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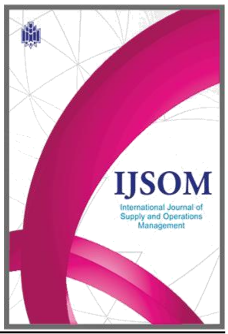

The Quadratic Approximation of an Inflationary Bi-objective Integrated Vendor-buyer Inventory Model with an Imperfect Manufacturing Process and Fixed and Variable Lead Time Crash Costs

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Abstract

In this paper, we develop an integrated bi-objective model of a two-stage supply chain composed of a vendor and a buyer under an imperfect production process. In the stochastic inflationary condition the first objective is to minimize the expected costs of the proposed supply chain model and the second objective is to minimize buyer's shortage variance. We assume lead time and ordering cost are controllable parameters and lead time crashing cost is considered as a function of both order quantity and reduced lead time. An effective solution procedure is developed to determine the optimal policy of the proposed model. Finally, a numerical example and sensitivity analysis are proposed to show the performance of the model.

Keywords: Supply chain; Integrated vendor-buyer inventory model; Lead time; Inflation; Stochastic; Multi-objective programming.

1. Introduction

Considering both classical economic order quantity (EOQ) and economic production quantity (EPQ) models, the vendor and buyer minimize their own inventory costs individually. This independent decision making for inventory problem usually cannot satisfy the optimal policy for both the vendor and buyer. However, the recent inventory control literature has shown that integrating the EOQ model of buyer and the EPQ model of vendor may lead to lower inventory total costs for the supply chain system rather than obtaining optimal policies for chain members separately.

The first paper on an integrated model of the vendor and buyer was written by Goyal (1976). Subsequently, many researchers have investigated this important issue considering various assumptions. For instance, Banerjee (1986) expanded the model of Goyal (1976) by assuming a finite vendor's production rate. Goyal (1988) assumed that the vendor's production quantity is a multiple of the order size of the buyer. Ha and Kim (1997) further extended Goyal (1988)'s model and proposed an integrated lot-slitting model for facilitating multiple shipments in small lots. Yang and Wee (2000) developed an integrated inventory model for deteriorating items. A novel method was proposed by Hans et al. (2006) wherein the joint economic lot size for the distribution system with multiple shipment policies is obtained. Hariga and Al-Ahmari (2013) constructed a two-layer supply chain model with hybrid vendor-managed inventory and consignment stock policy and stock dependent demand at retailer's shelf space. In the majority of the mentioned studies on integrated inventory models of vendor and buyer, researchers concentrated on the deterministic demand of the system.

However, when demand is considered as a stochastic variable, the lead time becomes an important issue and shortening it reduces safety stock as well as the stock-out loss and improves the customer service level so as to gain competitive advantages in business. If it is assumed that lead time can be decomposed into several components, such as setup time, process time, and queue time, it can be considered that each component might be reduced to its minimum duration with

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an extra crashing cost. Liao and Shyu (1991) were first who proposed a deterministic variable lead time inventory model. The authors assumed that lead time can be decomposed into its elements which in turn can be reduced to their minimum duration. Under the assumption that the demand is normally distributed, they calculated the optimal decision variable lead time and showed that reducing lead time may result in lower expected inventory costs. In another study, Pan and Yang (2002) considered lead time to be a decision variable and obtained a lower joint total expected cost and shorter lead time for a supply chain. Ouyang et al. (2004) extended Pan and Yang (2002)'s model considering an allowable shortage model where the reorder point is treated as a decision variable. Later, Pan and Hsiao (2005) proposed an integrated inventory model with backorder price discount and controllable lead time wherein lead time crashing cost is a function of both ordering quantities and reduced lead time. Yedes et al. (2012) studied a joint vendor-buyer inventory model wherein the production unit is assumed to randomly shift from an in-control to an out-of-control situation. They considered production, inventory, and maintenance policies simultaneously. On the other hand, almost all of the mentioned papers on the supply chain models did not investigate ordering/setup cost reduction. However, studies on ordering/setup cost reduction have shown that investment in ordering/setup cost can lower system's cost meaningfully. Porteus (1985) was first to present a mathematical model with setup cost reduction without backorders. Lin (2009) presented a model with backorder price discount and ordering cost reduction conditions. Glock (2012) considered a single-vendor single-buyer inventory model with lot-size-dependent lead time and lead time reduction and assumed vendor's set up cost can be decreased with an extra crash cost with a piece-wise linear function. Giri and Sharma (2017) studied a mathematical model with an imperfect production process wherein ordering cost can be reduced with extra investment.

Inflationary condition has been observed in many countries, especially in developing countries and considering the effect of this phenomenon in inventory/production models improves the performance of the models. Reviewing the literature on inflationary condition, it is observed that two methods have been applied to consider inflationary condition in inventory/production models. The first method computes optimal variables by minimizing the average annual cost and the second approach calculates optimal variables by optimizing the discounted future costs. Hadley (1964) showed that there is a negligible difference between the average annual cost and the discounted cost methods. Mirzazadeh (2011) studied an inventory model considering shortages and deterioration in uncertain environments. Sarkar et al. (2000) developed a supply chain model to determine an optimal ordering policy for deteriorating items under inflation, permissible delay in payment and allowable shortage. Lo et al. (2007) studied an integrated inflationary production-inventory model in which the deterioration rate follows the Weibull distribution. They used the discounted cash flow (DCF) approach and a classical optimization technique to determine the optimal production and replenishment policy. Mirzazadeh et al. (2009) considered an inventory model with shortage and finite time horizon and replenishment in which inflation is a random variable. Gholami-Qadikolaei et al. (2013) studied a stochastic (Q, r) inventory model under inflationary condition using an average annual cost method for buyer facility and proposed different inspection scenarios for the model. Ghoreishi et al. (2014) proposed an inventory system for deteriorating items and allowable shortage and considered inflationary conditions in the model. More recently, Kumar and Kumar (2016) built an economic order quantity model for a two-warehouse system with deterioration, inflation, and stock dependent demand. Nasrabadi and Mirzazadeh (2016) provided an inventory model with shortage and deterioration considering markovian inflationary conditions. Wan and Chen (2017) developed a supply chain model with perishable product under inflationary conditions.

One of assumptions of the studies is related to the quality of the product in a lot which is imperfect. In real environments, usually an arrived lot may contain some imperfect items. Therefore, if there are imperfect items in a lot, the firm may order a larger amount than the originally planned amount to meet the customer demand. Salameh and Jabber (2000) developed an EOQ model where imperfect quality items are salvaged at a discount price. The received raw material which includes imperfect quality items is a probabilistic variable. Huang (2002) extended the model in Salameh and Jaber (2000) and proposed a single-buyer single-vendor model for imperfect items. In the aforementioned study, it is considered that the defective rate is independent of lot-size. However, in some other research, it is assumed that defective unites is dependent on lot-size and follows beta-binomial random variables (for instance see Gholami-Qadikolaei et al (2012), Gholami-Qadikaolei et al. (2015), Ho et al. (2009)).

As stated before, often in practice, we observe inflationary conditions when making decisions about ordering policy of an inventory/production system; ignoring this important issue in modeling of systems in such areas might result in incorrect optimizations of decision variables and could cause great financial loss for the inventory/production systems. Hence, in this regard, proposing a mathematical model which considers inflationary conditions in order to simulate a real environment regarding an inventory/production system is very important. Also, integrating ordering decisions on supply chain different parties to diminish the total cost of inventories is of growing interest to inventory managers. Therefore, based on the above-mentioned conditions, we extend the previous research by proposing an inflationary bi-objective integrated vendor-buyer inventory model with stochastic demand and controllable lead time and ordering cost in which allowable shortage is a mixture of backorder and lost sale. Lead time crashing cost is considered a function of both lead time and order quantity. We also utilize the quadratic approximation for mathematically formulizing defective units in the proposed inflationary inventory system. We first minimize the joint expected annual cost as a main objective when ordering cost, ordering quantity, reorder point, number of shipment and lead time are the decision variables, and then reduce shortage variance as a second objective. We also propose a numerical example with sensitivity analysis to illustrate and validate the results of the proposed model.

2. Terminology

The following terminology is used throughout the paper:

Q	Buyer's order quantity (decision variable)
r	Buyer's Reorder point (decision variable)
k	Safety factor (decision variable)
L	Lead time (decision variable)
p	Defective rate in an order lot, $p\epsilon[0,1)$ (random variable)
$g(p)$	Probability density function (p.d.f.) of p
D	Annual demand for buyer (random variable)
n_V	Vendor's number of cycle (random variable)
n_b	Buyer's number of cycle (random variable)
T_b	Buyer's cycle time (random variable)
T_b^r	Reorder point time (random variable)
π	Buyer's stock out cost per unit short at the time zero
$\pi_\circ$	Buyer's Marginal profit per unit at the time zero
β	Backorder rate, $\beta\epsilon[0,1]$
C_{pr}	Vendor's production cost per unit at the time zero
C_{pu}	Buyer's purchasing cost per unit at the time zero
I	Inflation rate per unit time (random variable)
M	The number of lots in which the product is delivered from the vendor to the buyer in one production cycle, a positive integer, as a decision variable
h_V	Buyer's holding cost per year per unit at the time zero
h_b	Buyer's holding cost per year per unit at the time zero
S	Vendor's set up cost per set up at the time zero
$I(A)$	Buyer's capital investment required to achieve ordering cost A, $0 < A \leq A_0$
b	Percentage decrease in ordering cost A per dollar increase in investment $I(A)$
θ	Fractional opportunity cost of capital investment per year
A	Buyer's ordering cost per order at the time zero, as a decision variable
$C(L)$	Total lead time crashing cost per order at the time zero
Y	Number of imperfect items in an order lot (random variable)
X	Lead time demand (random variable)
X^+	Maximum value of x and 0
$E(\cdot)$	Mathematical expectation

3. The mathematical model

This investigation concentrates on a cooperative two-layer inventory model including a single vendor who provides one type of product to a single buyer. The buyer orders a lot of size Q and the vendor manufactures an order of size MQ with a finite production rate, P, at one set-up but ship in quantity Q to the buyer over M times. The vendor incurs a set-up cost, S, for each production run and the buyer incurs an ordering cost, A, for each order of quantity Q. This study assumes vendor's manufacturing system is imperfect and the number of defective items in an arriving lot, Y, is a beta-binomial random variable, where p $(0 \leq p \leq 1)$ shows the defective rate (see the appendix). Once an order quantity is reached, all items are inspected by the buyer and all of the defective ones are given back to the vendor at the same time. Thus, once the vendor produces the first Q units, he or she will deliver them to the buyer. After the buyers' inspection, the vendor plan deliveries every $\frac{Q-y}{D}$ units of time until the inventory level falls to zero. Therefore, the length of each production cycle for the vendor is equal to $T_V = \frac{M(Q-y)}{D}$ and the length of each ordering cycle is equal to $T_b = \frac{Q-y}{D}$ (see Fig. 1). Also, it is mentioned that inspection is error free and the inspection time is negligible. The cost of transportation from the vendor to the buyer and the buyer's inspection cost are constant and thus independent of the ordering quantity. So, we relinquish the total transportation and inspection costs of the model.

