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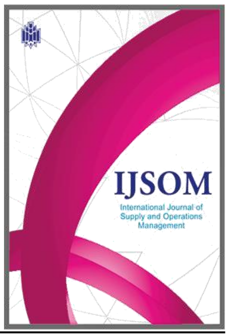

A Multi-objective Competitive Location Problem within Queuing Framework

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Abstract
This paper addresses a situation in which a firm is willing to locate several new multi-server facilities in a geographical area to provide a service to customers within a queue system. As a new assumption, it is also considered that there is already operating competitors in such system. This paper is going to find the location of facilities in a way that the market share of the entering firm is maximized. To this end, the simultaneous minimization of total cost and the maximum idle time in each facility are considered as two objective functions in the model. The total cost consists of two parts: (1) the fixed cost for opening a new facility, and (2) the operational costs for the customers, which depends on travel time to the facility and the waiting time at the facility. In addition, in order to make the problem more adapted to real-world situations, two new constraints on budget and the number of the servers in each facility are added to the model. Eventually, to tackle the suggested problem, a non-dominated sorting genetic algorithm (NSGA-II) and a non-dominated ranked genetic algorithm (NRGA) are utilized. Finally, the performance of algorithms is investigated by analyzing a set of test problems.

Keywords: Competitive location problem; $M/M/m/K$ queuing system; Multi-server facilities; Multi-objective modeling; NSGA-II, NRGA

1. Introduction and Literature review

The facility location problem (FLP) includes locating one or several facilities in one or more potential locations to minimize location and transportation costs and, at the same time, maximize the coverage of the demand and market share. This problem was introduced by Weber (1929) for the first time. He considered a single- facility problem in order to optimize the distance between the warehouse and customers' places. Afterwards, the facility location model was further extended by many other researchers. For instance, the p-median and p-center problems were introduced by Hakimi (1964), the set covering location problem by Toregas et al., (1971) and the maximum covering location problem by Church and ReVelle (1974).

One of the extensions of FLP is competitive location problem (CLP) in which the facility is located to provide service for clients in a condition that some competing facilities are available and ready for offering the same service. The aim of this problem is to find the location of the facilities according to the location of competitors so as to optimize the market share. The competitive location problem was firstly introduced by Hotelling (1929). In this problem, it is considered that customers receive the service from the nearest facility around them. Eiselt et al., (1993) studied CLP, provided a review, and divided the models into five subcategories: the number of players, the space, the pricing policy, the rules of the game, and the behavior of the customers. Benati (1999) studied the process of maximizing the market share with heterogeneous customers, wherein the entering firm p’s new facilities are located to cover the higher number of customers. Benati and Hansen (2002) studied an optimization model for finding the location of new facilities in competitive markets with random utility function. Drezner et al. (2002) proposed five heuristic methods to solve the multiple competitive facilities

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location problem (MCFLP) and their aim was to maximize the market share. Aboolin et al., (2007) studied a spatial interaction model that seeks to simultaneously optimize the location for a set of new facilities in a competitive environment. Bashiri and Hosseininezhad (2009) presented a multi-facility location problem to seek an optimal location for new facilities considering the fact that the competitor's facilities are available. Beresnev (2013) proposed a mathematical model generalizing the well-known facility location problem in order to maximize the market share, and used a branch and bound algorithm to solve the model. Biesinger et al. (2016) introduced a bi-level competitive facility location problem (CFLP) by studying possible customer behavior scenarios.

The CLP on network was introduced by Hakimi (1983). This problem consists of determining the locations of r facilities belonging to a firm in order to maximize its market share in a space where a competitor is already operating with p facilities $(r|xp)$. In addition, his model describes what happens when a firm locates its facilities in the area of competing facilities. Dobson and Karmarkar (1987) evaluated a network-based competitive facility location. In this model, customers at each node in the network choose an available facility to minimize the distance traveled. Marianov et al. (2008) investigated a CLP model to maximize the market share of the entering firm and introduced two key factors (the travel and waiting time) for the choice of facilities by the customers. In this research, the company aims to seek the location of facilities where there exists a competitor facility. Zarrinpoor and Seifbarghy (2011) proposed a new model in which a new entering firm desires to obtain a specific percentage of the market share in a way that the total costs be minimized. This model was solved with two heuristics based on genetic algorithm and tabu search. Some other researches including Brandeau and Chiu (1994), Shiode and Drezner (2003), Suarez-Vega et al. (2011), Rezapour et al. (2015), and Maleki et al. (2016) developed CLP on competitive location on a network. Table 1 shows a summary of the literature.

In all of above-mentioned research studies, the budget constraints have not been considered while due to this constraint, the firm cannot locate facility in all desired places in reality. Moreover, the number of servers in the problem is relaxed in the previous research studies, while this may be restricted in real situations. We also consider idle time in the CLP for the first time in the literature. The idle time is often an important performance criterion in planning process, and hence, it should not be neglected. To consider above suggestions, the model presented by Zarrinpoor and Seifbarghy (2011) is extended in this paper and a multi-objective model is proposed for the entering firm in a competitive environment. In this model, the objective function aims to maximize market share for entering firm and minimize the total cost including fixed cost for opening a new facility, traveling cost, waiting cost, and the maximum idle time in each facility. Moreover, the budget constraints and the upper bound of the server number in each facility are considered to represent the real problem.

The rest of the paper is ordered as follows. Section 2 defines and formulates the problem as an integer non-linear programming model. Section 3 describes the proposed NSGA-II and NRGA algorithms. Section 4 aims to adjust parameters of the algorithms and analyzes the computational results. Finally, in section 5, the conclusions and future research directions are provided.

2. Problem Definition

As mentioned before, we address the problem that a firm intends to locate in p multi-server facilities in a geographical area, where there is already one or several competing companies operating in the same region and offer the same service (represented as a network). Thus, each competitor firm tries to capture a large proportion of the demand as the possible or maximum market share. In this model, assuming that the customers decide which facility to patronize based on their nearness and the waiting times at the facilities (Marianov et al., 2008). To determine the percentage of customers attracted by the facility can be given by the logit functions of the time (McFadden, 1974) and each facility behaves as a $M/M/m/K$ queue system. As previously mentioned, the model proposed by Zarrinpoor and Seifbarghy (2011) is extended, in our work, to a multi-objective model for the entering firm in a competitive environment. In this model, the first objective function aims to maximize the market share for entering firm by minimizing total cost, and the second objective function minimizes the maximum idle time of each facility. Hajipour et al. (2014) used this criterion to reduce the average idle probabilities and this is the first time this criterion is used in competitive location problems. Also, the budget constraints and the upper bound of the server number are considered because in real world, the firms faced with the limited budget to establish a new facility with the given number of the server in each facility. The notations used in the model are displayed below.

Table 1. Summary of the literature

Papers	Integrated Concepts	Queuing System	Objective Function				Solution Method	Constraint	
			Single	Multiple	Cost	Idle Time		Budget	Number of server in facility
Dobson and Karmarkar (1987)	CLP	×	✓	×	×	×	Enumeration Algorithm	×	×
Brandeau and Chiu (1994)	CNLP	×	✓	×	✓	×	Exact Algorithm	×	×
Benati (1999)	CLP	×	✓	×	×	×	Lagrangian Relaxation	×	×
Drezner et al. (2002)	CLP	×	✓	×	✓	×	Simulated Annealing	×	×
Benati and Hansen (2002)	CLP	×	✓	×	✓	×	Branch and Bound	×	×
Aboolin et al. (2007)	CLP	×	✓	×	×	×	CPLEX Solver	×	×
Marianov et al. (2008)	CLP	*M/M/m/k*	✓	×	✓	×	Hoc Heuristic	×	×
Suarez-Vega et al. (2011)	CNLP	×	✓	×	×	×	Geographical Information Systems	×	×
Zarrinpoor and Seifbarghy (2011)	CLP	*M/M/m/k*	✓	×	✓	×	Genetic algorithm and Tabu Search	×	×
Beresnev (2013)	CLP	×	✓	×	×	×	Branch and Bound	×	×
Rezapour et al. (2015)	CNLP	×	✓	×	×	×	Exact Algorithm	×	×
Maleki et al. (2016)	CLP	×	✓	×	✓	×	Hybrid Algorithm	×	×
Biesinger et al. (2016)	CLP	×	✓	×	×	×	Evolutionary Algorithm	×	×
This paper	MCLP	*M/M/m/k*		✓	✓	✓	NSGA-II/NRGA	✓	✓

Indices:

i The customer index

j The facility index

Parameters:

N The set of candidate locations for the entering firm

N' The set of locations occupied by the competitor firm

h_i The average demand rate at each demand node i $(i \in N)$

λ_j The average arrival rate of customers to facility located at node j $(j \in N \cup N')$

$\tilde{\lambda}_j$ The effective arrival rate of customers to facility located at node j $(j \in N \cup N')$

μ_j The mean service rate at node j $(j \in N \cup N')$

π_{o_j} The probability that no customers being at the facility at node j $(j \in N)$

π_{n_j} The probability that n customers exist at the facility at node j $(j \in N)$

π_{k_j} The probability that k customers exist at the facility at node j $(j \in N)$

W_{q_j}	The average waiting time at the queue at node j ($j \in N$)
L_{q_j}	The average length of the queue at node at node j ($j \in N$)
t_{ij}	The travel time between customer at node i ($i \in N$) and candidate facility at node j ($j \in N \cup N'$)
f_j	The fixed cost per time unit to establish a new facility at node j ($j \in N$)
TC_j	The travel cost to node j per time unit ($j \in N$)
u	The maximum number of servers that can be used at each facility
θ_j	The waiting cost at node j per time unit ($j \in N$)
β	The minimum acceptable percentage of the market share for the entering firm
m_j	An integer number indicating the number of servers in server-site j
P	The maximum number of facilities that can be opened
BG	The available budget to design the system

