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# An Augmented Common Weight Data Envelopment Analysis for Material Selection in High-tech Industries

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**Abstract**

Material selection is a problematic issue in manufacturing processes. Inappropriate selected material may fail the manufacturing process or results in disaster for end users, especially in high-tech industries such as aircraft and shipping. A weighted linear optimization method (WLOM) in the class of data envelopment analysis is adopted to address material selection problem which deals with both qualitative and quantitative criteria, effectively. However, it is demonstrated that the adopted WLOM method is not able to produce a full ranking vector for the material selection problems borrowed from the literature. In this paper an augmented common weight data envelopment analysis model (ACWDEA) is developed with the aim of eliminating deficiencies of WLOM model. The proposed ACWDEA is able to produce full ranking vector in decision making problems with less computational complexity in superior to the WLOM. Also, the proposed ACWDEA determines the weight of qualitative and quantitative criteria precisely with solving a model without needing to any judgmental data. All the criteria will be involved in evaluation process with setting a lower bound for them. The presented ACWDEA can be used for any type of decision making problems as well as material selection problems. Finally, the robustness and effectiveness of the proposed ACWDEA method are evaluated through with solving two material selection problems and using Spearman's correlation tests.

**Keywords:** Material selection; Data envelopment analysis (DEA); Common weights; Multi-criteria decision making (MCDM).

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## 1. Introduction

Selecting the most appropriate material for the production processes plays an important role in the early stage of the product life cycle, especially in design and redesign processes (Edwards, 2005; Deng and Edwards, 2007). As there are a large number of criteria which ranges from mechanical to electrical characteristics, decision makers need to consider all the attributes simultaneously (Shanian and Savadogo, 2006.a). Material characteristics typically consist of mechanical, chemical, physical, magnetic, thermal and radiation, surface, and manufacturing properties in addition to availability, reliability, durability, cost, cultural aspect, and etc. (Crilly et al., 2004, Van Kesteren et al., 2007, Rao and Davim, 2008; Jahan et al., 2010). On the other hand, selecting improper material may result in failure in manufacturing processes, producing low-quality products, incurring extra costs, customer dissatisfaction, and decreasing operation performances (Torabi and Shokr, 2015; Jahan et al., 2011). Since both of the qualitative and quantitative criteria must be involved in material selection problems, it is a challenging issue to evaluate materials in the presence of different qualitative and quantitative criteria which are often in conflict together (Torabi and Shokr, 2015). It is noteworthy to mention that, criteria whose higher performance measures are preferable (i.e. quality) are known as positive criteria and criteria whose smaller values are desirable (i.e. cost) are called negative criteria. Criteria are often in conflict together. Therefore, it is vital to develop an efficient approach to determine the best material in engineering (Deng and Edwards, 2007) and production design stages (Cicek et al., 2010).

Multi-criteria decision making (MCDM) methods are widely-used techniques which can be applied as part of engineering design processes especially in material selection problems (Jahan and Edwards, 2015). Due to the simplicity and ease of applicability, MCDM techniques are often preferred to other methods for selecting the most proper material. A typical multi-attribute decision making (MADM) problem encompasses a number of alternatives and criteria both qualitative and quantitative. Alternatives should be evaluated after recognizing weight of criteria. Several MADM methods have been suggested by researchers to cope with material selection problems. Each of the MADM methods has some advantages and disadvantages and decision maker should select the best one case by case.

Jahan et el. (2011) proposed a new version of VIKOR method and applied it to material selection problems. They stated that their method could enhance the exactness of material selection results. Jeya and Vinodh (2012) applied fuzzy VIKOR for a material selection problem. As performance measures could be incomplete because of receiving in a linguistic manner from the decision maker, they used fuzzy numbers to overcome this difficulty. Jahan and Edwards (2012) studied the problem of material selection when a range of values for alternatives existed for each of criterion. For overcoming this difficulty they presented a VIKOR method with the ability of considering interval data. Jee and Kang (2000) exploited TOPSIS in order to obtain the most appropriate material for a flywheel material selection problem. Shanian and Savadago (2006.b) utilized Oidinary and Blok TOPSIS to enhance efficiency of their proposed method in a material selection problem. Rao and Davim (2008) utilized TOPSIS and AHP methods to present a procedure which is able to consider infinite number of qualitative and quantitative criteria in material selection problems. Chatterjee et al. (2009) utilized ELECTRE and VIKOR on the flywheel and the sailing boat material selection problems. Notably, ELECTRE and VIKOR belong to the outranking and compromising methods, respectively. Milani and Shanian (2006) employed ELECTRE III for the gear problem with consideration of incomplete data tradeoff and designers' preferences. Also, ELECTRE IV is applied by Shanian and Savadogo (2006.c) as a non-compensatory comprised solution to a material selection problem. ANP is utilized by Milati et al. (2013) on a case study to show how inner and outer dependencies among criteria and alternatives could affect the final ranking. Milani et al. (2005) investigated on the effect of different normalization techniques on the final rankings. Also, they exploited from Entropy and TOPSIS on a gear for power transmission material selection problem. Jahan et al. (2012) presented a new normalization method in addition to extending TOPSIS method. Their proposed method is able to find the best material where the current TOPSIS is deficient in ranking. Athawale et al. (2011) proposed a Utility Additive Method (UTA) as a mathematical programming in order to solve flywheel and sailboat material selection problems. Maniya and Bhatt (2010) developed preference selection index (PSI) method and represented its applicability on material selection problems. Mayyas et al. (2011) utilized a combination of quality function deployment (QFD) and AHP to find the best material in a vehicular structure material selection problem. QFD is exploited to collect customer needs and AHP is used to select the best material. Also, a combination of QFD and VIKOR is applied by Cavallini et al. (2013) to a material selection problem. Kumar and Singal (2015) employed AHP, TOPSIS, and a modified TOPSIS method in penstock and mild steel material selection problems. Chatterjee et al. (2011) proposed complex proportional assessment (COPRAS) and evaluation of mixed data (EVAMIX) as novel methods in MCDM. Applicability of the adopted methods is represented on material selection problems as well. Chatterjee and Chakraborty (2011) applied extended PROMETHEE II, COPRAS with gray relations, ORESTE, and operational competitiveness rating analysis (OCRA) as four MCDM methods on gear material problem. Chauhan and Vaish (2012) utilized Shannon's entropy method to find the weight of criteria and a combination of VIKOR and TOPSIS methods to determine the best material in a magnetic material selection problem. Extended TODIM, OCRA, ARS, and EVAMIX as MCDM methods are utilized by Dajri and Rao (2014.a) to find the best material in the sugar industry. Dajri and Rao (2014b) extended their previous work and compared four MCDM methods in the pipe material selection in the sugar industry in which Fuzzy AHP (FAHP) and TOPSIS, FAHP and VIKOR, FAHP and ELECTRE, and finally FAHP and PROMETHEE were applied. Liu et al. (2014) proposed a novel hybrid MCDM model with target based criteria consisting of a hybrid DEMATEL-ANP (DANP) and modified VIKOR. They showed that DEMATEL is a useful tool to model such problems when there are dependencies among the criteria in material selection problems.