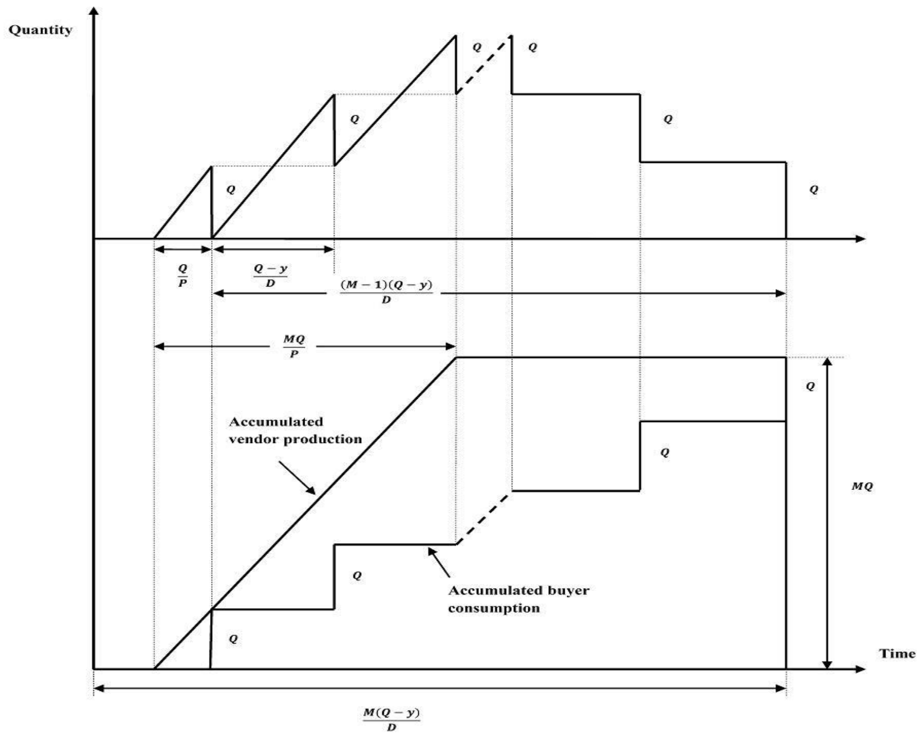

Figure 1. Inventory level for vendor

Buyer's lead time demand (LTD) follows a normal distribution, so that lead time demand, X, has a pdf, $f_x(X)$, with mean DL and standard deviation $\sigma_D\sqrt{L}$. Buyer's inventory is continuously reviewed and a lot-size is ordered whenever the inventory level hits the reorder point. Also, the replenishment rate is infinite. Reorder point, r , is equal to the sum of buyer's expected demand during lead time, DL, and safety stock, $k\sigma_D\sqrt{L}$, where k is safety factor. A partial backorder system is considered for unsatisfied demand and the expected number of backordered per cycle is equal to $(1-\beta)E(x-r)^+$. The backordering rate is dependent on the expected shortage quantity with a negative exponential function, $\beta = ve^{-\vartheta E(x-r)^+}$, wherein, $0 \le v \le 1,\ \vartheta \ge 0$, are backorder parameters. Ordering cost is controllable and capital investment, $I(A) = bln\left(\frac{A_0}{A}\right)$, is a logarithmic function of ordering cost for $A \in (0, A_0]$. under the assumptions that:

1. The lead time L consists of n mutually independent components. The zth component has a normal duration T_z and minimum duration t_z, $z = 1, 2, \ldots, n$.

2. For the zth component of lead time, the crashing cost per unit time c_z, depends on the ordering lot size Q and is described by $c_z = a_z + b_z Q$, where $a_z > 0$ is the fixed cost, and $b_z > 0$ is the unit variable cost, for $z = 1, 2, \ldots, n$.

3. For any two crash cost lines $c_z = a_z + b_z Q$ and $c_j = a_j + b_j Q$, where $a_z > a_j$, $b_z < b_j$, for $z \ne j$ and $z, j = 1,2, \ldots, n$, there is an intersection point Q^S such that $c_z = c_j$. These intersection points are arranged in ascending order so that $Q_0^S < Q_1^S < \cdots < Q_w^S < Q_{w+1}^S$, where $Q_0^S = 0, Q_{w+1}^S = \infty$ and $w \le n(n-1)/2$. For any order quantity range (Q_z^S, Q_{z+1}^S), $c_z s$ are arranged such that $c_1 \le c_2 \le \cdots \le c_n$, and the lead time components are crashed one at a time starting with the component of least c_z, and so on.

4. Let $L_0 \equiv \sum_{j=1}^{n} T_j$ and L_z be the length of lead time with components $1,2, \ldots, z$ crashed to their minimum duration, then L_z can be expressed as $L_z = L_0 - \sum_{j=1}^{z}(T_j - t_z), z = 1,2, \ldots, n$ and the lead time crashing cost per cycle $C(L)$ is given by $C(L) = c_z(L_{z-1} - L) + \sum_{j=1}^{z-1} c_j(T_j - t_j)$, where $L \epsilon [L_z, L_{z-1}]$, and $c_j = a_j + b_j Q$ for $j = 1,2, \ldots, z$.

This investigation considers that the lead time crashing cost is a function of both order quantity and reduced lead time.

3.1. Vendor's expected annual cost

As mentioned before, this paper aims to optimize decision variables of an integrated vendor-buyer inventory system in order to minimize the joint expected annual cost with inflation (*JEACWI*). Therefore, the calculation of vendor's yearly expected inventory cost with inflation is needed. The total inventory for the vendor is a random variable and can be expressed as follows:

$$\left[MQ\left(\frac{Q}{P}+(M-1)\frac{Q-y}{D}\right)-\frac{M^2Q^2}{2P}\right]-\left[\frac{Q(Q-y)}{D}\left(1+2+\cdots+(M-1)\right)\right] = \frac{MQ^2}{P}+\left[M(M-1)\frac{Q(Q-y)}{D}\right]-\frac{M^2Q^2}{2P}-\left[\frac{Q(Q-y)}{2D}M(M-1)\right] \tag{1}$$

Hence, dividing the total inventory per cycle into vendor's cycle time, $\frac{M(Q-y)}{D}$, the average inventory level can be calculated as follows:

$$\left\{\frac{MQ^2}{P}+\left[M(M-1)\frac{Q(Q-y)}{D}\right]-\frac{M^2Q^2}{2P}-\left[\frac{Q(Q-y)}{2D}M(M-1)\right]\right\}\Big/\frac{M(Q-y)}{D} = \frac{Q^2D}{P(Q-y)}-\frac{MQ^2D}{2P(Q-y)}+\frac{(M-1)Q}{2} \tag{2}$$

After taking the expected value, the expected average inventory level for the vendor is obtained as follows:

$$\frac{Q^2D}{P}E\left(\frac{1}{Q-y}\right)-\frac{MQ^2D}{2P}E\left(\frac{1}{Q-y}\right)+\frac{(M-1)Q}{2}=\left[\left(\frac{Q^2D}{P}E\left(\frac{1}{Q-y}\right)\left(1-\frac{M}{2}\right)\right)+\frac{(M-1)Q}{2}\right] \tag{3}$$

The vendor's total holding cost per year in the average annual cost method is dependent on the average inventory level. Thus, the buyer's total holding cost per year with inflation can be computed as follows $\left(T_V=\frac{1}{n_V}=\frac{M(Q-y)}{D}\right)$:

$$\left[\int_0^{n_VT_V} h_V\left[\left(\frac{Q^2D}{P}\left(\frac{1}{Q-y}\right)\left(1-\frac{M}{2}\right)\right)+\frac{(M-1)Q}{2}\right](1+it)dt\right] = h_V\left(1+\frac{i}{2}\right)\left[\left(\frac{Q^2D}{P}\left(\frac{1}{Q-y}\right)\left(1-\frac{M}{2}\right)\right)+\frac{(M-1)Q}{2}\right] \tag{4}$$

Considering that the probabilistic variables, (I,Y), are independent, the expected value of the holding cost per year with inflation (*EHCWI*$_V$) is computed by:

$$EHCWI_V = h_V\left(1+\frac{E(i)}{2}\right)\left[\left(\frac{Q^2D}{P}E\left(\frac{1}{Q-y}\right)\left(1-\frac{M}{2}\right)\right)+\frac{(M-1)Q}{2}\right] \tag{5}$$

According to the computation of the vendor's inventory costs with inflation, this study assumes the set-up cost for the vendor is paid at the beginning of the cycle. So, the total set-up cost can be calculated as follows $\left(T_V=\frac{1}{n_V}=\frac{M(Q-y)}{D}\right)$:

$$[S+S(1+T_Vi)+S(1+2T_Vi)+\cdots+S(1+(n_V-1)T_Vi)] = S\sum_{j=0}^{n_V-1}(1+T_Vij)=S\left[n+\frac{M(Q-y)}{D}i\times\frac{n_V(n_V-1)}{2}\right] \tag{6}$$