Decision variables:

y_j	A binary variable which is 1 if a facility is located at node j and zero otherwise

2.1. Queuing System

We consider that each facility behaves as a $M/M/m/K$ queue system, implying that Poisson arrivals with a mean rate λ exponentially distributed service time with mean μ, m servers to be working at each facility, and the queue capacity to be limited to K customers .The queuing equations for the $M/M/m/K$ queues are as the following (Hillier and Lieberman, 1986):

$$\pi_n = \begin{cases} \left(\frac{\rho^n}{n!}\right)\pi_0 & if\ n \leq m \\ \left(\frac{\rho^n}{n!}\right)m^{n-m}\pi_0 & if\ n \leq m \leq k \\ 0 & O.w \end{cases} \tag{1}$$

$$\pi_0 = \left[1 + \sum_{n=1}^{m} \frac{\rho^n}{n!} + \frac{\rho^n}{m!} \sum_{n=m+1}^{k} \left(\frac{\rho}{m}\right)^{n-m}\right]^{-1} \tag{2}$$

$$L_q = \sum_{n=m}^{k} (n-m)\,\pi_n \tag{3}$$

$$W_q = \frac{L_q}{\lambda} \tag{4}$$

$$\tilde{\lambda} = \lambda(1 - \pi_n) \tag{5}$$

where π_0 is the probability that no customers being at the facility, π_n is the probability that n customers being at the facility, L_q is the average length of the queue, W_q is the average waiting time at the queue, $\tilde{\lambda}$ is the effective arrival rate, and $\rho = \lambda/\mu$.

2.2. Customer Behavioral

The system under study is represented as a network and all nodes in the network are considered as candidates for the location of the new facilities. According to the previous research, different percentages of the demand at each demand node may choose different facilities to patronize (Marianov et al., 2008; Zarrinpoor and Seifbarghy, 2011). The percentage of customer share by each facility is given by a logit function of the time given by McFadden (1974). Therefore, the probability of a customer at node i choosing to go to the facility at node j (X_{ij}) is defined by Eq. 6 as follows:

$$X_{ij} = \frac{y_j e^{-\gamma t_{ij}}}{\sum_{k\in N} y_k e^{-\gamma t_{ij}} + \sum_{k\in N`} y_k e^{-\gamma t_{ij}}} \forall\, i \in N, j \in N \cup N' \tag{5}$$

where, γ is $\pi/\sigma\sqrt{6}$ and σ denotes the standard deviation of consumers' ‘‘taste’’ . If γ is large, all consumers at a demand node will always patronize the same facility. As γ decreases, the dispersion in facility choice increases. That is, the consumers at the demand node i will not always choose the same facility j, but they will use possibly all facilities, each one with a probability X_{ij}. This may happen because of the customer’s access to some types of information of the congestion at the same facility. Customers rank the open facilities by cost (travel and waiting times), and the higher the cost is, the smaller the probability X_{ij} of patronizing that particular facility would be (Marianov et al., 2008).

2.3. Mathematical Model

Marianov et al. (2008) and Zarrinpoor and Seifbarghy (2011) assume that the entering firm intends to locate p facilities in the area, represented as a network, where q competing facilities are already located. As previously mentioned, the model proposed by Zarrinpoor and Seifbarghy (2011) is extended as a multi-objective model for the entering firm in a competitive environment. In this model, the first objective function aims to maximize the market share for entering firm by minimizing the total cost which includes the fixed cost for opening a new facility, traveling cost, and waiting cost. The second objective function minimizes the maximum idle time in each facility. Also, the budget constraints and the upper bound number of the server in each facility are considered to show the real problem.

If the demand generation rate at each demand node i is the Poisson process with average demand rate h_i, the demand rate at node j can be written as (Marianov et al., 2008; Zarrinpoor and Seifbarghy, 2011):

$$\lambda_j = \sum_{i\in N} h_i X_{ij} \;\forall\, j \in N \cup N' \tag{7}$$

Thus the model can be formulated as:

$$Min\ Z_1 = \sum_{j\in N} f_j\, y_j + \sum_{i\in N}\sum_{j\in N} \lambda_j\, TC_j t_{ij} + \sum_{j\in N} \theta_j w_{q_j} \lambda_j \tag{8}$$

$$Min\ Z_2 = \max j = \{\pi_0 y_j \ \ \forall\ \ 1,2,\ldots,J\} \tag{9}$$

$s.t.$

$$\lambda_j = \sum_{i\in N} h_i X_{ij} \;\forall\, j \in N \cup N' \tag{10}$$

$$X_{ij} = \frac{y_j e^{-\gamma t_{ij}}}{\sum_{k\in N} y_k e^{-\gamma t_{ij}} + \sum_{k\in N`} y_k e^{-\gamma t_{ij}}} \forall\, i \in N, j \in N \cup N' \tag{11}$$

$$L_{q_j} = \sum_{n=m}^{k} (n-m)\, \pi_{n_j} \;\forall\, j \in N \tag{12}$$

$$W_{q_j} = \frac{L_{q_j}}{\lambda} \forall\, j \in N \tag{13}$$

$$\pi_{n_j} = \left(\frac{\rho^n}{n!}\right) m^{n-m} \pi_0 \forall\, j \in N \tag{14}$$

$$\pi_{k_j} = \left(\frac{\rho^k}{n!}\right) m^{k-m} \pi_0 \forall\, j \in N \tag{15}$$

$$\tilde{\lambda}_j = \lambda_j \left(1 - \pi_{k_j}\right) \quad \forall\, j \in N \tag{16}$$

$$\pi_0 = \left[1 + \sum_{n=1}^{m} \frac{\rho^n}{n!} + \frac{\rho^n}{m!} \sum_{n=m+1}^{k} \left(\frac{\rho}{m}\right)^{n-m}\right]^{-1} \tag{17}$$

$$\rho_j = \frac{\lambda_j}{\mu_j} \quad \forall\, j \in N \tag{18}$$

$$\sum_{j} y_j \ \le p \quad \forall\, j \in N \tag{19}$$

$$\sum_{j=1}^{J} f_j y_j + fs_j \times m_j \le BG \tag{20}$$

$$m_j \le u \times y_j \quad \forall\, j \in N \tag{21}$$

$$X_{ij} \le y_j \quad \forall i, j \in N \tag{22}$$

$$\sum_{j\in N\cup N`} X_{ij} = 1 \qquad \forall\, i \in N \tag{23}$$

$$\frac{\sum_{i\in N}\sum_{j\in N} h_i X_{ij}}{\sum_{i\in N}\sum_{j\in N\cup N'} h_i X_{ij}} \geq \beta \tag{24}$$

$$X_{ij} \in \{0,1\} \quad \forall\, i \in N, j \in N \cup N' \tag{25}$$

$$\lambda_j \geq 0 \quad \forall\, j \in N \cup N' \tag{26}$$

$$y_j \in \{0,1\}\, \forall j \in N\,, y_j = 1 \quad \forall j \in N' \tag{27}$$

$$m_j \geq 0, \ \text{integer} \tag{28}$$

The objective function (8) minimizes the total cost. The objective function (9) minimizes the maximum probability of a facility to be idle. Eq. (10) represents the average arrival rate of customers to the facility located at node j . Eq. (11) represents probability X_{ij} of customer at node i choosing to go to the facility at node$j$. Eq. (12) represents a waiting time in queue for facility j . Eq. (13) represents the queue length for facility j. Eq. (14) represents the probability of the existing n customer in facility j. Eq. (15) represents the probability of existing k customers in facility j. Constraint (16) represents the effective arrival rate of customers to the facility located at node j . Eq. (17) represents the probability of non-existent customer in facility j. Eq. (18) represent the effective arrival rate for facility j. Constraint (19) is used for preventing the number of open facilities that exceed the limit. Constraint (20) indicates the budget restriction on establishing the selected facilities plus server's staffing costs. Constraint (21) limits the number of servers at each facility. Constraint (22) shows the requirement that a customer can only be assigned to an open facility. Constraint (23) forces each customer to be assigned only to just one facility. Constraint (24) ensures reaching a minimum acceptable market share by the entering firm. Constraints (25) and (27) show the binary variables. Constraint (26) ensures the non-negativity of demand, and constraint (28) enforces the integer variable restrictions on the number of servers at each facility.

3. Multi-Objective Evolutionary Algorithms (MOEA)

Since the nonlinear mixed-integer programming (NLMIP) model presented in previous section is NP-hard and exact methods cannot be used to solve it, we need to use a meta-heuristic. To this end, two multi-objective evolutionary algorithms (MOEA) are implemented. One of these algorithms is called the non-dominated sorting genetic algorithm (NSGA-II), and another is called the non-dominated ranked genetic algorithm (NRGA). There are several examples in which such algorithms are used for NLMIP. Loghmanian et al. (2012) used NSGA-II to optimize neural network for dynamic system. Rahmati et al. (2014) used NSGA-II and NRGA to solve NP-hard problems. Vahdani et al. (2016) used NSGA-II to solve the multi-objective, multi-period location-routing models. Memari et al. (2016) used NSGA-II and NRGA to optimize the total cost and service level for a just-in-time distribution network.