Data Envelopment Analysis (DEA) is a well-known linear programming method which is able to calculate the efficiency of different decision making units (DMUs). There are several DEA-like models presented in the literature to evaluate DMU's efficiency scores in existence of exact or imprecise data (Cook et al., 1993; Cook et al., 1996; Zhu, 2003). Zhou et al. (2007) proposed a mathematical model which is able to produce the most favorable score for each DMU, but it has a poor discriminating power while assessing DMUs' efficiencies (Torabi and Hatefi, 2010). Torabi and Hatefi (2010) proposed a common weight DEA method with more discriminating power for evaluating DMUs' efficiencies. The proposed method by Torabi and Hatefi (2010) is not able to deal with qualitative criteria. Hatefi et al. (2014) proposed a new weighted linear optimization which is capable to cope with qualitative criteria as well as quantitative criteria.

In spite of several efforts to solve material selection problems, it is still necessary to develop appropriate methods to cope with qualitative criteria more precisely. Most of the reviewed MCDM methods on the material selection problems are not precise enough to deal with qualitative criteria. Typically Likert scale was used by previous reviewed methods to quantify the qualitative criteria. Likert scale is a simple method by which decision makers assign 1 to 9 to qualitative criteria in order to quantify them. Notably, evaluating DMUs with involving qualitative criteria more precisely is an important issue, especially in the material selection process in high-tech industries such as aircrafts, while improper material selection may lead to crisis for end-users.

Hatefi et al. (2014) proposed a weighted linear optimization method (WLOM) in the class of DEA-like models for evaluating efficiency of DMUs which is able to cope with qualitative criteria more precisely than other methods. Also, it requires less subjective data from decision makers (i.e. weight of each criterion) and is able to compute the weight of criteria with solving a mathematical model. Torabi and Shokr (2015) demonstrated that the WLOM (Hatefi et al., 2014) may be deficient with poor discriminating power in some cases of material selection problems in which full ranking vector might be not produced. Therefore, in this paper the WLOM model is modified to augment its discriminating power to be more efficient and practical. Table (1) represents the most important reviewed researches in the field of material selection problems.

**Table 1.** The most important reviewed methods

| Author | Problem | Quali-tative | Quant-itative | Quantifying | Weighting method | Assessing method |
|---|---|---|---|---|---|---|
| Khabbaz et al. (2009) | Spar of aircraft, Sailing boat | ✓ | ✓ | Fuzzy linguistic | Fuzzy linguistic | Fuzzy logic |
| Athawale et al. (2011) | Spar of aircraft, Sailing boat | ✓ | ✓ | Likert Scale | Likert Scale | Mathematical programming |
| Rao et al. (2008) | Metallic bipolar plate | ✓ | | | Comparison matrix | VIKOR |
| Chatterjee et al. (2009) | Flywheel, Sailing boat | ✓ | ✓ | Likert Scale | Comparison matrix | VIKOR, ELECTRE II |
| Jahan et al. (2011) | Metallic bipolar plate | ✓ | ✓ | Likert Scale | Delphi method | VIKOR |
| Jahan et al. (2010) | Flywheel, Spar of aircraft | ✓ | ✓ | Likert Scale | Comparison matrix | Linear assignment |
| Rao and Davim (2008). | Cryogenic storage tank | ✓ | ✓ | Likert Scale | Comparison matrix | AHP |
| Jee and Kang (2000) | Flywheel | ✓ | ✓ | Likert Scale | Subjective | TOPSIS |
| Shanian et al. (2006.b) | A bipolar plate | ✓ | | | Simos method | TOPSIS |
| Proposed ACWDEA | Spar of aircraft, Sailing boat | ✓ | ✓ | Mathematical concept | Mathematical programming | Mathematical programming |

