



Application of option games in investment analysis

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Abstract

This paper considers a popular problem in investment, the best time and size of investment, using methods of real options in a cooperative game setting. Moreover, it shows a combination of real option theory to invest, with a competitive game between two movers in the growth of a general-use asset and cooperative game theory between two movers to catch a network effect. In the model, two firms have similar and interacting investment opportunities. There is a real option for both firms to postpone the investment until they have proper price and production states. There are benefits to a first mover who can create a facility according to its own conditions. Also, there is a useful network effect of operating synergy if the first mover successfully motivates the second mover to start production instantaneously by sharing the production facility. So, the first mover has to discover when to create, what capacity to create, and the best economic rent for using the facility. The second mover has to discover whether to use the first mover's facility or create its own facility, and if it discovers to create its own, what time and size are better.

Keywords: Investment analysis; Uncertainty modelling; Real options analysis; Real options games; Bargaining games.

1. Introduction

Integrating real options theory with the game theory has been a dynamic field of research in the last decade. Game theory has been an important focus of attention over the last decades and has affected the growth of a broad range of research fields from economics, biology and mathematics to political science. Real options theory, on the other hand, appeared in the eighties as a valuation technique, mainly suitable for investments with high uncertainty. For researchers the attractiveness of modeling reasonable investment decisions by joining ideas from both theories is that such decisions in a reasonable market can be seen essentially as a "game" between firms, in the sense that in their decisions firms consider what they think will be the other firms' responses to their own actions, and they realize that their competitors think the same way. So, as one of the game theory's goals is to prepare an abstract for modelling conditions, including interdependent selections, a combination of the two theories is a logic step to take.

Corporate investment decisions sharing the features of irreversibility, uncertainty and scheduling are often considered in the traditional real options literature which declares that investments should be delayed until uncertainty is determined, by waiting for the best threshold. However, in the oil and gas industry, firms are often noticed to take part to become the first mover in investment by creating noticeable excess production capacity. These strategic firms not only select the best investment time, but also decide about the best investment size. Making correct decisions on these investments can either produce or demolish noticeable value, which is of interest to seniors in the firms. Such investment opportunities can be considered using real options theory and cooperative game theory.

The focused real option literature has created different balance models (Bertrand, Cournot, or Stackelberg) for firms' speculation choices under rivalry. This research gives another balance plausibility agreeable harmony in which firms share a regular generation office and both advantages from the network effect. Solid confirmation is given to demonstrate that organizations' speculation choices are key at any rate in the natural gas industry. Once in a while they rival each other by contributing prior to seizing others. Sometimes, they may collaborate with each other keeping in mind the end goal to produce the upside of network effect.

The decision amongst competition and cooperation may rely upon two elements including real option exercise price and the level of competition or the network effect. The higher option exercise cost will diminish the likelihood of collaboration, though higher network effect will expand the likelihood of participation. This paper examines the effect of interaction between firms' flexible investment decisions. Actually, this paper shows an equilibrium real options exercise game in which the investment cash flows are not exogenous to the firm, but endogenous in the purpose that the competitors behave according to the first-mover capacity selection and timing decisions. By considering the firms' behavior under a general setting of a sequential bargaining game of incomplete information on the existence of the positive externality, this paper shows that firms sometimes invest earlier than best and create excess production capacity not only for the preemptive result of a first mover benefit, but also for being able to derive rent from the follower.

This research additionally provides the exact confirmation for ventures with noteworthy open-door costs, for example, oil and gas industry. Firms in these ventures will not begin speculation once the NPV ascends to zero. Rather, their ventures are regularly postponed. The length the speculation delays is influenced by ware cost and the level of system impact. Product costs negatively affect the term of venture slack. The system impact positively influences the length of venture slack.

2. Review of the Literature

According to the real options analysis, firms should delay investment until a proper threshold for price or other stochastic variable is met. Shortsighted firms only appeal to the classical real option methods to discover the best time of investment without concerns about the future results of their current investment decisions. Nonetheless, the real options of distinct firms sometimes interact.

The first paper on the real options to analyze interactions between firms was Smets (Smets, F, 1993). Since Smets' work a new group of real options models, considering the interactions between firms, appeared including Grenadier (Grenadier, S, 1996), Smit and Trigeorgis (Smit, H. and L. Trigeorgis, 2006), Huisman (Huisman, K., 2001), Weeds (Weeds, H., 2002) Lambrecht and Perraudin (Lambrecht, B. and W. Perraudin, 2003), Huisman and Kort (Huisman, K. and P. Kort, 2003, Huisman, K. and P. Kort, 2004), Smit and Trigeorgis (Smit, H. and L. Trigeorgis, 2004), Paxson and Pinto (Paxson, D. and H. Pinto, 2005), Pawlina and Kort (Pawlina, G. and P. Kort, 2006), Kong and Kwon (Kong, J. and Y. Kwon, 2007) and Azevedo and Paxson (Azevedo, A. and D. Paxson, 2009) as good examples of these models.

In the literature on the real options, a "standard" real options game (ROG) model can be described as a model where the value of the investment is regarded as a state variable that follows a known process; time is examined as infinite and continuous; the investment cost is sunk, indivisible and fixed; firms are assumed to have plenty internal resources to make investments when it is best to do so; the investment game is played on a single project; the number of firms having the option to invest is usually two, and the emphasis of the analysis is bringing the firms' value functions and particular investment threshold under the assumption that both firms are risk-neutral (Leung, C.M. and Y.K. Kwok, 2012).

According to game theory, the three most essential parameters that distinguish a game are the players, their strategies and payoffs. Converting these to an ROG, the players are the firms that have the option of investing, the strategies are the choices of 'invest'/'defer', and the payoffs are the firms' value. Also, to be distinguished, a game still requires to be stated in terms of what sort of knowledge and information the players have at each point in time and considering the history of the game, what game is being played and whether mixed strategies are allowed.

Even though at a first look the adaptability of game theory to real options models appears clear and uncomplicated, there are some distinctions between a "standard" ROG and a "standard" game like those which are shown in basic game theory textbooks. Starting with the distinctions between a "standard" game in both theories, one distinction that is instantaneously identified considers the way the player's payoffs are given: in "standard" games used in most of the game theory textbooks, the player's payoffs are deterministic while in "standard" ROGs they are given by, sometimes, complex mathematical functions that rely on stochastic underlying variables (Ash, K., 2011, Deutsch, Y., B., 2011, Godinho, P. and J. Dias, 2013). This reality alters radically the rules under which the game equilibrium is mentioned. Also, another potential formal problem may also arise when we integrate real options and game theories. For example, the risk-neutral assumption often made in the real option literature, based on which firms' payoffs and their specific investment thresholds are gained, might not be logical in the world under which the principle of Nash equilibrium works.