Most real world problems cope with several conflicting objectives. Therefore, considering some conflicting objectives is vital to make an optimization problem more realistic. Consequently, bi-objective optimization problems have received more attention during the last decades (Rahmati et al., 2014). These problems are generally seeking the vectors $y = [y_1, y_2, \dots, y_J]^T$ of decision variables that simultaneously optimize two-objective functions $\{Z_1(y), Z_2(y)\}$ while satisfying the model's constraints. In a minimization problem, the solution X_i is said to dominate X_j solution if $\forall\, m \in 1,2, \dots, M, Z_m(X_i) \leq Z_m(X_j)$ and $\exists m \in 1,2, \dots, M, Z_m(X_i) < Z_m(X_j)$. If X_i dominates the solution X_j, X_i is called the non-dominated solution. Moreover, a vector of non-dominated solutions is called Pareto vector (Rahmati et al., 2014).

3.1. Non-dominated sorting genetic algorithms (NSGA-II)

NSGA-II, which is one of the most efficient and well-known MOEA, was proposed by Deb et al. (2000). More explicitly, the different steps of the algorithm are described in the rest of the subsections.

3.1.1. Initialization

Following notations are used to describe the algorithm:

- MaxGen: The maximum number of the generations
- Popsize: The number of the individuals' population
- p_c: Crossover operator ratio
- p_m: Mutation operator ratio

These values will be set in section of computational result.

3.1.2. Solution encoding

A good chromosome representation is a vital component for the efficient and effective search of the solution area. The encoding scheme in our problem is presented by Fig 1. It shows a vector $1 \times J$ in which J represents potential facility nodes. After decoding process, these cells determine open facilities.

Y_1	Y_2	Y_3	$\cdots$	Y_j	Y_{J-1}	Y_J	$Y_j \in \{0,1\}$

Figure 1. The solution representation

Now, the decoding process is explained through a simple example. Suppose the maximum number of opened facilities is $p = 4$. Thus, we generate a solution randomly as shown in Fig. 2.

1	**0**	**0**	**1**	**1**	**0**	**1**

Figure 2. A randomly produced solution

3.1.3. Evaluation

An important issue in constrained optimization is the method employed to handle the constraints to guide the optimization of the feasible regions. A common approach in case of the metaheuristic algorithms is applying the penalty functions (Michalewicz and Schoenauer, 1996). A penalty function (see Eq. (19)) is applied to unfeasible solutions to penalize that solution and generate a poor function value for it. In other words, some functions are used to penalize infeasible solutions by reducing their fitness values in proportion to the degree of their violation. Penalty method transforms constrained problem to unconstrained one (Rahmati et al., 2014). In this paper, additive penalty function is used as is shown in Eq. (19).

$$Z(y) = \begin{cases} Z(\tilde{y}) & \text{if } \tilde{y} \in \text{ feasible region} \\ Z(\tilde{y}) + p(\tilde{y}) & o.w. \end{cases} \tag{19}$$

Where $Z(\tilde{y})$ is the objective function and $p(\tilde{y})$ shows penalty value and means if no violation occurs, $p(\tilde{y})$ will be zero otherwise.

3.1.4. Selection and elitism

In order to rank the population in NSGA-II, two operators called fast non-dominated sorting ($FNDS$) and crowding distance (CD) are employed. $FNDS$ is used to assign a rank to each solution of the population. This ranking process is done according to a domination concept. At the end of the ranking, individuals with less value of $FNDS$ are better, and for the individuals with the same rank, CD is calculated. These metric estimates the density of solutions which are laid surrounding a particular solution in the population. The more value of CD shows a better individual or an individual which is placed in a less crowded area. According to its concept, CD is used for controlling diversity within the solutions of Pareto fronts during the evolution process.

Then, in order to build the mating pool, binary tournament selection method is implemented. In this special type of the tournament selection, after random selection of two individuals ($i\ and\ j$), they are compared according to their $FNDS$ ($i_{rank}\ and\ j_{rank}$), then the individual with the less $FNDS$ is selected. Then, if the solutions are from the same front (or with the same rank), the one with higher CD is selected. This selection method is also summarized as follows:

$$i < j \quad if\ (i_{rank} < j_{rank})\ or\ (i_{rank} = j_{rank})\ and\ (i_{distance} > j_{distance})$$

3.1.5. Crossover operator

In this paper, the crossover operator is a kind of uniform crossover on the two randomly selected parents. The steps of the crossover are as follows:

Step 1: Select two chromosomes randomly as the two parents.
Step 2: Select the same part (row) of both chromosomes randomly.
Step 3: Generate a row (α) with real random value between $(0,1)$ with the same length as the selected row of the parents in Step 2.
Step 4: Generate offspring by the Eqs. (20) and (21).

$$offspring_1 = \alpha \times parent_1 + (1 - \alpha) \times parent_2 \tag{20}$$

$$offspring_2 = \alpha \times parent_2 + (1-\alpha) \times parent_1 \tag{21}$$

The crossover process is shown in Fig. 3 schematically.

$parent_1$	1 0 0 1 1 0 1		$offspring_1$ 1 0 1 1 0 0 1
$parent_2$	0 1 1 1 0 0 1		
α	1 1 0 1 0 0 0		$offspring_2$ 0 1 0 1 1 0 1

Figure 3. Crossover operator

3.1.6. Mutation operator

In this paper, the flipping mutation operator is used as follows.
Step 1: Select a part of chromosome randomly
Step 2: Generate a random integer number $j \in (0,1)$
Step 3: Exchange cells of a part of chromosome with $1 - j$
Fig. 4 illustrates this operator graphically.

$X = parent$	1 0 0 1 1 0 1		1 0 0 0 1 0 1

Figure 4. Mutation operator

3.1.7. Offspring evaluation

After performing crossover and mutation operators, the fitness function can be calculated. As it is clear, the fitness of our algorithms is calculated according to our objective functions in the model.

3.1.8. Sort population and select the N first individuals

A distinctive step of NSGA-II is its elitism method during the evolution process. In its initial step, NSGA-II creates the offspring population Q_t from the parent population P_t by applying tournament selection, recombination, and mutation operators. Then, it combines two populations to form the entire population R_t of size $2N$, where N is population size. Finally, a non-dominated sorting is performed until the non-dominated sorting is over, and the new population is then established by solutions of different non-dominated fronts. The filling starts with the best non-dominated front (set F_1), continues with solutions from the set F_2 , followed by the solution of set F_3 , and so on. When the last front is being considered, there may exist more solutions in the last front than the remaining slots in the new population. In these situations, a crowding sort procedure is implemented to select the members of the last front such that a diverse set of solutions is chosen from this set. This scenario is depicted in Fig. 5 and its pseudo code is illustrated in Fig. 6.

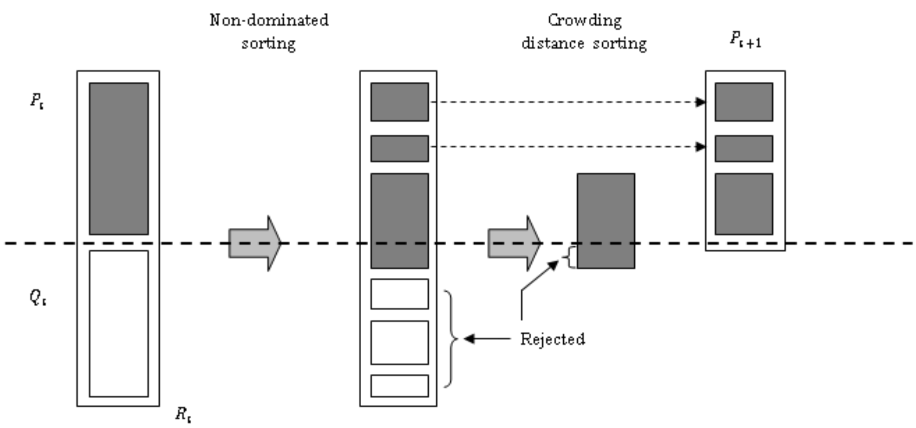

Figure 5. Schematic of the NSGA II procedure

1. Combine offspring and parent populations to create R_t
2. Perform a non-dominated sorting to R_t and identify different fronts F_i
3. Set new population $P_{t+1} = \emptyset$ and counter $i = 1$.
4. Perform $P_{t+1} = P_{t+1} \cup F_i$ and $i = i + 1$, till $|P_{t+1}| + |F_i| < N$.
5. Perform the crowding sort process and include the most widely spread $(N - |P_{t+1}|)$ solutions.
6. Create offspring population Q_{t+1} from P_{t+1} using the crowded tournament selection, crossover, and mutation operators.

Figure 6. NSGA II algorithm

3.2. Non-dominated ranking genetic algorithm (NRGA)

In this part, another MOEA called NRGA has been used to obtain Pareto Fronts. Al Jadaan et al. (2008) presented NRGA by transforming the NSGA-II selection strategy from the Tournament selection to the Roulette Wheel selection. In this method, first, the population is sorted according to $FNDS$ and the best solutions are chosen from the first ranked population. Then, according to their CD criteria, individuals of each front are ranked:

$$P_i = \frac{2 \times rank_i}{N_F(N_F + 1)} \qquad \forall\ i = 1,2, \dots, N_F \tag{22}$$

Where N_F shows the number of fronts and i shows rank of the front $i(i \in [1, N_F])$. In this equation, it is obvious that a front with the highest rank has the highest probability to be selected.

