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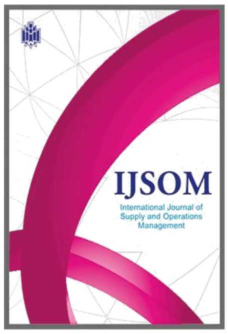

An Exploration of Evolutionary Algorithms for a Bi-Objective Competitive Facility Location Problem in Congested Systems

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Abstract
This paper presents a bi-objective competitive facility location model for congested systems in which the entering facilities compete with the competitors' facilities for capturing the market share. In the proposed model, customers can choose which facility to patronize based on the gravity function that depends on both the quality of service provider and the travel time to facilities. The proposed model attempts to simultaneously maximize the captured demand from each facility and minimize the total waiting time in the system. Two multi-objective evolutionary algorithms, involving a multi-objective harmony search algorithm (MOHS) and a non-dominated sorting genetic algorithm-II (NSGA-II), are designed to solve the proposed model. The performance of solution procedures are compared on the basis of different performance metrics including generational distance, spacing metric, diversification metric, and number of non-dominated solution. The computational results based on different test problems show that in general MOHS outperforms NSGA-II.

Keywords: Competitive facility location; Congested system; Gravity function; Multi- objective harmony search; NSGA-II.

1. Introduction

Facility location problems are among the strategic decisions in every organization which have been widely used in many practical contexts, such as supply chain planning, public service provision, and transportation infrastructure deployment. The earliest study of facility location problems dates back to the work of Weber (1902). After that, numerous research studies, covering both continuous and discrete spaces, have been developed for solving facility location problems in the literature (For further information, see Ortiz-Astorquiza et al., 2018, Ahmadi-Javid et al., 2017, Boonmee et al., 2017, Farahani et al., 2014).

Classical Facility location models are classified into different categories including p-median, p-center, p-dispersion, set covering, maximal covering, fixed charge, hub, and maxisum (Current et al., 2002). These models deal with the optimum location of facilities with respect to a set of customers, where the competition between the existing firms for patronizing customers is ignored. Competitive location models try to extend the classic location problems like p-median and maximal covering to the more complex environment, where companies offering the same service compete for their market share. In the competitive environment, customers can select a facility from the existing competitors or entering company to take the service based on the attractiveness of facilities which depends on travel time, waiting time, quality or reputation of service provider, service expense, etc.

In reality, competing service facilities are typically exposed to a high degree of uncertainty associated with customers' demand. Since the system has not sufficient resources available to serve all simultaneous demands immediately, the system may be congested in some situations (Boffey et al., 2007; Zarrinpoor et al., 2018). When the systems are

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congested, the customers have to wait for a long time to receive the services, and consequently, their willingness to patronize to the facility could decrease significantly. In some congested situations, customers may entirely give up the service and the system may suffer certain penalty. The unfavorable consequences of facility congestion, such as order delays and loss of market shares reveal the need for investigating congestion in competitive facility location problems. A prevalent way of considering congested situations is incorporating queuing theory into the facility location which attracts much attention in the recent decade. The examples of competitive location models considering congestion can be the location of automatic teller machines of a bank, post offices, gas stations, internet mirror sites, vending machines, web service providers, and the like in a given area, where competitors providing the same service have already located their facilities. The competitive location problem was addressed first by Hotteling (1929), who considered a model in which two competitors enter a linear market and the customers choose the nearest facility. The competitive facility location on a network was studied first by Hakimi (1983), who formulated the problem in which one company locates its facilities in an area, where competing facilities are already located. He also formulated another problem in which the firm locates its facilities before the location of competitors' facilities, but it takes the competitors' behavior into account. Drezner (1982) was the first researcher who examined the competitive location problem in the plane and assumed that the customer preference in competitive environment is based on distances. ReVelle (1986) presented the maximum capture problem to maximize the market capture on the basis of distances. Depalma et al. (1989) studied the location of several competitive firms and assumed that customers choose the closest facility among the different competitors with a certain probability. A good review of competitive location models can be found in Eiselt et al. (1993). They considered this type of problem based on the space, the number of competitors, the pricing policy, the rules of the game, and the behavior of the customers in the competitive environment. Benati (1999) introduced the maximum capture problem with a two-stage decision making process and assumed that customers select each facility according to the Logit function. Colome and Serra (2001) evaluated the customer's behavior with respect to distances or transportation costs in the basic maximum capture model. Shiode and Drezner (2003) addressed a model with stochastic weights on a tree network under the assumption that customers choose the nearest facility. Zarrinpoor and Seifbarghy (2011) proposed a competitive location model in which customers patronize each facility according to the Logit function that depends on the travel time. A leader-follower competitive location model subject to limited budgets is addressed by Drezner et al. (2015), where each facility attracts consumers within a sphere of influence. Zhang et al. (2016) considered disruption risks in a competitive facility location problem and used the Stackelberg game to model this problem. Qi et al. (2017) addressed the customer behavior in the competitive facility location model and assumed that the attractiveness of facilities decreases with distances and falls to zero if the distance is beyond a limitation. A bi-level linear programming model for capacitated competitive location model was introduced by Nasiri et al. (2018) in which the attractiveness of facilities is based on their distance from customers. Wang et al. (2018) addressed a competitive facility location problem for two facilities with distance-based attractiveness that can be full coverage, no coverage, or partial coverage of the demand point.

All of the aforementioned research studies assumed that the attractiveness of the competing facilities is only on the basis of distance. However, in reality, the customer behavior depends on the floor space of facilities, waiting time, price, reputation of service provider, and service quality. In this regard, Brandeau and Chiu (1994) proposed the competitive location problem on a tree network in which the customer preference is based on the distance and the waiting time. Brandeau et al. (1995) presented a good review of competitive location with market externalities. Benati and Hansen (2002) proposed a competitive location model, assuming that customers choose the facilities based on the random utility functions that depend on the quality of facility and the distance between facility and customer. Marianov et al. (2008) assumed that customers select the facilities based on the Logit function that depends on the travel and waiting time. A planar bi-objective competitive facility location and design problem was addressed by Redondo et al. (2015) who assumed that the utility function depends on the quality of the facility and distances. A planar single-facility competitive location was proposed by Fernandez et al. (2017) who assumed that customers split their demand among the firms in the market by patronizing only one facility with the highest utility from each firm. Beresnev and Melnikov (2018) presented a decision making process based on the Stackelberg's game framework for capacitated competitive facility location problem. Ljubic and Moreno (2018) presented a new procedure for solving the maximum capture problem with random utilities based on branch and cut approach enhanced by outer approximation and submodular cuts. The competitive hub location problems are introduced by Ghaffarinasab et al. (2018) who assumed that customers choose one firm based on the cost of service.