Anyway, in the real world, one firm's investment decision may affect the other firm's investment decision due to different factors such as the first mover benefit and the network effect. Recent preemptive real options literature records a tradeoff between the real options to delay the first-mover benefit. They use the intersection of industrial organization theory and real options to analyze firms' strategic preemptive investment decisions. Most of the articles expand a Bertrand, a Cournot, or a Stackelberg equilibrium relying on the competition assumed. Yet, regardless of the real development of

this literature, little consideration has been paid to the results of positive externality on firms' investment decision. The network effect is also offset by the first-mover benefit that supports early investment. The network effect derives from the cooperation that can produce operating synergy. The operating synergy may arrive in the form of the lower cost structure (Igartua, M.A., et al., 2011, Mejia, M., et al., 2011).

The real options games models address present day inquiries in investment analysis and give new answers to investment problems, contributing, in this way, to a superior comprehension of the complicated nature of firms' venture conduct in business sectors where vulnerability and rivalry hold. Due to the huge number of option games, it is difficult to provide a comprehensive classification of them. In the table below, you can see a fairly comprehensive classification of conventional option games in the subject literature.

Table1. Categorization of Real Options Games Models

Aspect of categorization of real options games	Types	Examples of research in this area
Information	Complete/ Incomplete Perfect/ Imperfect	1. Décamps and Mariotti (2004) 2. Hsu and Lambrecht (2003) 3. Lambrecht and Perraudin (2003) 4. Savva and Scholtes (2005) 5. K. J. Huisman (2013) 6. Bulan et al. (2009) 7. Scharph (2018) 8. Grullon et al. (2012) 9. Jeuland and Whittington (2014) 10. Wesseh and Lin (2016) 11. Bensoussan (2017) 12. Benaroch (2018)
Type of game	Winner takes all Large Cooperative/non-cooperative ex-ante symmetric/ asymmetric	1. Murto (2004) 2. Ruiz-Aliseda (2004) 3. Weyant and Yao (2005) 4. Wu and Tseng (2006) 5. Décamp and Mariotti (2004) 6. Hsu and Lambrecht (2003) 7. Mason and Weeds (2005) 8. Pawlina and Kort (2006) 9. Ruiz-Aliseda (2004) 10. Shackleton, et al (2004) 11. Milanes et al. (2014) 12. Rau and Spinler (2016) 13. Agaton and Karl (2018)
Number of firms	1 2 >2	1. Aguerrevere (2003) 2. Bouis, et al. (2005) 3. Kulatilaka and Perotti (1998) 4. Lambrecht and Perraudin (2003) 5. Maeland (2002) 6. Murto, et al. (2004) 7. Nielson (2002) 8. Odening, et al. (2007) 9. Wolbert-Haverkamp and Musshoff (2014) 10. Savolainen et al. (2017) 11. Benaroch (2018)
Leadership	Endogenous Exogenous	1. Grenadier (2000) 2. Grenadier (2002) 3. Mason and Weeds (2005) 4. Murto and Keppo (2002) 5. Odening, et al. (2007) 6. Shackleton, et al. (2004) 7. Thijssen, et al. (2004) 8. Weyant and Yao (2005) 9. Wang et al. (2015) 10. Favato and Vecchiato (2017) 11. Barth and de Beer (2018)

2.1 Some Real Options and Sequential Bargaining Game Models

In the oil and gas, the airline, and the real estate industry, investments usually need large capital to create or buy a production facility. Investment decisions in these industries include a two-phase game. In the first phase, firms (trying to arrest the first mover benefit) play a Bertrand game with a distinguished product or a Cournot game with a

homogeneous product. In the second phase, the leader wants to motivate the follower to start production earlier by suggesting rent part of the production facility to the follower, so the leader needs specifying the best economic rent and investment scale. The follower needs discovering whether to accept the leader's suggestion or to wait to create its own facility. Now, we formally build and analyze the real option bargaining model (Feri, F. and A. Gantner, 2011, Jin, N. and E. Tsang, 2011).

3. A model of real options and sequential bargaining game

3.1. Model Assumptions

Suppose there are two gas explorers, X and Y , who have adjoining features for gas exploration and production. There are two kinds of uncertainty.

Production uncertainty

The first is the technical uncertainty of the considered quantity of reserves on the property. Let $R_i(t)$ be producer i 's supposed remaining reserves dependent on information collected to time t and production up to time t .

$$dR_i = \mu_i(R_i)dt + \sigma_i(R_i)dz_i, \quad i \in \{X, Y\} \quad (1)$$

where the correlation $\rho_R = \text{corr}(dz_X, dz_Y)$.

Production at rate r_i does two things:

1. It reduces the reservoir at rate r_i ;
2. It prepares information that creates new information about total reserves. So $\sigma_i(r_i)$ is nondecreasing in r_i .

$$dR_i = -r_i dt + \sigma_i(r_i)dz_i \quad (2)$$

Price uncertainty

The price of gas P is a source of economic uncertainty. Assume it follows the diffusion

$$dP = \mu(P)dt + \sigma(P)dz_p \quad (3)$$

where the correlation between technical and economic uncertainty is zero. More, gas price is supposed to follow a Geometric Brownian Motion (GBM). The general standard deviation $\sigma(P)$ becomes a functional form $\sigma(P) = \sigma_p P$ and the drift rate $\mu(P) = \mu P$.

Building cost

The cost of building a gas plant with capacity of r_i^c has fixed and variable parts:

$$K(r_i^c) = x + yr_i^c \quad i \in \{X, Y\} \quad (4)$$

where the producers have the same building features $x, y > 0$.

3.2 The Players' Investment Decisions

Let f be the risk-free rate of return and s be the systematic risk factor. Assume the underlying asset is estimated by the capital asset pricing model. The investment asset is assumed to get a risk premium in ratio to the covariance between asset price variations and the risk factor, which proposes the following relationship:

$$\mu P + \delta(P, t) = fP + \lambda_p \beta(P) \quad (5)$$

where $\beta(P) = \frac{\text{cov}(dP, ds)}{\text{var}(dP)}$, λ_p is the risk premium for the systematic risk factor s , and $\delta(P, t)$ is the rate of

convenience yield of the underlying assets. The risk-neutral drift of price, $\hat{\mu}$ becomes $\mu P - \lambda_p \beta(P) = fP - \delta(P, t)$

. Since the price is assumed to follow the GBM, the future price P_t follows

$$P_t = P_{\tau_i} e^{(\hat{\mu} + \sigma_P^2 / 2)(t - \tau_i)} \tag{6}$$

The risk-neutral drift of R is: $\mu(R) - \lambda_R \beta(R) = -r_t$, where $\beta(R) = 0$ because the production rate $r_t = 0$ is zero before the early investment. After the production starts, the producers are price takers and their reserves are independent of market prices.

