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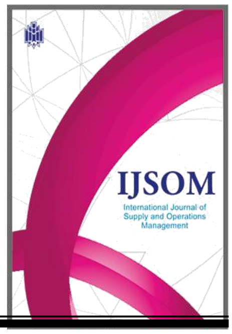

Using Metaheuristic Algorithms for Solving a Hub Location Problem: Application in Passive Optical Network Planning

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Abstract
Nowadays, fiber-optic are counted as one of the most important tools for data transfer due to having greater bandwidth and being more efficient compared with other similar technologies,. In this article, an integrated mathematical model for a three-level fiber-optic distribution network is presented with consideration of simultaneous backbone and local access networks the backbone network is a ring and the access networks has a star-star topology. The aim of the model is to determine the location of the central offices and splitters, the type of connection between central offices, and allocation of each demand node to a splitter or central office in a way that the wiring cost of fiber optical and concentrator installation are minimized. Moreover, each user's desired bandwidth should be provided efficiently. Then, the proposed model is validated by GAMS software in small-sized problems, afterwards the model is solved by two meta-heuristic methods including differential evolution (DE) and genetic algorithm (GA) in large-scaled problems and the results of two algorithms are compared with respect to computational time and objective function obtained value. Finally, a sensitivity analysis is provided.

Keywords: Fiber-optic; Telecommunication network; Hub-location; Passive splitter; Three-level network.

1. Introduction

As a result of technology advances in the last two decades, connection networks has been widely developed (Yoon and Current 2008). A connection network includes a set of nodes which transfer messages or information in form of data, audio, and video by means of fiber-optic, copper cables, radio or satellite communications (Saboury, et al., 2013).Nowadays, fiber-optic due to having greater bandwidth, less attenuation and being more cost efficient compared to other technologies, are known as an important tool for data transfer, and this technology created a revolution in the telecommunications industry (Jadhav and Shitole, 2013).In the past decades, high-speed fiber-optic was only used for long distances and data transfer among Central Offices (COs).But, today, in order to bring fiber-optic closer to the end-users, it is also being used in downstream layers which leads to new concepts such as Fiber-To-The-Home (FTTH), Fiber-To-The-Building (FTTB), Fiber-To-The-Cabinet (FTTC). Finding the number and location of each concentrator and allocation of each user to the concentrators are basic questions which should be answered in these problems (Fortz 2015).

Connection networks are usually based on two or more layers which have a hierarchical structure. This structure reduces costs of the system and enhances its flexibility. In fact, instead of forming a direct path between origin and destination, new alternative paths with the same origin but different destinations are formed, which results in a considerable reduction in costs (Ortiz-Garcia et al 2009).

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In access networks, fiber-optic is transmitted from CO to demand node either as Point to Point (P2P) or Passive Optical Network (PON). In P2P topology, demand points have a dedicated bandwidth and fiber-optic is connected directly from the Optical Line Terminal (OLT) located in CO to the Optical Network Unit (ONU) as the demand point. Each ONU can be representative for some existing demands in that area.

But in PON topology, bandwidth can be shared in the fiber-optic which passes through the splitter located in Distribution Center (DC).Therefore, multiple destinations can connect to the same cable. PON networks due to having concentrators, require less fiber-optic and tools in CO comparing to the P2P network. Nowadays, set of allowable split ratios is {1:2, 1:4, 1:8and so on}. The receiving bandwidth for users is an important issue in serving fiber-optic. Bandwidth or speed is measured based on Megabytes per second (Mb/sec). Every CO has a specific bandwidth. With the signal passes through the splitter type k (1:2^k), the bandwidth of each output of cable will be 2^ktimes of input cable (Yazar 2013). PON and P2P topologies are compared with each other in Figure 1. Speed for CO is considered equal to 2000 MB/s and the target value for each demand point is supposed at least 200 Mb/s.

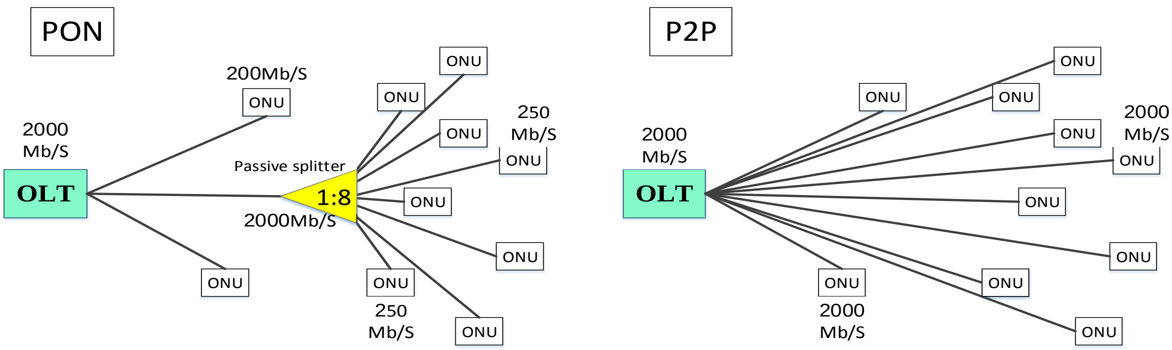

Figure 1. Comparison of PON and P2P topologies

The top layer and lower layers in hierarchical network structure can be different. The backbone networks consist of central station connection at the highest level and tributary or access network which feeds the connection between the end-user and backbone. Generally, due to the large size nature of applications of these problems in real world cases, the backbone and tributary networks are considered independent. Bely and Koch (2002) proposed an integer linear programming Model with a distinct formulation for Backbone and access networks considering survival and routing constraints in Backbone level. In this model, levels are not considered as an integrated approach, and sequential solving method is used and nodes are connected to each other with a tree structure.

Some studies tried to solve access network design without considering the backbone network. Gavish et al. (1992) used a heuristic method intending to solve the problem of single period access network design using tree/ring topology with respect to a capacity constraint. Other notable studies in access network design are (Balakrishnan et al. 1991; Chamberland 2010). Also, some studies considered backbone level individually such as (Din 2008; Ho et al. 2006).