Importance of material selection is described earlier. Every material has its own quantitative and qualitative characteristic which should be considered during assessing materials. On other hand quantifying qualitative criteria is an arguable issue in literature. Most of the reviewed methods used from Likert scale, judgmental data, and comparison matrices in order to quantify the qualitative criteria. Using these methods are not enough precise due to bias subjective data. On the other hand using methods such as comparison matrices increase complexity of problem, significantly. Thus, it is vital to use an efficient method which deals with qualitative criteria as precise as quantitative with fewer complexities. For this purpose the WLOM method (Hatefi et al., 2014) can be helpful. As it is shown in this paper the WLOM has poor discriminating power while ranking materials. For this purpose, WLOM is modified and the ACWDEA method is presented. Less computational complexity and stronger discriminating power are two main advantages of the proposed method. Also, any judgmental data are not necessary for weighting criteria in ACWDEA. Weights of criteria will be determined through solving model in a common weight approach.

The remainder of this paper is organized as follows. The proposed ACWDEA model with discussing on the WLOM model is presented in Section 2. In Section 3, two material selection problems borrowed from the literature are discussed. The robustness and effectiveness of the proposed ACWDEA are provided in Section 4 and finally concluding remarks are performed in Section 5.

## 2. Proposed MCDM method

As mentioned in previous section, the WLOM (Hatefi et al., 2014) has shortcoming in producing full ranking vector especially on material selection problems. In this section the WLOM is modified to eliminate its deficiency. Notably in this paper, exploited efficiency measures are not specific to a certain DMU, but common to all DMUs for utilizing common weights approach. Also, our presented method requires less computation complexity and is able to determine the efficiency of DMUs with one time running while the WLOM needs to be run multiple times. The presented ACWDEA can be categorized in common weight data envelopment analysis methods, which is named augmented common weight data envelopment analysis (ACWDEA) in this paper. Therefore, proposed ACWDEA has two main advantages: (*i*) improving the discriminating power among DMUs with unity efficiency in order to produce a full ranking vector and (*ii*) decreasing the number of times which the model should be solved.

Hatefi et al. (2014) assumed that there are $M_2$ qualitative criteria besides $M_1$ quantitative criteria, and proposed the following WLOM for evaluating efficiencies of $N$ DMUs. Following notations are used to develop WLOM:

*Indices:*

$n$ Index of DMUs ($n \in A=\{1, 2, \ldots, N\}$)

$i$ Index of linear programming (LP) model which should be solved ($1 \leq i \leq N$)

$j$ Index of quantitative criteria ($j = 1,2, \ldots, M_1$)

$r$ Index of qualitative criteria ($r = 1, 2, \ldots, M_2$)

$l$ Index of level for $r$-th qualitative criterion ($l = 1,2, \ldots, L_r$)

$A$ Set of DMUs

*Parameters:*

$y_{nj}$ Performance measure of *j*-th quantitative criterion for *n*-th DMU

$z_{ri}(n)$ Indicator for place of *r*-th qualitative criterion performance for item *n*

ε The first discriminating parameter

*Variables:*

$v_{ij}$ weight of *r*-th quantitative criterion when LP *i* runs

$w^{i}_{rl}$ weight of *l*-th level for *r*-th qualitative criterion when LP *i* is runs

$$f^{i} = Max\sum_{j=1}^{M_1} v_{ij}y_{ij} + \sum_{r=1}^{M_2}\sum_{l=1}^{L} w^{i}_{rl} z_{rl}(i) \tag{1}$$

$$\sum_{j=1}^{M1} v_{ij}y_{nj} + \sum_{r=1}^{M2}\sum_{l=1}^{L} w^{i}_{rl} z_{rl}(n) \leq 1, \qquad \forall n \tag{2}$$

$$w^{i}_{rl} - w^{i}_{r(l+1)} \geq \varepsilon, \qquad \forall r, l \in \{1, 2, \ldots, L_r - 1\} \tag{3}$$

$$w^{i}_{rL} \geq \varepsilon, \qquad \forall r \tag{4}$$

$$v_{ij} \geq \varepsilon, \qquad \forall j \tag{5}$$

The *i*-th WLOM model should be solved to determine the efficiency score of DMU *i* ($1 \leq i \leq N$). Also, $y_{rl(n)}$ is defined as follows:

$$y_{rl}(n) = \begin{cases} 1 & \text{if item } n \text{ is rated in the } l\text{-th level in respect to the } r\text{-th criteria} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Qualitative criteria can be classified into *L* levels. For instance, assume that price for materials as the first qualitative criterion is classified into three levels: low, medium and high. In this example *L* is equal to 3. Furthermore, assume that the performance measure of price with respect to item 4 is medium. Then we have $y_{11}(4) = 0$, $y_{12}(4) = 1$, *and* $y_{13}(4) = 0$. In model, $w^{i}_{rL}$ signifies the weight of *r*-th criterion at *l*-th level for *i*-th. Equation (2) denotes that DMUs efficiencies should be less than unity. Equations (3) and (4) denote and guarantee the acceptable set of weights for qualitative criteria and Equations (5) guarantees the lower bound for quantitative criteria. Also, $\varepsilon$ is a discrimination parameter which sets a lower bound for weight of criteria. Determining the proper value for $\varepsilon$ as the most discriminating power which maintains the model feasible is important. Hatefi et al. (2014) proposed using $\varepsilon_{max}$ instead of $\varepsilon$ so that it generates the most strength discrimination when ranking DMUs. They suggested using formulas (6) and (7) to calculate $\varepsilon_{max}$:

$$\varepsilon_{max} = min\left\{\frac{1}{\psi_n}, n = 1,2,\ldots,N\right\} \tag{7}$$

$$\psi_n = \sum_{j=1}^{M_1} y_{nj} + \sum_{r=1}^{M_2}\left(L_r - I_{nr} + 1\right) \tag{8}$$

In Equation (8), $\psi_n$ must be calculated for each DMU and $I_{nr}$ represents the place of *r*-th qualitative criterion performance for item *n*. Thus we have $y_{r(I\ w)}(n) = 1$ according to Equation (6).