Considering that the probabilistic variables, (I,Y), are independent, the expected value of the set-up cost per year for vendor (*ESUCWI*$_V$) is reduced to:

$$ESUCWI_V = S\left[\frac{D}{M}E\left(\frac{1}{Q-y}\right)\left(1+\frac{E(i)}{2}\right)-\frac{E(i)}{2}\right] \tag{7}$$

Also, this study assumes that the production cost is paid at the beginning of the vendor's cycle. Hence, the average production cost per unit is calculated as follows $\left(T_V=\frac{1}{n_V}=\frac{M(Q-y)}{D}\right)$:

$$\frac{\left(C_{pr} + C_{pr}(1 + T_V i) + C_{pr}(1 + 2T_V i) + \cdots + C_{pr}(1 + (n_V - 1)T_V i)\right)}{n_V} = \frac{C_{pr}}{n_V}\sum_{j=0}^{n_V-1}(1 + T_V ij) = C_{pr}\left[1 + \frac{i}{2}\left(1 - \frac{M(Q-y)}{D}\right)\right] \tag{8}$$

Considering that the probabilistic variables are independent, the vendor's expected production cost per year with inflation ($EPCWI_V$) is as follows:

$$EPCWI_V = \frac{C_{pr}D}{1 - E(p)}\left[1 + \frac{E(i)}{2}\left(1 - \frac{MQ\big(1 - E(p)\big)}{D}\right)\right] \tag{9}$$

Then, the expected annual cost for buyer with inflation is the sum of Eqs (5), (7) and (9) as follows:

$$EACWI_V(Q, M) = ESUCWI_V + EHCWI_b + EPCWI_V \tag{10}$$

3.2. Buyer's expected annual cost

In this subsection, the continuous review inventory model for the buyer is considered. The inventory level of an item for the buyer will be diminished due to the random demand and a partial backorder system is assumed for the model. The buyer places an order with the amount of Q and QY items are defective. The shortage quantity per cycle is probabilistic, because X (lead time demand) is probabilistic which is as follows.

$$(x - r)^+ = Max(x - r, 0) = \begin{cases} x - r, & x \geq r \\ 0\ , & x \leq r \end{cases} \tag{11}$$

We assume that the buyer's shortage cost is calculated at the end of cycle. Hence, the average shortage cost per unit with inflation is calculated by ($T_b = \frac{1}{n_b} = \frac{Q-y}{D}$):

$$\begin{aligned}&\Big(\big(\pi + \pi_\circ(1-\beta)\big)(x-r)^+(1+T_b i) + \big(\pi + \pi_\circ(1-\beta)\big)(x-r)^+(1+2T_b i) + \cdots \\ &+ \big(\pi + \pi_\circ(1-\beta)\big)(x-r)^+(1+n_bT_b i)\Big) = \big(\pi + \pi_\circ(1-\beta)\big)(x-r)^+\sum_{j=1}^{n_b}(1+T_b ji) \\ &= \big(\pi + \pi_\circ(1-\beta)\big)(x-r)^+\left[\frac{D}{Q-y}\left(1+\frac{i}{2}\right)+\frac{i}{2}\right]\end{aligned} \tag{12}$$

Considering that the probabilistic variables, (I, X, Y), are independent, ESCWI is transformed as:

$$ESCWI_b = \big(\pi + \pi_\circ(1-\beta)\big)\left[DE\left(\frac{1}{Q-y}\right)\left(1 + \frac{E(i)}{2}\right) + \frac{E(i)}{2}\right]E(x-r)^{`+} \tag{13}$$

This investigation assumes that the buyer's ordering cost will be paid at the time of replenishment. Hence, T_{r_b} is the reorder point time and can be expressed by:

$$T_b^r = \frac{I_{max} - r}{D}\ ,\ I_{max} = Q - y + r - x + (1-\beta)(x-r)^+ \rightarrow T_{r_b} = \frac{Q - y - x + (1-\beta)(x-r)^+}{D} \tag{14}$$

The buyer's total ordering cost per year paid at the time of order placing is computed as follows ($T_b = \frac{1}{n_b} = \frac{Q-y}{D}$):

$$\begin{aligned}&\left[\big(A + C(L)\big)(1 + T_b^r i) + \big(A + C(L)\big)(1 + (T_b^r + T_b)i) + \cdots + \big(A + C(L)\big)(1 + (T_b^r + (n_b - 1)T_b)i)\right] \\ &= \left[\big(A + C(L)\big)\sum_{j=0}^{n_b-1}(1 + T_b^r i + T_b ij)\right] = \left[\big(A + C(L)\big)\left(n_b + n_bT_b^r i + Ti\frac{n_b(n_b-1)}{2}\right)\right]\end{aligned} \tag{15}$$

Considering (I, X, Y) as independent variables, the expected ordering cost per year with inflation for buyer ($EOCWI_b$) can be expressed by:

$$EOCWI_b = \theta b ln\left(\frac{A_0}{A}\right) + [A + C(L)]\left[DE\left(\frac{1}{Q-y}\right)\left(1+\frac{E(i)}{2}\right) - \frac{E(i)}{2}\right] + [A + C(L)]\left[1 - \left((E(x) - (1-\beta)E(x-r)^+)E\left(\frac{1}{Q-y}\right)\right)\right]E(i) \tag{16}$$

where θ is the fractional opportunity cost of capital per year. The buyer's total holding cost with inflation based on the average annual cost method can be obtained as below($n_b = \frac{1}{T_b} = \frac{D}{Q-y}$):

$$\left[\int_0^{n_b T_b} h\left(\frac{Q-y}{2} + r - x + (1-\beta)(x-r)^+\right)(1+it)dt\right] = h\left(1+\frac{i}{2}\right)\left(\frac{Q-y}{2} + r - x + (1-\beta)(x-r)^+\right) \tag{17}$$

Hence, considering (I, X, Y) as independent, the expected value of the holding cost per year with inflation ($EHCWI_b$) for the buyer is computed by:

$$EHCWI_b = h\left(1+\frac{E(i)}{2}\right)\left[\frac{E(Q-y)}{2} + r - E(x) + (1-\beta)E(x-r)^+\right] \tag{18}$$

Similarly, the expected purchasing cost per year with inflation ($EPCWI_b$) is as follows:

$$EPCWI_b = C_{pu}D\left[1 + \frac{E(i)}{2}\left(1 - \frac{Q\big(1-E(p)\big)}{D}\right)\right] \tag{19}$$

So, the expected annual cost for the buyer with inflation is the sum of Eqs (13), (16), (18) and (19) as follows:

$$EACWI_b(Q, A, r, L) = ESCWI_b + EOCWI_b + EHCWI_b + EPCWI_b \tag{20}$$

3.3. Normal lead time demand distribution for buyer

We assume that lead time demand X follows a normal distribution with mean $E(x) = DL$ and standard deviation $\sigma_x = \sigma_D\sqrt{L}$, and having the reorder point $r = DL + k\sigma_D\sqrt{L}$, the expected shortage, $E(x-r)^+$, is as follows:

$$E(x-r)^+ = \int_r^\infty (x-r)f(x)dx \quad , \quad k = \frac{r - E(x)}{\sigma_x} \quad , \quad z = \frac{x - E(x)}{\sigma_x} \tag{21}$$

$$\rightarrow E(x-r)^+ = \sigma_x \int_k^\infty (z-k)f(z)dz$$

$$\rightarrow E(x-r)^+ = \sigma_D\sqrt{L}\left[\int_k^\infty zf(z)dz - k\overline{\Phi(k)}\right], \qquad \left[\int_k^\infty zf(z)dz - k\overline{\Phi(k)}\right] = \left[\phi(k) - k\overline{\Phi(k)}\right] = \psi(k)$$

$$\rightarrow E(x-r)^+ = \sigma_D\sqrt{L}\psi(k) \geq 0 \tag{22}$$

For the above formulation, ϕ denotes the probability density function of standard normal distribution and $\overline{\Phi}$ denotes the complementary cumulative function of standard normal distribution.

Considering $r = DL + k\sigma_D\sqrt{L}$, $E(x-r)^+ = \sigma_D\sqrt{L}\psi(k)$ and $\beta = \nu e^{-\vartheta\sigma_D\sqrt{L}\psi(k)}$, the expected stockout cost of the normal lead time demand distribution is changed as follows:

$$ESCWI_b{}^N = \left[\pi + \pi_\circ\left(1 - \nu e^{-\vartheta\sigma_D\sqrt{L}\psi(k)}\right)\right]\left[DE\left(\frac{1}{Q-y}\right)\left(1+\frac{E(i)}{2}\right) + \frac{E(i)}{2}\right]\sigma_D\sqrt{L}\psi(k) \tag{23}$$

Also, the expected holding cost with inflation of the normal distribution can be written as follows:

$$EHCWI_b{}^N = h\left(1+\frac{E(i)}{2}\right)\left[\frac{E(Q-y)}{2}+k\sigma_D\sqrt{L}+\left(1-\nu e^{-\vartheta\sigma_D\sqrt{L}\psi(k)}\right)\sigma_D\sqrt{L}\psi(k)\right] \quad (24)$$

Besides, the expected ordering cost with inflation is as follows:

$$EOCWI_b{}^N = \theta b ln\left(\frac{A_0}{A}\right)+[A+C(L)]\left[DE\left(\frac{1}{Q-y}\right)\left(1+\frac{E(i)}{2}\right)-\frac{E(i)}{2}\right]$$
$$+[A+C(L)]\left[1-\left(\left(DL-\left(1-\nu e^{-\vartheta\sigma_D\sqrt{L}\psi(k)}\right)\sigma_D\sqrt{L}\psi(k)\right)E\left(\frac{1}{Q-y}\right)\right)\right]E(i) \quad (25)$$

