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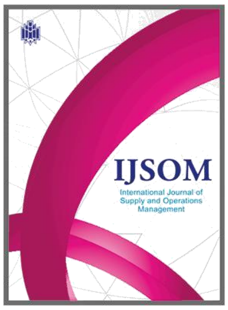

Determining an Optimal Maintenance Policy for Three State Machine Replacement Problem Using Dynamic Programming

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Abstract
In this article, we present a sequential sampling plan for a three-state machine replacement problem using dynamic programming model. We consider an application of the Bayesian Inferences in a machine replacement problem .The machine was studied at different states of good, medium and bad. Discount dynamic programming (DDP) was applied to solve the three-state machine replacement problem, mainly to provide a policy for maintenance by considering item quality and to determine an optimal threshold policy for maintenance in the finite time horizon. A decision tree based on the sequential sampling which included the decisions of renew, repair and do-nothing was implemented in order to achieve a threshold for making an optimized decision minimizing expected final cost. According to condition-based maintenance, where the point of defective item is placed in continuing sampling area, we decided to repair the machine or to continue sampling. A sensitivity analysis technique shows that the optimal policy can be very sensitive.

Keywords: Machine replacement; Dynamic programming; Sequential sampling plan; Maintenance.

1. Introduction

Maintenance is the set of actions conducted to keep a system into a situation where it can perform its function. Machine replacement problem is a principal issue in maintenance problems that can significantly affect the expenditures of the company to go lower. Many mathematical models based on cost objective function are presented for machine replacement problem not applying the quality of items produced by machine in such models can lead to incorrect decisions. Decision making about quality of machine based on inspecting the produced items usually results in the application of sampling plans. Two approaches have been applied to the machine maintenance problems. In the first approach, modeling helps to realize how the optimal maintenance policy depends on the intensity of technological change, the rate of capacity expansion, and the deteriorating rate. In the second approach, the maintenance policy that is being followed in practice is a combination of preventive and corrective maintenance (Niaki and Fallahnezhad, 2011).

A large body of literature has continuously focused on problems including maintenance decision optimization.Tagaras presented the joint process control and the machine maintenance problem of a Markovian deteriorating machine (Tagaras, 1988). Kuo applied an optimal adaptive control policy for a joint machine maintenance and product quality control (Kuo, 2006). Bowling et al. used a Markovian model to determine optimum process-target levels for a multi-stage serial production system (Bowling et. al, 2004). Goldstein et al. presented a planning horizon for the first optimal replacement in an infinite-horizon problem in which two types of machines are considered (Goldstein et. al, 1988). Ivy and Nembhard presented maintenance model for machine replacement problem by combining a statistical quality control technique with partially observable Markov decision processes (Ivy and Nembhard, 2005). Sethi et al. developed appropriate dynamic programming equations and established the existence of the solution using a verification theorem for optimality (Sethi et. al, 1997). Niaki and Fallahnezhad employed Bayesian inference and stochastic dynamic programming to design a decision-making framework in production environment (Niaki and Fallahnezhad, 2007). Further, Fallahnezhad et al. determined the optimal policy for two-machine replacement problem using Bayesian inference in the context of the finite mixture model –

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(Fallahnezhad et. al, 2007). Fallahnezhad and Niaki proposed a two-machine dynamic programming model (Fallahnezhad and Niaki, 2011). Aslam et al. proposed a new sampling system based on the constraints of first and second and type errors (Aslam et. al, 2013). In addition, Chun and Rinks assumed the Beta distribution for proportion defective and modified producer and consumer risks based on Bayes producer and consumer risks (Chun and Rinks, 1998). Fallahnezhad et al. presented a model of Markov chain approach in acceptance sampling plans based on the cumulative sum of the number of successive conforming items (Niaki and Fallahnezhad, 2012). Fallah Nezhad analyzed the acceptance sampling design by using minimum angle method (Fallahnezhad et.al, 2011). Aslam et al. proposed repetitive acceptance sampling plan with new decision rule (Aslam et.al, 2012). The other approach is to design a sampling system which minimizes the total cost of decision making. Niaki and Fallahnezhad proposed a stochastic dynamic programming and Bayesian inferences concept to design an optimum sampling plan (Niaki and Fallahnezhad, 2009). Fallahnezhad and Niaki proposed an economically optimal acceptance sampling policy based on number of successive conforming items (Fallahnezhad and Niaki, 2013). Fallahnezhad and Hosseini Nasab proposed a single-stage acceptance sampling plan by minimizing total cost of the system (Fallahnezhad and Hosseini Nasab, 2011). Fallahnezhad et al. proposed Bayesian acceptance sampling plan based on cost objective function (Fallahnezhad et.al, 2012).