A greater value for the objective function denotes a better performance. So, the efficiency value 1 will be assigned to the best DMU. As mentioned previously, considering qualitative criteria in parallel to quantitative criteria as precise as possible is the WLOP's advantage. It is able to calculate the weight of each criterion via solving model (1)-(5) and extra methods, such as AHP and comparative matrices, are not required to determine the criteria weights. In other words, fewer subjective and objective opinions are needed in comparison with other MCDM methods such as AHP, ANP, and TOPSIS for weightings.

Despite advantages, the proposed method by Hatefi et al. (2014) has two deficiencies. First, it is not able to produce a full ranking vector in some cases, which is shown in Section 3. Second, the current model needs to be solved for each alternative, separately. For instance, if a decision making problem contains *N* alternatives, it is required that the model be solved *N* times. To alleviate these deficiencies, an augmented common weight data envelopment analysis (ACWDEA) is proposed as follows.

Suppose that there are *N* entities as DMUs, $M_1$ quantitative criteria (i.e. Price), and $M_2$ qualitative criteria (i.e. Quality). Let $d_n$ indicates the deviation of DMU *n* from unity. To develop the ACWDEA following notations are defined in addition to previous defined sets and parameters.

*Parameters:*

$\xi$ The second discriminating parameter

*Variables:*

$d_n$ Deviation of *i*-th DMU from unity

$V_j$ Common weight of *j*-th quantitative criterion

$W_{rl}$ Common weight of *l*-th level for *r*-th qualitative criterion

Accordingly, model (1)-(5) is reformulated as follows:

$$f^n = Min\, d_n \tag{9}$$

$$\sum_{j=1}^{M_1} V_j y_{nj} + \sum_{r=1}^{M_2}\sum_{l=1}^{L} W_{rl} z_{rl}(n) + d_n = 1, \quad \forall n \tag{10}$$

$$W_{rl} - W_{r(l+1)} \geq \varepsilon, \quad r = 1,2,\ldots,M2\ ; l = 1,2,\ldots,L_r - 1 \tag{11}$$

$$W_{rL_r} \geq \varepsilon, \qquad r = 1, 2, \ldots, M2 \tag{12}$$

$$V_j \geq \varepsilon, \qquad j = 1, 2, \ldots, M1 \tag{13}$$

As defined with Equation (6), $y_{rl}(n)$ is equal to 1 if the DMU $n$ is rated in level $l$ for $r$-th qualitative criterion; otherwise is equal to 0. Also, $\varepsilon$ is a discriminating power and Equations (7) and (8) can be used to determine its value. Notably, efficiency of DMU $n$ is equal to $1\text{-}d_n$ which will be obtained after solving the model. The model (9)-(13) finds the minimum deviation for the DMU $n$ from unity. In other words, it aims to maximize the efficiency of the DMU $n$. Spirit of the model (9)-(13) is similar to the WLOM, but its discriminating power is not enhanced yet. Using above model still needs to be run for $N$ times to find DMUs' efficiencies. For overcoming these difficulties, the *minimax* approach is exploited to minimize the maximum deviations from unity among DMUs. Thus, the above model is reformulated again as follows:

$$Min\ \alpha \tag{14}$$

$$\alpha \geq d_n, \qquad n = 1, \ldots, N \tag{15}$$

*(10)-(13)*

It is assumed that $Max\{d_n\}$ is equal to $\alpha$. The modified model is able to determine the DMUs' efficiencies with one attempt of running the last model. Hereby, the computation complexity of WLOM is resolved, but this model might determine more than one efficient (i.e. the best) DMU with unity efficiency. In other words, it might not result in a full ranking vector. For overcoming this difficulty, the above model is reformulated as follows:

$$Min\ \alpha - \xi \sum_{n \in EF} d_n \tag{16}$$

$$\alpha \geq d_n, \qquad n = 1, \ldots, N \tag{17}$$

*(10)-(13)*

*EF* signifies the set of DMUs which their efficiencies are calculated as 1 after solving the model (10)-(15). $\xi$ is the second a discriminating parameter which can tolerate between 0 and 1 ($\xi \in [0,1]$). The above model can obtain a full ranking vector with involving the discriminating parameter ($\xi$). Decision maker can change $\xi$ from 0 to 1 with a preset step size (i.e. 0.1) until it coverage to a full ranking vector. Finally, the proposed ACWDEA method is as follows:

$$ACWDEA=\begin{cases} Min\ \alpha-\xi\sum\limits_{n\in EF} d_n \\ \alpha\geq d_n, \qquad n=1,...,N \\ \sum\limits_{j=1}^{M1} V_j y_{nj}+\sum\limits_{r=1}^{M2}\sum\limits_{l=1}^{L} W_{rl} z_{rl}(n)+d_n=1, \qquad n=1,...,N \\ W_{rl}-W_{r(l+1)}\geq\varepsilon, \qquad r=1,2,\ldots,M_2\ ; l=1,2,\ldots,L_r-1 \\ W_{rL_r}\geq\varepsilon, \qquad r=1,2,\ldots,M_2 \\ V_j\geq\varepsilon, \qquad j=1,2,\ldots,M_1 \end{cases} \tag{18}$$

