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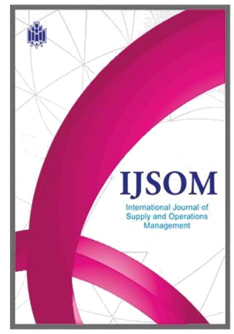

An Applicable Heuristic for Scheduling Multi-mode Resource Constraint Projects Using PERT Technique in the Presence of Uncertain Duration of Activities

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Abstract
In the process of project planning, resource over-allocation is a major shortcoming. The resource over-allocation causes the schedules not to be applicable in practice. Besides, in real projects, it is hard to predict the duration of activities since they may be changed due to lack of resources, delays in delivering resources, unskilled workers etc. that make activities not to be completed as predicted. Hence, it is important to develop a method that can schedule activities by considering different execution conditions. In this research, we focused on another aspect of solving resource over-allocation problem by considering uncertain activity duration. For this purpose, a mixed integer programming model is developed where the objective function is maximizing net present value of the project while duration of activities are not deterministic. Then a number of examples are solved using a heuristic algorithm. The results show that the proposed algorithm can effectively solve the case studies with no over-allocated resources. Afterward, the algorithm is solved using the data of constructing a hospital. The results reveal that the algorithm can be successfully used for real projects.

Keywords: Multi-mode Resource Constraint; Heuristic; Activity Duration Uncertainty; Resource Over-allocation.

1. Introduction

Projects are naturally risky. It is almost impossible to find a risk free project. Implementing activities of a project can yield profits or may lead to losses. The World Bank website which is one of the most important references reports that the amount of investing in national projects in the U.S has increased to 571 billion USD since 1947. Therefore, it is important to schedule projects, monitor them and take appropriate actions to reduce the risks as much as possible. Project management is organizational planning and controlling of projects. As we proceed, some important concepts and definitions that are aimed to be used in this research will be explained. In multi-mode resource constraint project scheduling problems (MRCPSP), activities are not forced to be executed in only one way. Instead, each of the activities can be compared in more than one way. In real practice, there are many activities that can be executed in more than one way. For example, roofing can be covered by ceramics or laminates, walls can be covered by wallpapers or be

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painted. It is obvious that various modes have different activity durations and required resources. Kolisch & Drexl (1997) proved that the MPRCPSP problem is a NP-hard problem. Having an improper plan for a project may cause big problems for project managers. Clearly, many of the problems of a project can be predicted during the planning phase. In this phase, normally schedules can be performed by considering scope of the project[ii]. There are many researches on the issues of time and cost which shows their importance to the researchers. Ignoring one of the above areas will cause delays in projects or wasting a big amount of resources and providing a low quality project (for example, building). Analyzing failed projects which waste an organization's money shows that for every US$1 billion spent on a failed project, US$135 million is lost forever and unrecoverable[iii]. Hence, as seen in the next section, dealing with the issue of resource scheduling is vital and cannot be ignored. The issue of resource over-allocating is a big concern for project engineers in the process of scheduling project activities. Resource over-allocations are frequently seen after scheduling a project in practice which causes the schedule to be useless. Modifying an over-allocated schedule is very complicated and needs a lot of effort and time. Resource over-allocating is considered a major shortcoming in scheduling problems. Unfortunately, in real projects resource over-allocation is usually seen after scheduling a project since planners are not able to focus on all resources in the scheduling process. Such resource over-allocation can cause delays in the completion time of a project which consequently leads to tardiness penalties. In most of the contracts, owners of projects set penalties for delays in delivering the projects deliverables. During scheduling a project in practice, the duration of some activities may be uncertain and depends on natural or technical conditions. This may increase or decrease the usage of resources and consequently may affect resource over-allocating. For example, cold weather causes an activity to be completed in more time than what it usually takes. (Implementing activities that use concrete as resource are very frequent for this problem). War, earthquake, flood, tornado, hot and cold weather, lack of materials, late material delivery, wrong plans and drawings, lack of money are examples that make activity durations not being considered deterministic and they must be a way to consider their uncertainty. On the other hand, due to lack of a methodology for scheduling, monitoring and controlling the activities during implementing projects and making different but not sufficient decisions in these conditions, nothing is obtained except big delays in completing projects or using too many resources which impose big losses of money. Thus, in the present study, a new method for scheduling resource constraint projects while durations of activities are not certain is proposed. The rest of the paper is divided into four sections. In the next section, the most important researches in scheduling resource constraint projects are reviewed. Then a mathematical programming model is presented, and a new method for solving the developed model is proposed. Finally, a number of case studies are solved to verify the proposed method.

2. Review of the literature

This section presents a review of mathematical programming models and techniques to solve the models. In fact, some novel researches are reviewed. The advantages and techniques are explained. Such approach helps us to choose the best method in the next sections. Below the researchers are divided into 3 sub-categories which are completion time, cost, profit. Afterward, some review papers are introduced for further studies. Well-known objective functions and constraint are explained, too.

2.1. Minimizing Completion Time of a Project ($\boldsymbol{C_{max}}$)

Minimizing completion time of a project is an attempt to finish a project as soon as possible. It is the most popular objective that is considered in many researches. For instance, Kim et al. (2005) developed a fuzzy programming based algorithm that called fuzzy logic controller with genetic algorithm which worked based on a serial programming method to schedule resource-constrained multiple-project scheduling problems. Ke & Liu (2010) also used fuzzy operator in a genetic algorithm for minimizing completion time in MRCPSP. Vanhoucke & Debels (2008) argued that activity duration should be considered variable. Then they developed a model to minimize project completion time by using activity preemption and rapid execution of activities. Van Peteghem & Vanhoucke (2010) focused on the impact of preemptive resources on minimizing the completion time of the MRCPSP problems. To solve the proposed problem, a genetic algorithm was proposed. Aidin Delgoshaei et al. (2015) addressed a serial programming approach for reducing the completion time of a project in backward mode by using remained and unused resources in working days. To solve the problem, a hybrid greedy and genetic algorithm was used. Aidin Delgoshaei et al. (2016c) used a dynamic forward approach for scheduling pre-emptive resource-constrained multimode projects using a genetic algorithm. Kreter et al. (2016) focused on RCPSP with general temporal constraints and break-calendars. They developed 3 linear models and solved them with CPLEX 16.0. Pérez et al. (2016) used a multi-modal genetic algorithm in RCPSP for generating solutions with good quality. Their algorithm was then chosen as the best solution regarding makespan and average percent delay. Last but not least, Vaez (2017) considered penalties for earliness and tardiness in the scheduling process.

2.2. Optimizing Robustness of Schedules

The aim of this section is to find schedules that are flexible enough to be used in practice. Normally, the schedules that are outcomes of mathematical algorithms are not flexible to be employed in real life cases. In this section, it is shown that how using some indexes or objective functions scientists have tried to provide more robust solutions in terms of quality of the schedules (Van de Vonder et al., 2005). Van de Vonder et al. (2006) focused on resource constraint impacts on determining trade-off values between quality-robustness and solution-robustness in RCPSP. A Delgoshaei & Ali (2019) reviewed role of human resources in planning systems. Abbasi et al. (2006) developed a model for minimizing completion time and increasing the float time. Chtourou & Haouari (2008) proposed a RCPSP model for minimizing completion time as a criterion for generating robust solutions in the stage when twelve indicators were used to choose best solutions. Hendricks et al. (2002) used degree of specialization for each of the human resources. This approach can help with evaluating the solutions that are provided by their method. Ward & Chapman (2003) argued that project risks should not been ignored in scheduling projects. This approach helps the authors be ready for confronting project risks before they are happening. Aidin Delgoshaei et al. (2017) dealt with scheduling systems while using outsource services. Castejón-Limas et al. (2011) focused on some other managerial factors to be considered for scheduling projects.

In MRCPSP problems, activities can be scheduled in more than one way and, therefore, the activities might have different durations and resources and consequently cash flows. Węglarz et al. (2011) published a review paper on the literature of the MRCPSPs. Laslo (2010) proposed a new method for minimizing negative dependent cash flows in the scheduling process. Financial aspects of scheduling projects should not be ignored as they may bring about infeasible or useless schedules. It should be mentioned that in the MRCPSP studies there are 2 main cash flows to be considered. Positive cash flows that refer to the earned money of the project while negative cash flows are those costs that should be considered for completing projects such as expert salary, worker's wage, as well as maintenance and services costs. Cash flows, whether positive or negative, can affect activity due date, completion time, resource availability, material purchasing etc. Yu et al. (2012) employed a genetic algorithm for selecting a multi-criteria project portfolio problem. Their goals were project interactions and preference information. Rabbani et al. (2017) used metaheuristics for providing robust solutions in a passive optical network planning. ZIAEE (2017) addressed flexible job-shop scheduling for production lines. This idea seems interesting in scheduling projects as well. Yan et al. (2009) focused on finding a solution for preparing a fast response to maritime disasters using heuristics. Metaheuristics are also used in many cases for solving MRCPSP problems. Of all metaheuristics, genetic algorithms are used more frequently than the others. Lin & Gen (2008). Kim et al. (2005) used the fuzzy based genetic algorithm for minimizing completion time and tardiness penalty. Naber & Kolisch (2014) proposed 4 RCPSP based profiles discrete-time mixed integer programming models with flexible resource. To solve the model, they offered preprocessing and priority-based heuristic methods. (Ke and Liu, 2010) also used a combination of fuzzy and genetic algorithm for optimizing project cost with respect to completion time (see also Chen & Askin (2009); Hartmann & Briskorn (2010). Particle swarm optimization method was also used by Jarboui et al. (2008) for solving MRCPSPs. Sharon & Dori (2015) proposed an applicable method for work breakdown structure in order to provide more robust solutions. Papke-Shields & Boyer-Wright (2017) focused on strategic characteristics in planning a project.