3.4. Joint expected annual cost with inflation

In this subsection, a situation is proposed in which the vendor and the buyer match their production and inventory strategies to specify their optimal strategy for the continuous review inventory supply chain system. Thus, the joint expected annual cost with inflation is obtained, as given below:

$$JEACWI(Q,A,k,L,M) \quad (26)$$
$$= EOCWI_b{}^N + EHCWI_b{}^N + ESCWI_b{}^N + EPCWI_b + EHCWI_V + ESUCW_V + EPCWI_V$$
$$Q \geq 0, k \geq 0, A\epsilon(0,A_0]\ and\ L\ \epsilon\ [L_z, L_{z-1}]$$

3.5. Reducing shortage variance as a second objective

In case of probabilistic demand, as a consequence of high demand uncertainty, it is important to control shortage variation in order to keep the service in higher level. The shortage variation can be reduced with increasing the safety factor and accordingly the reorder point level. Therefore, the paper aims to reduces buyer's shortage variance which is obtained as follows

$$Var(x-r)^+ = E\left[\left(Max(x-r,0)\right)^2\right]-\left[E\left(Max(x-r,0)\right)\right]^2 \quad (27)$$

$$E\left[\left(Max(x-r,0)\right)^2\right] = \int_r^\infty (x-r)^2 f(x)dx\,, k=\frac{r-E(x)}{\sigma_x}\,, z=\frac{x-E(x)}{\sigma_x}\,, P(x\leq r)=p(z\leq k)$$
$$\rightarrow \int_r^\infty (x-r)^2 f(x)dx = \int_k^\infty (z\sigma_x-k\sigma_x)^2 f(z)dz = \sigma_x{}^2\int_k^\infty (z-k)^2 f(z)dz \quad (28)$$

and

$$\left[E\left(Max(x-r,0)\right)\right]^2 = \sigma_x{}^2\left[\int_k^\infty zf(z)dz - k\overline{\Phi(k)}\right]^2 \quad (29)$$

Hence, variance of shortages is as follows:

$$Var(x-r)^+ = \sigma_x{}^2\left\{\int_k^\infty (z-k)^2 f(z)dz-\left[\int_k^\infty zf(z)dz-k\overline{\Phi(k)}\right]^2\right\} \quad (30)$$

$$\zeta(k)=\left\{\int_k^\infty (z-k)^2 f(z)dz-\left[\int_k^\infty zf(z)dz-k\overline{\Phi(k)}\right]^2\right\} \quad (31)$$

$$\rightarrow Var(x-r)^+ = \sigma_D{}^2 L\,\zeta(k) \geq 0 \quad (32)$$

3.6. The multi-objective model and the solution

As mentioned in the previous sections, the paper aims to minimize the joint expected annual cost of vendor and buyer with inflation as a first objective and shortage variance as a second objective as follows:

$$\begin{aligned}
&\min JEACWI(Q,A,k,L,M)\\
&= \theta b ln\left(\frac{A_0}{A}\right)\\
&+\left[A+a_i(L_{i-1}-L)+\sum_{j=1}^{i-1}a_i(T_j-t_j)\right.\\
&\left.+Q\left(b_i(L_{i-1}-L)+\sum_{j=1}^{i-1}b_i(T_j-t_j)\right)\right]\left[DE\left(\frac{1}{Q-y}\right)\left(1+\frac{E(i)}{2}\right)-\frac{E(i)}{2}\right]\\
&+\left[A+a_i(L_{i-1}-L)+\sum_{j=1}^{i-1}a_i(T_j-t_j)+Q\left(b_i(L_{i-1}-L)+\sum_{j=1}^{i-1}b_i(T_j-t_j)\right)\right]\left[1\right.\\
&\left.-\left(\left(DL-(1-ve^{-\vartheta\sigma_D\sqrt{L}\psi(k)})\sigma_D\sqrt{L}\psi(k)\right)E\left(\frac{1}{Q-y}\right)\right)\right]E(i)\\
&+h\left(1+\frac{E(i)}{2}\right)\left[\frac{E(Q-y)}{2}+k\sigma_D\sqrt{L}+(1-ve^{-\vartheta\sigma_D\sqrt{L}\psi(k)})\sigma_D\sqrt{L}\psi(k)\right]\\
&+\left[\pi+\pi_\circ(1-ve^{-\vartheta\sigma_D\sqrt{L}\psi(k)})\right]\left[DE\left(\frac{1}{Q-y}\right)\left(1+\frac{E(i)}{2}\right)+\frac{E(i)}{2}\right]\sigma_D\sqrt{L}\psi(k)\\
&+C_{pu}D\left[1+\frac{E(i)}{2}\left(1-\frac{E(Q-y)}{D}\right)\right]+\frac{C_{pr}D}{1-E(p)}\left[1+\frac{E(i)}{2}\left(1-\frac{ME(Q-y)}{D}\right)\right]\\
&+h_V\left(1+\frac{E(i)}{2}\right)\left[\left(\frac{Q^2D}{P}E\left(\frac{1}{Q-y}\right)\left(1-\frac{M}{2}\right)\right)+\frac{(M-1)Q}{2}\right]+S\left[\frac{D}{M}E\left(\frac{1}{Q-y}\right)\left(1+\frac{E(i)}{2}\right)-\frac{E(i)}{2}\right]\\
&\min Var(x-r)^+=\sigma_D{}^2L\,\zeta(k)
\end{aligned}$$

$$Q\geq 0, k\geq 0, A\epsilon(0,A_0]\ and\ L\ \epsilon\ [L_z,L_{z-1}] \tag{33}$$

Where

$$\psi(k)=[\phi(k)-k\overline{\Phi(k)}]$$

$$E\left(\frac{1}{Q-y}\right)\cong\frac{1}{Q(1-E(p))}+\frac{QE(p(1-p))+Q^2Var(p)}{\left(Q(1-E(p))\right)^3}$$

$$E\left(\frac{Q-y}{2}\right)=\frac{Q(1-E(p))}{2}$$

In order to solve the proposed multi-objective model in this paper, we present an algorithm where in the first step, the second objective (minimizing the variance of shortage) is eliminated. During this step, we obtain the minimum values of model (33) numerically because its partial derivatives are very complicated functions. It is noted that for any given (Q,A,k), $JEACWI(Q,A,k,L)$ is concave in $L\in[L_z,L_{z-1}]$, because

$$\frac{\partial^2 JEACWI(Q,A,k,L)}{\partial L^2}\leq 0 \tag{34}$$

Therefore, for fixed (Q,A,k), the minimum expected cost is at the end of interval$L\in[L_z,L_{z-1}]$. Thus, for fixed $L\in[L_z,L_{z-1}]$, the minimum value of $JEACWI(Q,A,k,L,M)$ will occur at the point (Q,A,k) which satisfies $\frac{\partial JEACWI(Q,A,k,L)}{\partial Q}=0$, $\frac{\partial JEACWI(Q,A,k,L)}{\partial k}=0$ and $\frac{\partial JEACWI(Q,A,k,L)}{\partial A}=0$ simultanuously.

Therefore, we can use the proposed algorithm for finding the optimal (Q^*,A^*,r^*,L^*,M^*).

Step 1. Set $M=1$.

Step 2. Compute the intersection points Q^s of the crash cost lines $c_z=a_z+b_zQ$ and $c_j=a_j+b_jQ$, for all z and j, where $a_z>a_j$, $b_z<b_j$, $z\neq j$ and $z,j=1,2,\dots,n$. Arrange these intersection points such that $Q_1{}^s<Q_2{}^s<\cdots<Q_w{}^s$ and let $Q_0{}^s=0$, $Q_{w+1}{}^s=\infty$

Step 3. For each order quantity range $(Q_{j-1}{}^s,Q_j{}^s)$, $j=1,2,\dots,w$, rearranged c_z such that $c_1\leq c_2\leq\cdots\leq c_n$, do:

Step 3.1. For each L_z, $z=0,1,2,\dots,n$, perform step 3-1-1 to step 3-1-3.

Step 3.1.1. Input the values of $A, S, D, h_b, h_V, k, \pi, \pi_0, C_{pu}, C_{pr}, i, , \theta, b, \nu, \vartheta$ and P.

Step 3.1.2. Compute $JEACWI(Q^k, A^k)$ in terms of different k and $\psi(k)$ and find $JEACWI(Q^{k_z}, A^{k_z}) = JEACWI(Q_z, A_z, k_z)$ as follows:

$$JEACWI(Q^{k_z}, A^{k_z}) < Min\{JEACWI(Q^{k_z-\xi}, A^{k_z-\xi}), JEACWI(Q^{k_z+\xi}, A^{k_z+\xi})\}$$

$$|JEACWI(Q^{k_z}, A^{k_z}) - JEACWI(Q^{k_z-\xi}, A^{k_z-\xi})| \leq \varepsilon$$

$$|JEACWI(Q^{k_z}, A^{k_z}) - JEACWI(Q^{k_z+\xi}, A^{k_z+\xi})| \leq \varepsilon$$

Step 3.1.3. If $Q_z \leq {Q_{j-1}}^s$, let $Q_z = {Q_{j-1}}^s$ and if ${Q_j}^s \leq Q_z$ let ${Q_j}^s = Q_z$. Using Q_z as a constant, compute $JEACWI\ (A_z, k_z)$.

Step 3.2. Put $MIN_{z=0,1,\ldots,n} JEACWI(Q_z, A_z, k_z, L_z, M) = JEACWI({Q_j}^*, {A_j}^*, {k_j}^*, {L_j}^*, M)$ for the order quantity range $({Q_{j-1}}^s, {Q_j}^s)$.

Step 4. Set $MIN_{j=1,2,..,w}\ JEACWI({Q_j}^*, {A_j}^*, {k_j}^*, {L_j}^* M) = JEACWI({Q_M}^*, {A_M}^*, {k_M}^*, {L_M}^*)$

Step 5. Set $M = M + 1$, perform Step 2 to 4 to find $JEACWI({Q_{M+1}}^*, {A_{M+1}}^*, {k_{M+1}}^*, {L_{M+1}}^*)$.

Step 6. If $({Q_{M+1}}^*, {A_{M+1}}^*, {k_{M+1}}^*, {L_{M+1}}^*) \geq JEACWI({Q_M}^*, {A_M}^*, {k_M}^*, {L_M}^*)$, then go to step 7, otherwise go to step 5.