3.2.1 Investment decisions with unique players

Suppose that neither producer originally has a gas processing facility. If the producers' characteristics are not adjoining, the problem for each producer would be a classic two-dimensional real option problem. The real option decisions are those that would be made by a monopolist owner of the project, without any analysis of interaction with the other producer. The best development option for producer $i \in \{X, Y\}$ has a threshold $\{(P^*(R_i), R_i) \mid R_i \in R^+\}$ where $P_i^* : R^+ \rightarrow R^+$ is the threshold development price if the considered reserves are R_i . That is, producer i expands the first time $(P_t, R_{i,t})$ are such that $P_t \geq P_i^*(R_i, t)$.

The cash flow for producer i at time t is $m_{i,t} : R^+ \times R^+ \rightarrow R$ given by $m_{i,t} = (P_t - C)r_{i,t}$, where C is the variable production cost. The expected payoff from an investment made by player i at time τ_i is:

$$W_i(P, R_i, \theta_i) = \hat{E}_{\theta_i} \int_{\theta_i}^{v_i} e^{-f(t - \theta_i)} m_{i,t} dt - K(r_i^c) \tag{7}$$

This develops as a spread process which may have a simplified threshold cash flow m^* . But this is not the case since the uncertainty and risk neutral growth rates in R and P may not be the same, so the profit may differ over the threshold border. These isolated producers are noncooperative in the sense that they do not have to analyze the strategic result from the investments by the competitors. As P and R are supposed to be uncorrelated, commonly, these noncooperative firms' real option values must fulfill the valuation PDE (Chen, X. and G. Hao, 2013, Epstein, L., et al., 2013, Li, D.-F., 2011, Thijssen, J.J.J., et al., 2012):

$$\frac{1}{2} [\sigma^2(R) V_{RR}(P, R) + \sigma^2(P) V_{PP}(P, R)] + V_R(P, R) \mu(R) + V_P(P, R) [\mu(P) - \lambda_P \beta(P)] + V_t = fV(P, R) \tag{8}$$

and the value-corresponding and smooth pasting border conditions:

$$\begin{aligned} V(P^*, R^*) &= W(P^*, R^*) \\ V_P(P^*, R^*) &= \frac{\partial W_i}{\partial P_{\theta_i}} = \hat{E}_{\theta_i} \left[\int_{\theta_i}^{v_i, trans} e^{(\hat{\mu} - f)(t - \theta_i)} r_i^c dt + \int_{v_i, trans}^{v_i} e^{(\hat{\mu} - f)(t - \theta_i) - \bar{\alpha}_i(t - v_i, trans)} r_i^c dt \right] \\ V_R(P^*, R^*) &= \frac{\partial W_i}{\partial R_i} = \hat{E}_{\theta_i} \left[\int_{v_i, trans}^{v_i} e^{-f(t - \theta_i)} (P_t - C) \bar{\alpha}_i e^{-\bar{\alpha}_i(t - v_i, trans)} dt \right] \end{aligned} \tag{9}$$

This equation can be simply solved numerically.

3.2.2. Adjoining players' investment decisions

Cooperative producers will follow a symmetric, subgame perfect equilibrium entrance strategy in which each producer's exercise strategy maximizes value dependent on the other's exercise strategy (Asteriadis, S., et al., 2012, Anshelevich, E., F.B., 2011). The solutions have two distinct exercise models: concurrent and sequential.

Equilibrium with simultaneous exercise

Suppose both producers have the same assumptions of beginning reserves on their own resources after the exploration. Denote F as the follower, and L as the leader, $F, L \in \{X, Y\}$. In this case, $P_X^*(R_X) = P_Y^*(R_Y) = P_F^*(R_F) = P_L^*(R_L)$, and both producers have the same trigger price. Once the price strikes the trigger, they both want to exercise the real option and create their own plant instantaneously (Godinho, P. and J. Dias, 2013) Whoever moves faster becomes the

common leader. Although given that the prices P and quantities R_X, R_Y are continually distributed and not correlated, this is a knife-border condition that only happens with probability zero if the producers do not interact.

Equilibrium with sequential exercise

Assume the leader has a larger early reserve and so lower best trigger price $P^*(R_L)$. In this condition, $P_L^*(R_L) < P_F^*(R_F)$ for $L, F \in \{X, Y\}, L \neq F$. The leader will enter alone, creating a gas processing plant to shield its own production only. Once its production volumes decrease, it will suggest an extra capacity to the follower at a rental rate r to be negotiated, remembering the follower's reservation cost of creating its own plant. So, there is a bargaining game played at and after the time the leader discovers to create the plant. This game discovers whether the follower starts production at the same time or with delay. If the follower accepts the rental, both producers start production concurrently and the game ends. If the follower rejects the rental, they play the same sequential bargaining game at following dates, where the leader suggests a rental rate and capacity, and the follower decides whether to accept the suggestion, create its own plant or delay further (Pavlova, Y. and G. Reniers, 2011, Said, M., 2011).

3.3. The sequential bargaining game with incomplete information for adjoining players

Designate θ_L as the first time (P, R_L) strikes the threshold $(P^*(R_L), R_L)$. The follower also solves for a threshold trigger price $P^*(R_F)$ that specifies the best condition under which it would create its own plant and start production. Designate the first striking time to the threshold $(P^*(R_F), R_F)$ by the stopping time $\theta_F \in [\theta_L, \infty)$. So the big follower exercises at $\theta_{F_b} = \theta_L$ because the big follower's beginning reserve is of the same size as the leader's. The small follower exercises at $\theta_{F_s} > \theta_{F_b}$ because the small follower's beginning reserve is smaller than the big follower's. The rental will start at $\theta_{rent} \in [\theta_L, \theta_{F_s}]$. The leader's maximum production time is θ_L . The big or small follower's maximum production time is θ_{F_b} or θ_{F_s} separately.

There are two players in the game, the leader and the follower. The product to be traded is the leader's excess processing capacity, where the leader sells capacity to the follower. The quantity of product to be traded is the contracted fixed rental production capacity each of the time r_{FL} . The network effect is the benefit of cooperation. The transfer is the rental payment l from the follower to the leader. The leader knows its cost of providing the excess capacity $K(\cdot)$. The follower has private information about its valuation $r_F \in \{\underline{r}_F, r_F^c\}$.

There are two kinds of buyers, the low kind buyer (the big follower, F_b) who values the rental at \underline{r}_F and the high kind buyer (the small follower, F_s) who values the rental at r_F . The leader does not know what buyer the follower is. So, there is a clash between efficiency and rent removal in mechanism design. The leader's strategy space is to suggest the rental at either r_F or \underline{r}_F . The follower's strategy space is to either accept or reject the leader's suggestion. If the follower accepts, the game ends. If the follower rejects, the leader will make another suggestion in the next period. The decision variables are the rental rate r , the cooperative and noncooperative plant capacity choices r_L^Ω , or r_L^c and r_F^c , which discover the construction costs $K(r_L^\Omega)$, or $K(r_L^c)$ and $K(r_F^c)$ and production volumes r_L and r_F [12, 71].