In this article, a dynamic programing model for three-state machine replacement problem is developed to determine an optimal maintenance policy. The main contribution of this paper is determining an optimal threshold for three-state machine replacement problem using dynamic programming based on the sequential sampling plan in a finite time horizon. Considering the state of machine in good, medium and bad is an innovative plan to model an optimal maintenance policy in machine replacement problem. In the second section, the model and it assumptions is described. The third section states Notations and formulation. A numerical example is illustrated to clarify the subject and the sensitivity analysis technique is used to determine the effect of any parameters on the results. Conclusion comes in section six.

2. Description of the model

A decision tree is a decision support tool that employs a set of decisions and their feasible results, it is one method to present an optimal solution algorithm. A decision tree is a flowchart-like framework in which internal node demonstrates one decision, each branch determines result of event or choosing a decision and each leaf node shows decision results [20].

A decision tree includes three types of nodes: 1) Decision nodes – mostly displayed by squares, 2) Chance nodes – represented by circles, 3) End nodes – represented by triangles [21]. In decision analysis, a decision tree is applied as an analytical decision support tool, where the expected costs of each decision are computed.

One application of decision trees is for classification, but we used the decision tree for choosing among alternatives not for classification where the applied decision tree is computed based on backward dynamic programming [22]. In this paper, a decision tree for a three-state machine replacement problem is used to determine outcomes of event or selecting a decision. The proposed decision tree includes decisions of renewing, repairing machine and continuing the production without any maintenance. When the stage variable is equal to n, there are n available decisions. Supposing that the repair decision is selected at stage n, according to backward dynamic programming approach, the decision in the stage n-1, is made based on the prior decisions. Figure 1 illustrates the decision tree for the model proposed.

The goal of the model is to minimize the cost recursive function in order to determine an optimal maintenance policy, more specifically in a finite time horizon. We can suppose that the machine states include bad, medium, good states. Assumptions and formulas of the model are illustrated using Bayesian inference method. A framework for the model was proposed by sequential sampling plan and partially observable Markov decision process (POMDP).

The POMDP approach is adopted to achieve the partial observations of machine state by sequential sampling. Sequential sampling plan is done so that the machine start to work and produce number of production; considering the framework presented, the random samples of lot is investigated; first the item is checked, then checking continues item by item. The process of sampling continues until the number of defective items is placed in the determined area according to sequential sampling plan. If the number of defective items is more than determined upper threshold, the optimal decisions is renewing. In addition, if the number of defective items is less than determined lower threshold, the optimal decisions is continuing the production without any maintenance. If the number of defective items is placed between the upper threshold and lower threshold, two approaches can be applied by considering condition-based maintenance: 1.the repair decision is selected. 2. Sampling continues until the number of defective items is more than determined upper threshold or the number of defective items less than determined lower threshold.

We can suppose that the machine can be placed in states good, medium and bad; the probability of being defective for machines depends on the amount of item produced. In fact, the case deals with a decision tree and optimal decision, which is selected, based on the produced parts quality.

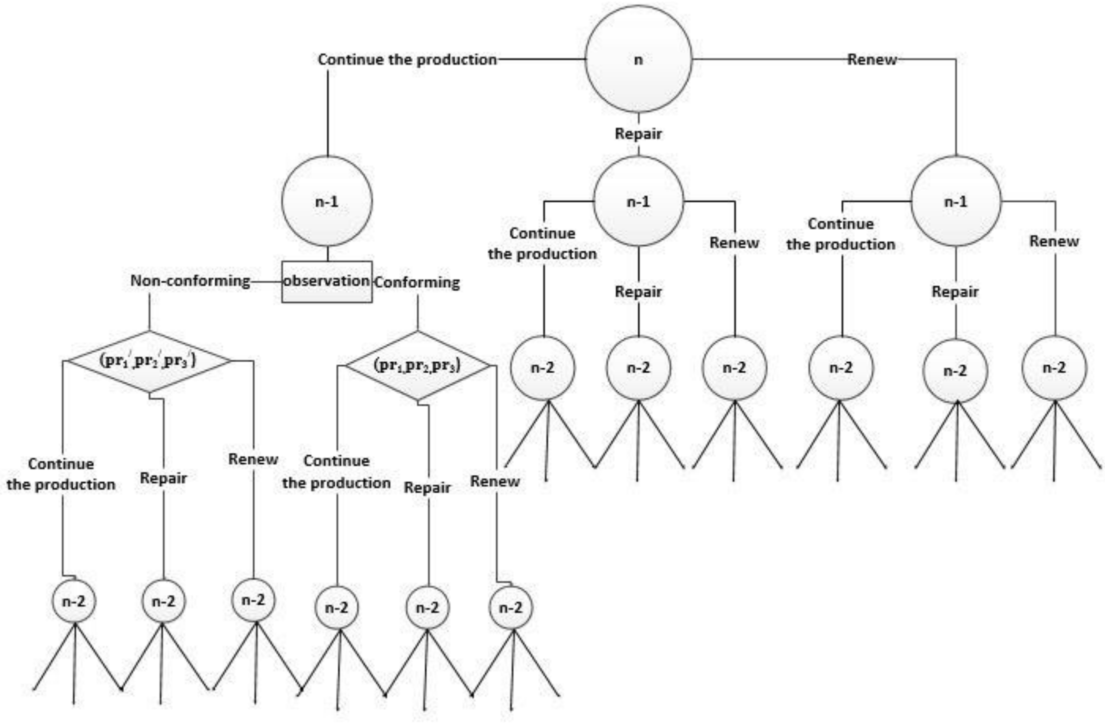

Figure1. Decision tree that includes the decisions of renew, repair or do nothing and continue