For the reason that levels interact with each other, the backbone and access layers must be considered simultaneously to ensure reaching to an optimal solution. Chung et al. (1992) proposed a binary quadratic programming model for a two-level fully interconnected/star problem. The aim of the model was minimizing the hub installation costs, allocating destination points to the hubs and also the connections between hubs. Another example of fully interconnected/star network is provided in Alumur and Kara (2008). Saboury et al. (2013) proposed a new mathematical formulation for a two-level network by full connection in each level using two hybrid algorithms. The basis of the proposed algorithms is adding Variable Neighborhood Search (VNS) to the general structure of simulated annealing (SA) and tabu search (TS) algorithms. Other studies with fully interconnection topology include (Thomadsen and Larsen 2007, Ortiz-Garcia et al. 2009). Helme and Magnati (1989) proposed a binary quadric programming for two-layer star/star satellite network connection. The aim of proposed models was to locate ground stations and allocate each demand point to one ground station with respect to capacity limit for ground station in such a way to minimize hub installation costs and allocating demand points to each hub. Thomadesnand Stidsen (2005) tackled two-level networks problem by using branch-and-price algorithm. The topology used in each of these levels was the ring type. The model seeks to minimize the constant link installation costs in addition to the link capacity costs. Another study that used ring topology is a paper written by(Labbe et al. 2004).Other topologies used in two-level network literature are tree/star (Kim and Tcha 1992), hub/line (Martins et al. 2015).

All of the aforementioned papers were related to the design of the network in one or two layers. Yaman (2009) presented a mixed integer programming (MIP) model in three layers in which full interconnection is used in the first layer and in the second and third layers star/star topology has been used. In this model, a number of hubs are predetermined and the model wants to minimize the cost of the route, by considering time constraints and quality of service. Also, Elmastas (2006) presented a three-level star-mesh-star network model to transit cargo between Istanbul and Ankara with consideration of time delivery constraint. The aim of the proposed model was to minimize the air and ground transit costs between all origin and destination points.

Location of fiber-optic equipment has been studied from various aspects. Yazar (2013) studied two different problems “green field” and “re-design” independently. In “green field”, the aim is to design the least cost fiber optical network that will provide high bandwidth from a given central hub to a set of demand nodes by considering insertion loss limitation. Moreover, in “re-design” the aim is to improve the current service level through adding fiber-optic to copper cable networks. Yazar et al. (2016) investigated problems originating from one of the largest Internet service providers operating in Turkey. The corporation essentially encounters two distinct design problems: the green field design and the copper field re-design. Mathematical models were presented for both problem specifications and extensive computational results based on realistic data were provided in this paper.

Li and Shen (2009) proposed a mathematical formulation to minimize the deployment cost of a single-stage topology based PON. Their presented model was non-linear optimization model. The authors supposed that the cost of a splitter is linearly proportional to the number of output ports of the corresponding splitter. They described their mathematical model as intractable due to its non-linearity and NP-complete nature.

Chardy et al. (2012) considered the formation of multiple PONs applying two splitting stage solutions for each PON in which the splitting ratio of each stage is determined. They addressed densely populated urban scenarios with available network. They proposed an integer linear programming (ILP) model with additional strengthening constraints and graph reduction method which can gain valid solutions with small gaps for real case problems.

Kim et al. (2011) considered the formation of PON access networks in which the goal is to minimize the network cost by suitable location of the optical splitters. They presented the mathematical model for the problem as a multi-level capacitated facility location problem (FLP) on an underlying tree topology with non-linear link costs.

Some other related studies concentrate on the location of regenerator in fiber-optic network. Albeit, fiber-optic is capable of transmission of an approximately infinite amount of bandwidth, it has limitations in the length of transmission. In other words, as we go away from the distribution source, the signal becomes weaker. To compensate this insertion loss, regenerators are used. Some examples are provided in (Chen et al.2010; Duarte et al. 2014).

In summary, details of some researches which are done in the field of multi-level network communication is are provided in Table 1.

Table 1. Comparison the papers in the literature of network design problem

Authors	Number of Layers	Facility type		Network topology		Capacity		Characteristics/note	Method
		single	multi	Backbone	access	Cap.	Uncap.		
Saboury, et al. (2013)	2	√	-	Complete	Complete	-	√	Quadratic integer programming model with its linearized version is proposed	SA_{VNS} and TS_{VNS}
Yaman, and Carello (2005)	2	√	-	Complete	Star	√	-	mixed integer programming formulation is proposed for the hub location problem with modular link capacities(HMLC)	Branch and cut/ Tabu search
Liefooghe et al. (2010)	2	√	-	Ring	Star	√	-	Bi-objective (assignment cost from non-visited nodes &ring cost) model proposed	combining SEEA and the non-iterative version of IBMOLS
Thomadsen and Larsen (2007)	2	√	-	Complete	Complete	-	√	development of two different formulations	Branch-and-price
Yaman (2009)	3	√	-	Complete	Star-Star	-	√	mixed integer programming model is proposed by considering service quality and delivery time restriction- the Number of hubs is predetermined	GAMS 22.5 and CPLEX 11.0.0
Thomadsen, and stidsen (2005)	2	√	-	Ring	Ring	√	-	both the fixed link establishment costs and the link capacity costs into account- Computational results are given for networks with up to 36 nodes	Branch and price
Yazar (2013)	2	-	√	-	Tree-star	√	-	It is used for fiber optical networks by bandwidth level and distance restriction	Gurobi 5.0.2 and Cplex 12.4
Elmastas (2006)	3	√	-	Star	Mesh-Star	-	√	mixed integer linear programs for cargo delivery network design proposed which uses both airplanes and truck with time restriction	CPLEX 9.1
Li and Shen (2009)	2	-	√	-	Star-Star	√	-	Nonlinear model for the deployment fiber-optic in a single-stage architecture based PON	Recursive Association & Relocation Algorithm (RARA)
Helme and Magnanti (1989)	2	√	-	-	Star-Star	-	√	formulated a zero-one quadratic programming model for satellite communication networks	branch and bound algorithm

Table 1. Continued

Authors	Number of Layers	Facility type		Network topology		Capacity		Characteristics/note	Method
		single	multi	Backbone	access	Cap.	Uncap.		
Kim et al. (2011)	2	√	-	-	Tree-star	√	-	deals with a physical access network design problem of fiber-to-the-home multi-stage architecture based PON	Objective function relaxation approach
This paper	3	-	√	Ring	Star-Star	√	-	It is used for designing fiber optical networks by bandwidth level restriction-predetermined clustering	Meta-heuristic Method(GA &DE)

All previous studies have addressed the fiber optic network design problem only in access level. But, to reach an optimal or near optimal solution, just like other communication networks, the access, and backbone levels should be considered integrated.