Step 7. Set $JEACWI({Q_M}^*, {A_M}^*, {k_M}^*, {L_{M+1}}^*) = JEACWI(Q^*, A^*, k^*, L^*, M^*)$. And hence the optimal reorder point is $r^* = \mu_D L^* + k^* \sigma_D \sqrt{L^*}$ and the optimal backorder rate is $\beta^* = \nu e^{-\vartheta \sigma_D \sqrt{L^*} \psi(k^*)}$.

Also, $JEACWI(Q, A, k, L, M)$ is convex in M, for fixed Q, A, k and L, because $\frac{\partial^2 JEACWI(Q,A,k,L)}{\partial M^2} \geq 0$.

Moreover, to ensure the convexity of the disscussed model, the optimal values of (Q^*, A^*, k^*, L^*) must satisfy the following sufficient conditions.
For a given value of $L \in [L_z, L_{z-1}]$, we obtain the Hessian matrix H for the objective function as follows:

$$H = \begin{bmatrix} \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q^2} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q \partial A} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q \partial k} \\ \frac{\partial^2 JEACWI(Q,A,k)}{\partial A \partial Q} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial A^2} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial A \partial k} \\ \frac{\partial^2 JEACWI(Q,A,k)}{\partial k \partial Q} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial k \partial A} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial k^2} \end{bmatrix}$$

The first, second, and third principal minors of *H* must be positive for the optimal values (Q^*, A^*, k^*) as follows:

$$|H_{11}| = \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q^2} \geq 0$$

$$|H_{22}| = \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q^2} \times \frac{\partial^2 JEACWI(Q,A,k)}{\partial A^2} - \left(\frac{\partial^2 JEACWI(Q,A,k)}{\partial A \partial Q}\right)^2 \geq 0$$

And

$$|H_{33}| = \begin{vmatrix} \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q^2} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q \partial A} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial Q \partial k} \\ \frac{\partial^2 JEACWI(Q,A,k)}{\partial A \partial Q} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial A^2} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial A \partial k} \\ \frac{\partial^2 JEACWI(Q,A,k)}{\partial k \partial Q} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial k \partial A} & \frac{\partial^2 JEACWI(Q,A,k)}{\partial k^2} \end{vmatrix} \geq 0$$

The above mentioned algorithm helps to find minimum values of the joint expected annual cost with inflation (*JEACWI*) as a first objective. However, to minimize the variance of shortage as a second goal of the corresponding inventory system, a heuristic method is proposed that reduces the shortage standard deviation in a distance of first objective's optimum point. As can be seen in the following, the standard deviation of shortage is dependent on the reorder point. The shortage standard deviation, therefore, can be reduced by increasing the reorder point in a distance (η) of optimum point of the first objective which is given below.

$$\min Var(x-r)^{+} = {\sigma_D}^2 L\, \zeta(k)$$

Subject to:

$$JEACWI \approx JEACWI^{*} + \eta$$

5. The numerical example

To illustrate the behavior of the model developed in this paper, we consider an inventory problem with the following data:
$D = 600\, unit/year$, $A_0 = \$200$, $h_b = \$20, \sigma_D = 7\, unit/week$
$\pi = \$50$, $\pi_0 = \$150$ $\nu = 1, \vartheta = 5$
$h_V = \$15, S = \$1000, b = 2800, \theta = 0.2, P = 2000\, unit/year$

Also, we consider $\alpha = 1$ and $\beta = 4$ for defective rate which follows the Beta distribution and the p.d.f of p is given by $g(p) = 4(1-p)^3$, $0 < p < 1$
Therefore, we have:
$$E(p) = \frac{\alpha}{\alpha+\beta} = 0.2\, , E(p^2) = \frac{\alpha(\alpha+\beta)}{(\alpha+\beta)(\alpha+\beta+1)} = 0.066\ and\ Var(p) = 0.0266$$
Besides, we take $C_{pr} = \$40$, $C_{pu} = \$60$.

Table 1. Lead time data

Lead time component z	1	2	3
Normal duration T_z (days)	20	20	16
Minimum duration t_z (days)	6	6	9
Unit fixed crash cost a_z (\$/day)	0.5	1.3	5.1
Unit variable crash cost b_z (\$/unit/day)	0.012	0.004	0.0012

Table 2. The values of Q^s, order quantity ranges and crash sequence

Inspection points (Q^s)	Order quantity range	Crash sequence of components
100	(0, 100]	1, 2, 3
426	(100, 426]	2, 1, 3
1357	(426, 1357]	2, 3, 1
---	(1357, ∞)	3, 2, 1

The data of lead time's components are listed in Table 1. The data are first used to evaluate the intersection points, order quantity range interval, and component crash priorities in each interval. Table 2 shows the crash sequence corresponding to each order quantity range. The inflation rate has a normal distribution function. The results of using the first step of the algorithm are shown in Table (3) for $E(i) = 0.00$, 0.04, 0.08, 0.12 and 0.16. Moreover, the summary of results are given in Table (4), and to see the effects of set-up cost reduction, we provide the optimal results of the fixed set-up cost model in the same table. The associated results reveal that the larger inflation rate, the larger order quantity (see Fig. 2). Also, from Table (4) and Fig. (3), it is observed that the cost performance of the fixed ordering cost model is improving as the inflation rate increases. Table 4 shows that the optimal number of shipments is decreased when the inflation rate grows. Besides, according to the solution procedure, the optimal JEACWI values obtained in Table 3 are tabulated in Table 5 for various safety factor (k) quantities. The outcomes show that JEACWI is convex in terms of various safety factor amounts (see Fig 4 for i=0.08, L=8, M=4). In addition, for instance, the optimal values obtained for the variable ordering cost model in Table 4 are tabulated for different lead times and respective crashing costs and order quantity ranges ($C_L(0{,}100], C_L(100{,}426]$) in Table 6. Considering $\xi = 10\$$ in the second step of the solution procedure, the effective solutions are tabulated in Table 7 for the variable ordering cost model. Moreover, performance of the multi-objective inventory model for variance of shortages is shown in Fig 5 where a larger amount of safety factor results in a smaller amount of variance of shortages.

This article assumes the quadratic approximation in order to show the influence of defective item in analyzing the first objective function. To illustrate that this obligation is satisfied in evaluating the first objective function, the values of $E\left(\frac{1}{Q-y}\right)$ and $E(Q-y)$ are given in Tables 3 and 8. Results reveal that using the corresponding approximation, effect of the standard deviation of the defective item on its mean is negligible and the computed expected values are precise, and the expected additional joint expected annual cost with inflation for considering defective units as a probabilistic variable is justifiable. For instance, in Table 3, for $E(i)$=0.00, M=4, the values are obtained as follows

$$E\left(\frac{1}{Q-y}\right) = 0.0125, E(Q-y) = Q - E(y) = 82.85 \; and \; E(y) = QE(p)$$

We know that:

$$\frac{Var(y)}{\left(E(Q-y)\right)^3} \cong E\left(\frac{1}{Q-y}\right) - \frac{1}{E(Q-y)} = 0.0125 - 0.0121 = 0.0004$$

Table 3. Results of first step of solution procedure (Lead time in weeks)

$E(i)$	M	(Q, A, L, k, r)	$E(Q-y)$, $E\left(\frac{1}{Q-y}\right)$	$EOCWI_b, EHCWI_b$, $ESCWI_b, EPCWI_b$	$ESUCWI_V, EHCWI_V$, $EPCWI_V$	$JEACWI$
0.00	1	299.61, 200.00, 6, 1.81, 109.03	239.69, 0.0043	612.51, 3020.93, 96.13, 36000	2606.66, 877.48, 30000	73213.73
	2	181.77, 130.28, 8, 1.98, 143.20	145.42, 0.0071	800.01, 2240.35, 105.33, 36000	2149.12, 1363.34, 30000	72658.17
	3	131.64, 94.31, 8, 2.08, 145.18	105.31, 0.0098	980.93, 1878.15, 99.42, 36000	1979.16, 1588.83, 30000	72526.51
	4	103.56, 74.16, 8, 2.14, 146.36	82.85, 0.0125	1115.53, 1676.91, 99.87, 36000	1887.68, 1722.80, 30000	72502.80*
	5	85.43, 61.15, 8, 2.19, 147.35	68.34, 0.0152	1223.52, 1551.45, 99.18, 36000	1831.35, 1811.14, 30000	72516.66
0.04	1	326.82, 200.00, 6, 1.78, 108.52	261.46, 0.0039	578.21, 3293.34, 100.28, 36406.24	2417.26, 976.28, 30338.53	74110.17
	2	202.78, 142.50, 6, 1.94, 111.26	162.22, 0.0064	865.93, 2335.31, 91.78, 36525.32	1944.80, 1551.30, 30275.54	73590.02
	3	145.51, 102.54, 8, 2.05, 144.58	116.41, 0.0089	934.09, 2016.97, 103.39, 36580.30	1806.08, 1791.40, 30250.75	73483.00
	4	114.53, 80.74, 8, 2.12, 145.97	91.62, 0.0113	1067.92, 1792.01, 99.96, 36610.04	1720.65, 1943.57, 30233.48	73467.64*
	5	94.57, 66.68, 8, 2.17, 146.96	75.66, 0.0137	1175.08, 1649.08, 99.23, 36629.20	1667.03, 2045.23, 30221.69	73486.56
0.08	1	361.39, 200.00, 6, 1.75, 108.00	289.11, 0.0036	540.01, 3635.28, 103.08, 36746.11	2207.21, 1100.64, 30621.76	74954.12
	2	227.26, 156.46, 6, 1.91, 110.74	181.81, 0.0057	808.14, 2574.36, 93.73, 37003.64	1747.34, 1772.67, 30472.74	74472.66
	3	163.71, 113.41, 8, 2.02, 143.99	130.97, 0.0079	877.68, 2195.82, 105.51, 37125.66	1614.64, 2055.06, 30414.15	74388.55
	4	129.11, 89.56, 8, 2.09, 145.37	103.29, 0.0100	1009.86, 1936.28, 102.04, 37192.09	1534.64, 2234.11, 30373.64	74382.09*
	5	106.76, 74.11, 8, 2.14, 146.36	85.41, 0.0122	1115.88, 1770.60, 101.27, 37235.01	1483.40, 2354.27, 30345.89	74406.35
0.12	1	407.79, 200.00 6, 1.71, 107.32	326.23, 0.0031	496.51, 4084.77, 107.12, 36985.55	1969.71, 1265.75, 30821.29	75730.72
	2	261.51, 176.13, 6, 1.86, 109.89	209.21, 0.0049	735.58, 2896.65, 100.03, 37406.83	1522.95, 2079.04, 30544.73	75285.84
	3	192.61, 130.34, 6 1.96, 111.60	154.09, 0.0067	924.28, 2347.78, 94.07, 37605.25	1373.16, 2464.44, 30413.14	75222.15*
	4	150.11, 102.36, 8, 2.03, 144.19	120.08, 0.0016	935.05, 2126.77, 113.25, 37727.68	1319.66, 2647.48, 30358.94	75228.85
0.16	1	474.89,200.00, 6, 1.66, 106.46	379.91, 0.0027	445.92, 4723.99, 111.11, 37056.41	1695.69, 1501.71, 30880.34	76415.22
	2	312.40, 200.00, 6, 1.80, 108.86	249.92, 0.0041	642.96, 3369.59, 106.07, 37680.34	1269.91, 2530.51, 30400.58	75999.98
	3	232.82, 154.10, 6, 1.90, 110.57	186.25, 0.0055	819.48, 2717.79, 99.57, 37985.95	1127.82, 3035.17, 30164.89	75950.71*
	4	186.28, 123.82, 6, 1.97, 111.77	149.02, 0.0069	957.99, 2340.96, 95.83, 38164.66	1052.43, 3347.77, 30015.56	75975.24