3.3.1. Two-stage optimization of adjoining players

Each player i have three decision variables over which it must be optimized. One is a function $P(R)$, rather than just a single variable. Player i must choose a manifold of prices and quantities that designate the trigger threshold for exercise. It is the same for both players that the player i expands as soon as the random variables (P, R_i) are such that $P \geq P(R_i)$. The second variable, $r_{i,t}^c$ is the capacity selected by the players. The third variable, r_i is the reservation rental rate for the players, that is, the highest rental rate the follower would accept or the lowest rate the leader would accept separately. Let

$V_{i,co}(P_i, R_i, r_{i,co}^\Omega, r_i; N_{co})$ be the total business value for player i when it is playing cooperatively, and

$V_{i,nc}(P_i, R_i, r_{i,nc}^c; N_{nc})$ be the total business value for player i when it plays noncooperatively or separately from the other player.

To optimize player i 's noncooperative business value, $V_{i,nc}$ is done in two stages.

- Stage 1: For each possible (P, R) , suppose the firm expands the field at that pair. Solve for the best capacity as

$$r_i^c = \arg \max_{r_i^c} V_{i,nc}(P_i, R_i, r_i^c; N_{nc}) \quad (10)$$

The solution is $r_i^{c*}(P_i, R_i)$ and the value is $V_{i,nc}$

- Stage 2: Given $V_{i,nc}(P_i, R_i, r_i^{c*}(P_i, R_i); N_{\theta_i})$ as a function of (P, R) solve for the best development threshold $P_{\theta_i}^*(R_i)$

Use this two-stage process to solve the problem of the follower, as a function of $(P; R)$ and the capacity the leader suggests and the rental rate it suggests for that capacity. This will give a reservation rental rate for each capacity and price-quantity pair to the extent the follower is just ordinary between accepting the rental and taking the noncooperative value (Jin, N. and E. Tsang, 2011, Singh, A., 2012).

3.3.2. The network effect- gains from cooperation

The network effect N is modelled as the demotion in pipeline charges, one element of the production cost that influences the players' cash flow. Economy of scale and network effect of pipeline appears because the average cost of transferring oil or gas in a pipeline reduces while total throughput grows. There are two groups of costs that cause network results. In the condition of the joint pipeline, they are long-run fixed operating costs and capital investment cost (Anshelevich, E., F.B. Shepherd, and G. Wilfong, 2011, Briglauer, W. and I. Vogelsang, 2011, Parag, Y., et al., 2013).

The pipeline company has to determine whether to create and, if it creates, at what capacity and charge rate. For clarity, we suppose that, based on the information about both producers' early reserve R_L, R_F and production rate r_L, r_F , the pipeline company can consider and create a pipeline to adjust the noncooperative total transportation throughput, $(r_{L,nc} + r_{F,nc})$, for the leader and the follower. The actual noncooperative pipeline throughput

$$= \begin{cases} r_{L,nc}(t) & , t < \theta_F \\ r_{L,nc}(t) + r_{F,nc}(t) & , t \geq \theta_F \end{cases} \quad (11)$$

This results in a higher pipeline charge rate for the leader before θ_F , and a lower pipeline charge rate (category 1 network effect N^1) for both producers after θ_F as the total throughput transported grows. If the rental contract is moderated successfully at $\theta_{rent} < \theta_F$ or even concurrently at θ_L , the pipeline company sees the producers' decision, and it will build a larger pipeline to adjust this larger cooperative total throughput, $r_{L,co}(\theta_{rent}) + r_{F,co}$, which will create the category 2 network effect, N^2 .

The actual cooperative pipeline throughput

$$= \begin{cases} r_{L,co}(t) & t < \theta_{rent} \\ r_{L,co}(t) + r_{F,co}(t) & t \geq \theta_{rent} \end{cases} \quad (12)$$

3.3.3. The follower's individual rationality constraint

Small follower F_s 's IR .

The small follower can either rent the capacity of the leader at θ_{rent} or delay further until θ_{F_s} to create its own plant. The small follower gets the network effect in both cases. The distinction is that if it selects to create its own plant, the

benefit of network effect comes only after θ_{F_s} and will end at v_L when the leader's production ends (Manapat, M.L., M.A. Nowak, and D.G., 2013, Zheng, X. and Y. Cheng, 2011). Designate this network benefit for a small follower that creates its own plant as $N_{\theta_{rent}}^{v_L} = N \cdot \int_{\theta_{rent}}^{v_L} r_L dt$. If it chooses to rent, the rental contract may permit the small follower to start production earlier than θ_{F_s} and small follower will get the network effect in the interval $[\theta_{rent}, v_L]$. The small follower will make the comparison of $V_{F_s,nc}$ and $V_{F_s,co}$ at the date after θ_L whenever the leader suggests a rental at rate r . This gives the small follower's involvement restriction:

$$V_{F_s,co}(P, R_{F_s}, \bar{r}_F; N_{\theta_{rent}}^{v_L}) \geq V_{F_s,nc}(P, R_{F_s}, r_{F_s}^{c*}; N_{\theta_{F_s}}^{v_L}) \quad (13)$$

which discovers the high type buyer's valuation of rental:

$$\bar{r}_F \equiv \sup\{r_F \in R^+ : V_{F_s,co} \geq V_{F_s,nc} | r_{F_s}^c = r_{F_s}^{c*}\} \quad (14)$$

Big follower F_b 's IR

The big follower expands the field simultaneously as the leader. The big follower's independent rationality restriction is:

$$V_{F_b,co}(P, R_{F_b}, \underline{r}_F; N_{\theta_{rent}=\theta_{F_b}}^{v_L}) \geq V_{F_b,nc}(P, R_{F_b}, r_{F_b}^{c*}; N_{\theta_{F_b}}^{v_L}) \quad (15)$$

For the big follower, $N_{\theta_{rent}}^{v_L} = N_{\theta_{F_b}}^{v_L}$ the rental does not increase its entire amount of network effect received, but reduces its capital cost. Therefore, the low type buyer's rental is:

$$\underline{r}_F \equiv \sup\{r_F \in R^+ : V_{F_b,co} \geq V_{F_b,nc} | r_{F_b}^c = r_{F_b}^{c*}\} \quad (16)$$

3.3.4 The leader's individual rationality constraints

At τ_L , the leader has a noncooperative best capacity r_L^c which maximizes its total noncooperative business value