In this paper, the design of optical fiber communication network is studied in three layers. The case study of this paper encompasses from some sectors and clustering of nodes are predetermined. In backbone network which provides the connection between COs, the ring topology has been used, since this topology is more efficient than other topologies such as mesh and fully-interconnect. Also, in ring structure each node is connected to two nodes and allows us to replace the other, in a case of failure, so the system has high reliability. In addition, access networks are considered to be single-level PON topology in which the splitters are connected directly to the CO and each demand node. The general schema of this problem is shown in Figure 2.

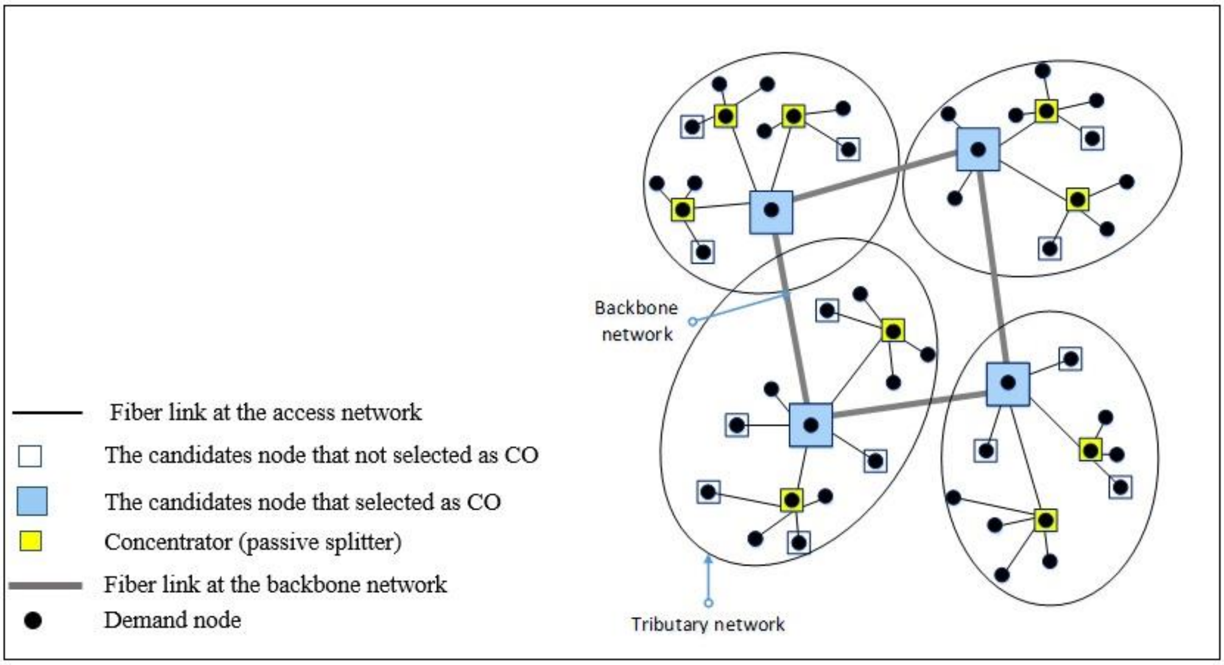

Figure 2. An example of three-layered ring backbone and star/star access network with 4 sector Predetermined

The rest of the paper is organized as follows: In Section 2, description of the mathematical model, and linearization of the proposed model are presented. In section 3, two proposed meta-heuristic algorithms are explained in detail and a simple way to code the problem is introduced. In section4, some test problems are conducted and the results of two algorithms are compared with each other. In section 5 a sensitivity analysis is done. Finally, conclusion remarks and future research are provided in section 6.

2. Problem description

In this research, a new mathematical model is presented for the three-level fiber-optic, by regarding simultaneous backbone and local access networks. The real case condition are considered in The model and it aims to locate CO’s and specify the connections between them, determining number and location of DC’s, allocating each demand point to either one of DC’s or CO and determining the type of splitter used in each DC in a way that the cost of fiber optic wires and costs associated with DC (installation and passive splitter) are minimized.

2.2. Assumptions

- The area which is studied here consists of some sectors and each demand point belongs to one of these sectors.
- Each sector has two or more candidate nodes for CO and one of them is selected as a CO.
- To enhance network’s reliability, each CO is connected to two COs (the edges between COs form a ring in a way that sub-tour is not created.)
- Each of demand nodes is the potential node for DC.
- CO’s bandwidth and requested bandwidth of each demand node is fixed and determined.
- Each demand point either feed directly from the CO or are allocated indirectly to one concentrator from the same sector.
- Each of concentrators , is connected directly to the CO in its own sector.
- The cost of establishment of DC is known.
- The cost of each type of splitter used in the DC is known.
- Cost of fiber optical wiring in each level is known.
- After passing through the splitter type *k*, $2^{(k-1)}$ outlet cables would exist that the bandwidth of each outlet cable is $2^{-(k-1)}$ times of inlet cables.

2.2. Indices

i Index of demand nodes ($i = 1, \ldots, I$),
k Index of passive splitter types ($k = 1, \ldots, K$),
p Index of sectors ($p = 1, \ldots, P$),

2.3. Parameters

I Number of nodes,
K Number of passive splitter types,
P Number of sectors,
COB Bandwidth at Central Office,
F_k Port number in the passive splitter *k*,
De_i Required bandwidth at demand node *i*,
C_1 Cost of transfer one unit of fiber optic between the demand nodes and DC,
C_2 Cost of transfer between the DC and CO,
C_3 Cost of transfer between CO,
Ces Fixed cost to establish a DC,
Cps_k Fixed cost to establish a DC,
d_{ij} Shortest distance between demand node *i* and *j*,
Y_{ip} 1; If node i is belonged to sector p, 0; otherwise,
Ca_{ip} 1; If node i is candidate as CO in sector p, 0; otherwise,
MM A big number,

2.4. Decision variables

A_{ij} 1 if demand node *I* is allocated to node *j*; otherwise 0.
S_{ik} 1if passive splitter type *k* is in node *i*; otherwise 0.
X_{pq} 1 if there is a connection between sector p and *q*; otherwise 0.
W_{ip} 1 if the node *i* in sector *p* is selected as a CO; otherwise 0.
M_i the speed at the node *i*

2.5. Mathematical formulation

$$MinZ = \sum_{p=1}^{P}\sum_{q=1}^{P}\sum_{i=1}^{I}\sum_{j=1}^{I} X_{pq} W_{ip} W_{jq} d_{ij} C_3 + \sum_{k=1}^{K}\sum_{q=1}^{P}\sum_{i=1}^{I}\sum_{j=1}^{I} S_{ik} Y_{ip} W_{jp} d_{ij} C_2 \tag{1}$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{I} A_{ij} d_{ij} C_1 + \sum_{k=1}^{K}\sum_{i=1}^{I} S_{ik} (Csp_k + Ces)$$

s.t.