Based on the statistical quality control techniques and partially observable Markov decision processes (POMDP), it is proposed that the probability of producing a defective product is determined based on the machine state. The statistical quality control is used to determine the probability distribution of defective items. If the machine is in bad state, the defective observation distribution follows Bernoulli distribution with parameter p_1 and if the machine is in medium state, the defective observation distribution follows Bernoulli distribution with parameter p_2. If the machine is in good state, the defective observation distribution follows Bernoulli distribution with parameter p_3. This assumption is shown in Figure 2.

To illustrate the model, some assumptions should be considered. It is assumed that the machine can be placed in three states: good, Medium and bad. The backward dynamic programming is used; π (the probability that the machine is in bad state) is considered as state variable and the number of the programming periods equals stage variable; and programming is done in finite time horizon.

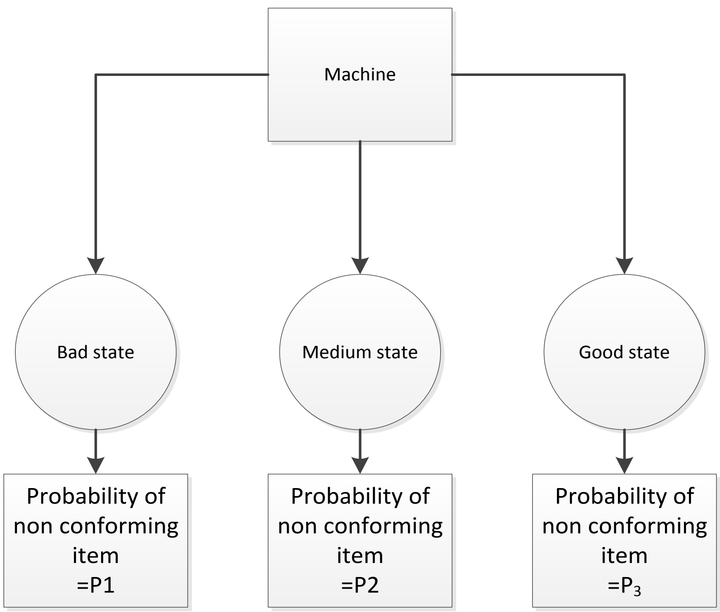

Figure 2. Determining the parameters of the Bernoulli distribution for producing one item

Parameters and formulas of the model are illustrated in the following; some of them are obtained using the bayesian inference method.

3. Notations

The notations required to model the problem at hand are given as:

π: probability of machine state (machine states includes bad, medium, good state)
Pr: Posterior probability of machine state when a conforming item is produced.
Pr': Posterior probability of machine state when a non-conforming item is produced.
π_1: Probabilty that the machine is in bad state; $\{s_t=0\}$.
π_2: Probability that the machine is in medium state; $\{s_t=1\}$.
π_3: Probability that the machine is in good state; $\{s_t=2\}$.
α: Discount factor; $\alpha \in [0,1]$
p_1: Probability that the observation is defective if the machine is in the bad state.
p_2: Probability that the observation is defective if the machine is in the medium state.
P_3: Probability that the observation is defective if the machine is in the good state.
z: Probability that the observation is defective.
L: The defective observation.
pr_1: The posterior probability that the machine is in the bad state when a conforming item is produced.
pr_2: The posterior probability that the machine is in the medium state when a conforming item is produced.
pr_3: The posterior probability that the machine is in the good state when a conforming item is produced.
pr_1': The posterior probability that the machine is in the bad state when a non-conforming item is produced.
pr_2': The posterior probability that the machine is in the medium state when a non-conforming item is produced.
pr_3': The posterior probability that the machine is in the good state when a non-conforming item is produced.
π_0: Probability of the machine state for new machine.
π_{01}: Probabilty that the machine is in the bad state after the machine is renewed.
π_{02}: Probabilty that the machine is in the medium state after the machine is renewed.
π_{03}: Probabilty that the machine is in the bad state after the machine is renewed.
n: The number of remained stages (the stage variable).
T: The coefficient for the cost of repair decision in different states.
T_1: The coefficient for the cost of repair decision when the machine is in the bad state.
T_2: The coefficient for the cost of repair decision when the machine is in the medium state.
T_3: The coefficient for the cost of repair decision when the machine is in the good state.
π_{11}: Probability that the machine is in the bad state after the machine is repaired.
π_{12}: Probability that the machine is in the medium state after the machine is repaired.
π_{13}: Probability that the machine is in the good state after the machine is repaired.
R: The fixed cost for renew decision
A: Profit of a conforming item.
C: Cost of one non-conforming item.
M: The coefficient for the salvage value of machine (the value of the machine when no stage is remaining and the process terminates.)
M_1: The coefficient for the salvage value when the machine is in the bad state.
M_2: The coefficient for the salvage value when the machine is in the medium state.
M_3: The coefficient for the salvage value when the machine is in the good state.
$V_0(\pi)$: The salvage value of the machine (the value of the machine when no stage is remaining and the process terminates).