In some competitive location problems, the customer's probabilistic behavior in a competitive environment is modeled based on the gravity function. The gravity function was first introduced by Huff (1964, 1966) who suggested that customers select facilities based on their attractiveness and distance. In Huff's original work, the facility size was used as the attractiveness measure. Nakanishi and Cooper (1974) replaced the facility size with a set of attractiveness attributes in Huff's model. An exhaustive review of the competitive spatial model can be found in Eiselt and Laporte (1989). Drezner et al. (2002) investigated several heuristic methods for solving the multiple competitive facility location problem based on the gravity model. Wu and Lin (2003) used the gravity function to extend the flow-capturing location-allocation problem in the competitive environment. McGarvey and Cavalier (2005) proposed the competitive location model under the budget constraints in the plane. They also introduced a new gravity model in which the capacity of facility is used as its attractiveness measure. Aboolian et al. (2007) addressed a competitive location model with elastic concave demand under the budget constraint. Redondo et al. (2012) proposed a continuous competitive location and design problem, and studied to what extent the assumption of variable demand has an impact on the location decision. Kung and Liao (2018) addressed a competitive facility location problem for profit maximization with endogenous consumer demands and network effects. Due to the diminishing marginal benefit effect, they modeled the effective demand by a non-decreasing concave function which depends on the benefits of constructed facilities. Drezner et al. (2018) extended the gravity model by allowing the attractiveness of the facilities to be randomly distributed. They presented two solution methods based on discretizing the attractiveness level distribution and the concept of effective attractiveness.

As the related literature shows, competitive location models in congested systems have gained less attention. However, two sources of uncertainty associated with demand and service (e.g. of the exact timing of demands and the time it takes to serve individual demands at service facilities) can be considered by combination of location problems with queuing theory (Castillo et al, 2009; Zarrinpoor et al., 2017). Moreover, the research studies on multi-objective competitive facility location problem are considerably fewer than the ones in single-objective competitive facility location problem. Nonetheless, the design of competitive facility location problems is naturally a multi-objective decision-making problem with several conflicting objectives from both service providers' and customers' perspectives.

Regarding the stated notes, this paper deals with the competitive facility location problem for congested systems in which facilities behave as the M/M/1/k queuing system. The proposed problem is investigated not only from the service provider's point of view, but also from the customer's perspective. From the service provider's point of view, the model attempts to maximize the captured demand by each facility. From the customer's perspective, the model increases the service level by minimizing the total waiting time in the system. The customer's patronizing behavior is modeled on the basis of gravity function that depends on the quality/reputation of service provider and the travel time to facilities. Since the derived model belongs to the NP-Hard class of optimization problem, two multi-objective evolutionary algorithms, namely multi-objective harmony search algorithm (MOHS) and non-dominated sorting genetic algorithm-II (NSGA-II), are implemented to solve it.

The rest of the paper is organized as follows. The mathematical formulation of the proposed model is developed in Section 2. Section 3 presents the details of the proposed MOHS and NSGA-II. In Section 4, different performance metrics are presented to evaluate the performance of solution procedures. The computational results are illustrated in Section 5. Some conclusions and possible directions for further research are given in Section 6.

2. Model formulation

In this section, a bi-objective competitive facility location problem for congested systems is proposed in which the entering firm wants to locate several facilities in the network, when there are already competitors operating in the same geographical area. It is assumed that customers decide to which facility patronize based on the gravity function. The investment budget for opening and operating facilities is limited. It is also assumed that facilities cannot be operated unless they achieve a minimum workload requirement. The objectives of the problem are to maximize the captured demand by each facility and increasing the service level by minimizing the total waiting time in the system. The sets, parameters, and decision variables are defined as follows:

Sets

N Set of network nodes
I Set of demand nodes
E Set of candidate locations for the entering facilities
$\bar{E}$ Set of existing facilities occupied by competitors

Parameters

f_j Fixed installation cost to establish facility *j*
c_{ij} Unit cost of serving customers at node *i* by facility *j*
a_i Number of customers residing in demand node *i*
h_i Average demand rate at demand node *i*
p Number of facilities that can be established by the entering firm
A_j Quality/reputation of service provider
t_{ij} Travel time from node *i* to facility *j* through the shortest path
γ Customer's sensivity to the quality/reputation of the service provider
β Customer's sensivity to the travel time
u_{ij} Attraction function according to which customers choose a specific facility
q_{ij} Probability of a customer at node *i* choosing the facility at node *j*
w_j Expected waiting time at node *j*
L_j Expected value of number of customers in the system at facility *j*

p_{nj} Probability of a number *n* of customers being at the facility at node *j*
λ_j Arrival rate of requests for services at facility *j*
$\bar{\lambda}_j$ Effective arrival rate of requests for services at facility *j*
L_{qj} Expected value of number of customers in the queue at facility *j*
k Queue capacity
μ_j Service rate of facility *j*
ϑ_j Minimum workload requirement for facility *j*
τ Upper limit of investment budget constraint

Decision variable

y_j Location variable which takes value 1 if a facility is located at node *j*, and zero otherwise.