$V_{L,nc}(P, R_L, r_L^c; N_{\theta_F}^{v_L})$, where $N_{\theta_F}^{v_L} = N \cdot \int_{\theta_F}^{v_L} r_L dt$

$$r_L^c = \arg \max_{r_L^c} V_{L,nc}(P, R_L, r_L^c; N_{\theta_F}^{v_L}) \quad (17)$$

A noncooperative leader is a leader who does not analyze the possibility of renting excess capacity to the follower in the future. Thus, the $V_{L,nc}$ function does not involve a rental rate r . The network effect $N_{\theta_F}^{v_L}$ happens when the follower's production starts at θ_F and ends at v_L . This is distinct from the leader's cooperative business value $V_{L,co}(P, R_L, r_L^\Omega, r; N_{\theta_{rent}}^{v_L})$, where $N_{\theta_{rent}}^{v_L} = N \cdot \int_{\theta_{rent}}^{v_L} r_L dt$. This early network effect $N_{\theta_{rent}}^{v_L}$ happens when the follower's production starts at θ_{rent} and ends at v_L . We now explain the leader's cooperative best capacity as:

$$r_L^{\Omega*} = \arg \max_{r_L^{\Omega}} V_{L,co}(P, R_L, r_L^{\Omega}, r; N_{\theta_{rent}}^{v_L}) \quad (18)$$

$$st. \quad \theta_{rent} \leq \theta_F$$

The leader will create cooperative capacity if the following individual rationality or involvement constraint $I(IR_l)$ is fulfilled:

$$V_{L,co}(P, R_L, r_L^{\Omega^*}, r; N_{\theta_{rent}}^{v_L}) \geq V_{L,nc}(P, R_L, r_L^c; N_{\theta_F}^{v_L}) \quad (19)$$

3.3.5. The leader's control set $\{r_L^{\Omega}, r\}$

Remember that r_L and r_F are determined as the leader's and the follower's production volume separately, r_L^c is the leader's noncooperative capacity and $\bar{\gamma}_L$ and $\bar{\gamma}_F$ are the maximum production rates that are set by a regulator or technological restrictions. The noncooperative leader and follower's production function as:

$$r_{L,nc}(t) = \begin{cases} r_L^c & t \in [\theta_L, v_{L,trans}] \\ \bar{\gamma}_L R_L(v_{L,trans}) e^{-\bar{\gamma}_L(t-v_{L,trans})} & t \in [v_{L,trans}, v_L] \end{cases} \quad (20)$$

and

$$r_{F,nc}(t) = \begin{cases} r_F^c & t \in [\tau_L, \theta_{F,trans}] \\ \bar{\gamma}_F R_F(v_{F,trans}) e^{-\bar{\gamma}_F(t-v_{F,trans})} & t \in [v_{F,trans}, v_F] \end{cases} \quad (21)$$

After $v_{L,trans}$, the noncooperative leader's capacity is not restricted, and it can suggest the follower its surplus processing capacity $r_L^c - r_L$ considering the follower has not created its individual plant yet. This gives the cooperative follower's production volume under rental:

$$r_{F,co} = \min\{r_L^c - r_L, \bar{\gamma}_F R_F\} \quad (22)$$

The results of the bargaining game relied on the amount of information accessible to the leader and the follower. The cooperative leader evaluates both producers' requirements and creates a gas plant with capacity $r_L^{\Omega} \geq r_L^c$. So, the above production functions become:

$$r_{F,co} = \min\{r_L^{\Omega} - r_{L,co}, \bar{\gamma}_F R_F\} \\ 0 \leq r_{L,co} \leq \min\{r_L^{\Omega}, \bar{\gamma}_L R_L\} \quad (23)$$

The cooperative leader has an excess capacity of $r_L^{\Omega} - r_{L,co}$, which will enlarge as the leader's production volume $r_{L,co}$ drops over time. Suppose the cooperative follower will use all the capacities suggested in the rental until reserves drop to restrict the production rate. That is, $r_{F,co} = \min\{r_{FL}, \bar{\gamma}_F R_F\}$. Once excess capacity achieves the contracted rental capacity r_{FL} at γ_{lease} , the rental can start. The cooperative production function is:

$$r_{L,co}(t) = \begin{cases} r_L^{\Omega} & t \in [\theta_L, v_{L,trans}] \\ \bar{\gamma}_L R_L(v_{L,trans}) e^{-\bar{\gamma}_L(t-v_{L,trans})} & t \in [v_{L,trans}, v_L] \end{cases} \quad (24)$$

and

$$r_{F,co}(t) = \begin{cases} r_{FL} & t \in [\theta_{rent}, v_{F,trans}] \\ \bar{\gamma}_F R_F(v_{F,trans}) e^{-\bar{\gamma}_F(t-v_{F,trans})} & t \in [v_{F,trans}, v_F] \end{cases} \quad (25)$$

The cooperative leader's choices of r_L^Ω and r will have reverse results on θ_{rent} . On the one hand, the cooperative leader can control an early or late θ_{rent} by restraining the size of its cooperative capacity r_L^Ω . When r_L^Ω is larger, the rental can happen earlier. The earlier rental will permit the cooperative leader to benefit from the network effect earlier than θ_F . On the other hand, the cooperative leader wants to charge the follower the highest rental rate up to \bar{r}_F for a small follower or \underline{r}_F for a big follower as determined by equations (14) and (16). So, the rental suggestion is inversely connected with the time the rental is accepted. The cooperative leader's purpose is to maintain an equilibrium among the incremental network effect benefit, the earlier rental fee, and the extra structure costs of $r_L^\Omega - r_L^c$, remembering that a higher rental rate leads to the follower's delay (Kulas, J.T., M. Komai, and P.J. Grossman, 2013, Rivas, M.F. and M. Sutter, 2011).

3.4. The Leader's and the follower's cash flows and expected payoff

Let C be the variable production cost for both the leader and the follower, involving the pipeline charges. The network effect N is the charge drop gained from transporting larger amounts of oil and gas with smaller unit breakeven charge rates.

3.4.1 Noncooperative leader and small follower

In this case, the leader and the small follower each build a gas plant to process their own gas individually. The leader creates a plant only large enough to process its own gas. The small follower enters later and creates its own plant. The leader does not get the network effect until the small follower has also started producing. The leader creates at the stopping time $\theta_L \geq 0$ and the small follower creates at $\theta_{F_s} \geq \theta_L$ (Antoniadou, E., C. Koulovatianos, and L.J. Mirman, 2013, Ishii, M., et al., 2013).