4. Computational Experience

4.1. Test problem generation

To compare the performance of NSGA-II and NRGA, three above-mentioned metric criteria are conducted on a network including 25 nodes with randomly generated demands at each nodes. The travel time between nodes t_{ij} is randomly generated from the interval$[0,300]$, and it is assumed that $t_{ij} = t_{ji}\ \forall$ all i, j. The nodes represent both demand concentrations and candidate facility locations. The upper bound of servers at each facility are assumed 5 and the maximum capacity of each queuing system at each facility is considered to be 10. The servers' service rate is assumed $1/_{12}$ for both existing competitor's facilities and the entering facilities. The travel cost between two arbitrary nodes and the waiting cost at each facility per time unit is considered equal to 3 and 4 , respectively. The fixed cost per time unit to establish a new facility is uniformly generated from the interval$[500, 1500]$. Moreover, the number of competitor firms, the minimum acceptable percentage of the market share of the entering firm (β), the available budget to design the system(BG), and the maximum number of facilities that can be opened (P) is considered equal to 6 and 8; 0.48 and 0.58; 8000 and 10000; 4 and 6, respectively. Then, 16 test problems are generated with different parameters as follows.

Problem 1= the number of competitor firms=6, $\beta = 0.48, BG = 8000, P = 4$
Problem 2= the number of competitor firms=6, $\beta = 0.58, BG = 8000, P = 4$
Problem 3=the number of competitor firms=6, $\beta = 0.48, BG = 10000, P = 4$
Problem 4=the number of competitor firms=6, $\beta = 0.58, BG = 10000, P = 4$
Problem 5=the number of competitor firms=6, $\beta = 0.48, BG = 10000, P = 6$
Problem 6= the number of competitor firms=6, $\beta = 0.58, BG = 8000, P = 6$
Problem 7=the number of competitor firms=6, $\beta = 0.48, BG = 10000, P = 6$
Problem 8=the number of competitor firms=6, $\beta = 0.58, BG = 10000, P = 6$
Problem 9= the number of competitor firms=8, $\beta = 0.48, BG = 8000, P = 4$
Problem 10=the number of competitor firms=8, $\beta = 0.58, BG = 8000, P = 4$
Problem 11=the number of competitor firms=8, $\beta = 0.48, BG = 10000, P = 4$
Problem 12= the number of competitor firms=8, $\beta = 0.58, BG = 10000, P = 4$
Problem 13=the number of competitor firms=8, $\beta = 0.48, BG = 8000, P = 6$
Problem 14=the number of competitor firms=8, $\beta = 0.58, BG = 8000, P = 6$
Problem 15= the number of competitor firms=8, $\beta = 0.48, BG = 10000, P = 6$
Problem 16= the number of competitor firms=8, $\beta = 058, BG = 10000, P = 6$

The standard deviation of consumer's "taste" γ varies between 0.1 and 1. The parameters of the proposed metaheuristics algorithm are population size (popsize), crossover probability (p_c), mutation probability (p_m), and number of generations (MaxGen). The parameters of the algorithms were obtained by some primary experiments. Table 3 shows the values of parameters.

Table 3. NSGA-II and NRGA parameter result

	NSGA-II	NRGA
Parameters	Range	Range
Popesize	30-50	30-60
P_c	0.7-0.75	0.65-0.85
P_m	0.25-0.4	0.08-0.40
MaxGen	200-300	250-300

4.2. Performance Measures

To compare various aspects of the obtained non-dominated fronts by the NSGA-II and NRGA, two performance metrics, i.e. normalized set coverage metric and spacing metric are utilized. The following subsections give brief descriptions of these metrics.

4.2.1. Normalized Set Coverage Metric($\bar{\mathbf{C}}$)

The set coverage metric was introduced by Zitzler (1998) for comparing two sets of non-dominant solutions. Considering a problem, two Pareto sets A and B are generated. Set coverage metric $C(A, B)$ calculates the fraction of solutions B that are weakly dominated by at least one solution in set A.

$$C(A,B) = \frac{|\{b \in B | \exists a \in A: a \preccurlyeq b\}|}{|B|} \tag{23}$$

Where a $\preccurlyeq$ b means that solution b is weakly dominated by solution a. $C(A, B) = 1$ means that all the generated non-dominated solutions in B are weakly dominated by set A. On the other hand, $C(A, B) = 0$ indicates that none of the solutions in set B can be weakly dominated by solutions in set A. It is worth to note that $C(A, B)$ is equal to $1 - C(B, A)$ only when the number of solutions in set A is equal to set B. To illustrate the comparison of the two Pareto sets, the normalized set coverage metric ($\bar{C}$) is proposed as it is formulated in equation (24). This equation indicates $\bar{C}(A, B) = 1 - \bar{C}(B, A)$ which makes the comparison of algorithm's performance more understandable than set coverage metric. Regarding the previous definition, $\bar{C}(A, B) \geq \bar{C}(B, A)$ demonstrates that set A has better coverage than set B.

$$\bar{C}(A,B) = \frac{C(A,B)}{C(A,B) + C(B,A)} \Rightarrow \bar{C}(A,B) = 1 - \bar{C}(B,A) \tag{24}$$

4.2.2. Spacing Metric ($\boldsymbol{\Delta}$)

Spacing metric, which was proposed by Deb (2001), assesses the spread of solutions of a Pareto set in the entire region by computing variance of distances of the neighboring solutions in the given Pareto set. In other words, it shows that how well the non-dominated solutions are distributed in the search space. The lower value of this metric means that the members of Pareto front are spread coherently.

$$\Delta = \sum_{i=1}^{|n|} \frac{|d_i - \bar{d}|}{|n|} \tag{25}$$

where$d_i = \min_{k \in n, k \neq i} \sqrt{\sum_{m=1}^{2} (f_m^i - f_m^k)^2}$, $\bar{d} = \sum_{i=1}^{n} \frac{d_i}{|n|}$ and n represents the number of non-dominated solutions in the Pareto set and f_m^i denotes the amount of m^{th} objective function for i^{th} non-dominated solution.

4.3. Analysis of Results

Tables 4-19 represent the result of NSGA-II and NRGA for problem 1 to problem 16, respectively. The final result of implementing the algorithms are given in Table 20 in which the first four columns corresponds to NSGA-II, the first and the second columns contain the first and the second objective function values ($Z_{1.best}.Z_{2.best}$), the third column shows set coverage metric ($\bar{c}$), and the last column represents the spacing metric (Δ). Similar patterns are repeated for NRGA in the second four columns.

Fig 7 and 8 illustrates comparisons between the first and the second objective function values ($Z_{1.best}.Z_{2.best}$), in NSGA-II and NRGA, respectively. Fig 9 and 10 illustrates comparisons between spacing metric and coverage metric, respectively.

Now, according to mentioned figures and tables, metrics can be assessed. The last row of Table 20 calculates the average values of each objective function and metric's outputs for all problems. According to the results presented by this row, NRGA has better performance on the first and the second objective function values and set covering metric, while NSGA-II has better performance on spacing metrics.

Table 4. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.48$, $BG = 8000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 1)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T[1].locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	6,713	0.4155	4,15,17,19	4	7,429	0.4153	1,16,18,21	4
2	0.2	6,493	0.3138	15,16,22	3	3,500	0.3695	5,11,25	3
3	0.3	9,710	0.2834	1,2,4,10	4	5,266	0.6405	12,17,18	3
4	0.4	6,883	0.4263	4,6,18,19	4	10,620	0.3247	11,13,25	3
5	0.5	8,851	0.2714	1,8,17,23	4	7,334	0.4894	20,21,23	3
6	0.6	8,491	0.5012	2,11,18	3	6,699	0.2029	5,14,17	3
7	0.7	10,603	0.1964	5,6,22,25	4	20,580	0.3581	7,15,22,23	4
8	0.8	11,858	0.1616	20,21,25	3	8,876	0.3758	6,9,16,17	4
9	0.9	4,441	0.2398	7,11,21	3	8,948	0.5269	4,14,25	3
10	1.0	10,987	0.3071	2,12,18	3	4,775	0.1991	4,9,14	3
Average		8,503	0.3616			8,402	0.4402		

Table 5. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.58$, $BG = 8000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 2)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	14,721	0.2110	5,9,17,23	4	9,690	0.0863	1,2,25	3
2	0.2	7,480	0.2967	7,15,22	3	8,450	0.3166	14,20,25	3
3	0.3	8,782	0.3311	3,8,14,18	4	10,519	0.1944	13,16,21,23	4
4	0.4	9,066	0.3312	1,10,21	3	5,601	0.2885	4,10,13,19	4
5	0.5	16,655	0.2411	8,11,16,25	4	5,534	0.3779	9,11,15,24	4
6	0.6	8,138	0.3086	3,5,22	3	6,816	0.1845	3,15,16,23	4
7	0.7	17,020	0.1668	9,16,21	3	13,115	0.2495	7,8,12,21	4
8	0.8	12,967	0.2853	3,14,19,23	4	12,280	0.3878	1,2,7,18	4
9	0.9	4,827	0.4021	4,8,15,25	4	21,289	0.1090	1,12,24	3
10	1.0	12,177	0.2297	2,12,16	3	8,569	0.3990	1,9,19	3
Average		11,183	0.3130			10,186	0.2593		