2.3. Maximizing Profit of the project (Profit/Net Present Value (NPV))

In many researches the goal was to increase the benefit of scheduling activities. Each activity has positive and negative cash flows that can be changed by executing mode of activity, scheduling time and different types of resources. Maximizing NPV is a logical way to choose a project or reject it. If NPV of a project is calculated and the result is a negative value, then it can be concluded that the project does not provide any benefit. Laslo (2010) considered negative cash flows to minimize the project completion cost as objective function of their model. Aidin Delgoshaei & Gomes (2016) proposed a new method for scheduling in the presence of uncertain cost. Mika et al. (2005) developed a mathematical model with a renewable and non-renewable resource constraint scheduling method. Their objective was to maximize NPV which was solved by a hybrid simulated annealing and Tabu search algorithms. Aidin Delgoshaei et al. (2014) proposed a new mathematical model for maximizing the net present values of the project while discounted positive cash flows are taken into account. Their model only considered finished to start relations. Recently, many researchers considered preemptive resources for showing the priority of activities in using resources. Aidin Delgoshaei et al. (2016a) focused on scheduling dynamic manufacturing systems using hybrid genetic and simulated annealing algorithms. Buddhakulsomsiri & Kim (2006) argued that preemptive resources can influence make span of the project and hence they must be considered during scheduling problems. Damay et al. (2007) used LP method where the RCPSP has preemptive resources. Ballestín et al. (2008) also developed a heuristic algorithm for a similar problem. Seifi & Tavakkoli-Moghaddam (2008) solved the maximizing NPV problem in 4 different payment methods. Van Peteghem & Vanhoucke (2010) developed a new method for minimizing completion time in an activity split allowed multi-mode resource constraint method. Aidin Delgoshaei et al. (2016d) proposed a new method for maximizing NPV in the multi-mode resource constrained scheduling problem. The aim of their model is to prevent resource over-allocation during the scheduling process. In their model all relation types between activities are considered. Reviewing the literature of project scheduling problems indicates that maximizing NPV of the MRCPSP's while durations of activities are uncertain has not been under consideration. Moreover, the negative cash flows are mainly not taken into account in maximizing NPV. Hence, this study continues Aidin Delgoshaei et al. (2016d)'s research as the base-paper and proceeds with their research by considering uncertain durations of activities, contracted time as a constraint for providing schedules, and contracted cost as a constraint for providing schedules. Table 1 lists the characteristics that are developed in this research in Section 2:

Table 1. List of the contributions of this research

No.	Idea	Status
1.	Considering Negative and positive cash flows in maximizing NPV	Partially new (found in a few researches)
2	Considering uncertain durations of activities in maximizing NPV	New in this research
3	Considering fixed contract time in schedules in maximizing NPV	New in this research
4	Considering Signed cost of the contract in providing schedules in maximizing NPV	New in this research
5	Developing Stochastic programming method for maximizing NPV	Partially new (found in a few researches)

In Sections 3 and 4 below, a mathematical programming method is developed, and an appropriate solving method is proposed to solve the model. Results are then evaluated with some performance measurements.

3. Research methodology

In this part, a non-linear mixed integer programming model is developed where the aim is to find the impact of uncertain activity duration in maximizing the NPV of the MRCPSP's while both positive and negative cash flows are taken into consideration. This research has continued pervious researches and developed some new ideas. In most of the previous studies, duration of activities were considered fixed, but in this research we deal with stochastic durations that mean the model is not a simple critical path method anymore and instead it is according to a stochastic method. The main points of the works can be denoted as follows:

1. Considering MRCPS models.
2. Considering renewable resources in maximizing profit of projects.
3. Considering split of activities. This part is similar to the split macro in Microsoft office project software.
4. Considering the restricted resource capacity
5. Scheduling with respect to the completion time in the contract.
6. Scheduling with respect to the total cost in the contract.
7. Considering both positive and negative cash flows
8. Considering uncertain executing duration of each activity mode.

The properties (assumptions) of the developed model are as follows:

1. The model is developed in multi-mode activities which means activities have various executing types.
2. All resources are considered work type. Noted that in project scheduling software as MSP work type refers to human resources and machines.
3. Durations of activities are not deterministic and may be varied due to natural or technical reasons.
4. For expressing durations of activities, triangular probability function is used.
5. Both positive cash flows and negative cash flows are taken into account.
6. The plans' total cost should not exceed what is signed in the contract.
7. Activities are allowed to be taken apart in order to increase the NPV.
8. No activity is allowed to be scheduled before all its precedence is scheduled.

3.1.1. Index

i is used for showing number of activities

k is used for showing number of resource types

t is used for showing time slots

m is used for showing modes of activities

3.1.2. Parameters of the Model

RC_k *shows the capacity of resource type K*

r(i, k) indicates the required resource type k for performing activity i

D(i,m) is duration of Activity i using m th mode that is estimated using triangular probability function:

$$D_{i,m} = \begin{cases} D(optimistic)_{i,m} & if\ P_l > \gamma \\ D(most\ probable)_{i,m} & if\ \delta \le P_l \le \gamma \\ D(pestimistic)_{i,m} & if\ P_l < \delta \end{cases} \tag{1}$$

γ *and* δ *are two random numbers between* (0,1)

TH = time horizon of the project

$P_{i,j}$ *indicates the precedence between activities*

LAG_{ij} shows the the lag time between activitiy i and j

$PCF(i,m) = positive$ cash flow of activity i using mth mode

$NCF(i,m) = negative$ cash flow of activity i using mth mode

TC = *Total cost of the project in the contract*

α= Interest rate

3.1.3. Decision Variables

$X(i,m,t) = \begin{cases} 1 & \text{if activity } i \text{ performs on mode } m \text{ during sub period } t \\ 0 & \text{otherwise} \end{cases}$ *(Binary)*

ES_i: Early Start of activity i $(Integer)$

LS_i: Late Start of activity i $(Integer)$

EF_i: Early Finish of activity i $(Integer)$

LF_i: Late Finish of activity i $(Integer)$

3.1.4. Mathematical Model:

In this part a NL-MIP model is developed for maximizing NPV of the project while both positive and negative cash flows are taken into account and durations of activities are considered uncertain.

$$Max: \sum_{t=1}^{TH}\sum_{i=1}^{n}\sum_{m=1}^{M} X\,.PCF_{(i,m,t)}.\,e^{(\alpha/t)} - \sum_{t=1}^{TH}\sum_{i=1}^{n}\sum_{m=1}^{M} X_{(i,m,t)}\,.NCF_{(i,m,t)}.\,e^{(\alpha/t)} \tag{2}$$

$S.T$:

$$ES_1 = 1 \tag{3}$$

$$ES_i = \min_{t=1:TH}\left(\left\{t.\left(X_{(i,m,t)} - X_{(i,m,t-1)}\right)\middle|X_{(i,m,t-1)} = 0\right\}\right) \quad \forall\ i = 1,\dots,n \tag{4}$$

$$ES_i > \max_{t=1:TH}\left\{t.X_{(j,m,t)}\middle|X_{(j,m,t)} = 1\right\};\ \forall\ (i,j)\ \in\ P_i \tag{5}$$

$$ES_j \ge ES_i + D_{(i,m)} + LAG_{ij}\ \forall\ \left((i,j)\ \in\ P_i\middle|\mathrm{FS}_{i,j} > 0\right) \qquad \forall\ D_{i,m}\sim tri\ (O,M,P) \tag{6}$$

$$ES_j \ge ES_i + LAG_{ij}\ \forall\ \left((i,j)\ \in\ P_i\middle|\mathrm{SS}_{i,j} > 0\right) \qquad \forall D_{i,m}\sim tri\ (O,M,P) \tag{7}$$

$$EF_j \ge EF_i + LAG_{\ ij} + D_{(i,m)}\ \forall\ \left((i,j)\ \in\ P_i\middle|\mathrm{FF}_{i,j} > 0\right) \qquad \forall\ D_{i,m}\sim tri\ (O,M,P) \tag{8}$$

$$EF_j \ge ES_i + D_{(i,m)} + LAG_{ij} + D_j\forall\ \left((i,j)\ \in\ P_i\middle|\mathrm{SF}_{i,j} > 0\right) \qquad \forall\ D_{i,m}\sim tri\ (O,M,P) \tag{9}$$

$$\max_{t=1:TH}\left\{t.X_{(n,m,t)}\middle|X_{(n,m,t)} = 1\right\} \le TH \tag{10}$$

$$\sum_{t=1}^{TH} X_{i,m,t} = D_{(i,m)};\ \forall\ i = 1,..,n\ \&\ m = 1,..,M \qquad \forall\ D_{i,m} \sim tri\ (O, M, P) \tag{11}$$

$$\sum_{m=1}^{M} X_{i,m,t} = 1;\ \forall\ i = 1,..,n\ \&\ t = 1,..,TH \tag{12}$$

$$\sum_{i=1}^{n} \sum_{m=1}^{M} r_{i,k} \le RC_k \quad ;\ \ t = 1,..,T\ \&\ k = 1,..,K \tag{13}$$

$$\sum_{t=1}^{TH} \sum_{i=1}^{n} \sum_{m=1}^{M} X_{(i,m,t)} \,.\, PCF_{(i,m,t)} .\, e^{(\alpha/t)} + \sum_{t=1}^{TH} \sum_{i=1}^{n} \sum_{m=1}^{M} X_{(i,m,t)} \,.\, NCF_{(i,m,t)} .\, e^{(\alpha/t)} \le TC \tag{14}$$

$$D_{i,m} = \begin{cases} D(optimistic)_{i,m} & if\ P_l > \gamma \\ D(most\ probable)_{i,m} & if\ \delta \le P_l \le \gamma \\ D(pestimistic)_{i,m} & if\ P_l < \delta \end{cases} \tag{15}$$

$$X_{i,m,t}: bin \quad \& \quad ES_i; LS_i; EF_i;\ LF_i\ \ : Integer \tag{16}$$

The objective function of this model contains calculating NPV using both positive and negative cash flows. The term $e^{(\alpha/t)}$ is a famous formula in engineering economy where the interest rate is calculated continuously. Hence, the term $\sum_{t=1}^{TH} \sum_{i=1}^{n} \sum_{m=1}^{M} X \,.\, PCF_{(i,m,t)} .\, e^{(\alpha/t)}$ is used to calculate the NPV for all activities in every scheduling period. Similarly, negative cash flows are calculated with the same logic. The model is developed in a way that activities can be scheduled in remained resources areas in every single day. Thus, the algorithm divides activities into smaller pieces considering their durations and then allocates every single activity through the calendar of the project.