Stage 1: $t \in (\theta_L, \theta_{F_s})$, only the leader produces

The leader has begun producing but the small follower is still waiting. The network effect does not occur at this stage because the pipeline can only charge the leader. The operating profit is

$$m_{L,nc,t|F_s}^{S1} = (P_t - C)r_{L,nc,t}, \quad t \in (\theta_L, \theta_{F_s}) \quad (26)$$

where $r_{L,nc,t}$ is determined in equation (20). The risk-neutral expected payoff to the leader is

$$W_{L,nc,\theta_L}^{S1} = \hat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-rt} m_{L,nc,t|F_s}^{S1} dt \quad (27)$$

where the \hat{E}_t is the risk-neutral expectation dependent on the information available at time t . The small follower has not built anything yet in this stage and so its cash flow is zero.

Stage 2: $t \in (\theta_{F_s}, v_L)$, the leader and the small follower both produce

The small follower enters at θ_{F_s} , but can only ship gas in the remaining area on the pipeline, which was built to adjust the noncooperative total yield. The leader and the small follower will get the network effect in this stage, and their cash flows will be:

$$\begin{aligned} m_{L,nc,t|F_s}^{S2} &= (P_t - C + N)r_{L,nc,t}, & t \in (\theta_{F_s}, v_L) \\ m_{F_s,nc,t}^{S2} &= (P_t - C + N)r_{F_s,nc,t}, & t \in (\theta_{F_s}, v_L) \end{aligned} \quad (28)$$

where $r_{F_s,nc,t}$ is determined in equation (21) by replacing F with F_s . The expected payoffs to the leader and the small follower respectively are:

$$W_{L,nc,\theta_{F_s}|F_s}^{S2} = \hat{E}_{\theta_{F_s}} \int_{\theta_{F_s}}^{v_L} e^{-ft} m_{L,nc,t|F_s}^{S2} dt \quad (29)$$

and

$$W_{F_s,nc,\theta_{F_s}}^{S2} = \hat{E}_{\theta_{F_s}} \int_{\theta_{F_s}}^{v_L} e^{-ft} m_{F_s,nc,t}^{S2} dt \quad (30)$$

Stage 3: $t \in (v_L, v_{F_s})$, the leader's production ends and only the small follower remains in production

The leader's production ends at v_L and the small follower's production ends at v_{F_s} . We suppose the leader and follower take the same amount of time to exhaust their fields. So, $v_L - \theta_L = v_{F_s} - \theta_{F_s}$. As the leader's production starts earlier, we have $v_L < v_{F_s}$. The follower's cash flow and expected payoff are:

$$m_{F_s,nc,t}^{S3} = (P_t - C)r_{F_s,nc,t}, \quad t \in (v_L, v_{F_s})$$

$$W_{F_s,nc,v_L}^{S3} = \hat{E}_{v_L} \int_{v_L}^{v_{F_s}} e^{-ft} m_{F_s,nc,t}^{S3} dt \quad (31)$$

To summarize the evidence, the noncooperative leader and small follower's total expected payoff from all three stages are:

$$W_{L,nc,\theta_L|F_s} = \hat{E}_0 \left(\hat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-ft} m_{L,nc,t|F_s}^{S1} dt + e^{-f(\theta_{F_s} - \theta_L)} \hat{E}_{\theta_{F_s}} \int_{\theta_{F_s}}^{v_L} e^{-ft} m_{L,nc,t|F_s}^{S2} dt - K(r_L^c) \right) \quad (32)$$

and

$$W_{F_s,nc,\theta_L} = \hat{E}_0 \left(e^{-f(\theta_{F_s} - \theta_L)} \hat{E}_{\theta_{F_s}} \int_{\theta_{F_s}}^{v_L} e^{-ft} m_{F_s,nc,t}^{S2} dt + e^{-f(v_L - \theta_L)} \hat{E}_{v_L} \int_{v_L}^{v_{F_s}} e^{-ft} m_{F_s,nc,t}^{S3} dt - K(r_{F_s}^c) \right) \quad (33)$$

3.4.2. Noncooperative leader and big follower

In this case, the leader and the big follower exercise their real option to invest concurrently at $\theta_L = \theta_{F_b}$. They each develop a gas plant to process their own gas individually. They will get the network effect during the entire production life, and their cash flows will be:

$$m_{L,nc,t|F_b} = (P_t - C + N)r_{L,nc,t}, \quad t \in (\theta_L, v_L)$$

$$m_{F_b,nc,t} = (P_t - C + N)r_{F_b,nc,t}, \quad t \in (\theta_{F_b}, v_L) \quad (34)$$

where $r_{F_b,nc,t}$ is determined in equation (21) if substituting F with F_b . The expected payoff to the leader and the big follower are:

$$W_{L,nc,\theta_L|F_b} = \hat{E}_0 \left(\hat{E}_{\theta_L} \int_{\theta_L}^{v_L} e^{-ft} m_{L,nc,t|F_b} dt - K(r_L^c) \right) \quad (35)$$

and

$$W_{F_b,nc,\theta_{F_b}} = \hat{E}_0 \left(\hat{E}_{\theta_{F_b}} \int_{\theta_{F_b}}^{v_L} e^{-ft} m_{F_b,nc,t} dt - K(r_{F_b}^c) \right) \quad (36)$$

3.4.3. Cooperative leader and small follower

Stage 1: $t \in (\theta_L, \theta_{lease})$, only the leader produces

As considered earlier, the leader may want to create a bigger gas plant of cooperative capacity r_L^Ω with construction costs $K(r_L^\Omega)$. It then for rental suggests the small follower the remaining processing capacity at a processing rate r . The leader's cash flow and risk-neutral expected payoff will be:

$$\begin{aligned} m_{L,co,t|F_s}^{S1} &= (P_t - C)r_{L,co,t}, \quad t \in (\theta_L, \theta_{lease}) \\ W_{L,co,\theta_L|F_s}^{S1} &= \hat{E}_{\theta_L} \int_{\theta_L}^{\theta_{rent}} e^{-ft} m_{L,co,t|F_s}^{S1} dt \end{aligned} \quad (37)$$

where $r_{L,co,t}$ is determined in equation (24). The rental has not started and the small follower is waiting on this stage.