To avoid any scaling problem we suggest normalizing the input data (i.e. Performance measures of each DMU) before applying the proposed ACWDEA method. In this way, the following normalization methods known as linear max-min approach are recommended (Milani et al., 2005; Jahan and Edwards, 2015):

$$R_{ij}=\frac{y_{ij}-min_{i=1,2,\ldots,N}\{y_{ij}\}}{max_{i=1,2,\ldots,N}\{y_{ij}\}-min_{i=1,2,\ldots,N}\{y_{ij}\}} \\ for\, i=1,2,\ldots,m \quad and \quad j=1,2,\ldots.,M1 \tag{19}$$

$$R_{ij}=\frac{max_{i=1,2,\ldots,N}\{y_{ij}\}-y_{ij}}{max_{i=1,2,\ldots,N}\{y_{ij}\}-min_{i=1,2,\ldots,N}\{y_{ij}\}} \\ for\, i=1,2,\ldots,m \quad and \quad j=1,2,\ldots.,M1 \tag{20}$$

Equation (19) and (20) should be used for beneficial and non-beneficial criteria, respectively. The following algorithm is presented to solve the ACWDEA model in order to evaluate the efficiency of materials and even any other type DMUs in MCDM problems:

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Normalize the performance measures using Equations (19)-(20).

$\xi\leftarrow 0$ , $StepSize\leftarrow 0.05\, or\, 0.1$ , $\Delta\leftarrow false$

**While** $\Delta = false$ **do**

Solve ACWDEA using model (18)

**If** full ranking vector obtained

Report the obtained results as final efficiency scores of DMUs

$\Delta\leftarrow true$

**Else**

$\xi\leftarrow\xi+StepSize$

**End while**

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Figure (1) represents the flowchart for solving the proposed ACWDEA and achieving a full ranking vector.

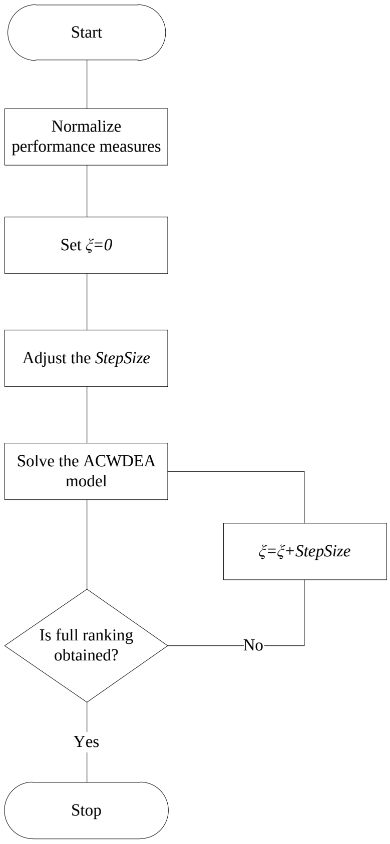


**Figure 1.** Flowchart for applying the ACWDEA method

## 3. Verification of the proposed ACWDEA method

Two case studies are borrowed from the literature to represent the applicability of the proposed ACWDEA method. Case studies are selected from high tech industries such as aircraft and shipping which is vital to select proper material in their manufacturing processes. The Spar of an aircraft`s wing and sailing boat material selection problems are two studied problems in this paper which the improper selected material might result in crisis for end-users.

### 3.1 The first case study: Spar of an aircraft`s wing material selection

The material selection problem for spar of an aircraft wing is borrowed from the literature (Mahmudi et al., 2000). Spar is one of the most important elements of a wing in aircrafts which act as a beam. Spar tolerates all aero dynamics and static loads which could be applied to the wing directly or indirectly. Selecting the appropriate material is a vital decision due to limitation of allowed weight in ultra-light structure of aircrafts (Khabbaz et al., 2009). The material properties are provided in Table (2). Lower values for price and density are favorable. Hence, price and density criteria are non-beneficial while the others are beneficial.

Mahmudi et al. (2000) concluded that attributes like tensile strength (MPa), young`s modulus (GPa), compressive strength (MPa) and density are the most favorable properties which should be considered in the spar material selection procedure. Khabbaz et al. (2009) suggested adding the price and the creep resistance to the problem. It is worth mentioning that the creep resistance is a qualitative criterion and the WLOM proposed by Hatefi et al. (2014), which accounts for both quantitative and qualitative criteria, can be useful in ranking the materials .

The WLOM (Hatefi et al., 2014) is applied to solve the Spar of an aircraft`s wing problem. Efficiency of each DMU while using WLOM is reported in Table (3).
It can be concluded from Table (3) that, WLOM is not able to obtain a full ranking vector even with applying normalization methods and DMUs M4 and M12 are determined as unity efficiency.

When the WLOM is not able to produce a full ranking vector, Torabi and Shokr (2015) suggested solving the problem under different normalization methods to alleviate obtaining non-full ranking vector on some problems. In our case, a full ranking vector is not obtained when the problem is solved with WLOM under normalized performance measures as it is obvious from Table (3).