$$\sum_{i=1}^{I} W_{ip} = 1 \ \forall p \tag{2}$$

$$W_{ip} \le Ca_{ip} \ \forall i, p \tag{3}$$

$$\sum_{q=1, q\ne p}^{P} X_{pq} = 1 \ \forall p \tag{4}$$

$$\sum_{p=1, p\ne q}^{P} X_{pq} = 1 \ \forall q \tag{5}$$

$$U_p - U_q + P.X_{pq} \le P - 1 \ \forall q, p; 2 \le p \ne q \le P \tag{6}$$

$$\sum_{j=1}^{I} A_{ij} = 1 \ \forall i \tag{7}$$

$$A_{ij} \le \sum_{k=1}^{K} S_{jk} + \sum_{p=1}^{P} W_{jp} \ \forall i, j \tag{8}$$

$$\left| Y_{ip} - Y_{jp} \right| \le MM (1 - A_{ij}) \ \forall i, j, p \tag{9}$$

$$\sum_{i=1}^{I} A_{ij} - MM (\sum_{p=1}^{P} w_{jp}) \le \sum_{k=1}^{K} S_{j,k} 2^{F_k} \ \forall j \tag{10}$$

$$\sum_{k=1}^{K} S_{ik} \le 1 \ \forall i \tag{11}$$

$$M_i \le COB \ \forall p \tag{12}$$

$$M_i \le \frac{COB}{2^{F_k}} + COB(1 - A_{ij}) + COB(1 - S_{jk}) \ \forall i, j, k \tag{13}$$

$$M_i \ge De_i \ \forall i \tag{14}$$

$$W_{ip}, X_{pq}, A_{ij}, S_{jk} \in \{0,1\}, M_i \ge 0, U_p \ge 0 Integer \tag{15}$$

Objective function (1) minimizes the total cost in 3-layers fiber optical network including the cost of connection between CO's in backbone network (i.e., term 1), the cost of connection between CO's and DC's (i.e., term 2), the cost of fiber-optic transition from DC's to the demand nodes (i.e., term 3), and finally the cost of establishment of DC and splitter used in them (i.e., term 4).

Equations (2) and (3) cause that one candidate node to be chosen as the CO in each sector. Equations (4) and (5) determine how are the connections between CO's (Condition to create a loop in backbone network). Equation (6) prevents a formation of sub-tour. Equations (7) and (8) ensure that each of demand node is allocated to one of DC's or CO for feeding. Equation (9) guarantees that each of demand nodes is allocated to a DC or CO belonging to its own sector Equation (10) declares that outlet cables from the splitter in each DC would not be less than demand nodes which are fed from this DC. Equation (11) ensures that each DC do not uses more than one type splitter Equation (12) shows that the speed in each node cannot be more than CO's bandwidth. Equation (13) calculates the maximum speed of each demand node considering the type of passive splitter used in its route. Equation (14) ensures that the speed of each demand node is not less than required bandwidth.

2.6. Linearization of the proposed model

In this section, we aim to linearize the proposed mathematical model. Equation (1) is a nonlinear integer equation; that is, in the first and second terms the binary variables are multiplied. To transform this equation into linear one, two positive variables ($L1_{ij}$,$L2_{ij}$) are defined and the objective function is rewritten as equation (16) and then constraints of (17)-(19) are added to the problem.

$$MinZ = \sum_{p=1}^{P}\sum_{q=1}^{P}\sum_{i=1}^{I}\sum_{j=1}^{I} L1_{ij} D_{ij} C_3 + \sum_{k=1}^{K}\sum_{q=1}^{P}\sum_{i=1}^{I}\sum_{j=1}^{I} L2_{ij} D_{ij} C_2 + \sum_{k=1}^{K}\sum_{i=1}^{I} S_{ik} (Csp_k + C_4) + \sum_{i=1}^{I}\sum_{j=1}^{I} A_{ij} D_{ij} C_1 \quad (16)$$

$$X_{pq} + W_{ip} + W_{jp} \le L1_{ij} + 2 \quad \forall i, j, p, q \quad (17)$$

$$S_{ik} + W_{jp} + Y_{ip} \le L2_{ij} + 2 \quad \forall i, j, p, k \quad (18)$$

$$L1_{ij}, L2_{ij} \ge 0 \quad \forall i, j \quad (19)$$

Also in equation (9) absolute term is used to linearize this constraint, we define the auxiliary variable $E1_{ij}$ and then replace equation (9) with the equations (20)-(23).

$$Y_{ip} - Y_{jp} \le E1_{ij} \quad \forall i, j, p \quad (20)$$

$$Y_{jp} - Y_{ip} \le E1_{ij} \quad \forall i, j, p \quad (21)$$

$$E1_{ij} \le MM(1 - A_{ij}) \quad \forall i, j \quad (22)$$

$$E1_{ij} \ge 0 \quad \forall i, j \quad (23)$$

3. Methodology

Due to the NP-hardness of the model (Abyazi-Sani and Ghanbari, 2016), first the problem is validated by GAMS software in small and medium scales, then model is solved by two meta-heuristic methods including Genetic Algorithm (GA) and Differential Evolution (DE) in larger scaled problems.GA is previously used by some researchers for similar problems (Kokangul and Ari, 2011; Hostage and Goodchild, 1986; Topcuoglu et al.2005). In this work, we will compare the performance of DE algorithm as the algorithm that has not used by other researchers in the literature for this type of problem, with the GA algorithm. We explain two algorithms in detail as follows:

3.1. Genetic algorithm

The basic principles of Genetic Algorithm (GA) were first proposed by John Holland (1975), and further described by Golberg (1989).GA is a one of the population based method which is applied extensively in optimization problems(Rabbani et al., 2016; Farrokhi-Asl et al., 2016; Rabbani et al., 2017). This algorithm is similar to -+gene functionally. Each chromosome is a series of numbers which is called a gene. In every iteration, the algorithm uses the mutation and crossover operators to create new chromosomes called "offspring". Then offsprings are evaluated, eligible chromosomes are chosen by using the selection operator and transferred to the next generation. GA main structure is shown in Figure 3.