Table 4. Summary of optimal solution for first step of solution procedure (Lead time in weeks)

Variable ordering cost model									Fixed ordering cost model (A=200)						
$E(i)$	Q^*	$r^*(k^*)$	A^*	L^*	m^*	β^*	$\zeta(k), Var(x-r)^+$	$JEACWI^*$	$E(\delta)$	Q^*	$r^*(k^*)$	L^*	m^*	β^*	$JEACWI^*$
0.00	103.56	146.36(2.14)	74.16	8	4	0.56	0.00375766, 1.47	72502.80	0.00	146.71	144.58(2.05)	8	3	0.47	72698.33
0.04	114.53	145.97(2.12)	80.74	8	4	0.54	0.00399181, 1.56	73467.64	0.04	160.59	143.99(2.02)	8	3	0.45	73615.01
0.08	129.11	145.37(2.09)	89.56	8	4	0.51	0.00436791, 1.71	74382.09	0.08	181.24	111.77(1.97)	6	3	0.45	74480.68
0.12	192.61	111.60(1.96)	130.34	6	3	0.44	0.00639634, 1.88	75222.15	0.12	205.98	111.09(1.93)	6	3	0.41	75271.89
0.16	232.82	110.57(1.90)	154.10	6	3	0.38	0.00759109, 2.23	75950.71	0.16	244.61	110.23(1.88)	6	3	0.36	75968.18

Table 5. Optimal JEACWI values in terms of different safety factor (k)

k	$\psi(k)$	$EACWI^{i=0.00}_{L=8,M=}$	k	$EACWI^{i=0.04}_{L=8,M=4}$	k	$EACWI^{i=0.08}_{L=8,M=4}$	k	$EACWI^{i=0.12}_{L=6,M=3}$	k	$EACWI^{i=0.16}_{L=6,M=3}$
0.00	0.398994	78655.44	0.00	79140.02	0.00	79512.02	0.00	78615.47	0.00	78818.88
0.40	0.230439	76462.97	0.40	77099.15	0.40	77642.92	0.40	77280.32	0.40	77658.98
0.80	0.120207	74721.99	0.80	75485.98	0.80	76174.96	0.80	76282.97	0.80	76803.12
1.20	0.056102	73520.06	1.20	74379.70	1.20	75177.01	1.20	75644.26	1.20	76272.80
1.40	0.036668	73112.56	1.40	74007.21	1.40	74844.62	1.40	75444.38	1.40	76111.41
1.60	0.023242	72808.84	1.60	73732.54	1.60	74602.14	1.60	75308.19	1.60	76005.63
1.80	0.014276	72614.77	1.80	73559.39	1.80	74452.67	1.70	75265.29	1.70	75974.51
2.00	0.008491	72519.35	2.00	73478.83	2.00	74387.87	1.90	75223.96	1.80	75956.46
2.10	0.006468	72504.20	2.08	73468.65	2.05	74382.95	1.92	75222.87	1.86	75951.57
2.11	0.006290	72503.57	2.09	73468.17	2.06	74382.51	1.93	75222.51	1.87	75951.19
2.12	0.006119	72503.19	2.10	73467.85	2.07	74382.23	1.94	75222.27	1.88	75950.91
2.13	0.005951	72502.91	2.11	73467.66	2.08	74382.10	1.95	75222.16	1.89	75950.75
2.14	**0.005787**	**72502.80***	**2.12**	**73467.64***	**2.09**	**74382.09***	**1.96**	**75222.15***	**1.90**	**75950.71***
2.15	0.005628	72502.82	2.13	73467.74	2.10	74382.23	1.97	75222.47	1.91	75950.77
2.16	0.005472	72502.97	2.14	73468.01	2.11	74382.46	1.98	75222.47	1.92	75950.94
2.17	0.005320	72503.37	2.15	73468.39	2.12	74382.91	1.99	75222.80	1.93	75951.21
2.20	0.004887	72505.24	2.20	73472.47	2.20	74390.81	2.00	75223.24	2.00	75956.87
2.40	0.002720	72539.03	2.40	73511.14	2.40	74434.80	2.10	75233.55	2.10	75970.39
2.60	0.001513	72599.22	2.60	73574.30	2.60	74501.12	2.20	75250.21	2.20	75991.22
2.80	0.000810	72669.68	2.80	73646.98	2.80	74576.11	2.40	75301.28	2.40	76046.78
3.00	0.000382	72744.40	3.00	73723.62	3.00	74654.29	3.00	75504.01	3.00	76255.12
3.50	0.000058	72939.43	3.50	73923.77	3.50	74858.69	3.50	75685.31	3.50	76349.60
4.00	0.000007	73137.58	4.00	74124.86	4.00	75063.24	4.00	75866.87	4.00	76624.17

Table 7. Results of second step of solution procedure (Lead time in weeks)

Variable ordering cost model								
$E(i)$	Q^F	$r^F(k^F)$	A^F	L^F	m^F	β^F	$\zeta(k), Var(x-r)^+$	$JEACWI^F$
0.00	102.63	148.94(2.27)	73.49	8	4	0.67	0.00251610, 0.98	75512.96
0.04	113.42	148.54(2.25)	79.96	8	4	0.65	0.00267871, 1.05	73478.10
0.08	127.87	147.75(2.21)	88.70	8	4	0.62	0.00303310, 1.18	74392.97
0.12	190.64	114.00(2.10)	129.02	6	3	0.57	0.00423912, 1.24	75232.90
0.16	230.00	112.97(2.04)	152.28	6	3	0.52	0.00506663, 1.48	75960.62

Table 6. Optimal values in terms of different lead time and respective crashing cost and order quantity range (Lead time in weeks)

	L_z	8		6		4		3	
$E(i)$	Range	(0,100]C_L	(100,426]C_L	(0,100]C_L	(100,426]C_L	(0,100]C_L	C_L (100,426]	(0,100]C_L	C_L (100,426]
0.00	Q^* $r^*(k^*)$ A^* m^* $JEACWI^*$	100 146.56(2.15) 71.60 4 72505.47	103.56 146.36(2.14) 74.16 4 **72502.80***	100 114.52(2.13) 71.60 4 72558.86	106.38 114.35(2.12) 76.19 4 72550.04	100 81.54(2.11) 71.60 4 72588.68	107.25 80.26(2.09) 76.81 4 72577.34	100 64.33(2.09) 71.60 4 72781.98	112.80 64.09(2.07) 80.79 4 72745.98
0.04	Q^* $r^*(k^*)$ A^* m^* $JEACWI^*$	100 146.56(2.15) 70.50 4 73505.81	114.53 145.97(2.12) 80.74 4 **73467.64***	100 114.52(2.13) 70.38 4 73560.71	117.66 113.83(2.09) 82.80 4 73504.41	100 81.54(2.11) 70.26 4 73592.18	118.47 80.98(2.07) 83.23 4 73529.89	100 64.33(2.09) 70.20 4 73790.12	124.57 63.73(2.04) 87.44 4 73679.11
0.08	Q^* $r^*(k^*)$ A^* m^* $JEACWI^*$	100 146.56(2.15) 69.43 4 74506.02	129.11 145.37(2.09) 89.56 4 **74382.09***	100 114.52(2.13) 69.20 4 74562.39	132.51 113.32(2.06) 91.60 4 74407.45	100 81.54(2.11) 68.97 4 74595.49	133.34 80.56(2.04) 91.86 4 74431.03	100 64.33(2.09) 68.85 4 74798.04	140.05 63.36(2.01) 96.30 4 74558.66
0.12	Q^* $r^*(k^*)$ A^* m^* $JEACWI^*$	100 146.56(2.15) 68.40 3 75954.22	189.52 143.00(1.97) 128.91 3 75225.99	100 114.52(2.13) 68.06 3 76010.58	192.26 111.60(1.96) 130.34 3 **75222.15***	100 81.54(2.11) 67.72 3 76045.28	193.16 79.16(1.94) 130.07 3 75241.20	100 64.33(2.09) 67.56 3 76252.42	200.31 62.27(1.92) 134.49 3 75309.21
0.16	Q^* $r^*(k^*)$ A^* m^* $JEACWI^*$	100 146.56(2.15) 67.39 3 77038.11	229.22 142.01(1.92) 152.75 3 75964.47	100 114.52(2.13) 66.95 3 77095.85	232.82 110.57(1.90) 154.10 3 **75950.71***	100 81.54(2.11) 66.52 3 77132.11	233.10 78.32(1.88) 153.31 3 75968.99	100 64.33(2.09) 66.31 3 77343.84	241.27 61.55(1.86) 158.06 3 76016.10