The system under study can be represented as a network with a set of nodes and a set of possible paths between nodes. Following the framework of the gravity-based spatial interaction model introduced by Huff (1964, 1966), it is assumed that the attraction function, u_{ij}, is affected by two factors including the quality/reputation of service provider and the travel time. The attraction function can formally be written as:

$$u_{ij} = \frac{A_j^{\gamma}}{t_{ij}^{\beta}} \qquad i \neq j \tag{1}$$

In competitive situations, different percentages at each demand node can select different facilities to patronize, including the facilities of the entering firm as well as the competitors' facilities. The probability q_{ij} of a customer at node *i* choosing the facility at node *j* can be expressed as:

$$q_{ij} = \frac{u_{ij}}{\sum_{l \in E} u_{il} + \sum_{l \in \bar{E}} u_{il}} \tag{2}$$

It is assumed that each facility behaves as an *M/M/1/k* queuing system, implying that Poisson arrivals with mean rate λ, exponentially distributed service time with mean μ, and the queue capacity to be limited to *k* customers. The average demand rate at each demand node is assumed to be the Poisson process with the mean h_i; thus, the demand rate at node *j* can be written as:

$$\lambda_j = \sum_{i \in N} h_i a_i q_{ij} \tag{3}$$

The stability of the queue must be satisfied as the following (Gross and Harris, 1998):

$$\bar{\lambda}_j \leq \mu_j \tag{4}$$

where $\bar{\lambda}_j$ is written as follows:

$$\bar{\lambda}_j = \lambda_j (1 - p_{kj}) \tag{5}$$

The average waiting time of customers for receiving service in this queuing system can be defined as follows (Gross and Harris, 1998):

$$w_j = \frac{L_j}{\bar{\lambda}_j} \tag{6}$$

where L_j can be stated as follows:

$$L_j = L_{qj} + (1 - p_{0j}) \tag{7}$$

Also the queuing equations for p_{nj}, L_{qj}, p_{0j}, and ρ_j for the *M/M/1/k* queues are as the following (Gross and Harris, 1998):

$$p_{nj} = \begin{cases} \dfrac{\left(1-\rho_j\right)\rho_j{}^{n}}{1-\rho_j{}^{k+1}} & \rho_j \neq 1 \\ \dfrac{1}{k+1} & \rho_j = 1 \end{cases} \tag{8}$$

$$L_{qj} = \begin{cases} \dfrac{\rho_j}{1-\rho_j} - \dfrac{\rho_j\left(k\rho_j{}^{k}+1\right)}{1-\rho_j{}^{k+1}} & \rho_j \neq 1 \\ \dfrac{k(k-1)}{2(k+1)} & \rho_j = 1 \end{cases} \tag{9}$$

$$p_{0j} = \begin{cases} \frac{1-\rho_j}{1-\rho_j{}^{k+1}} & \rho_j \neq 1 \\ \frac{1}{k+1} & \rho_j = 1 \end{cases} \tag{10}$$

$$\rho_j = \frac{\lambda_j\left(1-p_{kj}\right)}{\mu_j} \tag{11}$$

Regarding the aforementioned assumptions, the model can be formulated as follows:

$$\text{Max} \sum_{i\in N}\sum_{j\in E} h_i\, a_i\, q_{ij} \tag{12}$$

$$\text{Min} \sum_{j\in E} W_j y_j \tag{13}$$

s.t.

$$q_{ij} = \frac{y_j\, u_{ij}}{\sum_{l\in E} y_l\, u_{il} + \sum_{l\in \overline{E}} u_{il}}, \qquad \forall i \in N, j \in \overline{E} \cup E, \tag{14}$$

$$u_{ij} = \frac{A_j^{\gamma}}{t_{ij}^{\beta}}, \qquad \forall i \in N, j \in \overline{E} \cup E, \tag{15}$$

$$W_j = \frac{L_j}{\sum_{i\in N} h_i a_i q_{ij}\left(1-p_{kj}\right)}, \qquad \forall j \in E, \tag{16}$$

$$L_j = \frac{\rho_j}{1-\rho_j} - \frac{\rho_j\left(k\rho_j{}^{k}+1\right)}{1-\rho_j{}^{k+1}} + \left(1-\frac{1-\rho_j}{1-\rho_j{}^{k+1}}\right), \qquad \forall \left\{j \middle| j \in E \text{ and } \rho_j \neq 1\right\}, \tag{17}$$

$$L_j = \frac{k(k-1)}{2(k+1)} + \left(1-\frac{1}{k+1}\right), \qquad \forall \left\{j \middle| j \in E \text{ and } \rho_j = 1\right\}, \tag{18}$$

$$\rho_j = \frac{\sum_{i\in N} h_i\, a_i\left(1-p_{kj}\right)q_{ij}}{\mu_j}, \qquad \forall\, j \in E, \tag{19}$$

$$q_{ij} \leq y_j\,, \qquad \forall i \in N, j \in E, \tag{20}$$

$$\sum_{j\in E\cup\overline{E}} q_{ij} = 1, \qquad \forall i \in N, \tag{21}$$

$$\sum_{j\in E} y_j = p, \tag{22}$$

$$\sum_{i \in N} h_i\, a_i\,(1 - p_{kj}) q_{ij} \geq \vartheta_j y_j, \qquad \forall\, j \in E, \qquad (23)$$

$$\sum_{j \in E} f_j\, y_j + \sum_{i \in N} \sum_{j \in E} c_{ij} q_{ij} \leq \tau, \qquad (24)$$

$$\sum_{i \in N} h_i\, a_i\,(1 - p_{kj}) q_{ij} \leq \mu_j y_j, \qquad \forall\, j \in E, \qquad (25)$$

$$q_{ij} \geq 0, \qquad \forall i \in N, j \in \overline{E} \cup E, \qquad (26)$$

$$y_j = 0, 1, \qquad \forall\, j \in E. \qquad (27)$$

In this model, the first objective maximizes the captured demand by the entering competitor. The second objective provides the maximization of the service level by minimizing the total waiting time in the system. Constraints (14)–(19) are the definitions of the auxiliary parameters. Constraint (20) states that customers must be served only by open facilities. Constraint (21) insures that each customer is served by just one facility. Constraint (22) specifies the number of entering facilities to be located. Constraint (23) stipulates that facilities cannot be operated unless they achieve a minimum workload requirement. Constraint (24) limits the investment budget for opening and operating facilities. Constraint (25) forces the stability of the queue. Constraint (26) forbids negative q_{ij} and constraint (27) forces integrality of location variable.

3. Solution procedures

The proposed constrained bi-objective non-linear integer programming model belongs to NP-Hard class of problem and the exact methods are inefficient to solve it. Thus, two multi-objective evolutionary algorithms, namely multi-objective harmony search (MOHS) and non-dominated sorting genetic algorithm-II (NSGA- II) are applied to solve the problem**.**

3.1. Multi-objective harmony search

The harmony search (HS) algorithm, proposed by Geem et al. (2001), is a population-based meta-heuristic algorithm mimicking the improvisation of music players. In the music improvisation process, the members of the music group try to adjust the pitches of the instruments to obtain a better harmony which is analogous to finding global optimum. The details of the proposed algorithm are explained in the following.