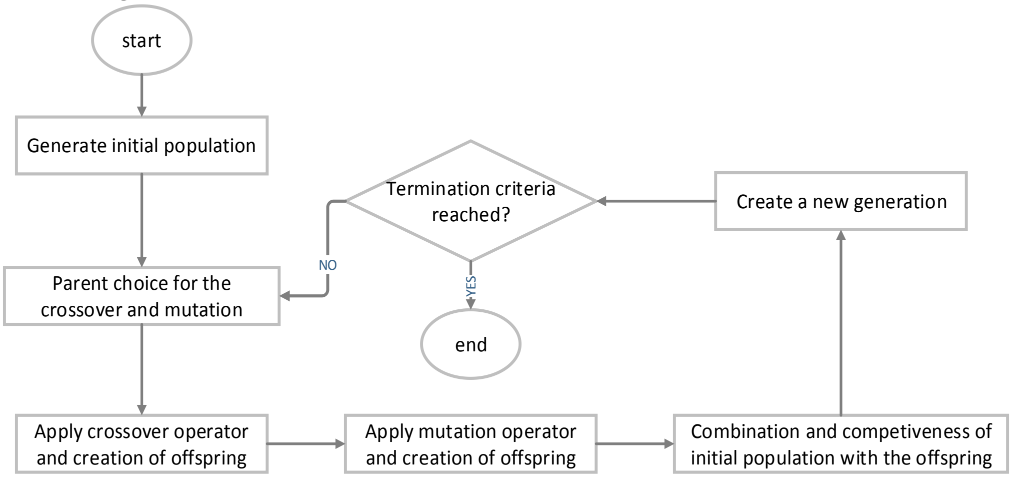

Figure 3. Flowchart of the GA

3.2. Differential evolution

The Differential Evolution (DE) algorithm is a type of evolutionary algorithm and population-based which is formed based on geometric search technique. This algorithm was first proposed by Storn and Price (1995). The difference between DE and rest of evolutionary algorithms is formation of test parameter vector which its survival or extinction in the next generation is defined by the selection operator. In this algorithm, the new member is formed by adding the weight difference vector between two population members to the third member. If the newly formed member has less objective function that a previously defined population member, the former will replace the latter. The main DE structure is shown in Figure 4.

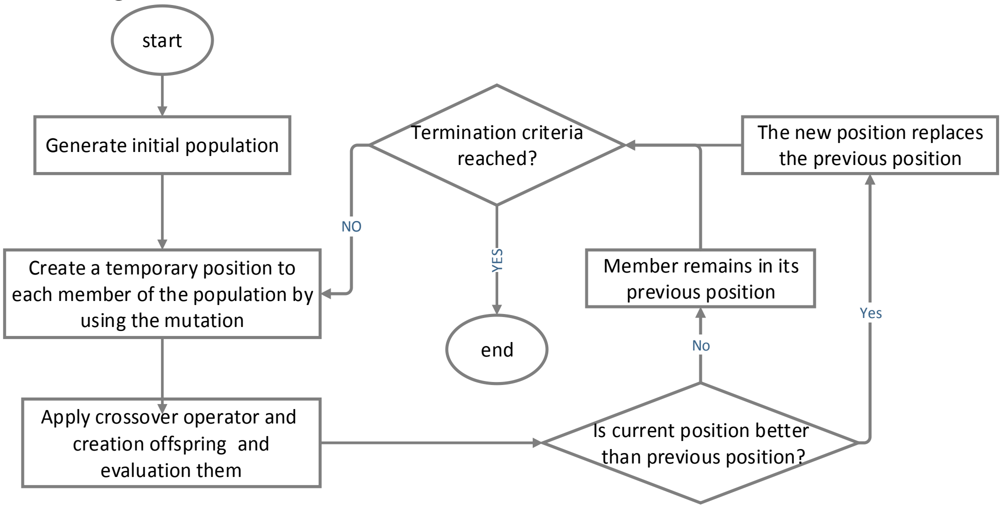

Figure 4. Flowchart of the DE

3.3. Solution representation

At first, we will define CO's location plus DC's quantity and location in a random fashion with the string of numbers. Then, we will define how the different sectors are connected, the type of splitter used in each DC and how the demand nodes will be allocated by using exact and heuristic methods in each step. To do this, first we define the auxiliary variable, V_i, as follows:

$$V_i = \sum_{k=1}^{K} S_{ik} \tag{24}$$

The variable V_i shows either the node i is DC or not. Then, we create a chromosome with I+1 cells in, where I represents the number of nodes and a number from the interval [0, 1] is assigned to each cell of (the) chromosome.

Then using the created random number in chromosome's last cell (θ), we can calculate the total number of DC's by equation (25).

$$\alpha = [\theta \times I] \tag{25}$$

Then first α number of cells which have less corresponding random number are selected and their corresponding V_i are set to 1 and V_i for rest of cells are set to 0.

Since no splitter is installed in CO node, in each sector among the candidate nodes for CO, the node which has the most quantity of random number will be chosen as the CO.

To determine the connection between sectors, we encounter with a type of Travelling Salesman problem (TSP). In TSP, some cities exist and the cost of directly travelling between each pair of nodes is known. We seek to determine the shortest path starting from one city, passing through all other cities just once, and then come back to the same city. In the sub-problem of model, the CO nodes are considered as cities also the distance between cities is equal to cost of direct travelling between them. And since the number of sector is not much, we use dynamic programming to calculate the exact solution of proposed TSP.

Each demand node is allocate to the nearest CO or DC of its own sector (A_{ij} is determined) and by considering allocated demand point to each concentrator, the type of required splitter in each DC will be determined (S_{ik} is determined). An example with consideration of I=12 and P=4 is shown in Table 2.

Table 2. Example of initial solution generation for model

		P=1			*P*=2			*P*=3			*P*=4		
Cell numbers	1	2	3	4	5	6	7	8	9	10	11	12	13
Random number	0.26	0.54	0.45	0.03	0.86	0.63	0.59	0.21	0.83	0.91	0.73	0.43	0.34
Sort random number &calculate α	3	6	5	1	11	8	7	2	10	12	9	4	$\alpha = 5$
V_i	1	0	1	1	0	0	0	1	0	0	0	1	-
Ca_{io}	1	1	0	1	1	0	1	0	1	0	1	1	-
W_{ip}	0	1	0	0	1	0	0	0	1	0	1	0	-

Finally, after finding the binary variables $A_{ij}(\forall\, i,j)$ and $S_{ik}(\forall\, i,k)$ by using equation (13),we can calculate $M_i(\forall i)$ variable. If we trespass from equations (14), a big penalty for objective function will be considered.