The first constraint is to show the beginning of the project by setting the day 1 for early start of the first activity. The second constraint shows the early start of the rest of the activities. The third constraint is used to show the finish to start relation between the activities. Similarly, equations 13, 14 and 15 are used to explain the SS, FF and SF relations, respectively. The equation number 16 is set to ensure that none of schedules exceed the completion time that is signed in the contract. The next constraint indicates that number of the scheduled days for an activity must not exceed its estimated duration. The equation number 18 is set to show that an activity can be choosing only one mode during its execution. The 9th constraint is used to prevent the algorithm from scheduling activities in over-allocated days. The 10th constraint is developed to ensure that all feasible solutions have total costs less than the contract cost. The equation number 21 is developed to show that durations of activities must be estimated using the triangular probability function. The last constraints indicate that the X must be binary and ES variables must be integer.

3.1.4. Precedence relations

For showing Finish to Start relation the formula below:

$$ES_j = ES_i + D_j + LAG_{ij} \tag{17}$$

For showing Start to Start relation the formula below:

$$ES_j = ES_i + LAG_{ij} \tag{18}$$

For showing Finish to Finish relation the formula below:

$$EF_j = EF_i + LAG_{\ ij} + D_j \tag{19}$$

For showing Start to Start relation the formula below:

$$EF_j = ES_i + D_i + LAG_{ij} + D_j \tag{20}$$

3.1.5. Using the triangular probability function for expressing the uncertain durations of activities:

In this research, the triangular probability function is used to express the durations of activities since it is well-known and proved in PERT SCHEDULING METHOD. Aidin Delgoshaei et al. (2016b) maintained that dynamic conditions can have serious impacts on the productivity of systems. The reason to use triangular probability function in this study is that this function can consider the 3 clusters for the variables domain. A triangular probability function can be expressed using the following formula:

$$X \sim tri(p, m, o) \quad (21)$$

Where p shows the pessimistic probability, o shows the optimistic probability and m indicates the most probable event. For example, suppose implementing cabling as an activity. This activity in normal conditions takes 10 days (50%). But for some reasons (like problem with drawings) it can take more time (let's say 15 days) in reality (with 30% chance). In the same way, the operators may finish the activity faster so they can finish it in 6 days. This situation has a probability of 20%. So this activity can be expressed by a triangular probability function as below:

$$Duration_{Cabling} = \begin{cases} 6; & if\ P_i < 20 \\ 10; & if\ 21 \le P_i \le 50 \\ 15; & if\ P_i > 51 \end{cases}$$

3.2. Heuristics and metaheuristics and reasons to use them

As illustrated in Section 2, most of the MRCPSP problems are too complicated and are mostly NP-hard in nature which make them too hard to solve with regular optimizing algorithms. It is found out that more than 80% of opted researches in Section 2 were solved by heuristics and meta-heuristics. It is decided to use MATLAB for developing the proposed heuristics and metaheuristics which are explained in the next section. Zhou & Askin (1998) found that while RCPSPs are expressed in terms of cash flows, the complexity of the problem increases. In the first step, the solving method that was offered generates an activity list in all iterations. Then the algorithm chooses the best activity that has more positive cash flows and less negative cash flows, and the duration of activity is estimated by the triangular probability function. Afterward, the algorithm takes apart the selected activity into separate single day parts and allocates the activity in the Gantt chart in a forward mode. If there is enough resources for scheduling the activity, the algorithm assigns it; otherwise, the algorithm selects another activity. While all activities are scheduled, the algorithm checks if the new schedule has completion time less than the contracted time and also a total cost less than the contracted cost. If so, the algorithm considers the schedule as a feasible solution. Then the algorithm checks if the new solution improves the total NPV and so this solution will be considered as a good feasible solution. The algorithm repeats this procedure until the end of the process. Figure 1 indicates the steps of the proposed heuristic.

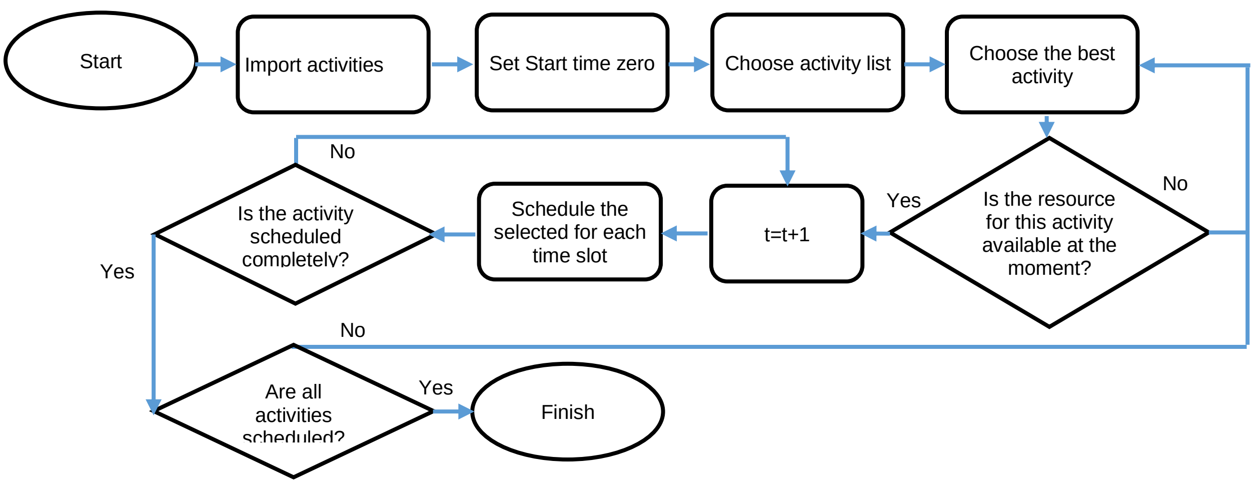

Figure 1. Flow chart of the proposed method

It is noted that mechanism of the proposed heuristic is scheduling activities in the forward mode by checking the resource availability and resource over-allocation in single time slots. Below, each of the steps is explained.

3.3. Contracted Time and Cost of the Project

In real projects, the time and cost of the project are determined by the owner of the project (with the help of the consultant of the project) and contractors must prepare their schedules by respecting these time and cost. Hence, it is necessary to consider them as the first step of solving the algorithm.

3.4. Create Activity List

The first step in using the algorithm is creating an activity list. This activity list is a set of activities that must be implemented in order to complete the project. The activity list must be entered by the decision maker and so it is an input data.

3.5. Choosing Modes of Activities

After generating the activity list, the algorithm should choose the best executing mode of activities. This process will be done based on positive and negative cash flows to find the desired mode of an activity. It should be noted that based on the proposed mathematical model in Section 3, the algorithm respects the selected mode of an activity during scheduling.

3.6. Duration Prediction Operator

Then each of the activities must be implemented using an execution mode (since we assumed the MRCPSP model). Choosing each activity mode causes 3 different activity durations which may happen according to the triangular probability function.

$$D_{i,m} = \begin{cases} D(optimistic)_{i,m} & if\ P_l > \gamma \\ D(most\ probable)_{i,m} & if\ \delta \leq P_l \leq \gamma \\ D(pestimistic)_{i,m} & if\ P_l < \delta \end{cases}$$

In this equation, γ *and* δ are two random parameters between 0 and 1 that can be estimated by decision maker according to the condition of the project (Figure 2).

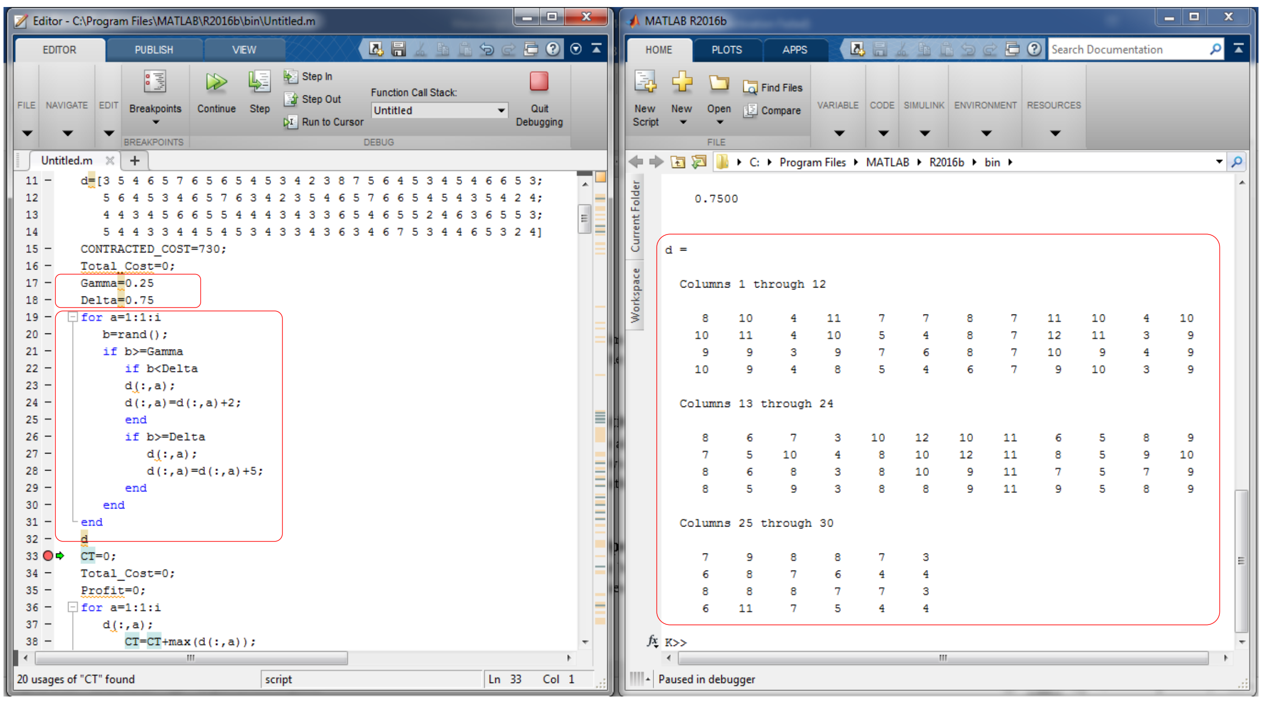

Figure 2. Duration prediction operator codes (Left Image) and results of running the operator (Right Image)

3.7. Create a Candidate List

This part of algorithm is developed to choose a set of activities that can be scheduled in every single day of the project. The activities in this list must have one condition: all the predecessors of the activity should be scheduled before so it has the chance to be executed (Figure 3).

