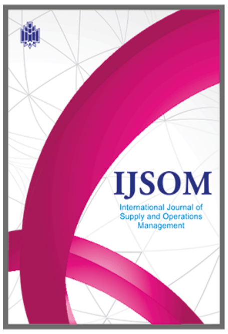
International Journal of Supply and Operations Management

IJSOM

May 2016, Volume 3, Issue 1, pp. 1192-1214
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com



The Inventory System Management under Uncertain Conditions and Time Value of Money

Mehri Nasrabadi[a] and Abolfazl Mirzazadeh[a*]

[a] *Department of Industrial Engineering, Kharazmi University, Tehran, Iran*

Abstract
This study develops an inventory model to determine ordering policy for deteriorating items with shortages under markovian inflationary conditions. Markov processes include process whose future behavior cannot be accurately predicted from its past behavior (except the current or present behavior) and which involves random chance or probability. Behavior of business or economy, flow of traffic, progress of an epidemic, all are examples of Markov processes. Since the far previous inflation rate don't have a great impact on the current inflation rate, so, it is logical to consider changes of the inflation rate as a markov process. In addition, it is assumed that the cost of the items changes as a Continuous – Time - Markov Process too. The inventory model is described by differential equations over the time horizon along with the present value method. The objective is minimization of the expected present value of costs over the time horizon. The numerical example and a sensitivity analysis are provided to analyze the effect of changes in the values of the different parameters on the optimal solution.

Keywords: Supply Chain, Inventory Management; Markovian Costs; Deteriorating Items.

1. Introduction

In real world problem, deterioration of many items such as chemicals, volatile liquids, blood banks, medicines and some other goods during storage period is non-negligible. Furthermore, both inflation and time value of money issues will have main effects in financial markets. Ghare and Schrader (1963) were the pioneers to establish an inventory model for deteriorating items.

Corresponding author email address: a.mirzazadeh@aut.ac.ir

Dave and Patel (1981) derived an EOQ (economic order quantity) model for deteriorating items with time-proportional demand and without shortage. Sachan (1984) extended the model of Dave and Patel (1981) by allowing shortages. Aggarwal and Jaggi (1995) proposed an inventory control model for deteriorating items in which shortage was not permitted. Hariga and Ben-daya (1996) developed lot-sizing problem with timedependent demand under inflationary conditions. Chen 1998, proposed an inflationary model with time proportional demand and Weibull distribution for deteriorating items using dynamic programming. Balkhi (2004a) presented a production lot-size inventory model where the production, demand and deterioration rates are known, continuous and differentiable functions of time. Shortages are allowed. Covert and Philip (2005) extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution.Tripathi (2014) developed a inventory Model for deteriorating Items with Four level System and Shortages. Sharmila and Uthayakumar (2015) presented a inventory model for deteriorating items involving fuzzy with shortages and exponential demand.

The mentioned studies have considered a constant inflation rate over the time horizon, but the effect of inflation and time-value of money cannot be ignored. Buzacott (1975) made the first attempt in this field that dealt with an economic order quantity (EOQ) model with inflation subject to different types of pricing policies. Aggarwal (1981) developed a purchase inventory decision model for inflationary conditions. Chen (1998) presented an inventory model for deteriorating items with time proportional demand and shortages under inflation and time discounting. Chung and Lin (2001) proposed an Economic Order Quantity (EOQ) model for deteriorating items, where the time value of money is taken into account. Liao and Chen (2003) surveyed a retailer's inventory control system for the optimal delay in payment time for initial stock dependent consumption rate when a wholesaler permits delay in payment. The effect of inflation rate, deterioration rate, initial stock-dependent consumption rate and a wholesaler's permissible delay in payment is discussed. Chang (2004) proposed an EOQ model for deteriorating items under inflation when the supplier offers a permissible delay to the purchase. Lo, Wee, and Huang (2007), developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. Loa et al. (2007) represented an integrated production inventory model for imperfect production with Weibull distribution deterioration under inflation. Mirzazadeh et al. (2009) presented stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon and the inflation dependent demand rate. The developed model, also, implicates to a finite replenishment rate, finite time horizon, deteriorating items with shortages. Mirzazadeh (2010b) proposed an inventory model with stochastic internal and external inflation rates for deteriorating items and allowable shortages. Alizadeh et al. (2011) developed a modified (S _ 1,S) inventory system for deteriorating items with Poisson demand and non-zero lead time. Lee and Chang (2012) proposed An Inventory Model for Deteriorating Items in a Supply Chain with System Dynamics Analysis. Ghoreishi et al. (2014) presented optimal pricing and ordering policy for non-instantaneous deteriorating items under inflation and customer returns. Pall et al. (2015) presented a production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness.