3.4. Crossover operator

Three crossover types including single-point, double-point and uniform are used in GA and DE algorithms and in every iteration one of them is chosen randomly. If we call the number of chromosome's cell as a "*length*", in single point crossover a number in [1, *length*-1] interval is chosen and two parents are cut away from the aforementioned point. Then they are mixed to generate the off springs. The double-point crossover performance is much like the single point except that parent chromosomes are cut in two points, then to generate off springs, parents are mixed together from cut points. In uniform crossover a randomly binary chromosome with the same size of parent chromosomes is created. This chromosome specifies from which parent each of genes will inherit two children. Figure 5 displays an example of each of crossover's performance.

Parent1	0.23	0.58	0.01	0.97	0.17	0.64	0.25	0.88	0.35	0.81	0.62	0.45
Parent2	0.39	0.71	0.53	0.69	0.11	0.42	0.04	0.73	0.18	0.27	0.22	0.55

Single-point crossover rand-number=3

Offspring1	0.23	0.58	0.01	0.69	0.11	0.42	0.04	0.73	0.18	0.27	0.22	0.55
Offspring2	0.39	0.71	0.53	0.97	0.17	0.64	0.25	0.88	0.35	0.81	0.62	0.45

double-point crossover rand-numbers=2,6

Offspring1	0.23	0.58	0.53	0.69	0.11	0.42	0.25	0.88	0.35	0.81	0.62	0.45
Offspring2	0.39	0.71	0.01	0.97	0.17	0.64	0.04	0.73	0.18	0.27	0.22	0.55

Uniform crossover	0	1	1	0	0	1	0	1	0	0	1	0
Offspring1	0.39	0.58	0.01	0.69	0.11	0.64	0.04	0.88	0.18	0.27	0.62	0.55
Offspring2	0.23	0.71	0.53	0.97	0.17	.0.42	0.25	0.73	0.35	0.81	0.22	0.45

Figure 5. Example of the single-point, double-point and uniform crossover operator

3.5. Mutation operator

The performance of mutation operators used in each DE and GA algorithms is based on displacement of parent chromosome cells. In every iteration, to generate a child one or more pairs from chromosome's cell is randomly chosen (based on input parameter relating to mutation) which will be substituted together. Figure 6 presents an example of mutation operator.

Parent	.350	.280	0.11	.850	0.25	0.51	0.72	0.93	0.28	0.81	0.71	0.48

$\lambda = .1 \rightarrow N = 2 \times \lfloor 0.1 \times 12 \rfloor = 2$ Rand-numbers= 2,8

offspring	.350	.930	0.11	.850	0.25	0.51	0.72	0.28	0.28	.810	0.71	0.48

$\lambda = .2 \rightarrow N = 2 \times \lfloor 0.2 \times 12 \rfloor = 4$ Rand-numbers= 1,6,3,10

offspring	.510	.280	0.81	.850	0.25	0.35	0.72	0.93	0.28	0.11	0.71	0.48

Figure6. Example of the mutation operator

4. Numerical experiments

Two algorithms are coding in MATLAB R2013a software on a personal computer ASUS Core i7, 2.2 GHz with 4GB RAM.16 test problem are conducted to verify the performance of this model, 8 of them are small and medium size and others are large-size. The parameters of the test problem are shown in Tables 3 and 4.

Table 3. Data of the test problems

Parameter	Value	Parameter	Value
C_1	2$	*COB*	2000
C_2	3$	*De*	U(80,120) Mb/s
C_3	4$	Cps_k	15(*k*-1) $
Ces	250$	-	-

Table 4. Problem sets for small and medium size problems

Small and medium size				**Large size**			
Test problem	Number of node (*I*)	Number of sector (*P*)	Number of candidate node in each sector ($\sum_i C_{ip}$)	Test problem	Number of node (*I*)	Number of sector (*P*)	Number of candidate node in each sector ($\sum_i C_{ip}$)
1	12	2	2	9	80	3	2
2	20	2	2	10	100	4	3
3	25	3	2	11	130	4	4
4	30	3	3	12	200	4	4
5	40	3	3	13	300	6	5
6	60	2	2	14	450	6	5
7	60	3	3	15	600	4	6
8	70	4	3	16	800	4	6

4.1. Parameters tuning

The optimum value of input parameters of DE and GA algorithms in small and large size problems is (-,are) determined by designing test in Minitab software with the application of TAGUCHI method. The final results are shown in Table 5.

4.2 Meta-heuristic algorithms comparison

In this part, the test problems are solved using DE and GA algorithms, then results of two algorithms are compared with respect to time and objective function value. Each test problem is run for 5 times and the average of computational time and objective function value in small, medium and large sizes are shown in Table 6 and Table 7, respectively. Figure 7 shows CPU times related to the GAMS, GA and DE for test problems. Also, Figure 8 shows the result of test problem 13 obtained by DE. It should be mentioned that GAMS software utilizes the branch and bound (B&B) algorithm which is a well-known exact algorithm to find optimal solution for mathematical formulation.

Table 5. Results of parameters tuning for GA and DE

GA	Population	Number of generation	Crossover rate	Mutation rate
Small-size	100	100	0.80	0.3
Large-size	80	100	0.70	0.2
DE	Population	Number of generation	Crossover Probability	Vector factor
Small-size	110	100	0.2	U(0.3,1)
Large-size	100	100	0.25	U(0.2,8)

Table 6. Comparison between B&B, DE and GA runs for small and medium-sized test problems

Test problem	*B&B*		*DE*			*GA*		
	Z^{best}	*CPU Time*	Z^{mean}	*CPU Time*	*GAP*	Z^{mean}	*CPU Time*	*GAP*
1	**3628**	00: 00: 01	**3628**	00: 00: 02	0	**3628**	00: 00: 03	0
2	**3800**	00: 00: 02	**3800**	00: 00: 04	0	**3800**	00: 00: 6	0
3	**7364**	00: 00: 03	**7364**	00: 00: 05	0	**7364**	00: 00: 08	0
4	**7655**	00: 00: 17	**7655**	00: 00: 07	0	7660	00: 00: 11	.0006
5	10213	00: 00: 23	10215	00: 00: 10	.0001	10219	00: 00: 14	.0005
6	8981	00: 10: 33	9007	00: 00: 17	.002	8995	00: 00: 25	.001
7	12047	00: 15: 21	12169	00: 00: 20	.010	1270	00: 00: 28	.010
8	11421	00: 16: 51	11533	00: 00: 24	.009	11547	00: 00: 33	.011

