optimal solutions of both models are equal. Based on these two prepositions, the proposed heuristic algorithm is shown in Table 2.

Table 2. Pseudo code of the heuristic algorithm

```
10   Begin
20   Objective= Inf;
30   WHILE any New Network exists
40     SET a New Network Specification
           Network.Hub=pH.SET ;
           Network.Size=N;
50     CREATE Network
           Allocate Nearest Nodes to Each Hub
60     SOLVE Network Sub_Program
          Solve Inventory-Production Planning and  Control MILP Program for Current_Network
         Sub.Obj = Optimum Solution of Sub Problem Solving
70     IF Sub.Obj > Objective THEN
80       Go to Line 20
             ELSE
```

```
            Objective = Sub.Obj
            Set Optimum_Network = Current_Network
        END WHILE
90      PRINT Objective
100     PRINT Optimum_Network
110     END;
```

Allocate Nearest Nodes to Each Hub method in the literature is also used (O'Kelly, 1987). However, the significant difference in the use of this method in other resources and this research is that, first, most studies have used this method in their research as the main contribution. While in this research, this method is used as one of the steps of the Heuristic algorithm.

Secondly, most researchers have used this method to reduce the iterations but in this research, this method has been used to convert the Mixed Integer non-linear Programming to the MIP.

Each of the steps of the algorithm is a mathematical programming sub-problem, which will be coded into the computer to be solved by a Solver application. Solving the sub-problems has precedence over solving the main problem because:

The sub-problems have far fewer variables and constraints than the main problem. It can be shown that in the sub-problems, constraints (4), (5), (6), (7), (8) and (9) will be inactive and $N^4 + 2N^2$ number of positive and binary variables will be removed.

The main problem is a mixed-integer quadratic (non-linear) programming problem (Expressions (1) to (13)), which has a long solution time, whereas the sub-problems are mixed-integer linear programming problems (Expressions (18) to (24)). Therefore, solving sub-problems by the use of heuristic algorithm significantly reduces the solution time and increases the accuracy of solution to be obtained from the Solver (by converting the quadratic problem to a linear problem). Table 2 provides the pseudo code of the heuristic algorithm presented in this paper.

Proposition 3 the maximum number of undirected edges in a hub-less network with n nodes is given by Eq.(32).

$$\textbf{Max Edges} = \frac{\mathbf{n(n-1)}}{\mathbf{2}} \tag{32}$$

Proof Eq. (32) is proven by the following inference. Using an undirected edge, each of the n nodes on the network can be linked to n-1 other nodes. Thus, according to counting principle, there can be no more than n (n-1) edges. An edge between nodes i and j is counted once as the edge from i to j and once as the edge from j to i, so each edge is counted twice. Therefore, the maximum number of undirected edges in a network with n nodes is $\frac{n(n-1)}{2}$.

Proposition 4 the maximum number of edges in a single allocation network with p hubs is given by Eq.(33).

$$\textbf{Max Edges} = \frac{\mathbf{2n + p^2 - 3p}}{\mathbf{2}} \tag{33}$$

Proof the edges of a hub network can be categorized into two groups: the hub-hub edges, and the hub-no hub edges. But, because of the simplicity of proving the proposition, it is neglected.

Proposition 5 The maximum number of edges in a hub-less network has a higher degree of complexity than a hub network where $p \ll n$.

Proof according to Proposition 3, the maximum number of undirected edges in a hub-less network with n nodes is $\frac{n(n-1)}{2}$, which can be simplified into $\frac{\mathrm{n}^2-\mathrm{n}}{2}$ and then to $\frac{1}{2}\mathrm{n}^2 - \frac{1}{2}\mathrm{n}$. This function has a complexity of $\mathcal{O}(n^2)$, so the maximum number of edges in a hub-less network is of $\mathcal{O}(n^2)$ complexity.

On the other hand, the maximum number of edges in a single allocation network with p hubs is $\frac{2n+p^2-3p}{2}$, which can be simplified into $n + \frac{1}{2}p^2 - \frac{3}{2}p$. Thus, assuming that $p \ll n$, this function has a complexity of $\mathcal{O}(n)$.