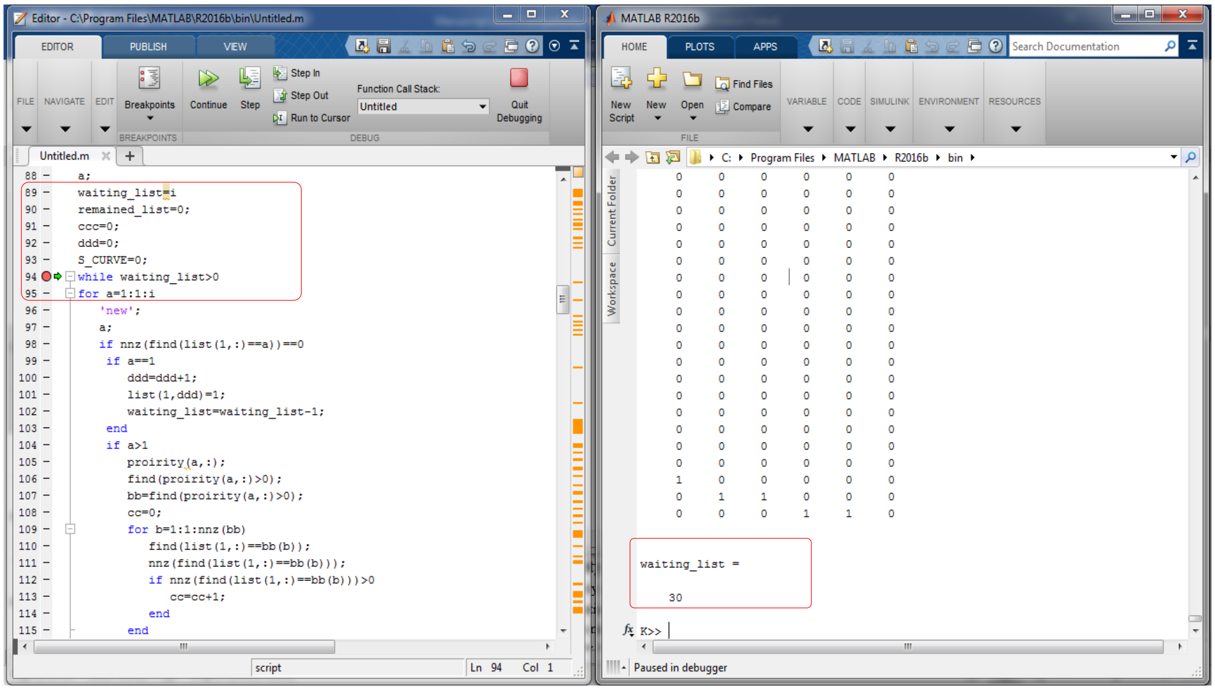

Figure 3. Candidate list codes in matlab (left image) and results of running the operator (right image)

3.8. Selecting Process: Choosing an activity in the candidate list to be scheduled

In this step, the algorithm finds the most suitable activity in the list to be scheduled.
To accomplish this purpose, two rules should be taken into consideration:
1- The activity with the higher positive cash flow and less negative cash flow is desired.
2- There must be enough available resources for the activity to be scheduled.
Since the activities can be separated, the algorithm can assign the activities to the most suitable places in order to increase the NPV of project.

3.9. Over-allocation Check

After choosing an activity, it is time to schedule it in the first available place that the resources are enough for that activity. For this purpose, the algorithm will count the cumulative resources in every single day and if there is enough resources, the activity will be scheduled; otherwise, the algorithm will consider another activity in the candidate list.

3.10. Objective Function

The objective function of the mathematical model that was developed in section 3.2 can be considered as a measuring tool for accepting or rejecting the generated solutions.

$$Max: \sum_{t=1}^{TH}\sum_{i=1}^{n}\sum_{m=1}^{M} X\,.PCF_{(i,m,t)}.\,e^{(\alpha/t)} - \sum_{t=1}^{TH}\sum_{i=1}^{n}\sum_{m=1}^{M} X_{(i,m,t)}\,.NCF_{(i,m,t)}.\,e^{\left(\frac{\alpha}{t}\right)} \qquad (22)$$

In this work, both positive and negative cash flows are taken into account in order to check the desirability of schedules. Thus, after generating a schedule, a solution may be accepted or rejected.

3.11. Termination Criteria

The program will be finished if any of the following modes happens:
1. If activities in the activity list are planned according to the calendar of the project.
2. The number of algorithm iterations is passed (for example, 100 runs).

3.12. Solution representing

In this study like many other researches, the solution representation is according to a zero one matrix where non-zero elements show the exact time of scheduled activities (Figure 4).

Activity Number									
A	A	A	0	0	0	0	0	0	0
B	0	0	B	B	0	0	0	0	0
C	0	0	0	0	C	0	0	0	0
D	0	0	0	0	0	D	D	D	D
Calendar	1	2	3	4	5	6	7	8	9

Figure 4. A representing scheme sample

For instance, in this matrix, activity A is scheduled in days 1 and 2, activity B in days 3 and 4, activity C in day 5, and activity D in days 6, 7, 8 and 9.

4. Experiments

After addressing the mathematical model and proposing a mechanism to schedule activities in order to maximize NPV, it is time to evaluate the efficiency of the heuristic. To this end, some examples are used (Delgoshaei et al. 2016). Note that our aim is to find the impact of the uncertain durations on maximizing NPV in the MRCPSPs. In the first step, 2 problems (in the medium and large scales) are illustrated in detail at first. The AON network diagram and Gantt chart are drawn and explained. Then the impact of uncertain durations is discussed for the problems. Note that the activities can be taken apart by the algorithm in order to utilize the use of resources in the calendar of the project. For this purpose, in each section, a problem is solved in the fixed time of activities and the results are compared with the condition of uncertain durations of activities in terms of completion time and number of resource over-allocation.

Experiment 1 (medium scale):

The first example contains 15 activities where each of the activities can be done in 4 modes. In addition, there are 3 types of constraint renewable resources. Figure 5 indicates the network of the project. Using figure 5, a binary matrix is drawn which can be entered into the Matlab program. Table 2 shows the priorities and lags between the activities. In order to test the performance of the heuristic, each of the examples in Section 4 are solved in 2 ways where in the first way the resources are not limited and in the second condition the resources are limited but solved by the proposed heuristic. After solving the example, the Gantt of the examples are shown by Figures 6 and 7. As a result, the completion time of the project is 41 days in the first mode and 36 working days in the second mode. While there are no limitations for the resources, the algorithm can easily schedule the project in 36 working days. But when the resources are not enough, the algorithm extends the completion time of the project in order to prevent resource over- allocation. This action is set by spreading those activities which are overlapped in a way that maximum net present value can be achieved. In this experiment, the limited resources also cause the makespan of the project to be extended from 36 working days in the first mode to 41 working days in the second mode. The Gantt chart in Figure 7 shows that activity 12 is decided to be split by the algorithm. The reason for such a split is to prevent resource over-allocation. Such separation makes using resources in working days 23, 24 and 25 to not exceed the resource level.After developing the Gantt chart, the S-curve of the schedule is drawn to see if the schedule can be used efficiently in practice or not. Note that this curve shows the usage of the resources in the project and while the curve is similar to the character “S”, the schedule is suitable to use. Figures 8 and 9 show the S-curve of the schedule in the lifespan of the project. The figures indicate that the schedule has smoothed resources and thus it can be used in practice.

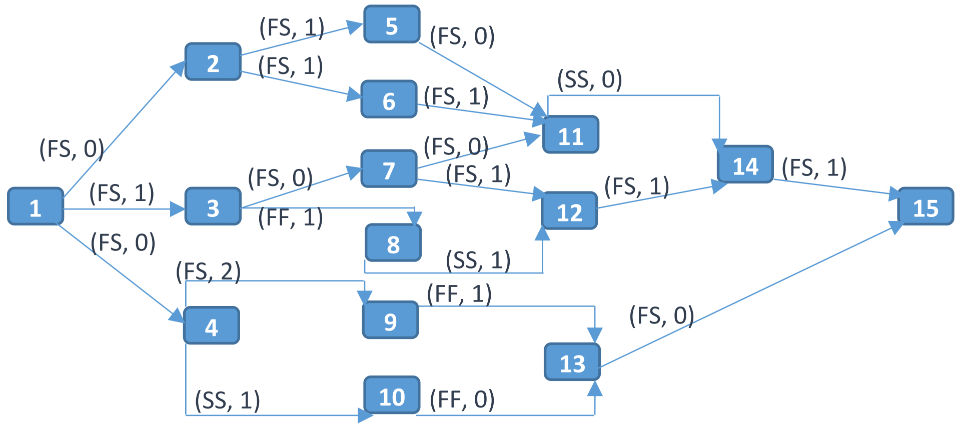

Figure 5. Network Diagram for Experiment number 1

Table 2. Priority Matirx for Experiment number 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	FS	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	FS+	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	FS	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	FS+	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	FS+	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	FS	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	FF+	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	FS+	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	SS+	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	FS	FS+	FS	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	FS+	SS+	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	FF+	FF	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	SS	FS+	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	FS	FS+	0