Step 1: Initialize the algorithm parameters

In this step, the MOHS algorithm parameters are specified including harmony memory size (*HMS*), harmony memory considering rate (*HMCR*), pitch adjustment rate (*PAR*), number of decision variables (*N*), and the maximum number of improvisations (*NI*). The *HMS, HMCR,* and *PAR* are the number of solution vectors in the harmony memory, the probability of utilizing each component of harmony memory in a new solution, and the probability of pitch adjusting the selected value, respectively. Moreover, *N* and *NI* are the number of decision variables and the maximum number of iterations to evaluate the objective function, respectively.

Step 2: Initialize the harmony memory

The harmony memory (*HM*) is a location to store all the solution vectors which can be filled with randomly generated feasible solutions as follows:

$$HM = \begin{pmatrix} x_1^1 & x_2^1 & \cdots & x_{N-1}^1 & x_N^1 \\ x_1^2 & x_2^2 & \cdots & x_{N-1}^2 & x_N^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \cdots & x_{N-1}^{HMS-1} & x_N^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_{N-1}^{HMS} & x_N^{HMS} \end{pmatrix} \qquad (28)$$

Step 3: Improvise a new harmony memory

In this step, three rules are utilized to generate a new harmony vector, $\acute{X} = (\acute{x_1}, \acute{x_2}, \dots, \acute{x_N})$, including memory consideration, pitch adjustment, and random selection. In the memory consideration rule, the values of decision variables $\acute{X}$ are selected from $(x^1 - x^{HMS})$, with probability of *HMCR* to construct a new vector. To improve the efficiency of algorithm, *HMCR* is generally set between 0.75 and 0.95 (Geem et al., 2001).

In pitch adjustment rule, the value of decision variable is adjusted with probability of *PAR* and it is kept without any change with probability of (*1 – PAR*). The typical value of *PAR* ranges from 0.3 to 0.99 (Taleizadeh et al., 2011). For the pitch adjustment, two different procedures are applied containing swap and reversion operators. In the swap operator, two arrays of the solution are selected randomly and the position of these arrays will be changed as shown in Figure 1. In the reversion operator, an array is randomly selected and the position of the selected cells is reversed. Figure 2 shows the used reversion procedure.

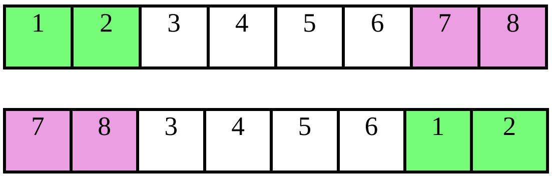

Figure 1. Pitch adjustment by swap operator

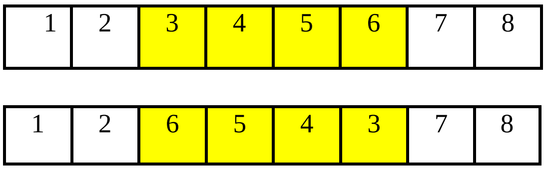

Figure 2. Pitch adjustment by inversion operator

Random selection prevents solutions from the local optimum by randomly selecting one value from the possible range of values with probability of $(1 - HMCR)$.

Step 4: Update the harmony memory

Since the proposed model belongs to the multi-objective optimization problem, updating the harmony memory differs from the basic HS algorithm. To find the Pareto-optimal solutions, the non-dominated sorting and crowding distance are utilized, following the scheme proposed by Sivasubramani and Swarup (2011).

In this step, the new harmony memory is combined with the existing harmony memory to form *2 × HMS* solution vectors. The non-dominated sorting is performed on the combined harmony memory to rank each individual based on the non-domination level. To form the best harmony memory, which is of size *HMS*, two individuals are compared according to their ranking and the one with smaller rank is selected. If individuals are from the same front, the crowding distance metric is used. The crowding distance is applied to evaluate the closeness of an individual to its neighbors. The crowding distance of *i*th individual, CD_i, is calculated as follows:

$$\mathrm{CD_i} = \frac{1}{r}\sum_{k=1}^{r}\left|f_{i+1}^{k} - f_{i-1}^{k}\right| \tag{29}$$

Where r is the number of objectives, f_{i+1}^{k} is the *k*th objective of the $(i+1)$th individual, and f_{i-1}^{k} is the *k*th objective of the $(i-1)$th individual. It should be noted when the number of non-dominated solutions exceeds *HMS*, solutions with smaller crowding distance must be removed.

Step 5: Check the stopping criteria

The algorithm stops when a fixed number of iterations is reached. Otherwise, steps 3 and 4 are repeated.
The flowchart of proposed MOHS is given in Figure 3.

3.2. Non-dominated sorting genetic algorithm-II

The Non-dominated sorting genetic algorithm-II (NSGA-II) was first introduced by Deb et al. (2002). This algorithm has been successfully applied in a wide range of combinatorial optimization problems. The details of NSGA- II used in this paper are explained bellow.

Step 1: Initialization

The random initialization for initial population is developed which creates the feasible solutions randomly. Since y_j is the only decision variable, a binary string with length *N* can represent each solution. Note that *N* is the number of nodes on the network.