Since the complexity of maximum number of edges in a hub-less network ($\mathcal{O}(n^2)$) is higher than that of single allocation network with p hubs assuming that $p \ll n$ ($\mathcal{O}(n)$), the proof is concluded.

The three Propositions proved in this section support the claim that following the hub approach reduces the network complexity. The lower complexity of a hub location problem compared with a typical network allows the of network and path creation and maintenance costs to be significantly reduced. In addition to cost reduction, this approach can improve the decision-making power of logistics management.

The complexity of the problem can be proved by several methods, some of which need extensive mathematical computation beyond the scope of this paper. Thus, here we provide a simple and easily understandable proof to cover this aspect of this discussion.

Proposition 6 The model problem presented in this paper is of NP-Hard complexity.

Proof First, we prove that the present model is a special case of a known standard model, and then prove that this standard model is NP-Hard. The following lemma is used for this purpose.

Lemma The presented model is a special case of the incapacitated single allocation p-hub median problem (UCSApHMP).

Proof of Lemma Consider the following three conditions in the presented model:

A) The objective function only contains its first term (all other terms are removed).
B) The number of periods is 1; therefore expression $\mathbf{t} \in \{\mathbf{1}, \dots, \boldsymbol{T}\}$ is turned into $\mathbf{t} \in \{\mathbf{1}\}$or $\boldsymbol{T} = \mathbf{1}$.
C) Variables $R_{ij}^{m_{\mathcal{H}}}(t),\ IR_i(t),\ SIR_i(t),\ IP_i(t),\ SIP_i(t),\ PR_i(t),\ PP_i(t)$ take fixed values proportional to the model.

If the conditions A, B and C apply on expressions (1) to (13), the result of the relationship is presented below.

$$\textbf{Min } \mathbb{Z} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \sum_{\mathrm{j}=1}^{\mathrm{N}} \sum_{\mathrm{t}=1}^{\mathrm{T}} \sum_{m_{\mathcal{H}}=1}^{\mathrm{N}} \boldsymbol{CTr}_{\mathrm{ij}} * \boldsymbol{P}_{\mathrm{ij}}^{\boldsymbol{m_{\mathcal{H}}}}(\mathbf{1}) \tag{34}$$

S. t.

Expression (2) always is satisfied. (35)

Expression (3) always is satisfied. (36)

$$\mathcal{X}_{\mathrm{i}m_{\mathcal{H}}} \leq \mathcal{X}_{m_{\mathcal{H}} m_{\mathcal{H}}} \quad , \forall \mathrm{i}, m_{\mathcal{H}} = 1 \cdots N \tag{37}$$

$$\textstyle\sum_{\mathrm{j}=1}^{\mathrm{N}} \mathcal{X}_{\mathrm{ij}} = 1 \quad , \forall \mathrm{i} = 1 \cdots \mathrm{N} \tag{38}$$

$$\textstyle\sum_{\mathrm{m}=1}^{\mathrm{N}} \mathcal{X}_{m_{\mathcal{H}} m_{\mathcal{H}}} = PH \tag{39}$$

$$\mathcal{X}_{\mathrm{i}m_{\mathcal{H}}} + \mathcal{X}_{\mathrm{j}n_{\mathcal{H}}} - 1 \leq z_{\mathrm{i}m_{\mathcal{H}} n_{\mathcal{H}} \mathrm{j}} \quad , \forall \mathrm{i}, m_{\mathcal{H}}, n_{\mathcal{H}}, (\mathrm{j} \neq \mathrm{i}) = 1 \cdots N \tag{40}$$

$$\textstyle\sum_{\mathrm{m}=1}^{\mathrm{N}} \sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{z}_{\mathrm{i}m_{\mathcal{H}} n_{\mathcal{H}} \mathrm{j}} = 1 \quad , \forall \mathrm{i} = 1 \cdots \mathrm{N} \ , \forall \mathrm{j} = \mathrm{i} \cdots N \tag{41}$$