Table 7. Comparison between B&B, DE and GA runs for large-sized test problems

Test problem	*B&B*		*DE*			*GA*		
	Z^{best}	*CPU Time*	Z^{mean}	*CPU Time*	*GAP*	Z^{mean}	*CPU Time*	*GAP*
9	15615	00: 37: 03	15832	00: 00: 31	.013	15782	00: 00: 46	.010
10	15892	01: 28: 51	16130	00: 00: 47	.015	16218	00: 01: 12	.020
11	N/A	N/A	17324	00: 01: 23	N/A	17588	00: 01: 56	N/A
12	N/A	N/A	25348	00: 03: 01	N/A	25151	00: 04: 51	N/A
13	N/A	N/A	39083	00: 07: 40	N/A	39145	00: 11: 23	N/A
14	N/A	N/A	56175	00: 17: 29	N/A	55824	00: 23: 31	N/A
15	N/A	N/A	68524	00: 28: 23	N/A	69216	00: 42: 57	N/A
16	N/A	N/A	84638	00: 55: 09	N/A	85994	01: 15: 12	N/A

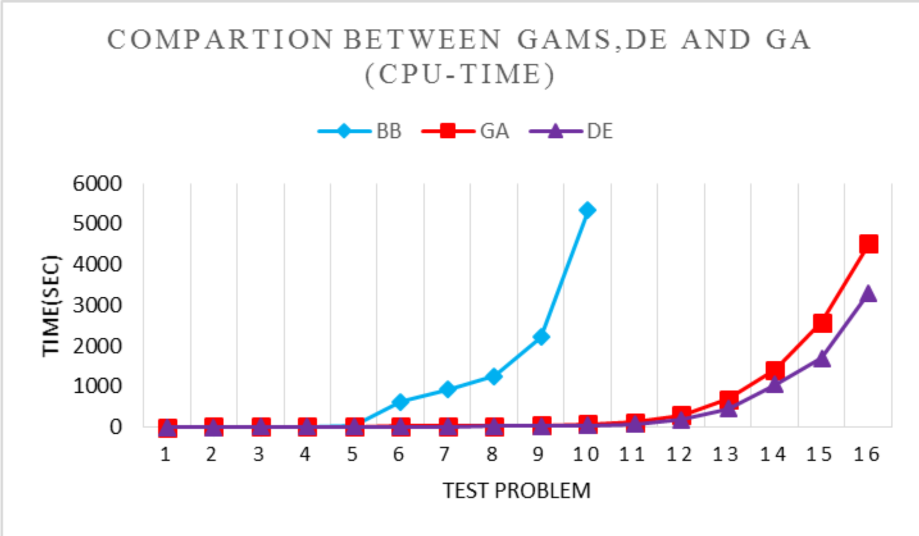

Figure7.Comparison between CPU times related to the B&B, GA and DE: small and medium-sized problems and large-sized problems

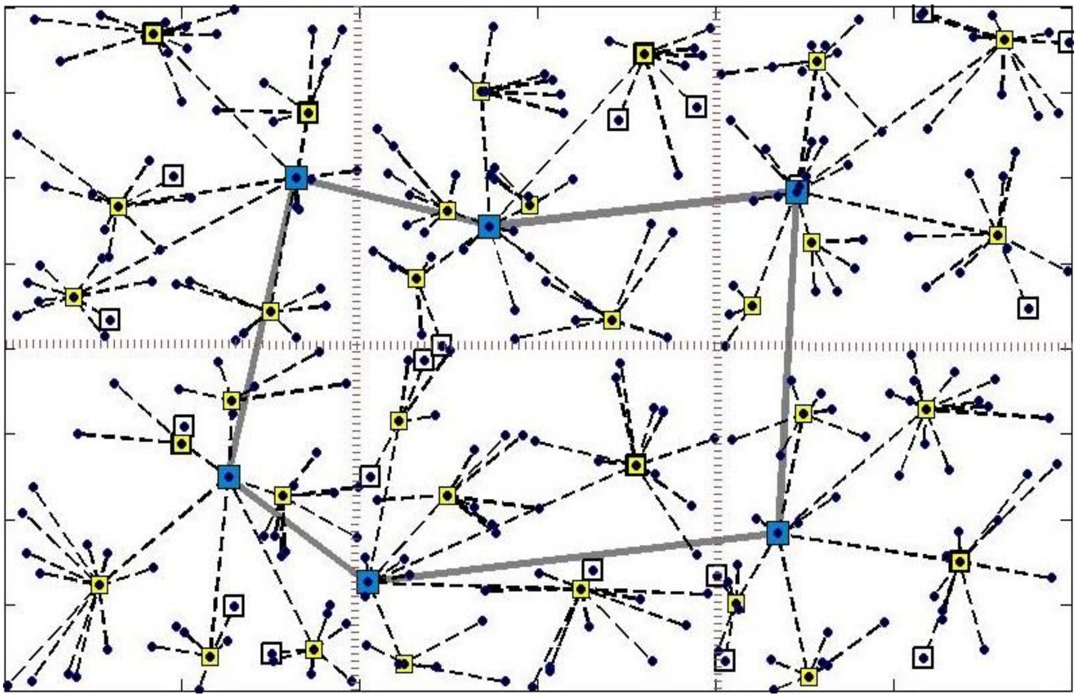

Figure 8. The result of test problem 13 obtained by DE

The result of experiment show that the DE algorithm in small and medium-sizes has a fairly better objective function value than the GA but in large-sizes the GA algorithm has a better performance, also the process time for DE algorithm is less than that of GA in problems of every sizes.

5. sensitivity analysis

As it is shown in Equations (1), in fiber optic network there are two types of separable costs. The first type is the costs associated with fiber optic wire (connection links in different levels), and the second type is the costs associated with DC (consisting the installation costs and passive splitters). In model analysis, we change the proportion between these costs using alpha (α) value. When alpha is equal to 1, it shows that costs are considered based on Table 3.As alpha ascends, the proportion of DC costs to fiber optic wireless increases. Reversely, as alpha descends toward zero, the associated DC costs fall down and finally are considered as zero. Sensitivity analysis results based on alpha changes for test problem 8 using GAMS software are shown in Table 8.