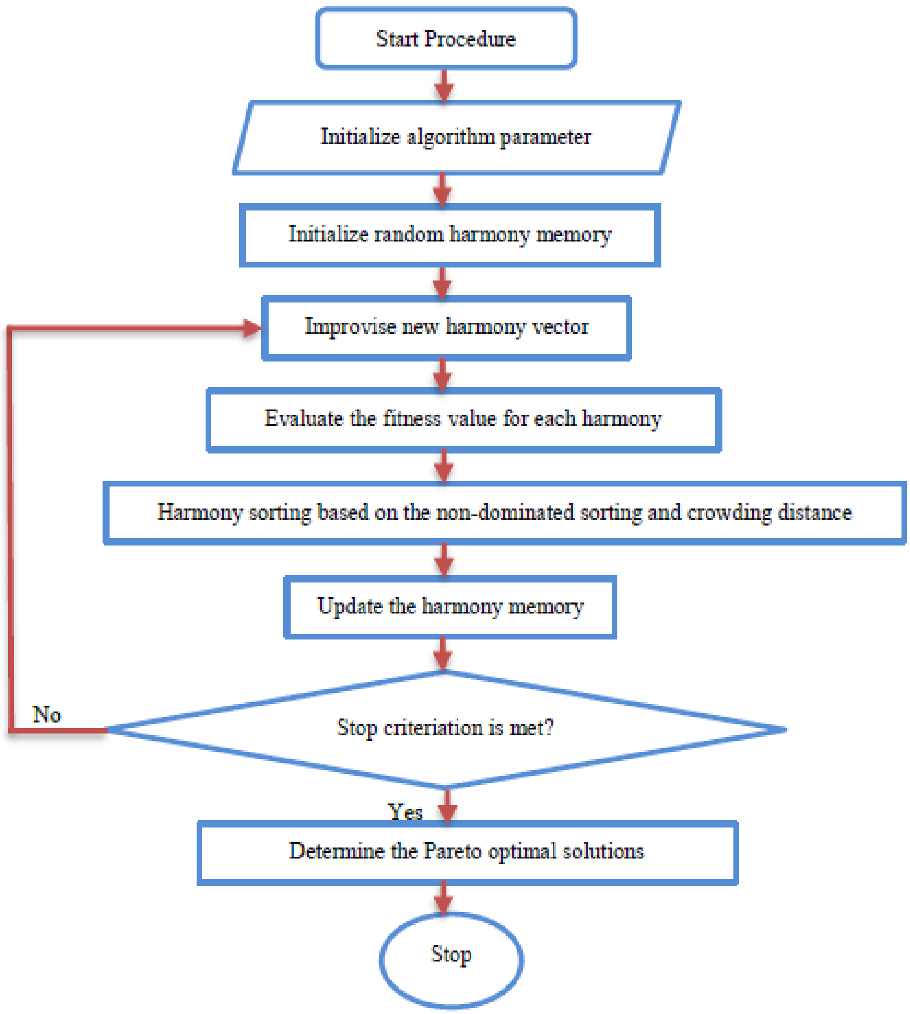

Figure 3. Flowchart of MOHS

Step 2: Non-dominated sorting

In NSGA-II, non-dominated sorting is used for fitness assignment and based on it, the initial population is ranked based on the non-domination level. Note that, the first front contains all individuals not dominated by any other individuals. The crowding distance is utilized for individuals to rank them in descending order in each front.

Step 3: Selection

The selection of individuals for mating is done using the "crowded tournament" method. In this method, two parents are selected randomly and their non-dominated rank and crowding distance are compared. The individuals with higher non-dominated rank must be selected for mating. When the individuals have the same rank, the selection operator chooses the individual with the grater crowding distance.

Step 4: Crossover operator

Crossover operators generate two offspring by recombining the genes of two parents which are selected based on the crowded tournament selection operator. In this paper, the simulated binary crossover is used to generate offspring near the parents. This operator generates a random number u_i between 0 and 1. The difference between the objective functions of parents and children, B_i , is calculated as follows:

$$B_i = \begin{cases} (2u_i)^{\frac{1}{\eta_c+1}} & \text{if } u_i \leq 0.5 \\ \left(\frac{1}{2(1-u_i)}\right)^{\frac{1}{\eta_c+1}} & \text{otherwise} \end{cases} \tag{30}$$

Where η_c is a positive constant which shows the difference between the objective functions of parents and children. After calculating B_i , the offsprings are generated as follows:

$$x_1^{child} = 0.5[(1 + B_i)x_1^{parent} + (1 - B_i)x_2^{parent}] \quad (31)$$

$$x_2^{child} = 0.5[(1 - B_i)x_1^{parent} + (1 + B_i)x_2^{parent}] \quad (32)$$

In the above equations, x_1^{child} and x_2^{child} represent the values of the first and second offsprings and x_1^{parent} and x_2^{parent} are the values of the first and the second parent chromosomes, respectively (Deb and Agrawal, 1995).

Step 5: Mutation operator

In this step, the mutation operator is applied to make small random changes to the binary strings so as to escape solutions from the local optimum. It also helps to expand the search space by generating new random chromosome. Following the work of Deb and Goyal (1996), the polynomial mutation operator is used. To utilize this operator, a random number r_i must be created between 0 and 1. Then, the mutation value is calculated as follows:

$$\delta_i = \begin{cases} (2r_i)^{\frac{1}{\eta_m+1}} - 1 & \text{if } r_i < 0.5 \\ 1 - [2(1 - r_i)^{\frac{1}{\eta_m+1}}] & \text{if } r_i \geq 0.5 \end{cases} \quad (33)$$

where η_m is the distribution constant. Then, δ_i parameter is added to the parent gene value as follows:

$$x_i^{child} = x_i^{parent} + \delta_i \quad (34)$$

Step 6: Recombination

In this step, all individuals including parent population and offspring are combined together and the individuals in the combined population are sorted based on non-domination level and crowding distance. The best population of size *N* is selected for the next generation.

Step 7: Stopping criteria

There are several possible conditions for stopping the algorithm. In this study, a predetermined number of iterations is selected as a stopping criteria. If this criterion is not satisfied, then steps 3 to 6 are repeated.

Figure 4 illustrates the flowchart of proposed NSGA-II.

4. Performance metrics

In this paper, four performance metrics are taken into account to evaluate the performances of proposed multi-objective algorithms. These metrics evaluate the uniformity and diversity among non-dominated solutions. The performance metrics used in this paper are as follows.

4.1. Generational distance

The generational distance, *GD*, determines the distance between the Pareto-optimal front and the solution set as shown below:

$$GD = \frac{\sum_{i=1}^{N} d_i}{N} \quad (35)$$

Where d_i is calculated based on the Euclidean distance between solution *i* and the nearest solution of the Pareto-optimal front as follows:

$$d_i = \min_{p \in PF}\left(\sqrt{\sum_{k=1}^{|m|} (f_k^i - f_k^p)}\right) \quad (36)$$

Where f_k^i , fe_k^p , and $|m|$ are the *k*th objective function of the solution *i* , the *k*th objective function of the Pareto-optimal front (PF), and the number of objective functions, respectively. A smaller *GD* value yields a better convergence towards the Pareto-optimal front (Veldhuizen, 1999).