$$CTr_{\mathrm{ij}} = \textstyle\sum_{\mathrm{m}=1}^{\mathrm{N}} \sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{z}_{\mathrm{i}m_{\mathcal{H}} n_{\mathcal{H}} \mathrm{j}} \left(CTd_{\mathrm{i}m_{\mathcal{H}}} + CTd_{m_{\mathcal{H}} n_{\mathcal{H}}} + CTd_{n_{\mathcal{H}} \mathrm{j}} \right) \quad , \forall \mathrm{i} = 1 \cdots N \ , \forall \mathrm{j} = \mathrm{i} \cdots N \tag{42}$$

Expression (10) always is satisfied. (43)

Expression (11) always is satisfied. (44)

Expression (12) always is satisfied. (45)

$$CTr_{\mathrm{ij}} , P_{\mathrm{ij}}^{m_{\mathcal{H}}}(1), \geq 0 \quad , \quad \mathcal{X}_{\mathrm{ij}} , \mathrm{z}_{\mathrm{i}m_{\mathcal{H}} n_{\mathcal{H}} \mathrm{j}} = \{0,1\} \tag{46}$$

The above expression show that if the conditions A, B and C apply, the model of this research is converted into a UCSApHMP.

With the above conditions, all model constraints related to integration of three-level supply chain (e.g. production, consumption, inventory, etc., namely constraints (2), (3), (9), (10), (11) and (12)) will be deactivated and play no role in the objective function value and solution. Naturally, the only constraints that will remain active are those related to the single allocation p-hub median problem. Comparing the remaining constraints (namely constraints (4), (5), (6), (7), and (8)) suggests that the objective function and constraints remaining after assuming the above three conditions are the same as objective function and constraints of UCSApHMP.

Table 3. Comparison of the present paper with Ilić, Urošević, Brimberg, and Mladenović (2010)

Number of Problems	N	PH	Number of feasible Solutions		Deviation of two Problems	
			Ilić et al. (2010)	current article	Number of Solutions	Other specs
48	10, 15, 20,25, 30, 40, 50, 60, 70,80, 90, 100	2, 3, 4, 5	3.32E+36	3.32E+36	-	-

4. Results of exact and heuristic solutions

The problem was coded and solved using GAMS-IDE V.24.0.1 and BARON Optimizer run on a computer with Corei5@2.33 GHz CPU and 4 GB of RAM using CAB data.

This dataset, which has been extracted from the flow of air passenger traffic between major cities in the United States, consists of 25 nodes, inter-node flows, and interactions, and the cost of transport between different locations (nodes) (Fotheringham, 1983).

The results obtained by solving the model are provided in Table 3. As shown in Table 3, due to time constraints and software limitations, only the results pertaining to the first 10 to 20 cities are presented.

The validity of the model is studied and proved using the following two methods:

In the first method, validity is confirmed by manually solving the small instances of the presented model (less than 10 sites and less than 5 hubs) and comparing the results with the expected values.

In the second method, validity is confirmed with the help of Proposition 7, which is proved based on the results of previous Propositions.

Proposition 7 The optimal value of the proposed model equals the optimal value of incapacitated single allocation p-hub median problem (UCSApHMP) if and only if the following conditions are met:

Variables $SIP_i(t)$ ، $SIR_i(t)$ ، $PP_i(t)$ ، $PR_i(t)$ ، CIP ، CIR ، CPR are removed or set to zero, and thus constraints (2), (3), (10), (11) and (12) are deactivated. The model is single-period (t = 1) and single-product (variables that are related to raw material and include the index R are removed and the related constraints are deactivated). The variable $\mathcal{P}_{ij}^{m}(t)$and the index m are removed and w_{ij} is replaced with its counterpart in typical hub location models (this causes other variables such as $CAPP_i$, $CAPR_i$and $RPco$ to lose their effect on solution).