**Table 2.** Performance measures for the Spar of an aircraft`s wing problem

| No. | Name | Price | Tensile strength (MPa) | Young's modulus (GPa) | Density (gr/cm3) | Compressive strength (MPa) | Creep resistance (25 °C) |
|---|---|---|---|---|---|---|---|
| M1 | Al 7075-T6 | 3.5 | 581 | 70 | 2.6 | 581 | Good |
| M2 | Al 2024-T4 | 3.5 | 425 | 72.5 | 2.6 | 425 | Good |
| M3 | Ti–6Al–4V | 21 | 1008 | 112 | 4.4 | 1008 | Excellent |
| M4 | Ti–2Fe–3Al–10V | 22 | 1295 | 120 | 4.5 | 1295 | Excellent |
| M5 | E-glass73%-Epoxy | 2.6 | 1642 | 55.9 | 2.17 | 410 | Average |
| M6 | E-glass56%-Epoxy | 2.5 | 1028 | 42.8 | 1.97 | 290 | Weak |
| M7 | E-glass65%-Polyester | 2.5 | 340 | 19.6 | 1.8 | 90 | Weak |
| M8 | S-glass70%-Epoxy continuous fibers | 9 | 2100 | 62.3 | 2.11 | 550 | Average |
| M9 | S-glass70%-Epoxy fabric | 8 | 680 | 22 | 2.11 | 180 | Average |
| M10 | Carbon 63%-Epoxy | 45 | 1725 | 158.7 | 1.61 | 900 | Average |
| M11 | Aramid 62%-Epoxy | 20 | 1311 | 82.7 | 1.38 | 300 | Average |
| M12 | Balsa | 6 | 28.5 | 7 | 0.22 | 17.5 | Average |

**Table 3.** Results of WLOM method for the Spar of an aircraft's wing problem

| **No.** | **M4** | **M12** | **M3** | **M1** | **M10** | **M2** |
|---|---|---|---|---|---|---|
| WLOM | 1 | 1 | 0.937 | 0.775 | 0.761 | 0.747 |
| **No.** | **M5** | **M8** | **M11** | **M6** | **M9** | **M7** |
| WLOM | 0.715 | 0.662 | 0.605 | 0.498 | 0.475 | 0.405 |

**Table 4.** Compared results between ACWDEA and that of WLOM

| No. | WLOM (Hatefi et al., 2014) | ACWDEA | |
|---|---|---|---|
| | | $\xi=0$ | $\xi=0.1$ |
| M4 | 1 | 1 | 1 |
| M12 | 1 | 1 | 0.5270284 |
| M3 | 0.936817 | 0.9368167 | 0.9362642 |
| M1 | 0.774748 | 0.7747484 | 0.7569822 |
| M10 | 0.760717 | 0.7607169 | 0.7170768 |
| M2 | 0.746555 | 0.7465546 | 0.7287884 |
| M5 | 0.715277 | 0.7152768 | 0.6891727 |
| M8 | 0.662474 | 0.6624741 | 0.6349363 |
| M11 | 0.604845 | 0.6048447 | 0.5498793 |
| M6 | 0.498333 | 0.4983331 | 0.4671105 |
| M9 | 0.474868 | 0.4748683 | 0.4473305 |
| M7 | 0.405331 | 0.4053312 | 0.3688638 |

**Table 5.** Comparative results with previous researchers for the Spar of an aircraft's wing problem

| | ACWDEA | Torabi and Shokr (2015) method | Khabbaz et al. (2009) method | Mahmudi et al. (2000) method |
|---|---|---|---|---|
| Rank 1 | M4 | M4 | M10 | M10 |
| Rank 2 | M3 | M3 | M3 | M4 |
| Rank 3 | M1 | M1 | M4 | M3 |
| Rank 4 | M2 | M2 | M8 | M8 |
| Rank 5 | M10 | M10 | M5 | M5 |
| Rank 6 | M5 | M5 | M1 | M1 |
| Rank 7 | M8 | M8 | M2 | M11 |
| Rank 8 | M11 | M12 | M11 | M2 |
| Rank 9 | M12 | M11 | M6 | M6 |
| Rank 10 | M6 | M6 | M9 | M9 |
| Rank 11 | M9 | M9 | M12 | M12 |
| Rank 12 | M7 | M7 | M7 | M7 |

To remove this deficiency, the proposed ACWDEA is applied to the problem. Results are reported in Table (4). The second discriminating parameter ($\xi$) is set to 0 at the first step of flowchart provided in Figure (1). In other words, first the problem should be solved without considering the second discriminating parameter into model ($\xi$). When the problem solved using ACWDEA with $\xi=0$, full ranking vector was not obtained similar to the results of WLOM. It is assumed pre step-size is equal to 0.05 in our example. Thus, the second discriminating parameter ($\xi$) will be increased in next iterations with this step-size. As can be seen from Table (4), full ranking vector is obtained after two iterations ($\xi=0.1$).

As can be seen from Table (4) the proposed ACWDEA method is able to produce full ranking vector in two steps while the WLOM is not. Also, the obtained results are compared to the previous researchers' results reported in Table (5). Also, as our applied method weights the qualitative criteria more precisely than the methods used by previous researches, therefore ACWDEA can perform better than other methods. Notably, the proposed method calculates the weights of criteria in parallel to solving the model and finding DMUs ranking. Thus, any judgmental data are not required for weighting criteria in ACWDEA, but previous researches weight the criteria with judgmental data. Hence the ACWDEA enable decision makers to rank DMUs more precisely without any biased subjective data.