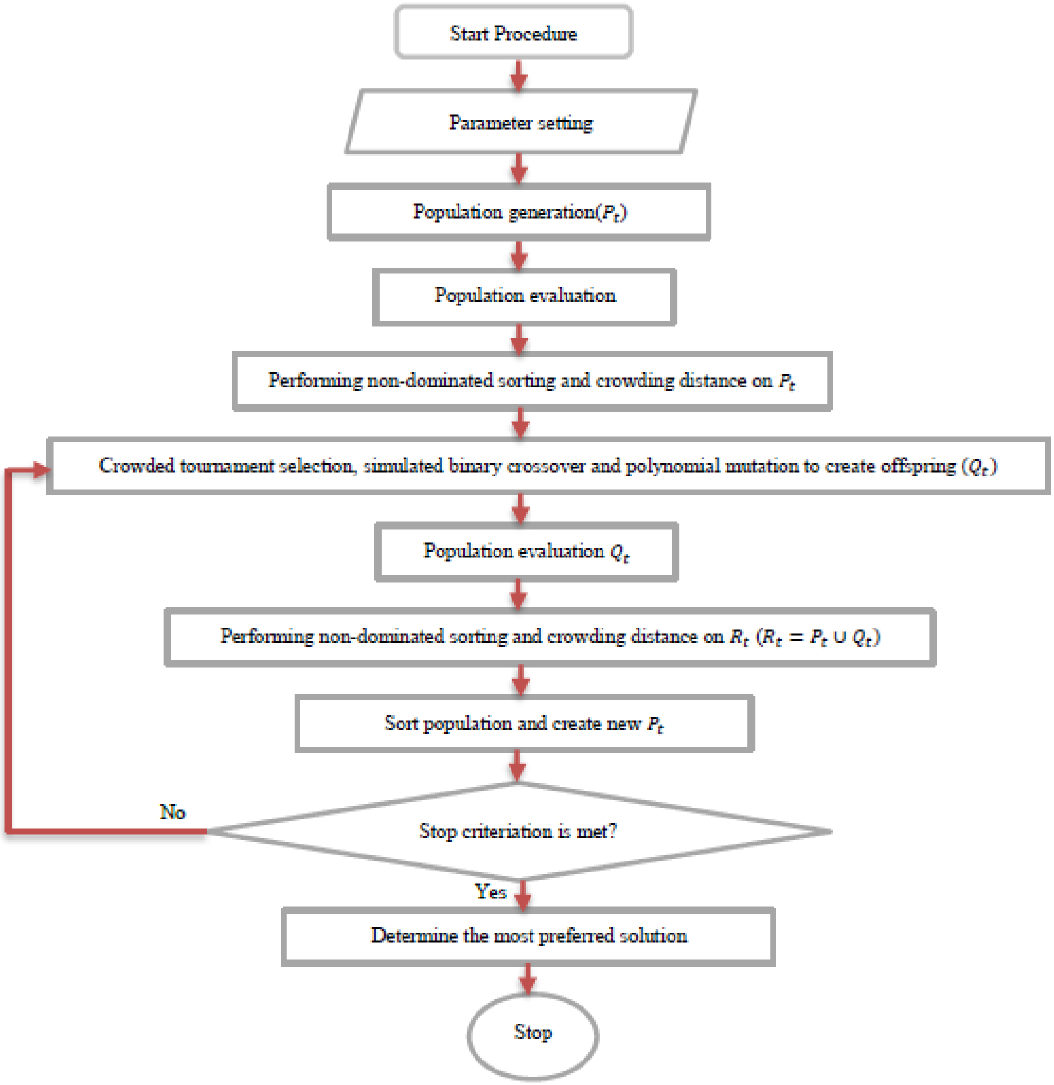

Figure 4. Flowchart of NSGA- II

4.2. Spacing metric

The spacing metric, *SM*, measures the uniformity of the spread of the points of the solution set as follows:

$$SM = \sqrt{\sum_{i=1}^{N} \frac{(d_i - \bar{d})^2}{N-1}} \tag{37}$$

Where $\bar{d}$ determines the mean value of all d_i and is calculated as follows:

$$\bar{d} = \frac{\sum_{i=1}^{N} d_i}{N} \tag{38}$$

The smaller the *SM* value yields the better the set of non-dominated solutions (Khalili-Damghani et al., 2013)

4.3. Diversification metric

The Diversification metric, *DM*, measures the spread of the solution set. The definition of this metric is as follows:

$$\mathrm{DM} = [\sum_{\mathrm{i}=1}^{\mathrm{N}} \max(\|\mathrm{x_i} - \mathrm{y_i}\|)]^{\frac{1}{2}} \tag{39}$$

where $\|x_i - y_i\|$ is the Euclidean distance between of the non-dominated solution x_i and the non-dominated solution of y_i (Khalili-Damghani et al., 2013).

4.4. Number of non-dominated solutions

The Number of non-dominated solutions, *NS*, shows the number of the Pareto solutions that each algorithm can find in the Pareto-optimal front.

5. Computational study

This section presents some numerical examples to study the performance of proposed algorithms. The randomly generated 30-node to 150-node network with a symmetric travel distance matrix is used in which the demands are randomly generated at each node. Nodes represent both demand concentrations and candidate facility locations. Competitors' facilities are located first on the network at random. The servers' service rate is assumed 1/20 for both existing competitors' facilities and the entering facilities. The demand arrival rates at each node are set to 4 customers per hour. The travel time is generated randomly in the interval [0, 2] hour. The utility functions are computed using different weights γ on the attractiveness of service provider and β on the travel time ranging from 0.1 to 1. The quality of service provider is assumed 2 for both existing competitors and entering facilities. Total budget allocated to suppliers is considered as $\tau = 15000$. The solution procedures are coded in MATLAB 7.0 and evaluated on a personal computer equipped an INTEL Core 2 CPU with 2.4 GHz clock speed and 2 GB of RAM. In order to select appropriate parameter setting for algorithm parameters, 20 independent trials are conducted. The values of the parameters of the MOHS turned out to be 0.8, 0.4, and 100 for *HMCR*, *PAR* and *NI*, respectively. The value of *HMS* is equal to the number of nodes on the network. The values of the parameters of the NSGA-II turned out to be 0.9, 0.1, 5, 20, and 100 for crossover rate, mutation rate, crossover index (η_c), mutation index(η_m), and maximum number of iterations, respectively. The population size is considered to be equal to the number of nodes on the network.

Initially, the solutions are considered for 30-node and 50-node networks and different numbers of entering and competitor's facilities. The results are presented in Tables 1 and 2. It is notable that, in these tables, q , MS, obj_1, and obj_2 represent the number of competitor facility, the market share of the entering firm, the first objective function, and the second objective function, respectively. The market share of the entering firm is computed based on the average captured demand in all the iterations of the solution procedure.