Proof comparing the model modified by the above said conditions and the incapacitated single allocation p-hub median problem (UCSApHMP) (Campbell, 1991; Ilić et al., 2010) shows that these models are the same. The same conclusion can also be made based on identical results obtained from these two models (Table 3), which practically confirms this argument.

Using Proposition 7, validity of modeling was checked by giving the inputs of UCSApHMPs whose results are available in the literature to the model and comparing the outputs. The equality of the solutions obtained from models confirms the validity of modeling. Table 3 shows an instance of above described comparison with the results provided by Ilić et al. (2010). This comparison was also made with other related references, but given the similarity of all results they are not presented in the paper.

Table 4. Comparison of the proposed heuristic method and exact solution based on CAB data

N	pH	Heuristic Sub Models	Exact Method		Heuristic Method				%GAP (with Best Known Solution)
			Best Solution	CPU Time	Solution		CPU Time		
					Max	Min	Sub Models (Mili Seconds)	Total (Min)	
10	2	45	5.563E+09	0:00.883	5.967E+09	5.563E+09	7.007	0:00.315	0.000%
	3	120	5.553E+09	0:01.547	5.962E+09	5.553E+09	8.890	0:01.067	0.000%
	4	210	5.504E+09	0:03.605	5.947E+09	5.504E+09	11.075	0:02.326	0.000%
	Mean		5.540E+09	0:02.012	5.959E+09	5.540E+09	8.991	0:01.236	0.000%
12	2	66	7.207E+09	0:02.408	4.041E+09	7.207E+09	7.231	0:00.507	0.000%
	3	220	7.195E+09	0:04.874	4.028E+09	7.195E+09	9.420	0:02.499	0.000%
	4	495	6.976E+09	0:17.354	7.975E+09	7.150E+09	11.824	0:08.465	2.489%
	Mean		7.126E+09	0:08.212	5.348E+09	7.184E+09	9.492	0:03.824	0.812%
14	2	91	8.851E+09	0:14.255	2.114E+09	8.851E+09	7.456	0:00.699	0.000%
	3	364	8.609E+09	0:21.036	2.094E+09	8.836E+09	9.951	0:03.932	2.638%
	4	1001	8.550E+09	1:24.122	1.000E+10	8.795E+09	12.574	0:14.605	2.870%
	Mean		8.670E+09	0:39.804	4.737E+09	8.827E+09	9.993	0:06.412	1.817%
15	2	105	9.666E+09	2:03.964	1.151E+09	9.673E+09	7.568	0:00.795	0.068%
	3	455	9.657E+09	2:31.069	1.127E+09	9.657E+09	10.216	0:04.648	0.000%
	4	1365	9.408E+09	9:52.971	1.102E+10	9.618E+09	12.948	0:17.674	2.228%
	Mean		9.577E+09	4:49.335	4.432E+09	9.649E+09	10.244	0:07.706	0.752%
16	2	120	1.095E+10	17:06.007	4.504E+09	1.106E+10	7.953	0:00.972	1.041%
	3	560	1.081E+10	23:42.334	4.412E+09	1.099E+10	11.069	0:06.696	1.626%
	4	1820	1.061E+10	106:53.038	1.227E+10	1.092E+10	13.960	0:29.953	2.987%
	Mean		1.079E+10	49:13.793	7.062E+09	1.099E+10	10.994	0:12.541	1.874%
18	2	153	1.384E+10	135:09.438	1.121E+10	1.384E+10	8.466	0:01.327	0.000%
	3	816	1.319E+10	200:39.479	1.098E+10	1.365E+10	12.207	0:10.793	3.530%
	4	3060	1.296E+10	1019:06.038	1.477E+10	1.353E+10	15.308	0:54.512	4.403%
	Mean		1.333E+10	451:38.318	1.232E+10	1.367E+10	11.994	0:22.211	2.591%
20	2	190	1.661E+10	Over 1 day	1.792E+10	1.661E+10	8.851	0:01.682	0.000%
	3	1140	1.631E+10	Over 1 day	1.755E+10	1.631E+10	13.061	0:14.889	0.000%
	4	4845	1.614E+10	Over 1 day	1.728E+10	1.614E+10	16.320	1:19.070	0.000%
	Mean		1.636E+10	Over 1 day	1.758E+10	1.636E+10	12.744	0:31.880	0.000%
22	2	231	1.822E+10	Over 1 day	1.977E+10	1.822E+10	10.145	0:02.459	0.000%
	3	1540	1.773E+10	Over 1 day	1.951E+10	1.773E+10	14.740	0:24.812	0.000%
	4	7315	1.753E+10	Over 1 day	1.921E+10	1.753E+10	19.061	2:44.697	0.000%
	Mean		1.783E+10	Over 1 day	1.950E+10	1.783E+10	14.649	1:03.989	0.000%
24	2	276	1.983E+10	Over 1 day	2.163E+10	1.983E+10	11.439	0:03.237	0.000%
	3	2024	1.914E+10	Over 1 day	2.146E+10	1.914E+10	16.420	0:34.735	0.000%
	4	10626	1.892E+10	Over 1 day	2.115E+10	1.892E+10	21.802	4:10.323	0.000%
	Mean		1.930E+10	Over 1 day	2.141E+10	1.930E+10	16.554	1:36.098	0.000%
25	2	300	2.063E+10	Over 1 day	2.256E+10	2.063E+10	12.086	0:03.626	0.000%
	3	2300	1.984E+10	Over 1 day	2.244E+10	1.984E+10	17.259	0:39.696	0.000%
	4	12650	1.962E+10	Over 1 day	2.211E+10	1.962E+10	23.173	4:53.136	0.000%
	Mean		2.003E+10	Over 1 day	2.237E+10	2.003E+10	17.506	1:52.153	0.000%