### 3.2 The second case study: Sailing boat material selection

The problem which is provided by Khabbaz et al. (2009) is to select the most appropriate material in sailing boat mast in the form of length 1000mm, which should have a total compressive axial force of 153 KN. At this application, it is a major issue to consider low specific density because of weight limitations in addition to have high yield strength and high elastic modulus to resist the plastic yielding and local and global buckling. The performance measures for each alternative are summarized in Table (6). The problem is solved by WLOM and results are reported in Table (7).

As it is obvious from the Table (7), the WLOM method is not able to produce full ranking vector in this case. Thus, the proposed ACWDEA method is applied to solve the problem. At first step the discriminating parameter *($\xi$)* is set to 0 and the problem is solved again. As can be seen from Table (8), full ranking vector is not obtained at the first step ($\xi=0$). Thus, the step size is selected to be 0.05 and the model is solved again according to the proposed algorithm in Section (2). Note, that the second discriminating power *($\xi$)* will be increased with the step size 0.05 until the full ranking vector obtains.

As can be concluded, a full ranking vector is obtained by ACWDEA method in two iterations. Also, It is worth mentioning that, ACWDEA method has produced results with more discriminating power than that of WLOM method with $\xi = 0$ in this case. Comparative results are provided in Table (9).

**Table 6.** The sailing boat material selection problem (Khabbaz et al., 2009)

| No | Material | Specific strength (MPa) | Specific modulus (GPa) | Corrosion resistance | Cost category |
|---|---|---|---|---|---|
| M1 | AISI-1020 | 35.9 | 26.9 | Poor | very low |
| M2 | AISI-1040 | 51.3 | 26.9 | Poor | very low |
| M3 | ASTM A242 type1 | 42.3 | 27.2 | Poor | very low |
| M4 | AISI 4130 | 194.9 | 27.2 | very good | moderate |
| M5 | AISI 316 | 25.6 | 25.1 | very good | moderate |
| M6 | AISI 416 (heat treated) | 57.1 | 28.1 | very good | moderate |
| M7 | AISI 431(heat treated) | 71.4 | 28.1 | very good | moderate |
| M8 | AA 6061 T6 | 101.9 | 25.8 | good | low |
| M9 | AA 2024 T6 | 141.9 | 26.1 | good | low |

**Table 6.** Continued

| No | Material | Specific strength (MPa) | Specific modulus (GPa) | Corrosion resistance | Cost category |
|---|---|---|---|---|---|
| M10 | AA 2014 T6 | 148.2 | 25.8 | good | Low |
| M11 | AA 7075 T6 | 180.4 | 25.9 | good | low |
| M12 | Ti-6Al-4V | 208.7 | 27.6 | excellent | very high |
| M13 | Epoxy-70%glass fabric | 604.8 | 28 | very good | high |
| M14 | Epoxy-63% carbon | 416.2 | 66.5 | very good | very high |
| M15 | Epoxy-62% aramid | 637.7 | 27.5 | very good | very high |

**Table 7.** Results when WLOM is applied to the sailing boat material selection problem

| **No.** | **M2** | **M4** | **M11** | **M12** | **M3** | **M1** | **M10** | **M9** |
|---|---|---|---|---|---|---|---|---|
| WLOM | 1 | 1 | 1 | 1 | 0.998 | 0.996 | 0.991 | 0.990 |
| **No.** | **M8** | **M7** | **M6** | **M13** | **M5** | **M4** | **M5** | |
| WLOM | 0.979 | 0.971 | 0.967 | 0.950 | 0.948 | 0.891 | 0.799 | |

**Table 8.** Comparison between the results of ACWDEA and that of WLOM

| No. | WLOM (Hatefi et al., 2014) | ACWDEA | |
|---|---|---|---|
| | | *$\xi$ =0* | *$\xi$ =0.05* |
| M2 | 1 | 0.9704837 | 0.9617771 |
| M4 | 1 | 1 | 1 |
| M11 | 1 | 1 | 0.9912933 |
| M12 | 1 | 1 | 0.8470452 |
| M3 | 0.9988214 | 0.9693052 | 0.9605985 |
| M1 | 0.9960237 | 0.9665074 | 0.9578008 |
| M10 | 0.9913042 | 0.9913042 | 0.9825975 |
| M9 | 0.9908228 | 0.9908228 | 0.9821161 |
| M8 | 0.9793495 | 0.9793495 | 0.9706428 |
| M7 | 0.9715479 | 0.9715479 | 0.9715479 |
| M6 | 0.9678556 | 0.9678556 | 0.9678556 |
| M13 | 0.9508457 | 0.9508457 | 0.9508457 |
| M5 | 0.9482698 | 0.9482698 | 0.9482698 |
| M14 | 0.8910782 | 0.8910782 | 0.8910782 |
| M15 | 0.7993868 | 0.7993868 | 0.7993868 |

**Table 9.** Comparison between the results of ACWDEA and those previous researchers

| | ACWDEA | Athawale et al. (2011) method [22] | Khabbaz et al. (2009) method [7] |
|---|---|---|---|
| Rank 1 | M4 | M14 | M14 |
| Rank 2 | M11 | M15 | M13 |
| Rank 3 | M10 | M13 | M15 |