[1] Total

Table 6. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.48$, $BG = 10000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 3)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	12,830	0.2562	6,7,11,17	4	7,606	0.2889	6,8,14	3
2	0.2	10,249	0.2412	2,4,5,14	4	9,968	0.1457	17,21,24	3
3	0.3	8,883	0.2223	1,14,15,24	4	9,219	0.2160	1,11,12	3
4	0.4	8,222	0.2246	1,14,24	3	11,679	0.2857	4,11,12	3
5	0.5	4,777	0.2717	7,14,18,25	4	9,340	0.1406	2,4,7	3
6	0.6	11,876	0.1455	8,9,20,22	4	4,759	0.3512	4,19,22	3
7	0.7	8,019	0.3217	7,8,23	3	13,700	0.2392	4,6,10,19	4
8	0.8	7,486	0.2829	9,11,22	3	6,189	0.3291	8,9,14	3
9	0.9	14,947	0.1692	1,10,21	3	9,361	0.1721	6,12,14,23	4
10	1.0	15,271	0.1967	5,11,22,24	4	18,245	0.1515	5,21,24	3
Average		10,256	0.2645			10,006	0.2320		

Table 7. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.58$, $BG = 10000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 4)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	16,604	0.3679	6,7,11,17	4	11,699	0.3013	10,17,20,23	4
2	0.2	14,328	0.1689	2,4,5,14	4	15,838	0.2858	10,12,14,25	4
3	0.3	15,850	0.2446	1,14,15,24	4	15,960	0.2868	6,11,17,18	4
4	0.4	16,982	0.2681	1,14,24	3	8,842	0.2221	2,3,6,23	4
5	0.5	11,941	0.1767	7,14,18,25	4	7,365	0.2173	1,18,22,24	4
6	0.6	15,867	0.2526	8,9,20,22	4	10,951	0.2329	1,2,5,24	4
7	0.7	11,194	0.1195	7,8,23	3	5,923	0.2824	1,8,14,23	4
8	0.8	7,133	0.2032	9,11,22	3	10,900	0.3159	4,14,17,21	4
9	0.9	5,096	0.3794	1,10,21	3	4,393	0.3330	1,2,4	3
10	1.0	14,407	0.2500	5,11,22,24	4	12,707	0.2681	1,4,18,25	4
Average		12,940	0.2430			10,457	0.2745		

Table 8. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.48$, $BG = 8000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 5)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	8,907	0.3087	9,11,13,14	4	6,842	0.2550	13,14,21,22	4
2	0.2	14,569	0.3055	2,3,9,10,14	5	11,028	0.1186	6,11,23,24	4
3	0.3	8,284	0.1802	5,16,25	3	7,971	0.2416	5,23,24,25	4
4	0.4	11,432	0.2228	1,6,7,23	4	10,118	0.0991	7,21,22	3
5	0.5	12,206	0.2160	3,4,7,9	4	17,711	0.3897	5,7,11,12,13	5
6	0.6	9,646	0.2403	8,23,25	3	9,894	0.1169	2,3,4,12,25	5
7	0.7	15,080	0.2477	1,3,4,21	4	7,623	0.3768	11,13,16	3
8	0.8	5,531	0.3136	5,17,22	3	7,221	0.3771	10,12,23	3
9	0.9	15,344	0.2196	3,5,9,11,18	5	3,435	0.3659	9,10,15	3
10	1.0	6,317	0.1755	5,11,14	3	11,064	0.1471	2,6,7,10	4
Average		10,731	0.2429			9,290	0.2487		

Table 9. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.58$, $BG = 8000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 6)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	16,363	0.2916	1,3,4,5,12	5	13,651	0.2859	12,13,20,24,25	5
2	0.2	19,671	0.0700	13,16,17,21	4	10,015	0.3419	9,17,21	3
3	0.3	12,852	0.2306	3,18,22,24	4	12,666	0.2917	1,5,7,10	4
4	0.4	9,068	0.0937	3,6,13	3	19,248	0.1635	7,14,17,21,22	5
5	0.5	9,086	0.3360	7,9,10	3	5,201	0.3616	1,7,13	3
6	0.6	12,985	0.1544	4,23,24,25	4	11,769	0.1514	2,3,15	3
7	0.7	14,097	0.2666	3,6,8,21	4	14,028	0.2250	2,6,13,16	4
8	0.8	23,596	0.1776	6,9,10,19,24	5	9,403	0.2758	5,7,9,21,22	5
9	0.9	9,240	0.3032	2,18,22	3	8,707	0.2393	7,18,22	3
10	1.0	12,040	0.3182	1,3,4,5,12	3	6,160	0.1151	22,24,25	3
Average		13,998	0.2241			11,084	0.2451		

Table 10. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.48$, $BG = 10000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 7)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	6,966	0.2619	14,20,25	3	11,239	0.1607	5,11,14,18	4
2	0.2	3,961	0.1915	3,16,22	3	5,833	0.3666	8,22,25	3
3	0.3	11,295	0.2276	9,19,22,24	4	12,230	0.2639	3,8,23,25	4
4	0.4	16,458	0.1385	1,14,17,21,25	5	14,745	0.2173	5,8,17,19,22	5
5	0.5	9,726	0.2742	8,10,21	3	7,240	0.1824	9,17,20	3
6	0.6	11,104	0.2070	15,20,23	3	4,563	0.1730	9,14,17	3
7	0.7	5,875	0.3846	3,12,19	3	10,897	0.3097	3,4,11,19,20	5
8	0.8	15,280	0.3667	4,17,19,21	5	11,366	0.3549	4,9,19,21,24	5
9	0.9	15,072	0.2146	1,5,9,10	4	13,175	0.2577	7,12,13,16,20	5
10	1.0	6,854	0.3582	1,2,11	3	18,130	0.1928	1,2,6,17,21,22	3
Average		10,258	0.2604			10,941	0.2479		

Table 11. Results from NSGA-II and NRGA when the number of competitors' facilities is 6, $\beta = 0.58$, $BG = 10000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 8)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	11,005	0.2303	12,15,17,22,24	5	15,158	0.2849	4,7,13,16,18,20	6
2	0.2	17,204	0.1513	1,4,7,12,22,25	6	8,518	0.1769	14,16,19,21,22	5
3	0.3	12,782	0.2510	3,5,7,9,18,25	6	6,603	0.3477	5,11,22	3
4	0.4	10,658	0.3524	6,9,10,12,17	5	6,406	0.1009	3,8,14,25	4
5	0.5	14,865	0.3647	1,2,3,6,8	5	10,088	0.2777	2,3,5,10,13	5
6	0.6	13,291	0.2021	5,10,11,20	4	11,515	0.1833	1,7,8,9	4
7	0.7	10,633	0.1674	9,13,17	3	14,814	0.2868	2,3,16,17	4
8	0.8	10,970	0.3163	1,4,10,20	4	9,699	0.1815	4,18,19,24	4
9	0.9	15,923	0.2478	5,9,12,17	4	7,647	0.1991	4,18,19,22	4
10	1.0	14,267	0.3108	11,17,19,21	4	17,122	0.1955	6,7,9,12,21	5
Average		13,159	0.2594			10,757	0.2234		

Table 12. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.48$, $BG = 8000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 9)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	14,254	0.6671	4,14,19,20	4	11,641	0.2905	1,3,16,22	4
2	0.2	13,358	0.3295	1,6,19,22	4	12,662	0.3588	10,15,17	3
3	0.3	8,676	0.4526	1,3,25	3	6,816	0.3616	1,3,17,23	4
4	0.4	5,835	0.3104	2,4,20	3	11,687	0.2647	1,2,14,21	4
5	0.5	7,294	0.1510	14,16,18	3	5,116	0.3378	1,10,16,23	4
6	0.6	5,205	0.3753	2,3,4,9	4	7,060	0.1643	4,14,18	3
7	0.7	17,389	0.1984	14,15,22,23	4	8,907	0.1955	2,6,17,22	4
8	0.8	4,090	0.2495	3,8,11	3	8,876	0.2646	1,5,20,21	4
9	0.9	8,235	0.3219	1,5,10,21	4	11,396	0.2023	2,5,20,23	4
10	1.0	11,462	0.2149	2,3,6,11	4	8,244	0.3189	6,19,24,25	4
Average		9,579	0.3270			9,240	0.2759		

Table 13. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.58$, $BG = 8000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 10)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	5,923	0.2372	2,8,20	3	6,716	0.5171	1,5,21,25	4
2	0.2	10,939	0.4322	18,21,22,23	4	22,580	0.3159	4,5,15,22	4
3	0.3	13,162	0.2307	4,12,21,24	4	13,347	0.2623	1,11,21,24	4
4	0.4	7,301	0.3361	1,8,15	3	12,575	0.2630	1,4,12,22	4
5	0.5	11,093	0.3242	1,2,8,24	4	8,860	0.3310	1,4,18,24	4
6	0.6	15,050	0.2686	12,16,17,20	4	13,673	0.3376	10,11,19,24	4
7	0.7	12,259	0.2540	1,7,9,20	4	12,694	0.2759	3,14,17,24	4
8	0.8	7,086	0.3346	1,2,3,8	4	15,536	0.3332	1,4,10,20	4
9	0.9	13,646	0.3265	13,15,22,25	4	8,940	0.3345	2,10,21,23	4
10	1.0	19,651	0.2632	3,9,17,19	4	12,343	0.2428	1,19,22,25	4
Average		11,611	0.3007			12,726	0.3213		