As shown in Table 4, due to time constraints and software limitations, only the results pertaining to the first 10 to 20 cities are presented. For comparison, the problems of size $n \in \{10,12,14,15,16,18,20,22,24,25\}$ were extracted the CAB dataset. For each given n, several problems with hub numbers of $\mathcal{PH} \in \{2,3,4\}$ were developed. Increasing the time for solving sub-models by proposed heuristic algorithm is shown in Figure 7.

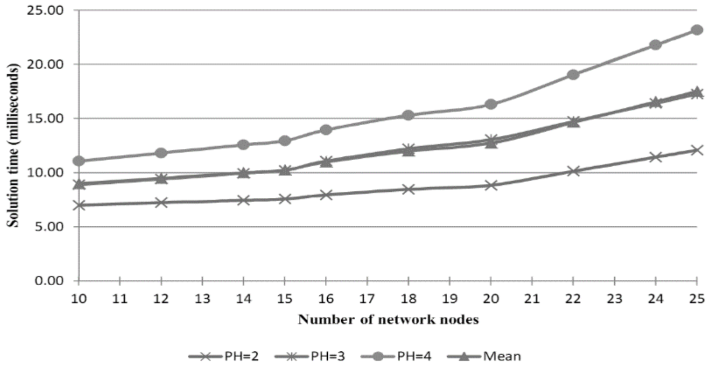

Figure 7. Time to solve sub-models by proposed heuristic algorithm

The time to solve the model by proposed heuristic algorithm is shown in Figure 8.

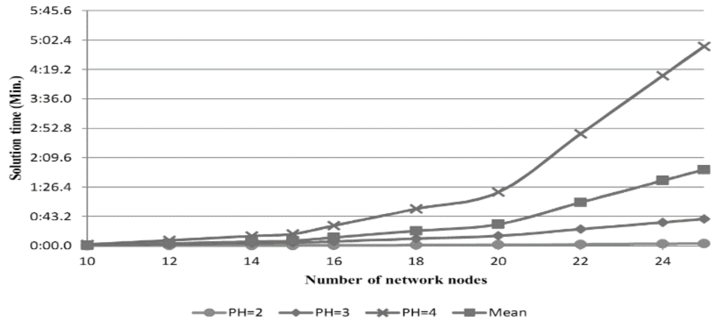

Figure 8. Time to solve the model by proposed heuristic algorithm

The time used to solve the problem using exact method is shown in Figure 9.