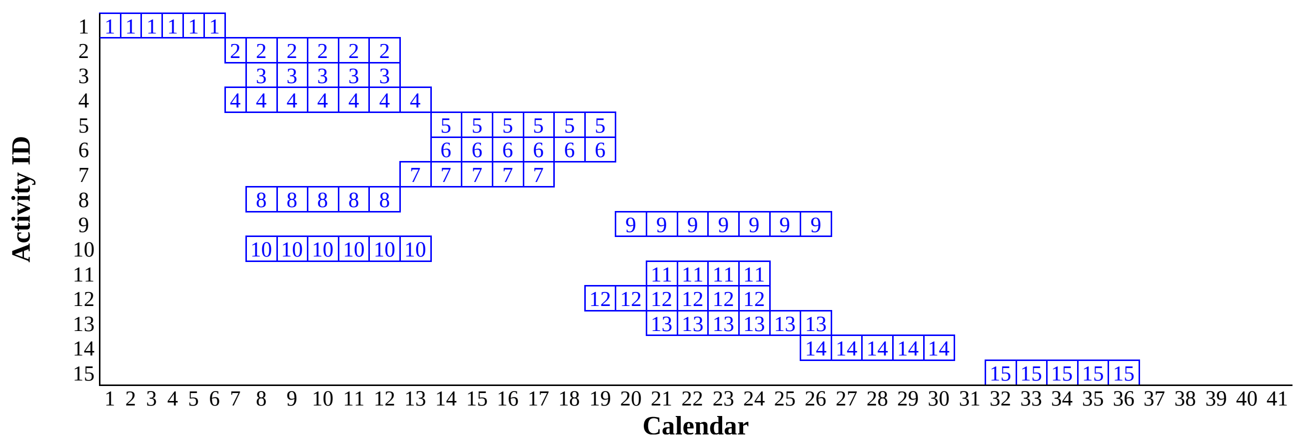

Figure 6. The Gantt of the first experiment with no resource limitations

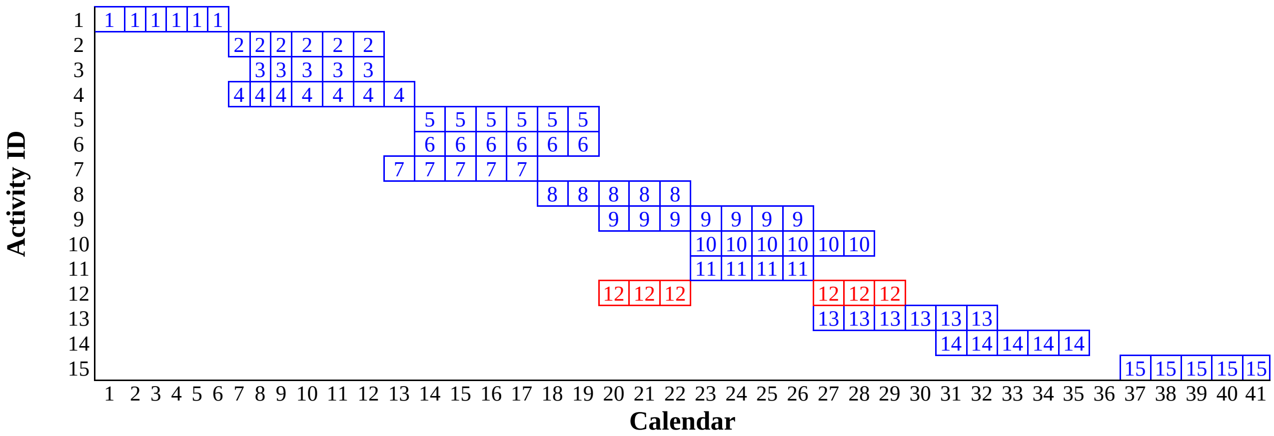

Figure 7. The Gantt of the first experiment after using the heuristic in the condition of confronting with resource limitations

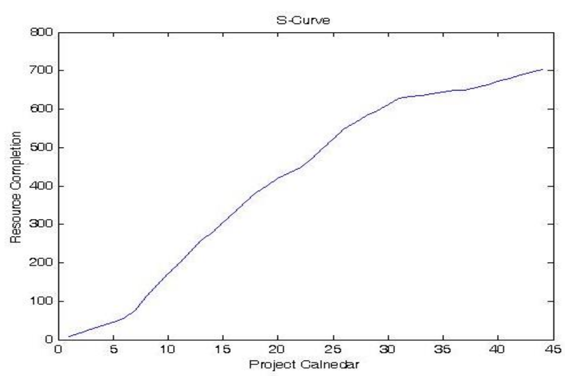

Figure 8. S-Curve of the first Experiment (Relaxed Mode)

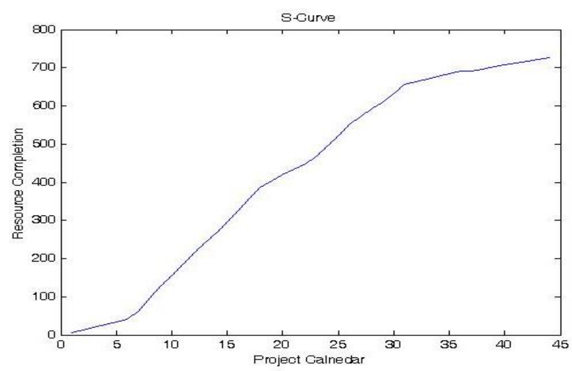

Figure 9. S-Curve of the first Experiment (Resource Constrained Mode)

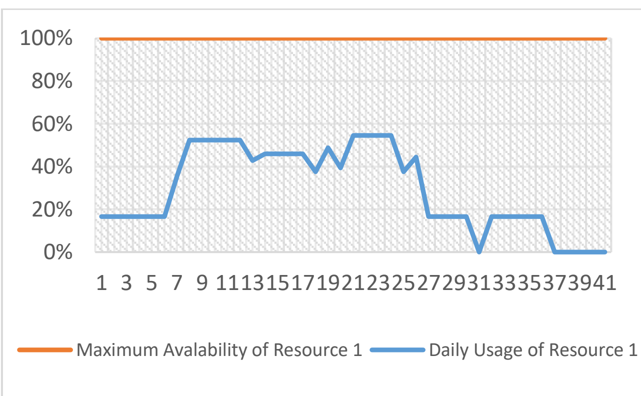

Figure 10. Graph of the first resource consumption before using the heuristic

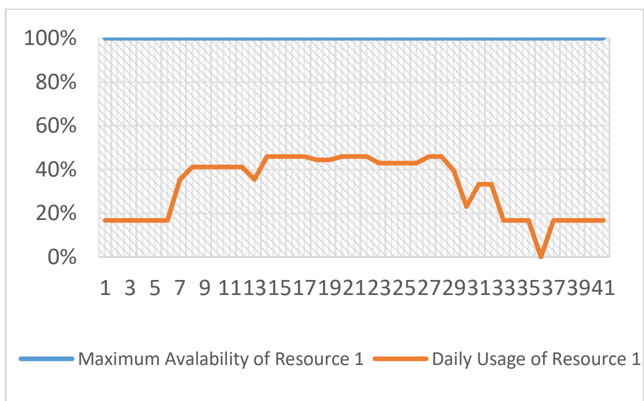

Figure 11. Graph of the first resource consumption after using the heuristic

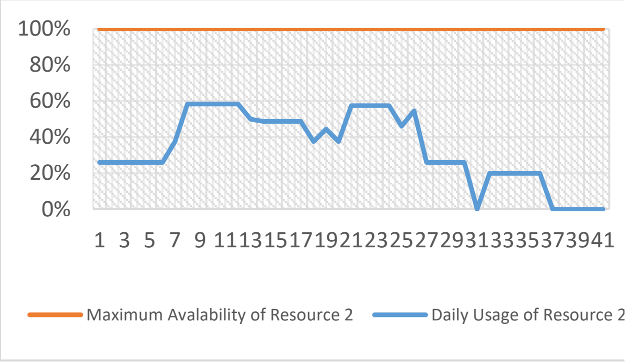

Figure 12. Graph of the second resource consumption before using the heuristic

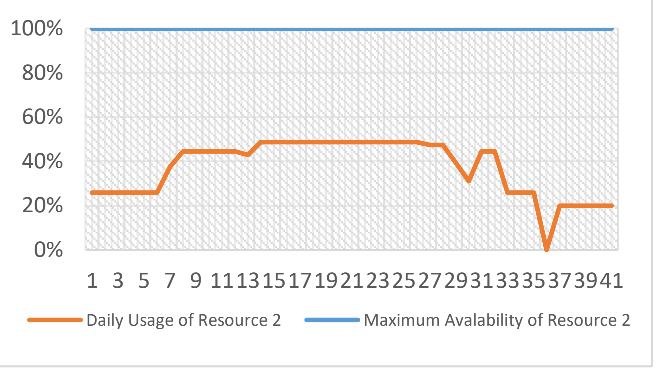

Figure 13. Graph of the second resource consumption before using the heuristic

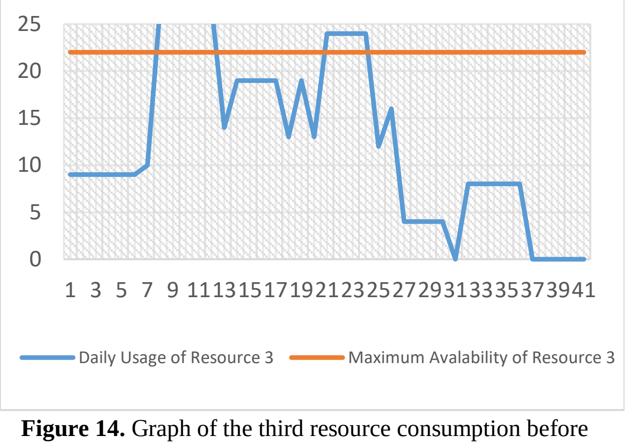

Figure 14. Graph of the third resource consumption before using the heuristic

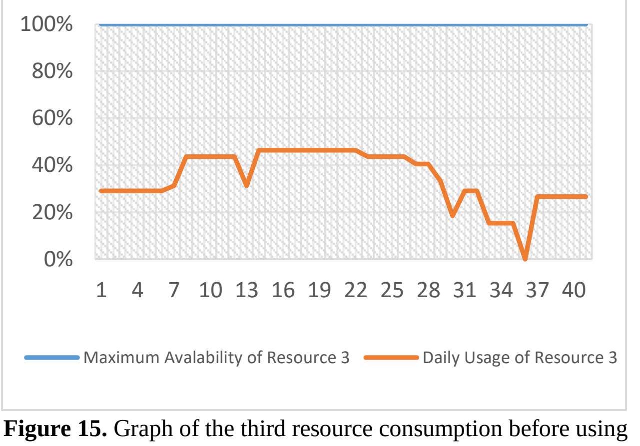

Figure 15. Graph of the third resource consumption before using the heuristic

The radar charts are drawn for showing the resource over-allocation in the calendar of the project. The green lines in each chart shows the resource limits and the blue line indicates the resource usage in the working days. When a resource is over-allocated, the blue line passes the red line that reveals the solution is not feasible (Figures 10, 12 and 14). The outcomes show that the proposed heuristic is able to remove all resource over-allocations and, therefore, the schedule is correct and can be used (Figures 11, 13 and 15).