The optimality equation is illustrated as following:

$$V_n(\pi) = V_n(\pi_1, \pi_2, \pi_3) = \min\{\mathrm{R} + \alpha\, \mathrm{V}_{n-1}(\pi_{01}, \pi_{02}, \pi_{03}),$$
$$T_1\pi_1 + T_2\pi_2 + T_3\pi_3 + \alpha V_{n-1}(\pi_{11}, \pi_{12}, \pi_{13}), ZC - (1-Z)A + \alpha V_{n-1}(pr_1, pr_2, pr_3)\,\mathrm{Z}$$
$$+\alpha\, \mathrm{V}_{n-1}(pr_1', pr_2', pr_3')(1-Z)\} \quad (1)$$

where

$$\pi = (\pi_1, \pi_2, \pi_3) \tag{2}$$

$$\pi_0 = (\pi_{01}, \pi_{02}, \pi_{03}) \tag{3}$$

$$M = (M_1, M_2, M_3) \tag{4}$$

$$V_0(\pi) = M\pi \tag{5}$$

$$T = (T_1, T_2, T_3) \tag{6}$$

$$pr = (pr_1, pr_2, pr_3) \tag{7}$$

(8)

$$pr' = (pr_1', pr_2', pr_3') \tag{9}$$

$$\pi_1 = (\pi_{11}, \pi_{12}, \pi_{13}) \tag{10}$$

$$Z = \mathrm{P}(L \mid s_t = 0)\mathrm{P}(s_t = 0) + \mathrm{P}(L \mid s_t = 1)\mathrm{P}(s_t = 1) + \mathrm{P}(L \mid s_t = 2)\mathrm{P}(s_t = 2)$$
$$= \pi_1 p_1 + \pi_2 p_2 + \pi_3 p_3 = \pi_1 p_1 + \pi_2 p_2 + (1 - \pi_1 - \pi_2) p_3$$

$$pr_1 = \mathrm{P}(s_t = 0 \mid L) = \frac{\mathrm{P}(L \mid s_t = 0)\mathrm{P}(s_t = 0)}{\mathrm{P}(L)} = \frac{p_1 \pi_1}{Z} \tag{11}$$

$$pr_2 = \mathrm{P}(s_t = 1 \mid L) = \frac{\mathrm{P}(L \mid s_t = 1)\mathrm{P}(s_t = 1)}{\mathrm{P}(L)} = \frac{p_2 \pi_2}{Z} \tag{12}$$

$$pr_3 = \mathrm{P}(s_t = 2 \mid L) = \frac{\mathrm{P}(L \mid s_t = 2)\mathrm{P}(s_t = 2)}{\mathrm{P}(L)} = \frac{p_3 \pi_3}{Z} \tag{13}$$

$$Pr_1' = P(s_t = 0 \mid L^c) = \frac{P(L^c \mid s_t = 0)P(s_t = 0)}{P(L^c)} = \frac{(1 - p_1)\pi_1}{1 - Z} \tag{14}$$

$$Pr_2' = P(s_t = 1 \mid L^c) = \frac{P(L^c \mid s_t = 1)P(s_t = 1)}{P(L^c)} = \frac{(1 - p_2)\pi_2}{1 - Z} \tag{15}$$

$$Pr_3' = P(s_t = 2 \mid L^c) = \frac{P(L^c \mid s_t = 2)P(s_t = 2)}{P(L^c)} = \frac{(1 - p_3)\pi_3}{1 - Z} \tag{16}$$

The condition-based maintenance (CBM) and sequential sampling plan are used to illustrate the model proposed. CBM is used so that the point is placed in continue sampling area then the decisions of repairing the machine or continuing sampling can be chosen until the point is placed in rejection area and decisions of the renew are selected; if the point is placed in accept area then the decisions of do-nothing is selected. Figure. 3 clearly shows sequential sampling method in machine replacement problem.