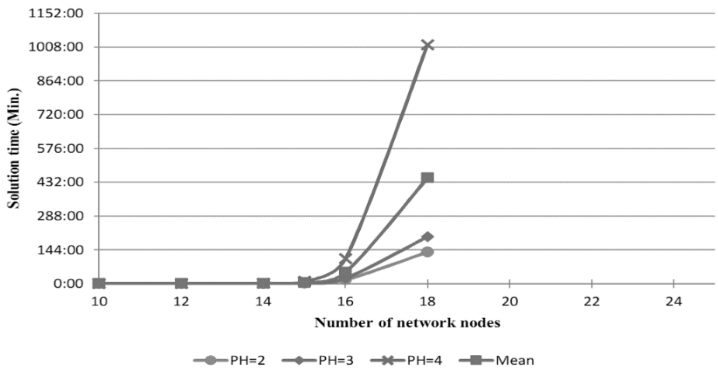

Figure 9. Time to solve the model by GAMS (CPLEX Solver)

Accordingly, the total time required for solving all problems listed in the table using the heuristic method can be expected to remain less than 5 minutes (Figure 8). Meanwhile, Figure 9 shows an exponential growth in the time of the solution by CPLEX. As can be expected for an NP-Hard problem and is shown in Figure 10, there is a quick exponential growth in the solution time achieved by CPLEX for larger problems, whereas the increase in solution time of the proposed heuristic algorithm is much slower. Thus, this heuristic method can be relied upon to solve the larger problems.

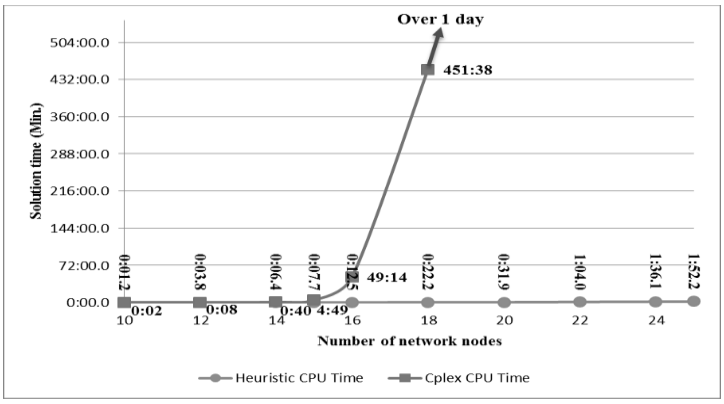

Figure 10. Comparison of the mean times of the CPLEX model and proposed heuristic algorithm

5. Conclusion

Present study aims to merge the hub location concept with the classical supply chain models. This paper addresses a three-level supply chain consisting of suppliers, producers, and consumers, which are linked in a single allocation p-hub median network. This problem is formulated as a mixed integer quadratic mathematical programming model, and then coded and solved in the software GAMS. The results show that due to close association between different aspects of supply chain elements, using a combined model can leads to more realistic solutions. Propositions, proofs, and results presented in this paper confirm the accuracy and validity of the model presented in the paper. The model presented in this paper provides an opportunity to merge the supply chain elements of “production and inventory planning and control” on the one hand, and “location and transportation” on the other hand with the help of mathematical programming. The provided formulation showed that the mentioned gap with regard to integration of mathematical modeling of supply chain drivers can be filled by examination of the interaction between different aspects of the supply chain. More efforts in this field and particularly establishing a reliable relationship between the concepts of supply chain and hub in a comprehensive study can open up new horizons for future researchers, and addressing this issue and achieving positive results in this regard can lead to significant breakthroughs in both areas of study. The use of fuzzy and stochastic decision variables or other decision-making practices, such as MADM for a modeling similar to the one carried out in this paper can yield significant results for future researches.

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