4.1. Solving the Small, Medium and Large Scale Examples

In this section, some experiments from the small (5 activities) to large scales (up to 1000 activities) are solved. Each experiment is solved 2 times when in the first time, there is no resource limit and in the second time, in contrast, there is resource limits at least in one resource. Table 3 lists the outcomes of solving the algorithm with different δ and γ. The outcomes show that when the difference of δ and γ is about 0.5 the best results will be achieved. Table 4 shows the information of uncertain durations based on the triangular function which is the main concept of this research. Table 5 presents the results of solving 14 experiments. The outcomes are compared in terms of completion time, number of split activities, CPU time and total cost. The results reveal that the algorithm needs more time to schedule the experiments in the condition of confronting insufficient resources (Figure 18). The total cost is also decreased compared to the condition that there is no resource limits (Figure 19). Figure 20 depicts the compared number of the activities that are split in both modes. It is shown that in all case studies the completion time is increased while the resources are not sufficient (Figure 21).

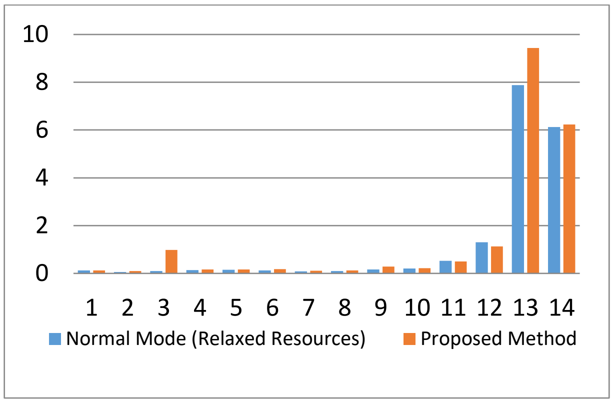

Figure 16. Comparing the CPU time of solving the Algorithms

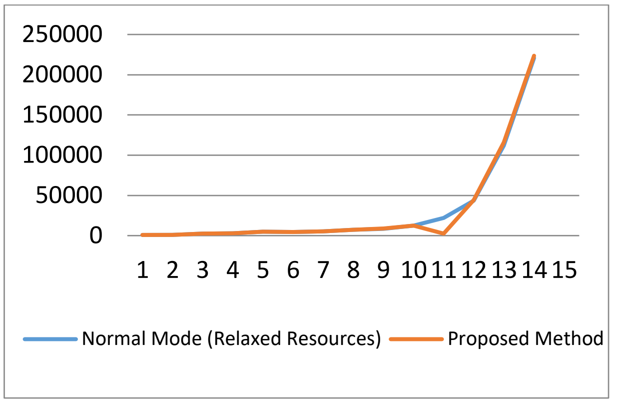

Figure 17. Comparing the Total Cost of Experiments

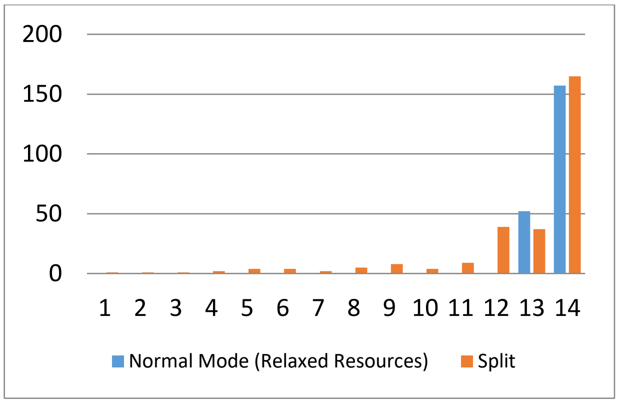

Figure 18. Number of Activity Splits

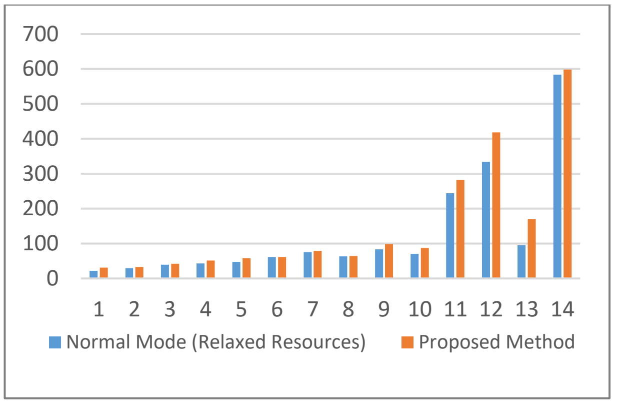

Figure 19. Comparing the Completion Time of the Experiments

Table 3. Impact of the Triangular Probability Function on the Completion Time

No.	δ	γ	Completion Time	Split Activity
1	0.45	0.48	102	4
2	0.45	0.5	111	6
3	0.45	0.55	101	9
4	0.4	0.6	101	4
5	0.35	0.65	93	8
6	0.3	0.7	105	8
7	0.25	0.75	87	7
8	0.2	0.8	91	8
9	0.15	0.85	92	2
10	0.1	0.9	94	3

Table 4. Duration of Activity Database for Solving Experiments from the Literature

No.	Durations				Stochastic	
	Number of Activities	Optimistic	Most Probable	Pessimistic	δ	γ
1	5	3	7	14	0.2	0.5
2	6	3	7	14	0.2	0.5
3	13	3	7	14	0.2	0.5
4	15	6	8	14	0.2	0.5
5	18	4	8	14	0.2	0.5
6	20	3	9	14	0.2	0.5
7	25	4	7	14	0.2	0.5
8	30	3	7	14	0.2	0.5
9	40	7	10	14	0.2	0.5
10	50	3	7	15	0.2	0.5
11	100	6	7	14	0.2	0.5
12	200	3	7	14	0.2	0.5
13	500	3	7	14	0.2	0.5
14	1000	9	10	16	0.2	0.5

Table 5. Results of solving 14 sets of experiments in 2 conditions of normal mode and using the proposed heuristic*

No.	Activity	Resource	Mode	Normal Mode (Relaxed Resources)				The Proposed Heuristic			
				Completion Time	Split	NPV	CPU Time	Completion Time	Split	NPV	CPU Time
1	5	3	2	22	0	837.21	0.128	31	1	835.6842	0.127083
2	6	2	2	29	0	1102.1	0.061	33	1	1101.1	0.0945
3	13	2	2	39	0	2438.7	0.095	42	1	2437.3	0.979
4	15	3	4	43	0	2944.5	0.144	51	2	2943.6	0.16
5	18	3	3	48	0	4735.8	0.146	58	4	4734.6	0.165
6	20	3	4	61	0	4500.7	0.125	61	4	4498.9	0.173
7	25	2	3	75	0	5446.2	0.088	79	2	5445.2	0.116
8	30	2	4	63	0	7297.3	0.096	64	5	7295.9	0.128
9	40	4	3	83	0	8629.2	0.166	98	8	8778.6	0.289
10	50	5	4	71	0	12389	0.211	87	4	12381	0.218
11	100	10	4	244	0	22159	0.527	282	9	2330	0.506
12	200	20	3	334	0	43573	1.303	418	39	44198	1.129
13	500	50	5	95	52	111780	7.88	170	37	116030	9.43
14	1000	100	5	584	157	220780	61.22	598	165	223590	62.28

* The precedence matrices are shown as appendices

4.2. Measuring the Efficiency of the Proposed Method

As mentioned in Section 3, after developing the solving method and solving the experiments, the performance of the algorithm can be evaluated using mathematical indexes. In this study, 2 indexes are used which are described in Section 2 (Table 6).

Having solved the problem using the heuristic, it is found that the proposed method can successfully resolve the resource over-allocation problem. In addition, uncertain durations of the activities can increase the completion time of the solved experiments up to 79%. Moreover, as mentioned before, since resource limitations increase the makespan of the project, the NPV is supposed to decrease drastically. But the proposed algorithm can prevent sudden dropping of the NPV by splitting the activities and using the resources in the time horizon of the project. As seen, in all cases the NPV is not decreased more than 4%.

Table 6. Measuring the performance of the Solving Algorithm

Experi ment	Resource Over allocation	Completi on time	Experiment	Resource Over allocation	Completi on time
1	0.183	40.91	15	0.183	40.91
2	0.091	13.79	16	0.091	13.79
3	0.057	7.69	17	0.057	7.69
4	0.031	18.60	18	0.031	18.60
5	0.025	20.83	19	0.025	20.83
6	0.040	0.00	20	0.040	0.00
7	0.018	5.33	21	0.018	5.33
8	0.019	1.59	22	0.019	1.59
9	-1.702	18.07	23	-1.702	18.07
10	0.065	22.54	24	0.065	22.54
11	-4.897	15.57	25	-4.897	15.57
12	-1.414	25.15	26	-1.414	25.15
13	-3.663	78.95	27	-3.663	78.95
14	-1.257	2.40	28	-1.257	2.40

5. Validating the Proposed Method: Constructing a Hospital

The following case study is constructing a hospital in 2014. The list of activities is shown in Table 7. Table 8 indicates the precedence network between the activities.

Table 7. List of Activities for Constructing a Hospital

I	Activity	D		I	Activity	D		ID	Activity	Du
1	Shop Preparedness and	7		2	Installing Coverage for Ducts	7		43	Windows	3
2	Foundation	21		2	FCU(Duct)	6		44	Wooden Doors	2
3	Structure	40		2	FCU (Cold, Hot and Return	3		45	Plastering	7
4	Flooring	10		2	FCU (Drainage Piping)	4		46	Tilling	2
5	Trench	2		2	FCU (Machine)	2		47	Operating Rooms	2
6	Wall Erection	12		2	FCU (Grille)	2		48	Installing Ceramics	4
7	Roofing	5		2	Structure of Chillers	2		49	Painting	4
8	Window Frames	3		2	Install Chillers	4		50	Installing Wooden	2
9	Door Frames	3		3	Installing Gas Supply System	2		51	Installing Radiology	1
1	Fire Box Frames	1		3	Installing Cable Trunk	5		52	Locker Room	2
1	Installing False Ceiling	10		3	Install Cable Tray	5		53	Installing Hospital Beds	1
1	Roof Insulation	2		3	Cabling	7		54	Sliding Door	2
1	Installing False Ceiling	10		3	Installing Lightning Cables	5		55	Installing Surgical Beds	1
1	Rain Water Piping	3		3	Installing Lightning Lamps	2		56	Area	30
1	Install Piping Tray	4		3	Installing of Electrical Panels	7		57	Building Facades	30
1	Water Piping	5		3	Cable Connecting for Electrical	2		58	Testing And Site	2
1	Return Piping	5		3	Installing of Plugs & Sockets	2				
1	Drainage Piping	5		3	Connect Cables and Pipes of	1		*D	Duration	
1	Floor Drains	1		4	Installing CCTVs	2				
2	Installing Supports for	10		4	Installing Nurse Call System	2				
2	Installing Ducts	7		4	Installing Paging System	2				