**Table 9.** Continued

| | ACWDEA | Athawale et al. (2011) method [22] | Khabbaz et al. (2009) method [7] |
|---|---|---|---|
| Rank 4 | M9 | M4 | M4 |
| Rank 5 | M7 | M11 | M11 |
| Rank 6 | M8 | M10 | M9 |
| Rank 7 | M6 | M9 | M10 |
| Rank 8 | M2 | M8 | M8 |
| Rank 9 | M3 | M12 | M12 |
| Rank 10 | M1 | M7 | M7 |
| Rank 11 | M13 | M6 | M6 |
| Rank 12 | M5 | M5 | M5 |
| Rank 13 | M14 | M2 | M2 |
| Rank 14 | M12 | M3 | M3 |
| Rank 15 | M15 | M1 | M1 |

As can be seen from Table (9), there is a deviation between the results of ACWDEA and that of previous researchers. ACWDEA determined M4 as the best material, but Athawale et al. (2011) method and Khabbaz et al. (2009) method reported M14 as the best one. The previous methods utilized from the Likert scale to quantify the qualitative criteria which is not as precise as the proposed ACWDEA method. It means 0 to 9 scores will be assigned to qualitative criteria in order to quantify them in previous methods, but ACWDEA does not need to receive judgmental views to weight criteria and weights of criteria will be calculated with solving the model.

## 4. Discussion

To verify the proposed model, the robustness and effectiveness of the ACWDEA method is assessed. The Spearman’s correlation test is applied, which readers may refer to (Sheskin, 2003) for more details. The Spearman’s correlation test shows whether or not there is a positive correlation between the obtained results of ACWDEA and that of WLOM. For this purpose the Following hypothesis is tested:

$$\begin{cases} H_0 : r = 0 \\ H_1 : r > 0 \end{cases}$$

The null hypothesis ($H_0$) indicates that there is no any correlation ($r$) between the results of the two methods and alternative ($H_1$) indicates that there is a positive correlation between them.

The Spearman’s rank correlation coefficient for the first example is equal to 0.9999. Since the number of DMUs in both of the example is greater than 10, t-test is exploited. T-test follows from a normal distribution with mean 0:

$$t = \frac{r}{\sqrt{(1-r^2)/(n-2)}} \sim T(n-2)$$

Which $r$ is the correlation coefficient of data and $n$ is the number of data (DMUs). Thus, the value of the observant $t$ is 223.59 ($t_{obs}$) and the critical value of $t$ distribution at the confidence level 0.999 is equal to 4.144 ($t_{crit}$). Since $t_{crit} < t_{obs}$, the null hypothesis is rejected.

Also, The Spearman's rank correlation coefficient for the second example is equal to 0.91332 and the $H_0$ is rejected in a similar way with the confidence level 0.999. Thus, there is a high correlation of ranks between the proposed ACWDEA and the WLOM. According to the results, the robustness of the proposed ACWDEA method is verified.

## 5. Concluding remarks

Material selection is a challenging issue in production processes and manufacturing environments, especially in high-tech industries such as aircraft and shipping. Each material has its own performances in the presence of qualitative and quantitative criteria. Quantifying qualitative criteria is an arguable problem in MCDM literatures. Most of the previous methods quantify the qualitative criteria with using Liker scale which is not precise enough. Therefore, evaluating material with an efficient approach in the presence of qualitative criteria is vital.

Hatefi et al. (2014) proposed WLOM which can determine efficiency of DMUs in presence of qualitative and quantitative criteria in an effective way through solving a mathematical model. Torabi and Shokr (2015) demonstrated how WLOM can be deficient in some material selection problems where it is not able to produce full ranking vector and determine the best material. For overcoming this difficulty, we have modified the WLOM and converted it to a common weight DEA-like model. The discriminating power is enhanced to produce a full ranking vector with fewer computational complexities. In summary, the proposed augmented common weight DEA (ACWDEA) has the following merits:

(1) It is capable to produce full ranking vectors where other DEA likes model such as WLOM (Hatefi, et al., 2014) are deficient in producing a full ranking vector. In other words, the proposed ACWDEA has more discriminating power than WLOM and will coverage to a single best DMU.
(2) The presented ACWDEA finds the efficiency of DMUs in just one time running, but the WLOM needs to be run $N$ times. $N$ is the number of DMUs.
(3) ACWDEA is able to determine weight of qualitative and quantitative criteria through solving the model and extra subjective data are not required from decision makers. It calculates weight of qualitative and quantitative criteria with a common weight approach.
(4) The presented method involves all of the criteria in evaluation process while some previous DEA-like models may ignore some criteria by calculating the weight of zero for them. In ACWDEA all of the common weights are greater than $\varepsilon\ (V_j \geq \varepsilon; \forall j\ ; W_{rl} \geq \varepsilon; \forall r\ )$.
(5) The proposed ACWDEA method can be applied on any other decision making problems as well as material selection problems where it is essential to consider qualitative criteria precisely.

To demonstrate the applicability of the proposed ACWDEA, two material selection problems are borrowed from the literature and represented that the WLOM is not able to produce full ranking vectors. Thus, the presented ACWDEA method is applied to demonstrate that it is able to obtain full ranking vector where the WLOM is not. Finally the robustness and effectiveness of the presented ACWDEA method are evaluated and verified by the Spearman's rank correlation coefficient test.

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