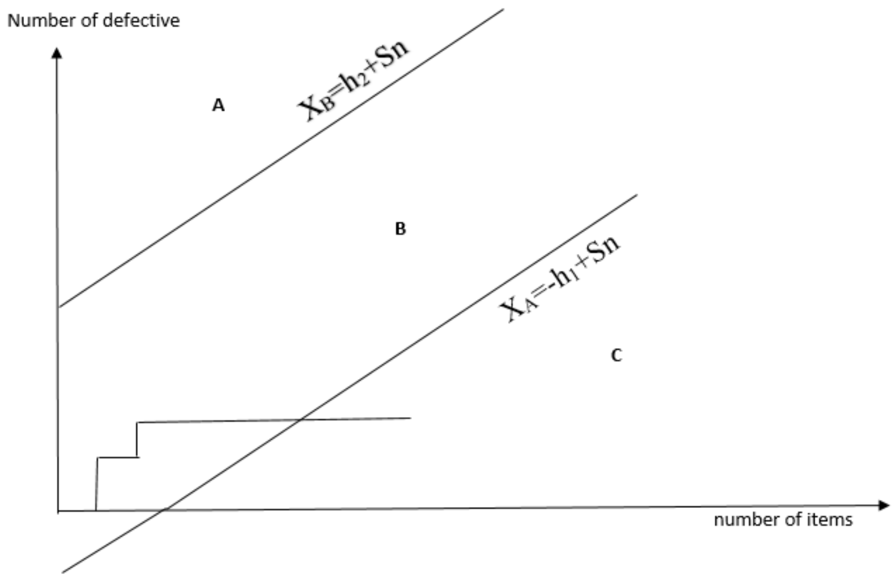

Figure 3. Sequential sampling plan for machine replacement problem

- A: Renew machine.
- B: Repair machine
- C: Continue the production without any maintenance action.

4. Numerical example

A numerical example is solved for illustrating the application of proposed methodology. Input data of the problem is as following:

$$\begin{pmatrix} R=30, \alpha=0.95, \pi_{01}=0.03, \\ \pi_{02}=0.27, \pi_{03}=0.7, \\ T_1=15, T_2=10, T_3=8, \\ \pi_{11}=0.2, \pi_{12}=0.3, \pi_{13}=0.5 \\ p_1=0.8, p_2=0.1, p_3=0.1, \\ C=15, A=5, M_1=2, M_2=6 \\ M_3=8 \end{pmatrix}$$

Assumptions and equations used in this model are simulated by MATLAB software.
For example, if n=5; decision making stages are available then the results for different values of state variable (π_1, π_2, π_3) are reported in Table 1.

Table 1. Total expected costs for each (π_1, π_2, π_3) and its optimal decision

(π_1, π_2, π_3)	Cost(Renew)	Cost(Repair)	Cost(Continue the production)	$V_n(\pi_1, \pi_2, \pi_3)$	Decision
(0,0,1)	26.54022	11.35199	-7.3829	-7.3829	Continue the production
(0,0.1,0.9)	26.54022	11.55199	-7.53765	-7.53765	Continue the production
(0,0.2,0.8)	26.54022	11.75199	-7.69241	-7.69241	Continue the production
(0,0.3,0.7)	26.54022	11.95199	-7.84716	-7.84716	Continue the production
(0,0.4,0.6)	26.54022	12.15199	-8.00192	-8.00192	Continue the production
(0,0.5,0.5)	26.54022	12.35199	-8.15668	-8.15668	Continue the production
(0,0.6,0.4)	26.54022	12.55199	-8.31143	-8.31143	Continue the production
(0,0.7,0.3)	26.54022	12.75199	-8.46619	-8.46619	Continue the production
(0,0.8,0.2)	26.54022	12.95199	-8.62095	-8.62095	Continue the production
(0,0.9,0.1)	26.54022	13.15199	-8.7757	-8.7757	Continue the production
(0,1,0)	26.54022	13.35199	-8.93046	-8.93046	Continue the production
(0.1,0,0.9)	26.54022	12.05199	-2.15346	-2.15346	Continue the production
(0.1,0.1,0.8)	26.54022	12.25199	-2.30486	-2.30486	Continue the production
(0.1,0.2,0.7)	26.54022	12.45199	-2.45627	-2.45627	Continue the production
(0.1,0.3,0.6)	26.54022	12.65199	-2.60767	-2.60767	Continue the production
(0.1,0.4,0.5)	26.54022	12.85199	-2.75907	-2.75907	Continue the production
(0.1,0.5,0.4)	26.54022	13.05199	-2.91048	-2.91048	Continue the production
(0.1,0.6,0.3)	26.54022	13.25199	-3.06188	-3.06188	Continue the production
(0.1,0.7,0.2)	26.54022	13.45199	-3.21328	-3.21328	Continue the production
(0,1,0.8,0.1)	26.54022	13.65199	-3.36469	-3.36469	Continue the production
(0.1,0.9,0)	26.54022	13.85199	-3.51609	-3.51609	Continue the production