Table 14. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.48$, $BG = 10000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 11)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.location s
1	0.1	13,858	0.3327	1,13,22,24	4	12,117	0.3289	5,7,10,25	4
2	0.2	8,338	0.3738	1,8,25	3	8,959	0.2765	4,5,11,23	4
3	0.3	6,612	0.3201	2,3,15	3	11,315	0.1481	1,2,5,8	4
4	0.4	4,419	0.3999	7,17,24	3	15,640	0.2301	1,3,13,22	4
5	0.5	13,148	0.2461	2,6,14,20	4	8,128	0.3344	3,13,25	3
6	0.6	10,444	0.2755	1,16,20,25	4	12,759	0.2658	3,13,15,16	4
7	0.7	14,362	0.3179	1,5,21,24	4	11,625	0.3137	3,6,21	3
8	0.8	8,953	0.3601	5,12,16,23	4	8,194	0.2970	10,16,22	3
9	0.9	10,774	0.2920	14,16,19,21	4	14,671	0.2028	9,10,16,25	4
10	1.0	12,732	0.3065	2,3,14,25	4	18,358	0.2155	10,12,20,23	4
Average		10,364	0.3224			12,176	0.2612		

Table 15. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.58$, $BG = 10000$, $P = 4$ and γ is varied between 0.1 and 1. (Problem 12)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.location s
1	0.1	7,371	0.2853	2,18,24	3	17,908	0.4182	9,13,15,22	4
2	0.2	14,500	0.1458	3,7,13,15	4	13,671	0.1701	6,8,12,18	4
3	0.3	12,441	0.3866	6,13,15,16	4	10,645	0.3797	8,15,18,21	4
4	0.4	7,466	0.2136	2,12,15	3	12,803	0.3963	1,2,11,22	4
5	0.5	17,820	0.3460	1,4,11,23	4	11,161	0.3816	2,3,5,22	4
6	0.6	10,755	0.2207	2,3,5,22	4	9,778	0.3828	1,4,5,21	4
7	0.7	10,582	0.3112	6,12,14,16	4	12,791	0.3604	8,13,14,21	4
8	0.8	8,139	0.3986	1,13,25	3	10,221	0.3870	6,16,17,25	4
9	0.9	15,376	0.3828	3,16,18,21	4	19,256	0.3450	3,10,15,23	4
10	1.0	13,074	0.1917	13,17,20,23	4	18,870	0.3164	8,14,20,25	4
Average		11,752	0.2855			13,710	0.3537		

Table 16. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.48$, $BG = 8000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 13)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	12,059	0.2696	1,14,17,23	4	11,355	0.2583	1,5,12	3
2	0.2	19,044	0.2286	3,9,16,19	4	12,184	0.2886	8,15,18,19	4
3	0.3	14,836	0.3569	7,9,15	3	9,979	0.3298	1,7,19,23,24	5
4	0.4	12,810	0.2974	4,5,6,8	4	13,951	0.2154	4,15,18,19,25	5
5	0.5	8,811	0.3116	7,8,15,19,23	5	6,036	0.2501	3,11,16,24	4
6	0.6	14,860	0.3356	1,3,4,12,21	5	17,880	0.2766	1,6,7,10,25	5
7	0.7	16,917	0.3739	4,18,19,20,23	5	18,792	0.2799	2,4,13,17,25	5
8	0.8	10,309	0.2856	10,12,14,16	4	9,360	0.2505	6,17,20,25	4
9	0.9	15,434	0.2645	4,9,17,24	4	6,136	0.2199	1,6,17	3
10	1.0	9,819	0.2842	2,12,14,24	4	5,780	0.3178	1,11,21	3
Average		13,489	0.3007			11,145	0.2686		

Table 17. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.58$, $BG = 8000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 14)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	11,670	0.3426	7,11,20,22	4	9,047	0.3393	1,2,5,7,22	5
2	0.2	16,525	0.2158	1,2,6,14,22	5	17,355	0.2812	3,14,17,22,23	5
3	0.3	8,181	0.0820	2,19,22	3	18,285	0.3334	1,9,13,19,25	5
4	0.4	19,439	0.1825	4,6,15,24,25	5	16,598	0.3088	3,10,15,20,21	5
5	0.5	11,550	0.2176	10,11,17,20,21	5	16,147	0.2333	2,3,9,16	4
6	0.6	10,913	0.2606	7,15,20,25	4	14,377	0.2038	2,9,12,17,25	5
7	0.7	8,804	0.2383	5,14,17,18	4	8,224	0.2537	5,11,18,19,21	5
8	0.8	17,994	0.1796	4,14,15,20,25	5	13,127	0.3123	2,5,10,15,21	5
9	0.9	11,265	0.2543	1,3,8,13,15	5	10,015	0.3037	8,10,25	3
10	1.0	17,366	0.2494	5,11,21,25	4	19,976	0.2732	1,8,11,12,25	5
Average		13,369	0.2222			14,315	0.2842		

Table 18. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.48$, $BG = 10000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 15)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	15,054	0.3046	1,15,21,22,23,24	6	10,175	0.2578	8,15,16,19	4
2	0.2	12,221	0.1828	12,16,24,25	4	18,433	0.3549	4,16,19,23,25	5
3	0.3	9,396	0.2688	13,24,25	3	8,778	0.2843	6,22,24	3
4	0.4	18,465	0.2351	5,10,11,15,19,21	6	9,694	0.2409	2,3,4,15,20,25	6
5	0.5	10,712	0.2299	2,3,13	3	6,098	0.2402	2,6,17,19	4
6	0.6	9,792	0.2380	4,6,23	3	11,484	0.2668	2,3,17,23	4
7	0.7	16,081	0.2406	1,6,9,21,23,25	6	10,922	0.3165	2,11,18 ,21	4
8	0.8	14,837	0.2441	1,3,9,17	4	19,586	0.4700	2,6,8,17,21,25	6
9	0.9	9,045	0.3379	8,16,22,25	4	9,138	0.3059	11,22,24,25	4
10	1.0	9,685	0.4508	1,5,11,16	4	16,457	0.5366	1,12,18,25	4
Average		12,528	0.2732			12,076	0.3273		

Table 19. Results from NSGA-II and NRGA when the number of competitors' facilities is 8, $\beta = 0.58$, $BG = 10000$, $P = 6$ and γ is varied between 0.1 and 1. (Problem 16)

No	γ	NSGA-II				NRGA			
		$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations	$Z_{1.best}$	$Z_{2.best}$	Location no.	T.locations
1	0.1	17,764	0.3809	1,4,5,11,21,22	6	17,947	0.5611	2,3,5,15,18,25	6
2	0.2	18,202	0.5115	5,14,16,17,18	5	20,758	0.4834	7,9,14,19,22,24	6
3	0.3	7,000	0.2851	9,17,21,25	4	17,836	0.3451	9,13,15,16	4
4	0.4	16,196	0.5621	1,3,5,12,19	5	23,886	0.3887	1,3,11,14,23,24	6
5	0.5	23,119	0.2330	4,7,11,24,25	4	14,619	0.3847	4,8,13,19,25	5
6	0.6	7,096	0.3939	3,10,18	3	13,749	0.2505	6,16,18,25	4
7	0.7	11,756	0.4159	2,4,5,10,17,21	6	10,959	0.2914	7,16,17,21,25	5
8	0.8	16,492	0.2646	3,17,20,24,25	5	20,093	0.3020	11,12,14,19,24	5
9	0.9	10,274	0.4584	2,3,12,19	4	22,239	0.3179	1,3,6,17,21,22	6
10	1.0	24,714	0.5683	2,8,9,12,13,21	6	7,794	0.2963	1,2,11	3
Average		15,261	0.4073			16,988	0.3621		

Table 20. Compare the performance of NSGA-II and NRGA

Problem	NSGA-II				NRGA			
	$Z_{1.best}$	$Z_{2.best}$	$\bar{C}$	Δ	$Z_{1.best}$	$Z_{2.best}$	$\bar{C}$	Δ
1	8,503	0.3616	1	0.6717	8,402	0.4402	0	0.7376
2	9,579	0.3270	1	0.5567	9,240	0.2759	0	0.5962
3	11,611	0.3270	0	0.1429	12,726	0.3213	1	0.5340
4	11,183	0.3130	1	0.8534	10,186	0.2593	0	0.7615
5	10,256	0.2645	1	0.6598	10,006	0.2320	0	0.6724
6	12,940	0.2430	0	0.6612	10,457	0.2745	1	0.7217
7	11,752	0.2855	0	0.8360	13,710	0.3537	1	0.7530
8	10,364	0.3224	1	0.4280	12,176	0.2612	0	0.7294
9	10,731	0.2429	0	0.5820	9,290	0.2487	1	0.9521
10	13,988	0.2241	0.6	0.6710	11,084	0.2451	0.4	0.7654
11	10,258	0.2604	0	0.7155	10,941	0.2479	1	0.6697
12	13,159	0.2594	0	0.5559	10,757	0.2234	1	0.7321
13	13,489	0.3007	1	0.6664	11,145	0.2689	0	0.8188
14	13,369	0.2222	0	0.5804	14,315	0.2842	1	0.9230
15	12,528	0.2732	0	0.5387	12,076	0.3273	1	0.6468
16	15,261	0.4073	1	0.6760	16,988	0.3621	0	0.6307
Average	11,623	0.2896	0.475	0.5702	11,468	0.2891	0.525	0.727