Table 8. The Activity Precedence Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																				
2	FS																			
3		FS																		
4			FS																	
5				SS+1																
6				FS+1																
7						FS														
8						FS														
9						FS														
10						FS														
11							FS													
12						FS	FS				FS									
13																FS	FS	FS		
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	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
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58	FS	FS											FS		FS			

Table 9 shows the resource usage of activities:

Table 9. Resource usage of activities

Activity ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Civil Worker	Mechanical Worker	Electrical Worker	Pipe man	Electrical Technician	Mechanical Technician	Civil Technician	Welder	Painter	Carpenter	Blacksmith	Civil Engineer	Mechanical Engineer	Electrical Engineer	Mason
1	1	1	1		1	1	1					1	1	1	
2	5						1					1			
3	5						1					1			
4	4						1					1			
5	1						1								
6	5						1					1			1
7	4						1					1			1
8	2										1				
9	2										1				
10	2										1				
11	4						1	2			1	1			

Table 10. Continued

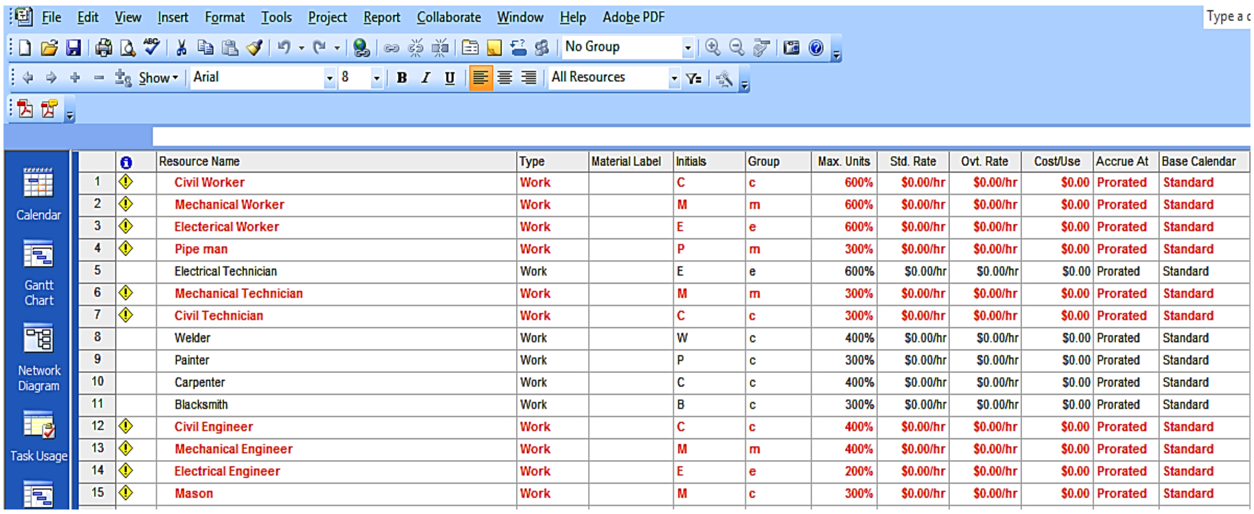
Activity ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Civil Worker	Mechanical Worker	Electrical Worker	Pipe man	Electrical Technician	Mechanical Technician	Civil Technician	Welder	Painter	Carpenter	Blacksmith	Civil Engineer	Mechanical Engineer	Electrical Engineer	Mason
12	1						1								1
13	4						1	2			1	1			
14		2		2		1							1		
15		4		2		1		2					1		
16		4		2		1							1		
17		4		2		1							1		
18		2		1		1							1		
19		2		1											
20		4				1							1		
21		4				1							1		
22		2				1									
23		2				1							1		
24		4				1							1		
25		1				1							1		
26		2				1							1		
27		1				1									
28		1				1		1					1		
29		2				1							1		
30		2				1							1		
31			4		1			1						1	
32			4		1			1						1	
33			2		1									1	
34			2		1									1	
35			2		1									1	
36			2		1									1	
37			2		1									1	
38			2		1									1	
39			2		1									1	
40			2		1									1	
41			2		1									1	
42			2		1									1	
43	2						1					1			
44	2						1			2		1			
45	4						1					1			2
46	4						1					1			2
47	2						1					1			1
48	4						1					1			2
49							1		2			1			
50	2						1			2		1			
51	2						1			1		1			
52	2						1					1			
53	2						1					1			
54	2						1					1			
55	2						1					1			
56	2						1					1			1
57	4						1					1			1
58					1	1	1					1	1	1	

Finally, Table 10 shows the maximum available resource:

Table 11. Maximum Available Resource

Resource	**Civil Worker**	**Mechanical Worker**	**Electrical Worker**	**Pipe man**	**Electrical Technician**	**Mechanical Technician**	**Civil Technician**	**Welder**	**Painter**	**Carpenter**	**Blacksmith**	**Civil Engineer**	**Mechanical Engineer**	**Electrical Engineer**	**Mason**
Maximum Level	**800%**	**800%**	**800%**	**200%**	**200%**	**200%**	**200%**	**200%**	**200%**	**100%**	**200%**	**100%**	**100%**	**100%**	**200%**

In the first step, we schedule the problem using MSP® 2016. As expected, the project is unacceptable since most of the resources are over-allocated (Figure 22).



		Resource Name	Type	Material Label	Initials	Group	Max. Units	Std. Rate	Ovt. Rate	Cost/Use	Accrue At	Base Calendar
1		Civil Worker	Work		C	c	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
2		Mechanical Worker	Work		M	m	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
3		Electerical Worker	Work		E	e	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
4		Pipe man	Work		P	m	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
5		Electrical Technician	Work		E	e	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
6		Mechanical Technician	Work		M	m	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
7		Civil Technician	Work		C	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
8		Welder	Work		W	c	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
9		Painter	Work		P	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
10		Carpenter	Work		C	c	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
11		Blacksmith	Work		B	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
12		Civil Engineer	Work		C	c	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
13		Mechanical Engineer	Work		M	m	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
14		Electrical Engineer	Work		E	e	200%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard
15		Mason	Work		M	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard

Figure 20. The Resource sheet of MSP® 2016

The Gantt chart of the resource over-allocated schedule is depicted in Figure 23.

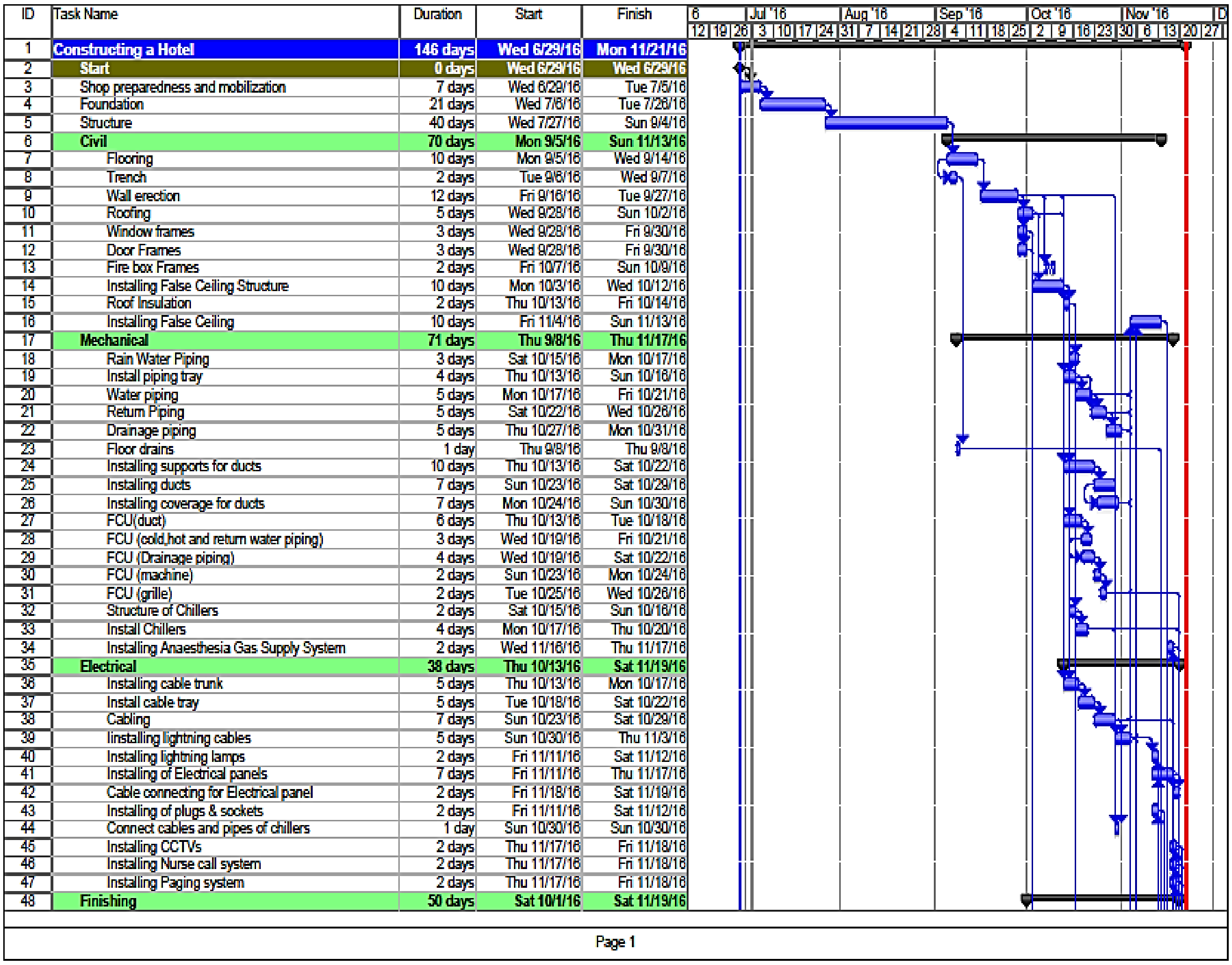
After using the resource leveling module in MSP® 2016, it can be seen that there are still some resources that remain over-allocated which means the module could not resolve the problem completely (Figure 24).