Table 1. Continued

(π_1, π_2, π_3)	**Cost(Renew)**	**Cost(Repair)**	**Cost(Continue the production)**	**V$_n$** (π_1, π_2, π_3)	**Decision**
(0.2,0,0.8)	26.54022	12.75199	2.172761	2.172761	Continue the production
(0.2,0.1,0.7)	26.54022	12.95199	2.05248	2.05248	Continue the production
(0.2,0.2,0.6)	26.54022	13.15199	1.9322	1.9322	Continue the production
(0.2,0.3,0.5)	26.54022	13.35199	1.811919	1.811919	Continue the production
(0.2,0.4,0.4)	26.54022	13.55199	1.691638	1.691638	Continue the production
(0.2,0.5,0.3)	26.54022	13.75199	1.571358	1.571358	Continue the production
(0.2,0.6,0.2)	26.54022	13.95199	1.451077	1.451077	Continue the production
(0.2,0.7,0.1)	26.54022	14.15199	1.330797	1.330797	Continue the production
(0.2,0.8,0)	26.54022	14.35199	1.210516	1.210516	Continue the production
(0.3,0,0.7)	26.54022	13.45199	6.108511	6.108511	Continue the production
(0.3,0.1,0.6)	26.54022	13.65199	5.991166	5.991166	Continue the production
(0.3,0.2,0.5)	26.54022	13.85199	5.873822	5.873822	Continue the production
(0.3,0.3,0.4)	26.54022	14.05199	5.756477	5.756477	Continue the production
(0.3,0.4,0.3)	26.54022	14.25199	5.639133	5.639133	Continue the production
(0.3,0.5,0.2)	26.54022	14.45199	5.521788	5.521788	Continue the production
(0.3,0.6,0.1)	26.54022	14.65199	5.404444	5.404444	Continue the production
(0.4,0,0.6)	26.54022	14.15199	10.03251	10.03251	Continue the production
(0.4,0.1,0.5)	26.54022	14.35199	9.915163	9.915163	Continue the production
(0.4,0.2,0.4)	26.54022	14.55199	9.797818	9.797818	Continue the production
(0.4,0.3,0.3)	26.54022	14.75199	9.680474	9.680474	Continue the production
(0.4,0.4,0.2)	26.54022	14.95199	9.563129	9.563129	Continue the production

Table 1. Continued

(π_1, π_2, π_3)	**Cost(Renew)**	**Cost(Repair)**	**Cost(Continue the production)**	**V_n** (π_1, π_2, π_3)	**Decision**
(0.4,0.5,0.1)	26.54022	15.15199	9.445785	9.445785	Continue the production
(0.5,0,0.5)	26.54022	14.85199	13.71193	13.71193	Continue the production
(0.5,0.1,0.4)	26.54022	15.05199	13.62182	13.62182	Continue the production
(0.5,0.2,0.3)	26.54022	15.25199	13.53171	13.53171	Continue the production
(0.5,0.3,0.2)	26.54022	15.45199	13.44161	13.44161	Continue the production
(0.5,0.4,0.1)	26.54022	15.65199	13.3515	13.3515	Continue the production
(0.5,0.5,0)	26.54022	15.85199	13.26139	13.26139	Continue the production
(0.6,0,0.4)	**26.54022**	**15.55199**	**17.3677**	**15.55199**	**Repair**
(0.6,0.1,0.3)	26.54022	15.75199	17.27759	15.75199	Repair
(0.6,0.2,0.2)	26.54022	15.95199	17.18748	15.95199	Repair
(0.6,0.3,0.1)	26.54022	16.15199	17.09738	16.15199	Repair
(0.7,0,0.3)	26.54022	16.25199	21.02347	16.25199	Repair
(0.7,0.1,0.2)	26.54022	16.45199	20.93336	16.45199	Repair
(0.7,0.2,0.1)	26.54022	16.65199	20.84325	16.65199	Repair
(0.8,0,0.2)	26.54022	16.95199	24.67924	16.95199	Repair
(0.8,0.1,0.1)	26.54022	17.15199	24.58913	17.15199	Repair
(0.9,0,0.1)	26.54022	17.65199	27.46757	17.65199	Repair
(1,0,0)	26.54022	18.35199	29.53257	18.35199	Repair

As can be seen, the optimal policy is in the form of a control threshold policy. When (π_1^*=0.6, π_2^*=0, π_3^*=0.4), then optimal decision is changed from continue-the-production decision to the repair decision; Fig 4 clearly shows this issue. According to input data, the renew decision is not recommended. By changing input data, the renew decision is applied which is further illustrated in the subsequent section.