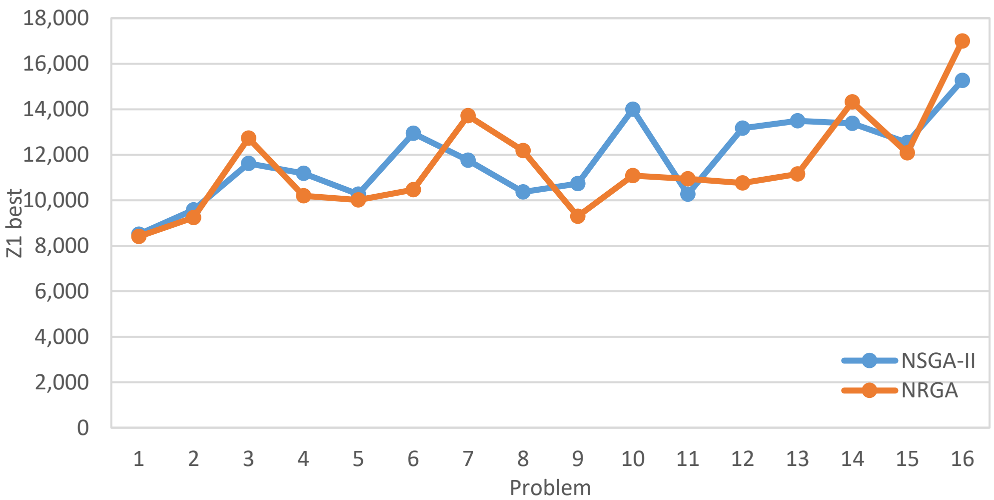

Figure 7. Comparisons between the first objectives function values of NSGA-II and NRGA in Table 20

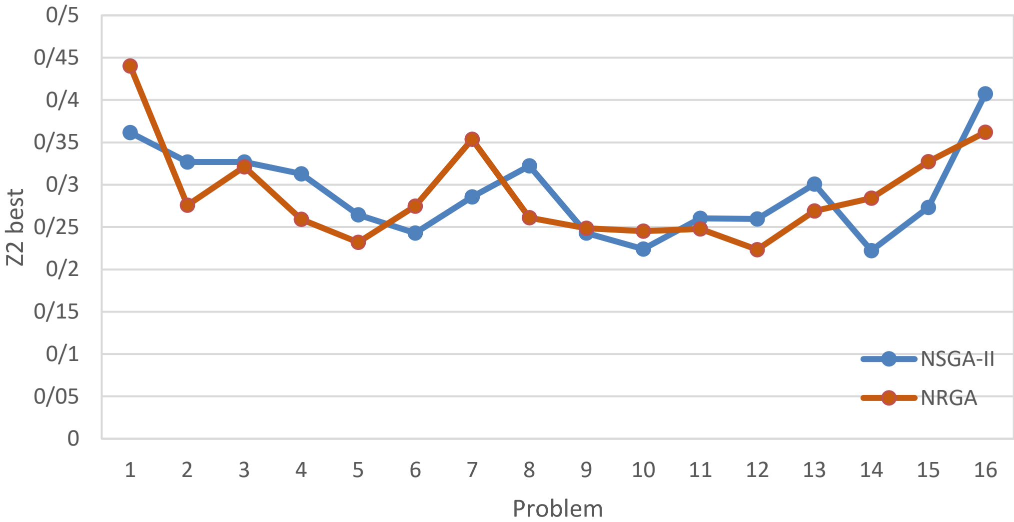

Figure 8. Comparisons between the second objectives function values of NSGA-II and NRGA in Table 20

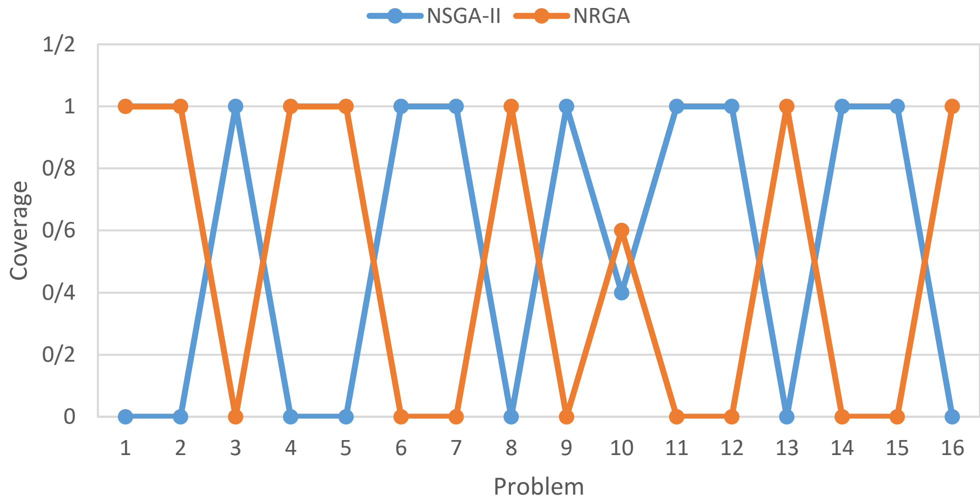

Figure 9. Detailed comparison of coverage metrics on different 16 test problems of NSGA-II and NRGA in Table 20

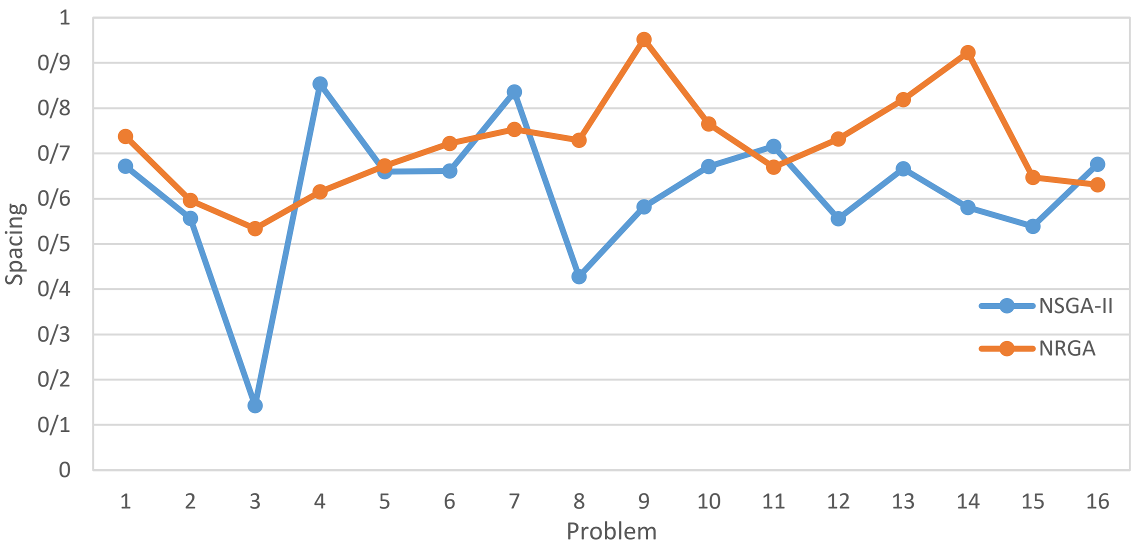

Figure 10. Detailed comparison of spacing metrics on different 16 test problems of NSGA-II and NRGA in Table 20

Figures 11-14 illustrate the convergence of NSGA-II and NRGA for problem 1. According to Fig. 11, before iteration 40, the value of the first objective function is decreasing and after iteration 40, this value is converged. According to Fig. 12, before iteration 10, the value of the second objective function is decreasing and after iteration 10, this value is converged.