As expected, for a fixed number of competitors' facilities, the captured demand of the entering firm is increasing as it locates more facilities. When the number of entering facilities, p, is fixed, the captured demand of the entering firm reduces as the number of competitor facilities increases. This means that the captured demand of the entering firm depends on the number of entering facilities as well as the number of competitor's facilities. The more the captured demand, the longer the total waiting time in the system would be. It is due to the fact that as the entering facilities capture more customers, the facilities become more congested and the waiting time in the system increases. Nevertheless, the solution procedures choose the locations that lead to more captured demand and shorter waiting time in the system on the basis of attractiveness measure.

Table 1. Results for 30-node network,$\gamma = 0.4, \beta = 0.2, \alpha = 0.9,$ and $b = 2$

P	Q	NSGA-II			MOHS		
		obj_1	obj_2	MS (%)	obj_1	obj_2	MS (%)
3	3	31.41	0.3249	55.09	34.35	0.3848	56.30
3	4	27.94	0.3167	52.46	31.61	0.3283	53.05
3	5	24.37	0.3011	49.78	28.73	0.3029	51.27
4	3	34.90	0.3638	58.59	37.18	0.4045	59.33
4	4	32.02	0.3462	56.74	32.28	0.3419	56.83
4	5	26.31	0.3064	53.07	29.69	0.3315	54.67
5	3	40.61	0.4519	60.39	43.48	0.4611	62.26
5	4	33.04	0.4335	57.99	34.69	0.3931	59.02
5	5	28.21	0.3263	55.85	30.88	0.3689	56.49

In order to design more numerical examples, 10 test problems are generated by varying the number of nodes on the network and the number of entering and competitors' facilities. The computational results of employing the algorithms are given in Table 3.

Table 2. Results for 50-node network, $\gamma = 0.4, \beta = 0.2, \alpha = 0.9, and\ b = 2$

P	q	NSGA-II			MOHS		
		obj_1	obj_2	MS (%)	obj_1	obj_2	MS (%)
4	4	46.01	0.5145	56.97	48.12	0.5392	57.52
4	5	42.34	0.4688	53.52	46.32	0.5241	55.43
4	6	38.05	0.4043	51.92	40.59	0.4128	52.85
5	4	51.96	0.5402	58.31	52.44	0.5444	58.87
5	5	46.33	0.4921	55.91	48.05	0.5131	56.63
5	6	39.97	0.4659	53.94	42.11	0.4379	54.33
6	4	53.33	0.5916	59.25	54.63	0.5902	61.17
6	5	48.94	0.5161	57.38	49.27	0.5182	58.04
6	6	41.53	0.4778	56.88	44.59	0.4853	57.82

Table 3. Results for 60-node to 150-node network, $\gamma = 0.4, \beta = 0.2, \alpha = 0.9\ and\ b = 2$

N	p	q	NSGA-II			MOHS		
			obj_1	obj_2	MS (%)	obj_1	obj_2	MS (%)
60	7	5	55.95	0.5819	60.27	56.03	0.5988	61.86
70	8	6	59.13	0.6065	62.52	59.97	0.6102	62.69
80	8	8	60.52	0.6338	59.51	61.15	0.6559	60.61
90	7	9	62.38	0.6423	58.42	65.51	0.6781	59.97
100	10	8	68.76	0.7424	60.64	69.16	0.7518	61.83
110	12	10	74.52	0.8178	62.32	76.92	0.8263	63.92
120	13	12	78.57	0.8519	60.14	78.83	0.8564	60.53
130	16	13	87.32	0.9421	64.56	88.07	0.9592	65.87
140	14	16	89.19	0.9568	59.71	89.54	0.9571	59.75
150	17	15	94.65	1.0162	62.68	95.28	1.0274	63.34

To measure the efficiency of each solution procedure, the multi-objective algorithm performance metrics are utilized. The performance metrics are computed by considering different levels of *p* and *q* in 30-node to 150-node network. The characteristics of each test set are illustrated in Table 4.

Table 4. Characteristics of test sets for the experiments

Test set	N	p	Q
1	30	5	3
2	50	6	6
3	60	7	5
4	70	8	6
5	80	8	8
6	90	7	9
7	100	10	8
8	110	12	10
9	120	13	12
10	130	16	13
11	140	14	16
12	150	17	15

The average values of each metrics are reflected in Table 5. It can be concluded that the average value of the *GD* metric in MOHS is smaller than NSGA-II. This result indicates that the distance between the Pareto-optimal front and the generated solution set obtained by MOHS is short. It also can be concluded that MOHS provides non-dominated solutions that have lower average values for the *SM* metric. Therefore, the non-dominated solutions obtained by MOHS are more uniformly distributed in comparison with those obtained by NSGA-II. The average for the *DM* metric in NSGA-II has a smaller value in comparison with MOHS. In other words, NSGA-II could find non-dominated solutions with a narrower spread. Overall, it can be concluded that NSGA-II works better than MOHS in *NS* metric, but in other metrics MOHS outperforms NSGA-II.

The results are evaluated statistically by 2-sample t-test to check whether the average difference between the performance of two algorithms is significant or if it is due to random chance.The statistical results are presented in Table 6. The confidence levels for all experiments are set to 95%. The results declare that there is not enough evidence to reject the hypothesis of equal means for *SM* and *NS* metrics. It can also conclude that there is enough evidence to reject the equal means for *GD* and *DM* metrics.