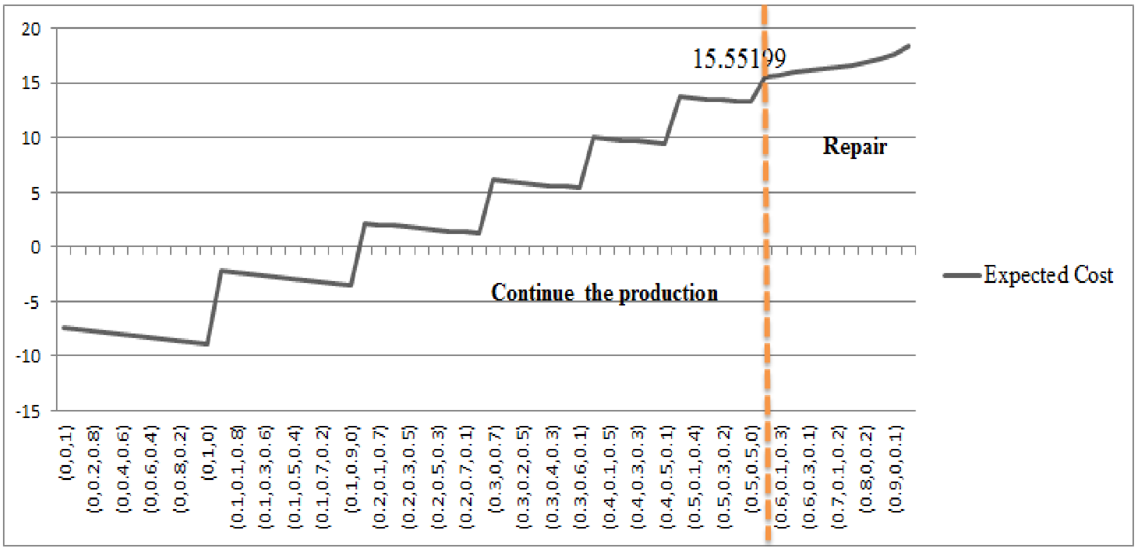

Figure 4. Diagram of the expected costs for each (π_1, π_2, π_3)

5. Sensitivity Analysis

A sensitivity analysis is used to analyze the effects of changing parameters on the optimal solution. In each case of sensitivity analysis, one parameter of the model is altered. It is necessary to adjust the parameter value in a level so that one can easily interpret its behavior. The decision numbers for decisions of Renew, Repair and continue the production are 1, 2 and 3 respectively. For example, 3→2 means that the optimal decisions change from continue- the production-decision to repair decision based on the cost objective function. Also 3→1 means that the optimal decisions changed from continue-the production-decision to renew decisions in the optimal threshold policy. The results are shown in Table 2.

Table 2 shows that the optimal threshold changes by changing parameters of the model. For example, by increasing R and C, the repair decision area decreases. In other words, the optimal threshold shifts to the right, as shown more clearly in Fig 5 and Fig 6. The result of sensitivity analysis for parametes π_{01}, π_{02}, π_{03}, M_1, M_2, M_3 shows that changing of these parameters does not affect optimal threshold. In addition, changing parameters p_2, p_3 does not follow a regular pattern.

Table 2. The results of sensitivity analysis for the proposed sampling plan

Parameters	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**
R	0	(0.2,0,0.8) 3→2	10	(0.5,0,0.5) 3→2	40	**(0.6,0,0.4) 3→2**
α	0.8	(0.7,0,0.3) 3→2	0.9	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
p_1	0	(1,0,0) 3	0.5	(0.6,0,0.4) 3→2	1	**(0.5,0,0.5) 3→2 (0.5,0.3,0.2) 2→3 (0.6,0,0.4) 3→2**
P_2	0	(0.5,0,0.5) 3→2 (0.5,0.3,0.2) 2→3 (0.6,0,0.4) 3→2	0.5	irregular	1	**Irregular**

Table 2. Continued

Parameters	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**
P_3	0	(0.5,0.3,0.2) 3→2	0.5	irregular	1	**Irregular**
T_1	0	(0.5,0,0.5) 3→2	20	(0.6,0,0.4) 3→2	40	**(0.7,0,0.3) 3→1**
T_2	0	(0.5,0.2,0.3) 3→2	20	(0.6,0,0.4) 3→2 (0.6,0.2,0.2) 2→3 (0.7,0,0.3) 3→2	40	**(0.8,0,0.2) 3→2**
T_3	0	(0.5,0,0.5) 3→2 (0.5,0.3,0.2) 2→3 (0.6,0,0.4) 3→2	20	(0.7,0,0.3) 3→2	40	**(0.7,0.2,0.1) 3→2**
π_{01}	0	(0.6,0,0.4) 3→2	0.5	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
π_{02}	0	(0.6,0,0.4) 3→2	0.5	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
π_{03}	0	(0.6,0,0.4) 3→2	0.5	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
π_{11}	0	(0.3,0,0.7) 3→2 (0.3,0.2,0.5) 2→3 (0.4,0,0.6) 3→2	0.5	(0.7,0,0.3) 3→1	1	**(0.7,0,0.3) 3→1**
π_{12}	0	(0.5,0,0.5) 3→1	0.5	(0.7,0,0.3) 3→1	1	**(0.7,0,0.3) 3→1**
π_{13}	0	(0.5,0,0.5) 3→1	0.5	(0.6,0,0.4) 3→1	1	**(0.7,0,0.3) 3→1**
A	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**
C	0	(1,0,0) 3	10	(0.7,0,0.3) 3→2	20	**(0.5,0,0.5) 3→2**
M_1	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**
M_2	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**
M_3	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**