Stage 2: $t \in (\theta_{rent}, v_L)$ the rental starts, the leader and the small follower both produce

In this stage, the small follower decides to rent the plant capacity from the leader. They both produce and receive the network effect. The cash flows to the leader and the small follower are:

$$\begin{aligned} m_{L,co,t|F_s}^{S2} &= (P_t - C + N)r_{L,co,t} + r_{FL}r, \quad t \in (\theta_{rent}, v_L) \\ m_{F_s,co,t}^{S2} &= (P_t - C + N)r_{F_s,co,t} - r_{FL}r, \quad t \in (\theta_{lease}, v_L) \end{aligned} \quad (38)$$

where $r_{F_s,co,t}$ is determined in equation (25) if substituting F with F_s . Their expected payoffs are:

$$W_{L,co,\theta_{rent}|F_s}^{S2} = \hat{E}_{\theta_{rent}} \int_{\theta_{rent}}^{v_L} e^{-ft} m_{L,co,t|F_s}^{S2} dt \quad (39)$$

and

$$W_{F_s,co,\theta_{rent}}^{S2} = \hat{E}_{\theta_{rent}} \int_{\theta_{rent}}^{v_L} e^{-ft} m_{F_s,co,t}^{S2} dt \quad (40)$$

Stage 3: $t \in (v_L, v_{F_s})$, the leader's production ends and only the small follower produces

The leader's production ends at v_L , and the small follower continues until v_{F_s} . They do not receive the network effect. The leader still receives the rental charge. The leader's cash flow and expected payoff are:

$$\begin{aligned} m_{L,co,t|F_s}^{S3} &= r_{FL}r \\ W_{L,co,v_L|F_s}^{S3} &= \int_{v_L}^{v_{F_s}} e^{-ft} m_{L,co,t|F_s}^{S3} dt = \int_{v_L}^{v_{F_s}} e^{-ft} r_{FL}r dt \end{aligned} \quad (41)$$

The small follower's cash flow and expected payoff are:

$$\begin{aligned} m_{L,co,t|F_s}^{S3} &= (P_t - C)r_{F_s,co,t} - r_{FL}r, \quad t \in (v_L, v_{F_s}) \\ W_{F_s,co,v_L}^{S3} &= \hat{E}_{v_L} \int_{v_L}^{v_{F_s}} e^{-ft} m_{F_s,co,t}^{S3} dt \end{aligned} \quad (42)$$

To summarize the evidence, the cooperative leader and small follower's total expected payoff from all three stages are: (43)

$$W_{L,co,\theta_L|F_s} = \hat{E}_0 \left(\int_{\theta_L}^{\theta_{rent}} e^{-ft} m_{L,co,t}^{S1} dt \right) + e^{-f(\theta_{rent}-\theta_L)} \hat{E}_{\theta_{rent}} \int_{\theta_{rent}}^{v_L} e^{-ft} m_{L,co,t}^{S2} dt + e^{-f(v_L-\theta_L)} \int_{v_L}^{v_{F_s}} e^{-ft} r_{FL} r dt - K(r_L^\Omega)$$

and

$$W_{F_s,co,\theta_L} = \hat{E}_0 \left(e^{-f(\theta_{rent}-\theta_L)} \hat{E}_{\theta_{rent}} \int_{\theta_{rent}}^{v_L} e^{-ft} m_{F_s,co,t}^{S2} dt + e^{-f(v_L-\theta_L)} \hat{E}_{v_L} \int_{v_L}^{v_{F_s}} e^{-ft} m_{F_s,co,t}^{S3} dt \right) \tag{44}$$

3.4.4. Cooperative leader and big follower

The big follower’s IR constraint verifies $v_{rent} \leq \theta_L$. So the stage (τ_L, θ_L) intersects stage (v_{rent}, θ_L) in equilibrium. In this stage, the big follower decides to rent the plant capacity from the leader. They both produce and receive the network effect. The cash flows to the leader and the big follower are:

$$\begin{aligned} m_{L,co,t|F_b} &= (P_t - C + N)r_{L,co,t} + r_{FL}r, & t \in (\theta_{rent}, v_L) \\ m_{F_b,co,t} &= (P_t - C + N)r_{F_b,co,t} + r_{FL}r, & t \in (\theta_{rent}, v_L) \end{aligned} \tag{45}$$

where $r_{F_b,co,t}$ is determined in equation (25) if substituting F with F_b . Their expected payoffs are [81]:

$$W_{L,co,\theta_{rent}|F_b} = \hat{E}_{\theta_{rent}} \int_{\theta_{rent}}^{v_L} e^{-ft} m_{L,co,t|F_b} dt \tag{46}$$

and

$$W_{F_b,co,\theta_{rent}} = \hat{E}_{\theta_{rent}} \int_{\theta_{rent}}^{v_L} e^{-ft} m_{F_b,co,t} dt \tag{47}$$

4. Case Study

Firms’ optimal investment decision under vulnerability has been a subject of inquiry for quite a while because of the observed deviation from the zero NPV limit. In the standard real options literature, including Brennan and Schwartz (Mejia, M., et al., 2011); Dixit and Pindyck (Dixit, A.K. and R.S. Pindyck, 1994); Dixit (Dixit, A.K. and R.S. Pindyck, 1995); Capozza and Sick (Capozza, D.R. and G.A. Sick, 1991); Sick (Sick, G., 1995); Trigeorgis (Trigeorgis, L., 1996), it has been asserted that speculations ought to be deferred until the vulnerability is settled or sit tight for the ideal limit. However, in the aggressive real options literature, including Fudenberg and Tirole (Fudenberg, D. and J. Tirole, 1985); Mason and Weeds (Mason, R. and H. Weeds, 2005); Garlappi (Garlappi, L., 2001); Lambrecht and Perraudin (Lambrecht, B. and W. Perraudin, 2003); Huisman and Kort (Huisman, K.J. and P.M. Kort, 2004); Thijssen et al. (Thijssen, J.J., K.J. Huisman, and P.M. Kort, 2012); Smit and Trigeorgis (Smit, H.T. and L. Trigeorgis, 2004), it has been contended that opposition decreases the real options esteems and mitigates speculation delays, in this manner, with adequate rivalry, firms’ venture edge might be pushed back to zero net present esteem (NPV).

This section tests whether firms will think about collaborating with their rivals when the opposition turns out to be excessively furious utilizing the venture level information from Iran’s natural gas exploration and processing industry. As mentioned in Sick and Li (Sick, G. and Y. Li, 2007), in enterprises with economies of scale or network effects, firms may profit from collaboration by keeping away from the disintegration impact of rivalry on real option value.

Developing a natural gas field can be a long-term process. To start with, in the investigation phase, firms need to gather land review, seismic and gravitational information keeping in mind the end goal to inspect the surface structure of the earth and decide about the conceivable areas of the gas reservoir. Second, in the drilling stage, firms need to bore a few revelation wells to decide about the surmised profundity and the amount of the gas repository. Considering the natural gas ware cost and the evaluated hold amount, firms might sit tight for a considerable length of time before they begin

the actual generation. This holding up period between the enrollment dates of the disclosure well and the generation well is characterized as the investment slack in this paper.

Likewise, natural gas fields contrast with each other by the sort, depth, age and area of the underground store and the geography of the zone. Typically, natural gas is extricated from unadulterated gas wells and from condensate wells where there is practically no raw petroleum. Such gas is called non-related gas. Sometimes, petroleum gas is also found in oil wells where it could be either separate from or broke down in the unrefined petroleum in the underground arrangement. These gases are called related gas. To evade the potential heterogeneity issue in the gas field supplies, just the non-related gas fields are incorporated into the example.