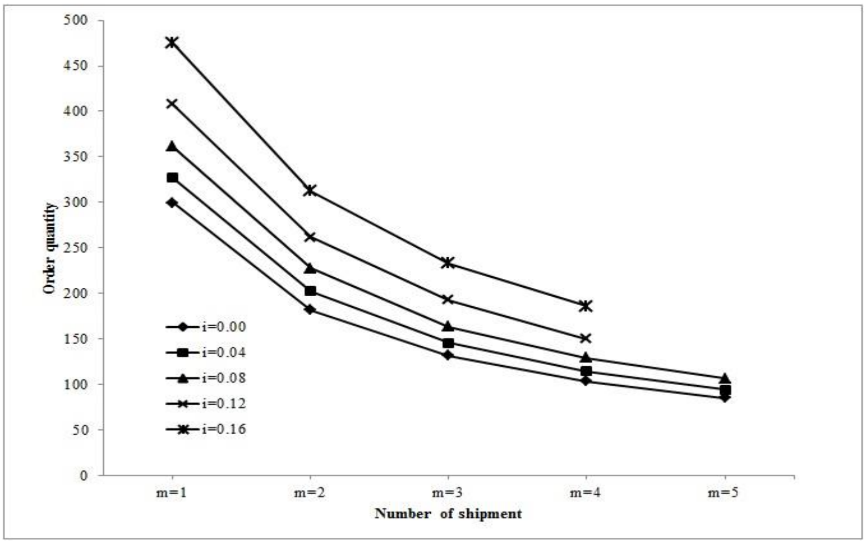

Figure 2. Optimal order quantity in terms of various inflation rates

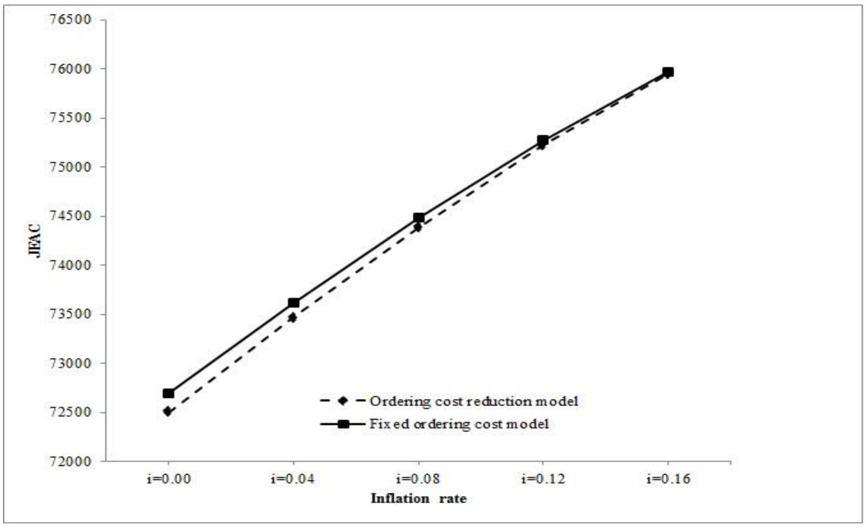

Figure 3. Effect of changes in inflation rates for ordering cost reduction and fixed ordering cost model

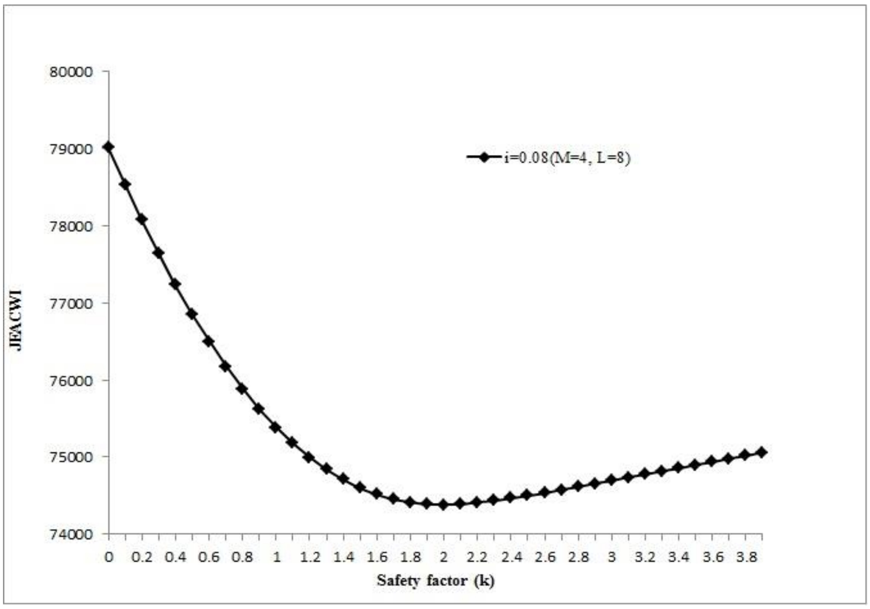

Figure 4. JEACWI values in terms of different safety factors (k)

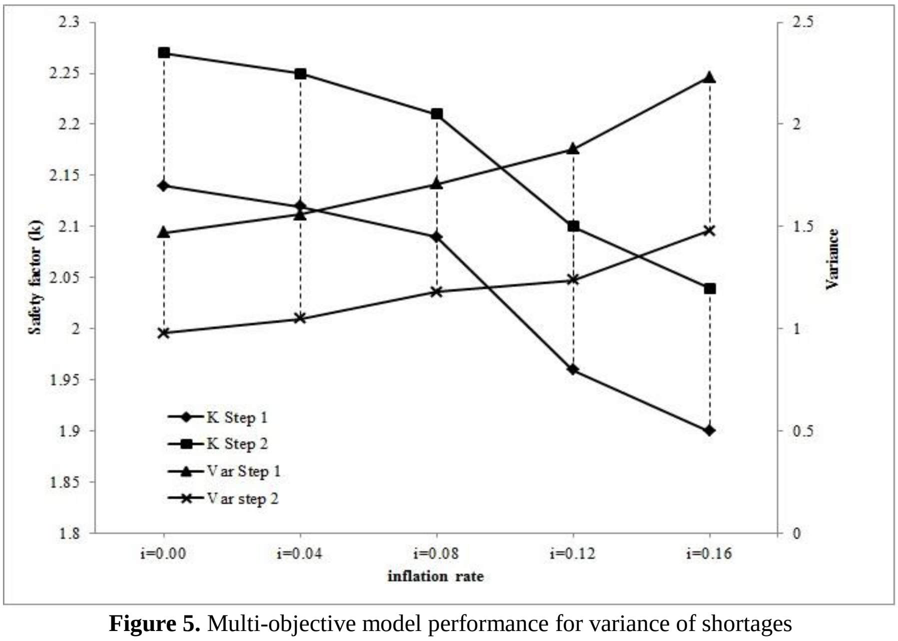

Figure 5. Multi-objective model performance for variance of shortages

In order to study how various parameters affect the optimal solution of the proposed inventory model, a sensitivity analysis is performed. Holding all other parameters fixed and varying a single parameter at a time, we analyze the results. Results of the sensitivity analysis are shown in Table 8. The following interpretations are obtained from it:

Table 8. Effect of changes in various parameters (Lead time in weeks)