Thus, the case study is solved by the proposed solving algorithm while all resources constraints are considered relaxed. The same results are completely in accordance with the results gained by MSP® 2016.

Table 12. Comparing results of scheduling the hospital using MSP, Classic Branch and Bound and the proposed algorithm

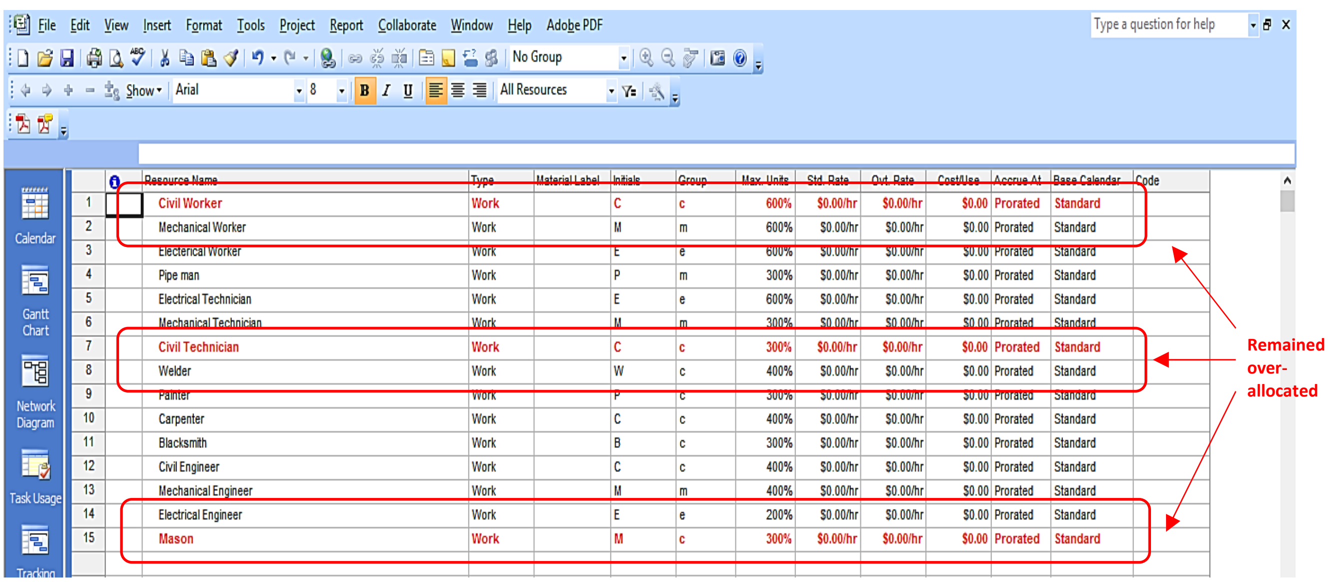
	MSP® 2016 normal schedule	**MSP® 2016 After Using resource leveling module**	**Classic Branch and Bound**	**Proposed Algorithm**
Number of Activities	58	58	58	58
Number of Resources	15	15	15	15
Number of over allocated	10	3	10	0
Number of Split Times	0	13	0	5
Makespan	146	161	146	204
NPV	-	-	77564$	77560$

Below, the proposed algorithm is used to modify the over-allocated schedule. The results show that the modified schedule does not suffer from any over-allocated resource so it is trustable and can be used for the constructing phase. Table 11 compares the information of the activities before and after using the proposed method. Results indicate that the proposed algorithm can successfully modify the over-allocated resources in a reasonable computing time. Figure 23 depicts a view of the corridor of the hospital before and after installing the false ceilings.



ID	Task Name	Duration	Start	Finish
1	Constructing a Hotel	146 days	Wed 6/29/16	Mon 11/21/16
2	Start	0 days	Wed 6/29/16	Wed 6/29/16
3	Shop preparedness and mobilization	7 days	Wed 6/29/16	Tue 7/5/16
4	Foundation	21 days	Wed 7/6/16	Tue 7/26/16
5	Structure	40 days	Wed 7/27/16	Sun 9/4/16
6	Civil	70 days	Mon 9/5/16	Sun 11/13/16
7	Flooring	10 days	Mon 9/5/16	Wed 9/14/16
8	Trench	2 days	Tue 9/6/16	Wed 9/7/16
9	Wall erection	12 days	Fri 9/16/16	Tue 9/27/16
10	Roofing	5 days	Wed 9/28/16	Sun 10/2/16
11	Window frames	3 days	Wed 9/28/16	Fri 9/30/16
12	Door Frames	3 days	Wed 9/28/16	Fri 9/30/16
13	Fire box Frames	2 days	Fri 10/7/16	Sun 10/9/16
14	Installing False Ceiling Structure	10 days	Mon 10/3/16	Wed 10/12/16
15	Roof Insulation	2 days	Thu 10/13/16	Fri 10/14/16
16	Installing False Ceiling	10 days	Fri 11/4/16	Sun 11/13/16
17	Mechanical	71 days	Thu 9/8/16	Thu 11/17/16
18	Rain Water Piping	3 days	Sat 10/15/16	Mon 10/17/16
19	Install piping tray	4 days	Thu 10/13/16	Sun 10/16/16
20	Water piping	5 days	Mon 10/17/16	Fri 10/21/16
21	Return Piping	5 days	Sat 10/22/16	Wed 10/26/16
22	Drainage piping	5 days	Thu 10/27/16	Mon 10/31/16
23	Floor drains	1 day	Thu 9/8/16	Thu 9/8/16
24	Installing supports for ducts	10 days	Thu 10/13/16	Sat 10/22/16
25	Installing ducts	7 days	Sun 10/23/16	Sat 10/29/16
26	Installing coverage for ducts	7 days	Mon 10/24/16	Sun 10/30/16
27	FCU(duct)	6 days	Thu 10/13/16	Tue 10/18/16
28	FCU (cold,hot and return water piping)	3 days	Wed 10/19/16	Fri 10/21/16
29	FCU (Drainage piping)	4 days	Wed 10/19/16	Sat 10/22/16
30	FCU (machine)	2 days	Sun 10/23/16	Mon 10/24/16
31	FCU (grille)	2 days	Tue 10/25/16	Wed 10/26/16
32	Structure of Chillers	2 days	Sat 10/15/16	Sun 10/16/16
33	Install Chillers	4 days	Mon 10/17/16	Thu 10/20/16
34	Installing Anaesthesia Gas Supply System	2 days	Wed 11/16/16	Thu 11/17/16
35	Electrical	38 days	Thu 10/13/16	Sat 11/19/16
36	Installing cable trunk	5 days	Thu 10/13/16	Mon 10/17/16
37	Install cable tray	5 days	Tue 10/18/16	Sat 10/22/16
38	Cabling	7 days	Sun 10/23/16	Sat 10/29/16
39	Iinstalling lightning cables	5 days	Sun 10/30/16	Thu 11/3/16
40	Installing lightning lamps	2 days	Fri 11/11/16	Sat 11/12/16
41	Installing of Electrical panels	7 days	Fri 11/11/16	Thu 11/17/16
42	Cable connecting for Electrical panel	2 days	Fri 11/18/16	Sat 11/19/16
43	Installing of plugs & sockets	2 days	Fri 11/11/16	Sat 11/12/16
44	Connect cables and pipes of chillers	1 day	Sun 10/30/16	Sun 10/30/16
45	Installing CCTVs	2 days	Thu 11/17/16	Fri 11/18/16
46	Installing Nurse call system	2 days	Thu 11/17/16	Fri 11/18/16
47	Installing Paging system	2 days	Thu 11/17/16	Fri 11/18/16
48	Finishing	50 days	Sat 10/1/16	Sat 11/19/16

Figure 21. Gantt Chart of MSP® 2016 for the case study (Continued)



	Resource Name	Type	Material Label	Initials	Group	Max. Units	Std. Rate	Ovt. Rate	Cost/Use	Accrue At	Base Calendar	Code
1	Civil Worker	Work		C	c	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
2	Mechanical Worker	Work		M	m	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
3	Electerical Worker	Work		E	e	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
4	Pipe man	Work		P	m	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
5	Electrical Technician	Work		E	e	600%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
6	Mechanical Technician	Work		M	m	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
7	Civil Technician	Work		C	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
8	Welder	Work		W	c	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
9	Painter	Work		P	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
10	Carpenter	Work		C	c	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
11	Blacksmith	Work		B	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
12	Civil Engineer	Work		C	c	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
13	Mechanical Engineer	Work		M	m	400%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
14	Electrical Engineer	Work		E	e	200%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	
15	Mason	Work		M	c	300%	$0.00/hr	$0.00/hr	$0.00	Prorated	Standard	

Figure 22. After using the resource level module even when the effort driven option is turned on, there are still some resources that remain overallocated


Figure 23. The project before and after installing the false ceilings

6. Conclusions

In the present study, the problem of resource over-allocation was taken into consideration. The primary goal was to investigate the impact of uncertain durations of activities on the degree of resource over-allocation. To this end, a non-linear mixed integer programming model was developed. In the model, all durations of activities was considered uncertain which can be changed due to the conditions of the project. Therefore, the triangular probability function was considered for estimating the durations of activities. Then a heuristic method was proposed based on forward serial programming with respect to the activities' precedence relations. The algorithm then selected the most suitable activity that caused more positive and negative cash flows simultaneously. Such method can increase the profit of the model which was expressed by the net present value in this research. Our findings show that when durations of activities are uncertain, the resource over-allocation infeasibility increases. To solve this problem, it is better to predict all possible conditions for the duration of activities in the algorithm before executing such activities. The outcomes also indicate that the proposed method can raise the NPV under the constant discounted rate. To increase the NPV, the algorithm uses the activity split ability. Such ability helps the algorithm to find the unused resources in the Gantt chart and fills them by parts of other activities. In addition, it is found out that while the resources availability is considered limited, reducing the resource over-allocation leads to increases in the make span of the studied projects. Another important feature of the proposed model is considering the cost of the project as a constraint in the scheduling process. Such constraint helps the algorithm not to propose schedules with the cost more than the project cost. Out results indicate that the proposed heuristic can successfully develop schedules with lower costs or costs equal to the signed ones in all the studied cases. The S-curve of all the studied cases reveals that the proposed heuristic provides schedules with smoothed resources that means schedules are good enough to be used in practice. This research can be expanded by using multi-objective scheduling for resource constraint projects. Using other metaheuristics like the genetic algorithm and ant colony optimization is suggested, too.

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