The information gathering work was a slow learning process which included extensive literature reading, information arranging, and meeting with industry professionals. I went by different information sources and organizations. I might want to inspect how strategies and decisions of National Iranian Gas Company fit with my model if the following information were accessible:

- The improvement cost of a gas property and the development cost of the preparing plant, which must be fixed to the limit.
- The development expenses of pipelines that are expected to transport the gas from wells to the plants.
- Any reports and examinations of the transactions between the National Iranian Gas Company and different organizations.

I began to think about gathering a sample of oil and gas speculation extends in focused settings and using it to perform an observational test of the expectations of my hypothetical model. To do the observational test of the real options exercise, amusement display, I collected information from in excess of 30 natural gas fields and 8 natural gas processing plants in Iran.

A cooperative gas processing plant is characterized as one plant serving or historically having served different natural gas repository handles that are working with numerous field administrators. A non-agreeable gas, preparing plant is a plant serving one field or different fields worked by one field administrator, or historically never served various fields worked with numerous field administrators. The variable, COOP, shows whether the gas, preparing plant is agreeable. On the off chance that a plant is enrolled to process gas from numerous fields, it is an agreeable plant and COOP is equivalent to one. Otherwise, it is a non-agreeable plant and COOP has an estimation of zero. The logical factors are recorded in this vector.

{PD; PP; RES; WD; DU; D; C}

PD is the natural gas cost at the revelation time

PP is the natural gas cost at the season of generation

RES is the underlying store amount of the field.

WD is the aggregate number of disclosure wells in a specific field, representing the level of network effect.

DU measures the venture slack between disclosure time and generation time.

D is the normal profundity of all generation wells inside specific fields, representing the drilling costs.

C is the plant’s day by day handling limit, proxying the development cost of the plant.

The fundamental element of the information and factors are summarized in Table 2.

Table 2. Summary Statistics

Variable	obs	mean	Std dev	min	max
Coop	452	0.7	0.56	0	1.15
PD	508	0.55	0.9	0.06	8.43
PP	452	2.13	1.77	0.12	8.43
RES	590	12958.8	22902.7	1.15	89452
WD	590	120.69	206.31	1.15	1228
DU	588	36.05	29.2	0	118
D	588	1559.85	925.6	281.55	4815
C	445	1359.34	2480.29	13.69	13733

4.1 The Empirical Models and Results

To examine firms’ strategic real option investment decisions under rivalry, I build up a logit model to test whether firms may think about the choice of coordinating with their rivals when confronting serious rivalry, or basically be power to contribute when NPV breaks even with zero. The choice of collaboration is the aftereffect of a consecutive dealing amusement. On the off chance that organizations choose to coordinate, they manufacture a helpful gas plant with greater

ability to process gas from various fields. If organizations were not ready to concur on the rental rate, the pioneer would begin the speculation and creation along, the adherent would hold up until its own threshold reaches.

We could foresee the following:

1. Firms' reservation rent rates are curved in the commodity price, and the balance collaboration run is diminishing in ware cost once the genuine alternative to contribute is worked out.
2. Firms' reservation rent rates are not exceptionally delicate to the underlying store level inside the non-practicing locale.
3. Inside the practicing locale, a larger network effect diminishes the leader's and the follower's reservation rent rate.

These expectations yield three testable ramifications for the logit model of participation.

Hypothesis 1 The gas cost has a non-monotonic impact on the likelihood of participation. It expands the participation likelihood inside the nonexercising district, which is not detectable in the data sample.

Hypothesis 2 Initial hold amount is required to negatively affect the likelihood of participation inside the practicing locale.

Hypothesis 3 The impact of network effect or rivalry impact on the likelihood of participation is blended. On the off chance that the opposition impact overwhelms the system impact (economies of scale), firms will probably assemble noncooperative plants.

The logit regression equation is introduced as the log of the chances proportion for collaboration the proportion of the likelihood that organizations demonstration helpfully to the likelihood that organizations' demonstration non-cooperatively:

$$\ln\left(\frac{P}{1-P}\right) = \alpha + \beta^{pd} PD + \beta^{pp} PP + \beta^{res} RES + \beta^d D + \beta^c C + \beta^w WD + \beta^{du} DU + \varepsilon$$

where $P=E(COOP=1/X)$

and $X=\{PD, PP, RES, D, C, WD, DU\}$

Table 2 reports the results from the logit regression of cooperation.

Table 3. Logit models of cooperation. It provides the logit model estimates for cooperation.

Variables simple logit			
Coop	coef	Std. err.	z
PD	0.639	0.4898	1.5
PP	-0.6207	0.1527	-4.67
RES	-1.70E-05	0.00E+00	-1.44E+00
WD	-0.0023	0.0025	-1.05
DU	0.008	0.0166	0.55
D	0.0005	0.0002	1.9
C	0.0005	0.0002	3.16
Constant	0.75487	0.5299	1.63
N	371		
Chi-square	80.3		
Adj. R ²	0.1919		

In the model, the negative coefficients of PP are reliable with Hypothesis 1. The real option exercise cost negatively affects the likelihood of participation. In this way the follower's readiness to play helpfully diminishes since it has a better possibility of building its own particular plant. On the off chance that the pioneer does not bring down the rent rate in a similar manner, the bargaining game may end in a non-cooperative equilibrium, which decreases the likelihood of participation.

The coefficient of WD is certain, which affirms Hypothesis 3. The competition impact is commanded by the system impact and the quantity of disclosure wells positively affects collaboration. Hypothesis 2 is not firmly affirmed since the

coefficient of stores is not measurably huge in both of these two models. Be that as it may, the negative indication of saving coefficient indicates the correct course anticipated by Hypothesis 2. The limit is found to have constructive effect on the likelihood of participation in the basic logit show as it were. Item costs negatively affect the length of speculation slack. The network effect positively affects the term of investment lag.

5. Conclusion

The competitive real option literature has provided various equilibrium models for firms' investment decisions under competition. This research suggests another equilibrium possibility - the cooperative equilibrium in which firms share usual production facility and benefit from the network effect. Strong confirmation is offered to show that firms' investment decisions are strategic at least in the natural gas industry. Occasionally they contend with each other by investing earlier to preempt others. Sometimes, they may cooperate with each other to have the benefit of network effects. The choice between competition and cooperation may rely on two factors, namely real option exercise price and the competition or the network effect. Higher option exercise price will decrease the possibility of cooperation, whereas higher network effect will increase the possibility of cooperation.

The present study also provides the empirical evidence for other similar researches. That is, in industries with significant opportunity costs such as oil and gas industry, supplier heterogeneity or network effect may offset the erosion effect of competition on real option value to delay the investment. Firms in these industries will not start investment once the NPV rises to zero. Instead, their investments are typically delayed. The duration of the investment delay is influenced by commodity price and the level of network effect.

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