Table 5. Comparison of algorithms for 30-node to 150-node network

Test	NSGA-II				MOHS			
	GD	SM	DM	NS	GD	SM	DM	NS
1	0.4381	0.2316	4.36	19.8	0.4043	0.2115	5.94	17.3
2	0.4196	0.1982	5.01	27.4	0.3704	0.1845	7.35	25.2
3	0.4998	0.3035	6.22	29.5	0.4508	0.2776	8.53	28.5
4	0.4552	0.2151	6.41	31.8	0.4193	0.2063	7.79	29.7
5	0.4088	0.2166	6.89	33.0	0.3907	0.1927	8.24	30.3
6	0.4356	0.2086	8.08	30.7	0.4168	0.1946	8.92	29.2
7	0.4163	0.2394	8.31	32.7	0.3667	0.2192	9.55	30.5
8	0.4562	0.2449	6.51	28.5	0.4183	0.2096	8.13	25.7
9	0.5159	0.2356	7.12	31.3	0.4991	0.1905	8.58	29.5
10	0.4507	0.2543	7.26	34.5	0.3887	0.2421	9.43	31.1
11	0.4640	0.2096	7.87	32	0.3733	0.1903	8.92	30.4
12	0.4151	0.2283	8.79	32.5	0.3703	0.2145	9.17	30.8
Ave	0.4479	0.2321	6.9	30.3	0.4057	0.2111	8.38	28.8

Table 6. Statistical results of performance metrics

Performance metric	Algorithm	Mean	Standard deviation	P-value
GD	NSGA-II	0.4479	0.0335	0.010
	MOHS	0.4057	0.0391	
SM	NSGA-II	0.2321	0.0279	0.072
	MOHS	0.2111	0.0264	
DM	NSGA-II	6.9	1.31	0.006
	MOHS	8.38	1.01	
NS	NSGA-II	30.31	3.87	0.195
	MOHS	28.18	3.92	

The effect of different parameters including quality/reputation of service provider and travel time to each facility is studied. It is considered the 50-node network and the case in which the number of entering and competitors' facility is equal to 5. All the parameters are the same as before. Since, in general, the performance of MOHS is better than NSGA-II, MOHS is utilized for parametric analysis. The MOHS is executed 20 times and the analysis results with respect to the average market share and total waiting time in the competitive environment are presented. Figure 5 shows the results when the importance of the travel time is increased. This figure indicates that as customers become more sensitive to the travel time, the market share of the entering firm decreases. This is because this effect makes the entering facilities less attractive and, consequently, the market share reduces. This effect also decreases the total waiting time of customers in the system.

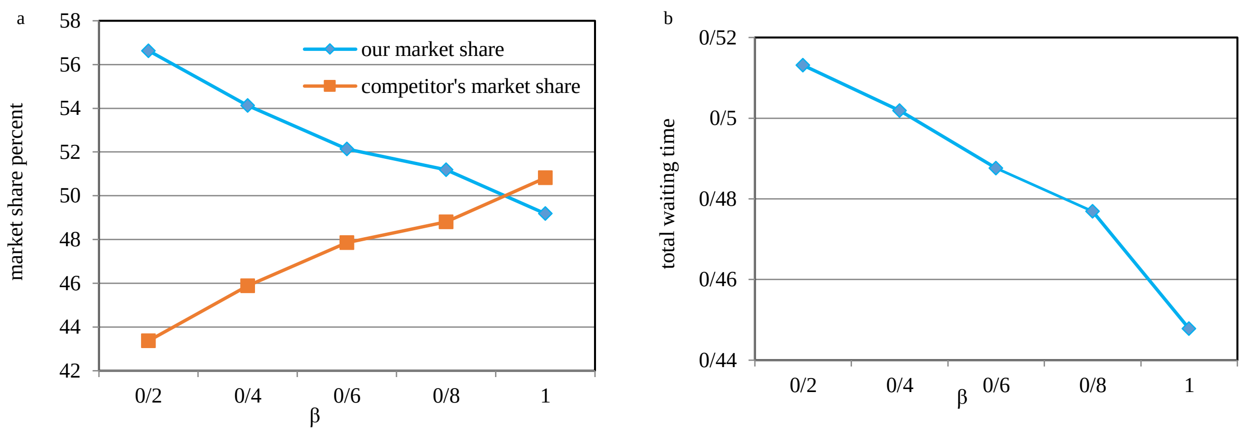

Figure 5. Sensitivity analysis with respect to β

Figure 6 shows when the importance of the quality/reputation of service provider becomes more important, the market share of the entering facilities increases. Since the quality of service provider is considered the same for both the entering and competitor facilities, the competitor market share is not decreased very much. This effect also increases total waiting time of customers in the system.

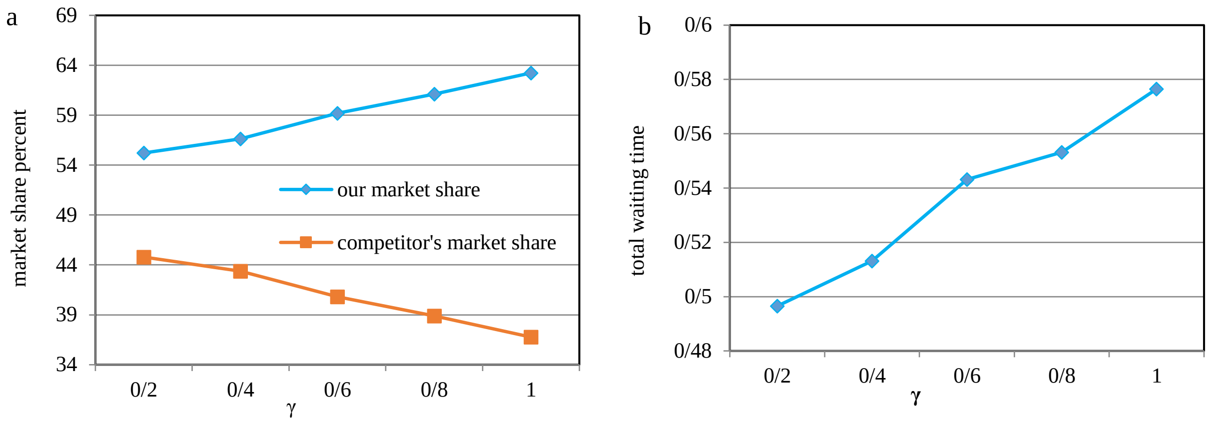

Figure 6. Sensitivity analysis with respect to γ

6. Conclusion

This paper presents a bi-objective competitive location model for congested systems with immobile servers and stochastic demand. The proposed model is based on the assumption that customers patronize facilities with minimum travel time and maximum quality/reputation of service provider. The objective functions are to maximize the total captured demand by each facility and to minimize the total waiting time of customers to receive the service. To cope with the computational burden, two multi-objective evolutionary algorithms, namely NSGA-II and MOHS, are developed to solve the model. The proposed algorithms are evaluated based on generational distance, spacing metric, diversification metric, and number of non-dominated solutions. Based on the computational results, it can be concluded that MOHS works better than NSGA-II. As future research, the risk of facility disruptions can be considered in the proposed model. Also it would be interesting to investigate the multi-server at each facility or consider different attractiveness factors, such as the floor space of facilities, waiting time, and price.

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