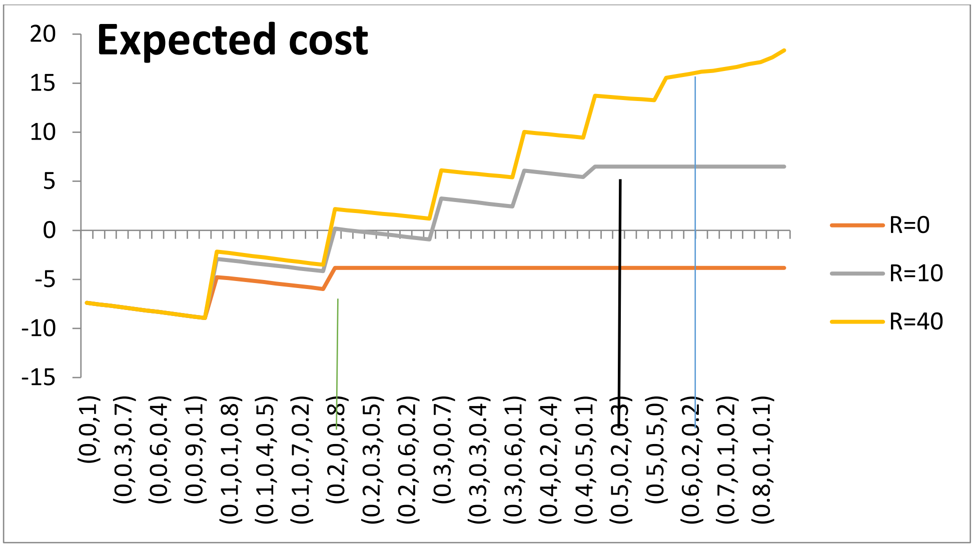

Figure 5. The results of sensitivity analysis for parameters R

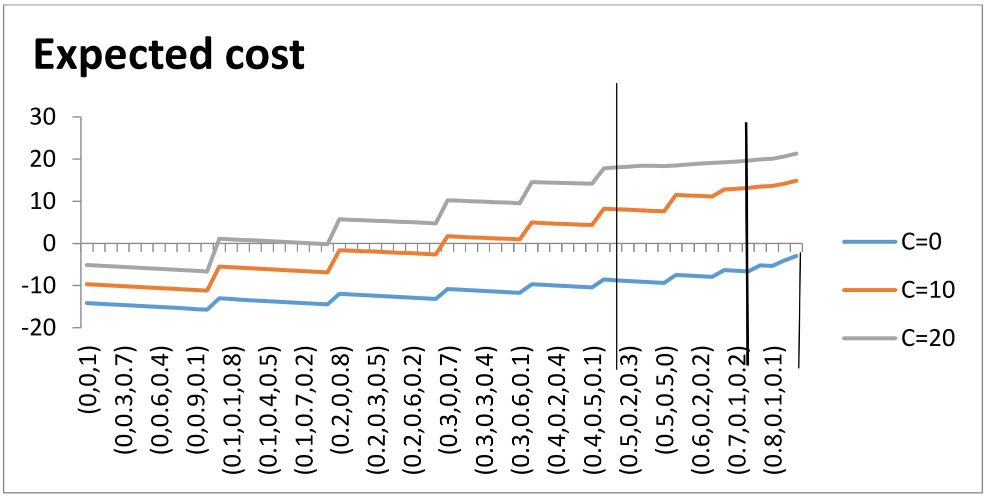

Figure 6. The results of sensitivity analysis for parameters C

6. Conclusion

In this article, we presented a backward dynamic programming model for three-state machine replacement problems in a finite time horizon in order to determine a control threshold policy using POMDP technique and sequential sampling plan. This model is applied for optimizing expected cost in machine replacement problem based on the methods of sequential sampling and Bayesian inferences. A decision tree is implemented to determine which decision can be chosen; if each decision is chosen the related cost is applied. A cost objective function including the costs of replacement and repair, and the cost of defectives. The presented model can be used in the production departments in which machine deterioration is monitored using the quality of produced items. In this paper, the medium state of machine is considered and the results show that the proposed model presents an exact and optimal maintenance policy and develops prior researches. In addition, the sensitivity analysis demonstrates that changing the input data significantly influences the optimal solution.

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