M	$(Q^*, A^*, L^*, k^*, r^*, \beta^*, M^*)$	$E(Q-y)$, $E\left(\frac{1}{Q-y}\right)$	$EOCWI_b, EHCWI_b$, $ESCWI_b, EPCWI_b$	$ESUCWI_V, EHCWI_V$, $EPCWI_V$	$JEACWI^*$
ϑ					
0	148.08, 104.35, 8, 1.44, 132.51, 1.00,3	118.47,0.0087	924.30, 1790.01, 179.47, 36577.83	1774.33, 1823.06, 30244.58	73313.61
1	114.89, 80.99, 8, 1.92, 142.01, 0.81,4	91.91, 0.0113	1066.17, 1713.83, 112.99, 36609.69	1715.19, 1949.67, 30232.33	73399.90
5	114.53, 80.76, 8, 2.12, 145.97, 0.54,4	91.62, 0.0113	1067.92, 1792.01, 99.96, 36610.04	1720.65, 1943.57, 30233.48	73467.64
20	114.44, 80.68, 8, 2.26, 148.74, 0.22,4	91.55, 0.0113	1068.36, 1848.02, 96.67, 36610.13	1722.03, 1942.03, 30233.77	73521.03
200	114.37, 80.63, 8, 2.33, 150.13,0.00,4	1.50, 0.0113	1068.70, 1940.85, 94.14, 36610.19	1723.09, 1940.85, 30233.99	73546.75
∞	114.37, 80.63, 8, 2.33, 150.13,0.00,4	991.50,0.0113	1068.70, 1940.85, 94.14, 36610.19	1723.09, 1940.85, 30233.99	73546.75
σ_D					
3	112.94, 79.62, 8, 2.04, 121.31, 0.72,4	90.35, 0.0115	1075.73, 1275.13, 41.88, 36611.57	1745.19, 1916.57, 30238.56	72904.67
5	113.72, 80.17, 8, 2.09, 133.55, 0.62,4	90.97, 0.0114	1071.91, 1531.65, 70.27, 36610.82	1733.14, 1929.73, 30236.08	73183.64
7	114.53, 80.76, 8, 2.12, 145.97, 0.54,4	91.62, 0.0113	1067.92, 1792.01, 99.96, 36610.04	1720.65, 1943.57, 30233.48	73467.64
9	149.53, 105.01, 4, 2.05, 88.90, 0.51,3	119.63,0.0087	1233.77, 1974.31, 87.92, 36576.44	1756.91, 1840.91, 30241.10	73711.40
12	150.65, 105.79, 4, 2.07, 101.68, 0.43,3	120.52,0.0086	1228.64, 2244.74, 121.26, 36575.37	1743.76, 1854.62, 30238.43	74006.86
b					
2000	90.81, 45.73, 8, 2.19, 147.35, 0.60,5	72.65, 0.0143	990.15, 1626.40, 95.37, 36632.81	1737.03, 1936.84, 30236.73	73282.36
2500	112.82, 71.02, 8, 2.13, 146.17, 0.55,4	90.26, 0.0115	1017.67, 1782.06, 97.53, 36611.68	1747.04, 1914.57, 30238.94	73409.51
2800	114.53, 80.76, 8, 2.12, 145.97, 0.54,4	91.62, 0.0113	1067.92, 1792.01, 99.96, 36610.04	1720.65, 1943.57, 30233.48	73467.64
4000	153.69, 154.70, 8, 2.04, 144.38, 0.47,3	122.95,0.0084	1005.43, 2079.74, 101.73, 36572.45	1708.78, 1892.12, 30231.12	73591.39
5000	160.59, 200.00, 8, 2.02, 143.99, 0.45,3	128.47,0.0081	989.93, 2128.09, 105.08, 36565.83	1634.45, 1977.04, 30214.57	73615.01
h_V					
7	107.13, 72.52, 8, 2.15, 146.56, 0.57,7	87.70, 0.0121	1105.32, 1743.53, 94.89, 36617.15	1043.55, 1547.09, 30000.07	72151.62
10	115.55, 81.46, 8, 2.13, 146.17, 0.55,5	92.44, 0.0112	1062.98, 1804.28, 95.23, 36609.06	1360.26, 1666.15, 31137.78	72735.78
15	114.53, 80.76, 8, 2.12, 145.97, 0.54,4	91.62, 0.0113	1067.92, 1792.01, 99.96, 36610.04	1720.65, 1943.57, 30233.48	73467.64
20	128.60, 90.64, 8, 2.09, 145.37, 0.51,3	102.88,0.0101	1003.14, 1894.84, 100.17, 36596.54	2046.62, 2110.81, 30291.35	74043.49
26	164.14, 115.63, 8, 2.01, 143.79, 0.44,2	131.31,0.0079	866.81, 2153.11, 106.78, 36562.42	2407.92, 2176.53, 30337.37	74610.96
S					
500	109.59, 77.24, 8, 2.13, 146.17, 0.55,3	87.65, 0.0118	1092.73, 1755.44, 100.43, 36614.81	1203.16, 1348.68, 30337.03	72452.32
700	125.31, 88.33, 8, 2.10, 145.57, 0.52,3	100.25,0.0103	1017.60, 1872.03, 98.84, 36599.69	1470.60, 1542.67, 30299.23	72900.69
1000	114.53, 80.76, 8, 2.12, 145.97, 0.54,4	91.62, 0.0113	1067.92, 1792.01, 99.96, 36610.04	1420.65, 1943.57, 30233.48	73467.64
1300	128.04, 90.25, 8, 2.09, 145.37, 0.51,4	102.43,0.0101	1005.56, 1890.32, 100.60, 36597.07	1997.69, 2172.98, 30190.24	73954.49
1700	118.58, 83.59, 8, 2.12, 145.97, 0.54,5	94.86, 0.0109	1048.52, 1825.00, 96.56, 36606.16	2252.43, 2564.76, 30125.67	74519.12

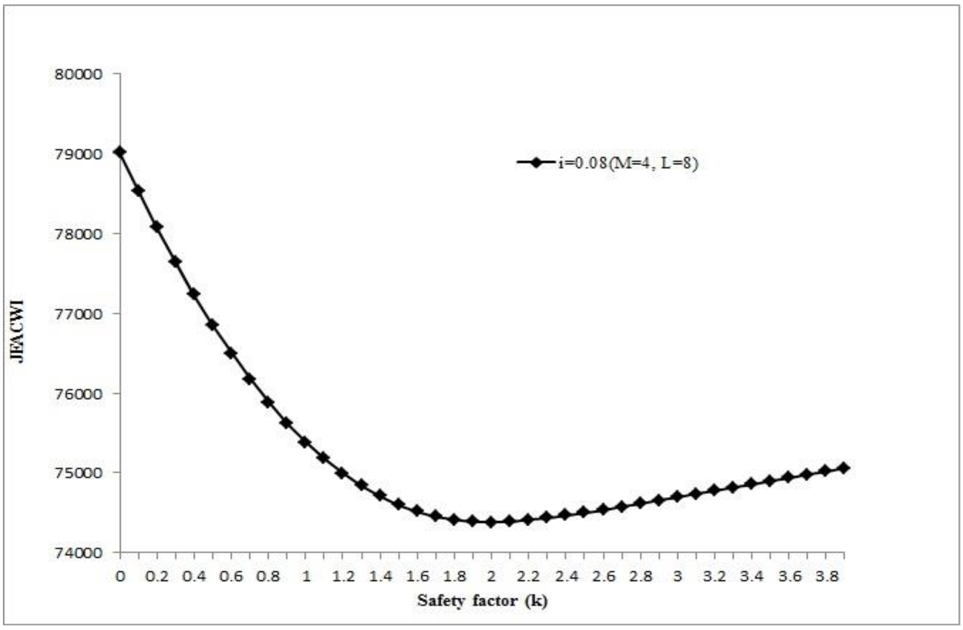

Figure 6. Optimal joint expected annual cost and backorder rate in terms of various backorder parameters ϑ

1. If the value of ϑ increases, the backorder rate, β, decreases, then the optimal reorder point, r^*, and the optimal expected annual cost, $JEACWI^*$, are increasing simultaneously. A simple economic viewpoint is as follows. A smaller backorder rate amount shows a larger shortage cost. Therefore, the optimal reorder point, r^*, and the optimal expected annual cost, $JEACWI^*$, get larger because a smaller backorder rate indicates a larger lost sales (see Fig 6).

2. For a fixed lead time, $L \epsilon [L_z, L_{z-1}]$, a higher standard deviation of demand, σ_D, increases the safety stock. Also the order quantity and the ordering cost escalate with increasing the standard deviation.

3. Due to higher, b, the ordering quantity and the ordering cost are higher, resulting in an increasing joint expected annual cost with inflation, $JEACWI^*$. Also, safety factor and reorder point decrease with increasing, b.

4. A higher vendor's holding cost reduces the number of shipment. Also, as most of the cost factors are higher, the joint expected annual cost with inflation, $JEACWI^*$, increases automatically.

5. A higher vendor's ordering cost reduces the number of shipment. Also, For a fixed number of shipment, with an escalation in vendor's ordering cost, order quantity and buyer's ordering cost increase, but the safety factor and safety stock are reduced.

6. Conclusion

With the current high inflation in many business environments, especially in developing countries, it is very important to adapt mathematical models in this direction especially in uncertain areas. To achieve this goal, we adjusted the lot-size reorder-point inventory model with stochastic demand with inflation where the lead time is a function of both reduced lead time and order quantity, and considered an integrated vendor-buyer cooperative inventory model instead of analyzing the optimal policy of one facility. The behavior of our model was illustrated in a numerical example. A sensitivity analysis with respect to the important system parameters was also performed. As regards future research, one line of inquiry would be to include some constraints in the model. We are also interested in finding optimal policy for the periodic review inventory model. Investigating optimal policies when LTD follows other distribution functions or a mixture of distributions would also be a challenging task for researchers.

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Appendix
It is assumed that imperfect items in a lot is a probabilistic variable, with beta-binomial distribution with parameters (Q, α, β) as follows.

$$P(y|p) = \binom{Q}{y} p^y (1-p)^{Q-y} \ , \qquad y = 0,1,2,\dots,Q \tag{A.1}$$

$$f(p) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{(\alpha-1)}(1-p)^{\beta-1}, \qquad 0 \le p \le 1 \tag{A.2}$$

$$f(y) = \int_0^1 f(p)P(y|p)dp = \binom{Q}{y}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_0^1 p^{y+\alpha-1}(1-p)^{Q-y+\beta-1}\,dp$$

$$\rightarrow f(y) = \binom{Q}{y}\frac{\Gamma(\alpha+\beta)\Gamma(\alpha+y)\Gamma(Q+\beta-y)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(Q+\alpha+\beta)} \quad , \ y = 0,1,2,\dots,Q \tag{A.3}$$

Therefore:

$$E(y|p) = Qp \tag{A.4}$$

$$Var(y|p) = Qp(1-p) \tag{A.5}$$

Thus, the expected value of Y is computed by:

$$E(y) = E\big(E(y|p)\big) = QE(p) \tag{A.6}$$

Also, the variance of Y is calculated as follows:

$$Var(y) = E\big(Var(y|p)\big) + Var\big(E(y|p)\big) \tag{A.7}$$

$$E\big(Var(y|p)\big) = E\Big(Q\big(p(1-p)\big)\Big) = QE\big(p(1-p)\big) \tag{A.8}$$

$$Var\big(E(y|p)\big) = E(Q^2p^2) - \big(E(p)\big)^2 = Q^2E(p^2) - Q^2E^2(p) = Q^2Var(p) \tag{A.9}$$

$$\rightarrow Var(y) = QE\big(p(1-p)\big) + Q^2Var(p) \tag{A.10}$$

Considering the quadratic approximation, $E\big(g(x)\big) \cong g(\mu) + \frac{g''(\mu)}{2}{\sigma_x}^2$, the expected value of $\frac{1}{Q-y}$ is calculated by:

$$g(y) = \frac{1}{Q-y}\ , g''(y) = \frac{2}{(Q-y)^3}$$

$$E\left(\frac{1}{Q-y}\right) \cong \frac{1}{Q\big(1-E(p)\big)} + \frac{Var(y) = QE\big(p(1-p)\big) + Q^2Var(p)}{\Big(Q\big(1-E(p)\big)\Big)^3} \tag{A.11}$$

It is mentioned that the above approximation is accurate when the standard deviation is small.
Besides, the expected value of $\frac{Q-y}{2}$ is:

$$E\left(\frac{Q-y}{2}\right) = \frac{Q\big(1-E(p)\big)}{2} \tag{A.12}$$

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