Table 8. Sensitivity analysis based on value of α

α	Number of DC	Passive splitter usage					CPU Time	fiber optical wiring cost	Objective function value
		1:2	1:4	1:8	1:16	1:32			
10	0	-	-	-	-	-	00: 00: 03	13486	13486
5	0	-	-	-	-	-	00: 00: 04	13486	13486
3	1	-	-	1	-	-	00: 00: 21	12800	12935
2	2	-	-	1	1	-	00: 01: 08	12169	12379
1.5	3	-	-	2	1	-	00: 09: 32	11732	11957
1	4	-	-	2	2	-	00: 16: 51	11211	11421
0.8	5	-	1	3	1	-	00: 19: 12	10731	10911
0.6	5	-	1	3	1	-	00: 36: 35	10731	10866
0.5	6	-	1	4	1	-	00: 41: 21	10568	10703
0.4	7	-	2	4	1	-	00: 58: 33	10403	10523
0.3	8	-	3	4	1	-	01: 36: 43	10196	10295
0.2	8	-	3	4	1	-	01: 43: 11	10196	10262
0.1	10	2	3	4	1	-	02: 16: 15	9787	9823
0.05	12	3	4	4	1	-	02: 48: 52	9495	9515
0	12	3	4	4	1	-	03: 04: 01	9495	9495

As results show, the minimum CPU time is 3 seconds which is related to $\alpha = 10$. But as the DC-related costs decrease the problem will be more complicated, therefore the CPU time increases greatly and the latest CPU times is about 3 hours.
For alpha greater or equal to 5there is no economic aspect in DC establishment so the fiber optic is connected directly from CO in each sector, by a star structure, to the demand nodes as shown in Figure 9.

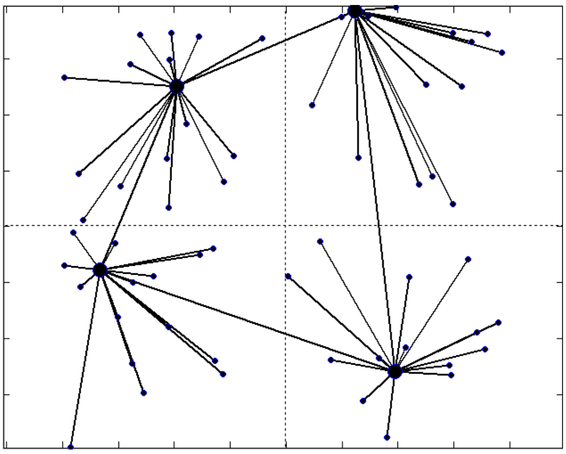

Figure9. Illustration of the example with $\alpha = 5$

The objective function value in $\alpha \geq 5$ is 13486, but as alpha drops further from 5, installing concentrators becomes more and more economically and demand nodes connect indirectly to their CO after passing splitters. Such as in $\alpha = 1$ that one DC is established in each sector and objective function value is decreased 2065 units comparing to the case when concentrators are not used. As shown in Figure 10.

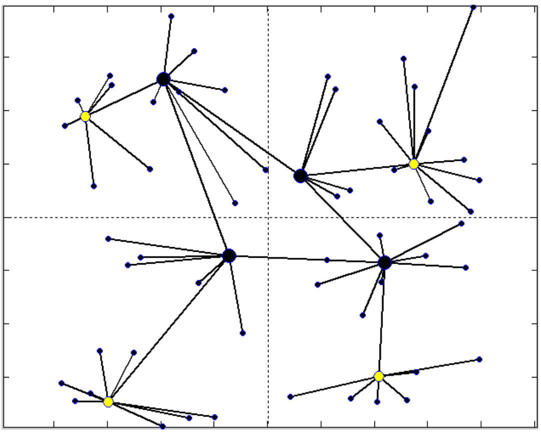

Figure10. Illustration of the example with $\alpha = 1$

Also, we can see as alpha decreases, the number of DC's are increased gradually. Therefore objective function value is decreased. In $\alpha = 0.05$ the number of DCs is 12 (as shown in Figure 11) and fiber optical wiring cost decreases (by 3991 units) comparing to the case when concentrators are not used. But, as alpha drops further than 0.05, this would not affect the network structure and fiber optical wiring costs.A notable aspect is that the passive splitter decreases the bandwidth at the demand node.

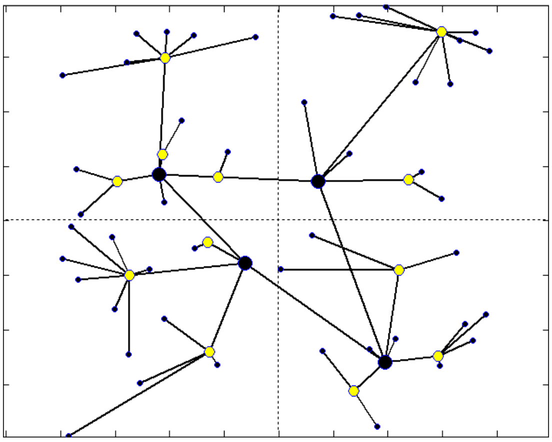

Figure 11. Illustration of the example with $\alpha = 0.05$

6. conclusion

In the two recent decades, use of fiber-optic due to having more speed and capacity and also being more cost-efficient compared to the other information transfer tools has widely grown. In this study, linear mathematical model was presented in three layers to transmit fiber-optic to demand points, considering simultaneous backbone and access networks, in such a way that backbone is ring and the access has a single-stage architecture based PON. The aim of proposed model is to locate the CO's and concentrators in each sector, allocate each demand node to either DC or CO and also determine the type of connection between CO's in a way that the costs associated with connection links and installation of DCs be minimized and each user's desired bandwidth is also provided. Due to the NP-Hardness of the model, first the problem is validated by GAMS software in small-sized problems then, two meta-heuristic algorithms, DE and GA, are presented to solve the problems of large sizes and an easy method to code the problem is proposed. To validate the problem, 16 test problems in small and large-sizes were conducted. Two algorithms were applied to solve a problem and the obtained results were compared with each other. The result showed that the DE algorithm is proper for small and medium sizes and for large sizes GA is preferred. Also DE algorithm has less CPU time than GA.
The results of sensitivity analysis on costs showed that, reducing the concentrators' costs will result in increasing the number of DC's and consequently fiber optic costs and also the total costs will decrease. However, this will add to the complexity and calculation time.
For future researches considering other topologies like loop and complete connection in local and backbone level to create more reliability in network, developing the problem for multi-period and dynamic models, adding signal power limitation and locating regenerators to the proposed model also designing the model using other heuristic and meta-heuristic solution approaches are proposed.

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