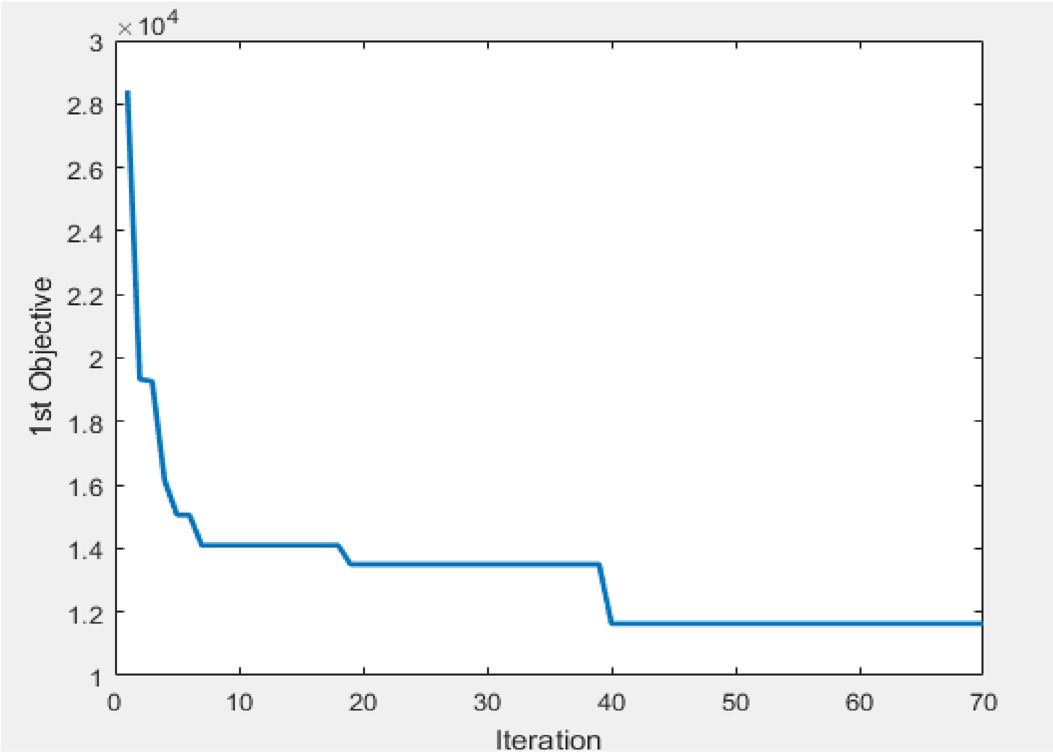

Figure 11. The convergence of NSGA-II on first objective function

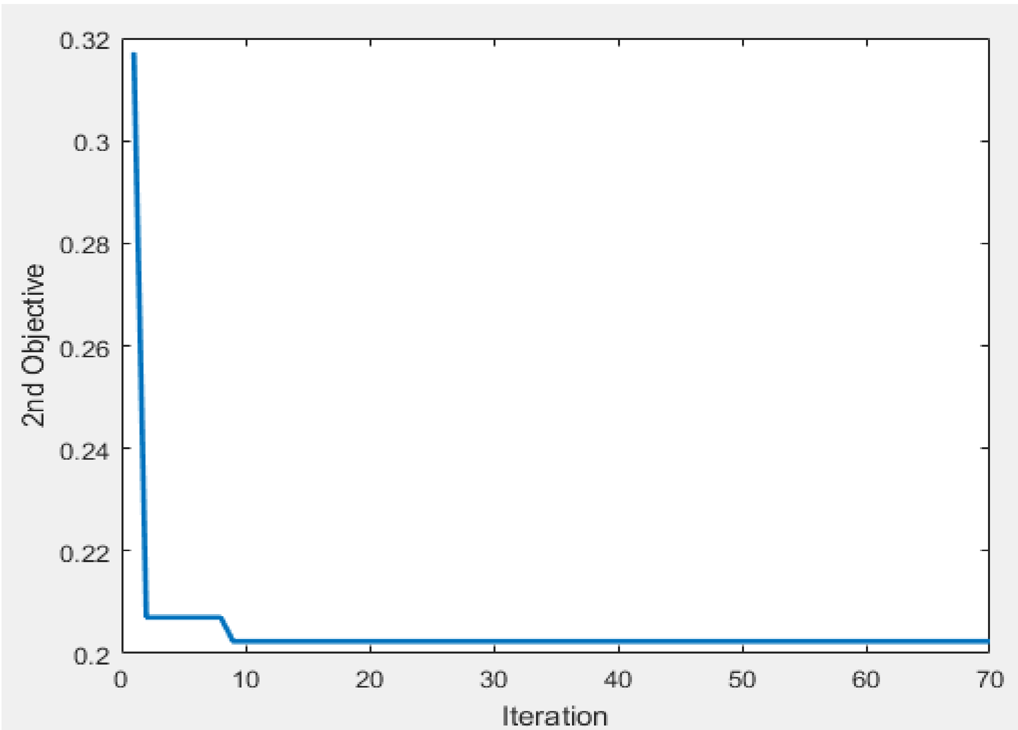

Figure 12. The convergence of NSGA-II on second objective function

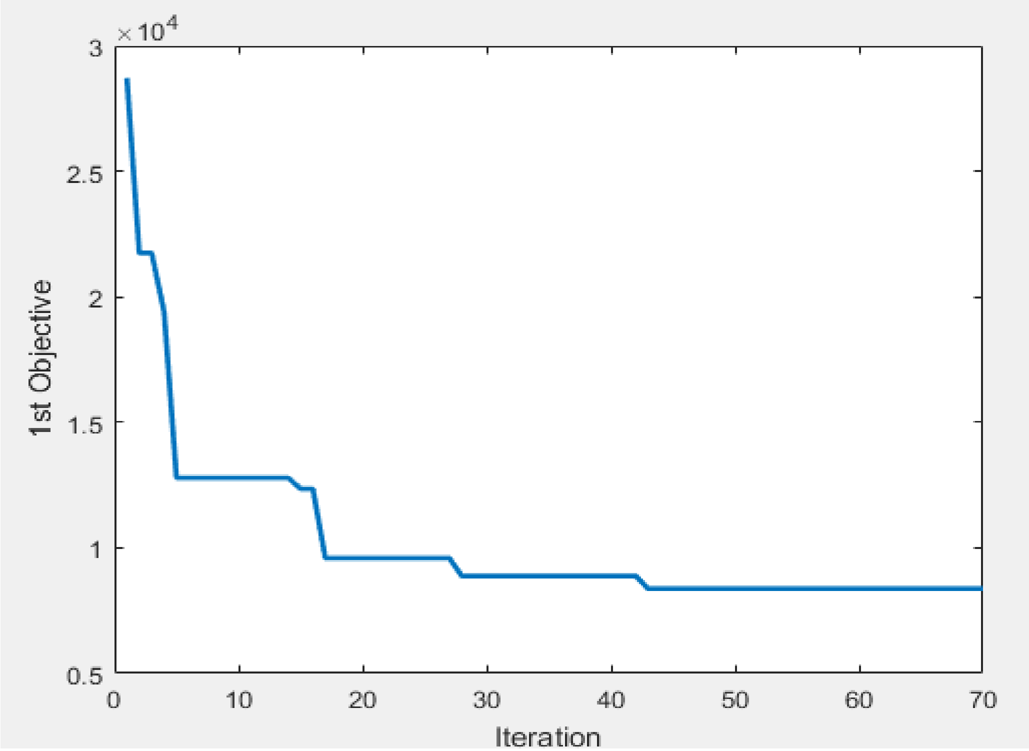

Figure 13. The convergence of NRGA on first objective function

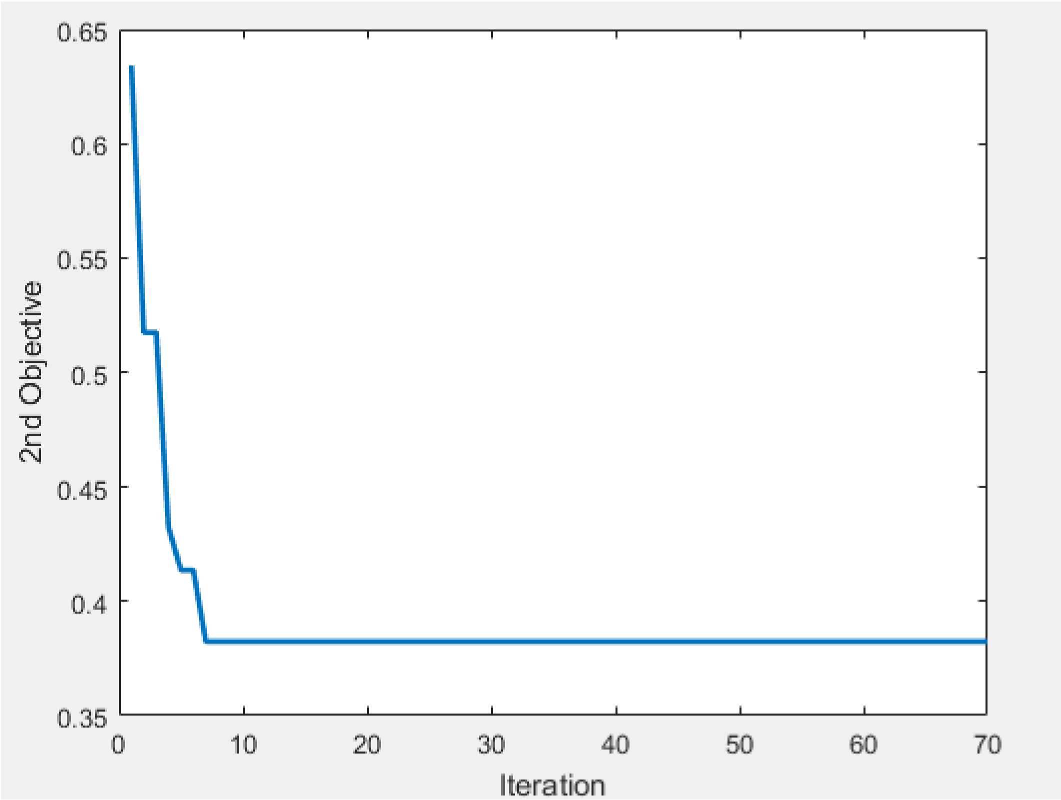

Figure 14. The convergence of NRGA on second objective function

5. Conclusion

In this paper, a multi-objective competitive location problem was developed with $M/M/m/k$ queue system for the entering firms in competitive environment, which aims to maximize the market share of the entering firm by minimizing total cost. The costs include fixed cost for opening a new facility, traveling cost, waiting cost and minimize the maximum idle time in each facility. Moreover, since the model belongs to an NP-hard class of problems, two multi-objective algorithms called NSGA-II and NRGA were developed to solve the problem. To evaluate the problem, 16 problems were considered. Finally, the performance of the algorithms was statistically analyzed by means of the value of objective functions, set coverage, and spacing metrics. The results showed that the average total costs of the first objective function in NSGA-II and NRGA were 11,623 and 11,468, respectively. The average values of the second objective function in NSGA-II and NRGA were 0.2896 and 0.2891, respectively. The average values of the set covering metric in NSGA-II and NRGA were obtained 0.475 and 0.525, respectively. Moreover, the values of the spacing metric in NSGA-II and NRGA became 0.570 and 0.727, respectively. As a result, it has been revealed that NRGA has better performance on the first and the second objective function values and set covering metric, while NSGA-II has better performance on spacing metrics.

For future research, we can classify the customers into two groups and consider auxiliary facility as the target for the new facility to provide a service for all customer and maximize the market share. Moreover, auxiliary facility can provide service for some special customers (i.e., ATM provides service for first type customer) and the facility (i.e., main branches) can provide service for all customers.

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