Markov chain is concerned with a particular kind of dependence of random variables involved.

When random variables are observed in sequence, the distribution of a random variable depends only on the immediate preceding observed random variable and not on those before it. In other words, given the current state, the probability of the chain's future behavior is not altered by any additional knowledge of its past behavior. This is the so-called Markovian property. Markov chain, a well-known subject introduced by Markov in 1906, has been studied by a host of researchers for many years Chung (1960), Doob (1953), Kushner & Yin (1997). Markovian formulations (see Chiang (1980), Yang, Yin, Yin, & Zhang (2002), Yin , Zhang, Yang, & Yin (2001), Yin & Zhang (1997), Yin, Yin, & Zhang (1995) and the references there in) are useful in solving a number of real-world problems under uncertainties such as determining the inventory levels for retailers, maintenance scheduling for manufacturers, and scheduling and planning in production management. Chakravarthy and Daniel (2004) presented a (s,S) inventory system for continuous decaying items with phase type replenishment time in which backorders of demand are allowed. They modeled their system according to Markovian Arrival Process (MAP). (s,S) inventory policy and exponentially distributed lead time were assumed by Sivakumar (2009). In addition, it was considered that demands occurring during the stock-out periods enter into an orbit. The orbiting demands send out signal according to exponential distribution to compete for their demand. The system has been analyzed as a Markov process. Larsen and Turkensteen (2014) presented a vendor managed inventory model using continuous approximations for route length estimates and Markov chain modeling for cost estimates. Diaz et al (2016) developed analyzing a lost-sale stochastic inventory model with Markov - modulated demands: A simulation-based optimization study.

As mentioned above, it's clear that in the real world, especially, for long-term investment and forecasting, the fluctuations in the inflation rate cannot be disregarded. Therefore there is a need to consider the inflationary changes and inventory control problem in a fluctuating inflation rate environment, and a Markovian inflationary modeling approach provides an effective mechanism to address this problem. In none of the previous models, it's not used markov properties for studying changes of inflation rate. The main contribution of this paper is to establish the mathematical model for deteriorating items with considering changes of inflation rate as a Markov chain in addition, it surveys the changes of cost of the items in each cycle except for the last cycle as a Continuous Time Markov Process. Our main objective is minimization of the expected present value of costs over the time horizon.

The rest of the paper is designed as follows: Section 2, includes the assumptions, notations and description of the inventory system. In Section 3, mathematical model is shown in Section 4, the objective of the problem is derived. Section 5, explains the solution procedure. Section 6, provides the numerical example to clarify how the proposed model is applied. In section 7 the sensitivity analysis has been provided for validation of the theoretical results. The final section is devoted to the conclusion.

2. The Notations and Assumptions

The following notations and assumptions are used in this paper:

2.1. Notations

r The discount rate.
P The constant annual production rate.
θ The constant deterioration rate per unit time, where ($0 \leq \theta \leq 1$).
λ Parameter of exponential distribution.
C Per unit cost of the item at time zero.
S The ordering cost per order at time zero.
H The fixed time horizon.
f(i) The stationary distribution of inflation rate

2.2. Assumptions

(1) Inflation rate changes as a discrete time Markov chain, over the time horizon.
(2) The demand rate is constant.
(3) The replenishment rate is finite and lead time is zero.
(4) A constant fraction of the on-hand inventory deteriorates per unit time.
(5) The production rate is higher than the rates of consumption and deterioration. On the other hand, the inventory level will increase as the production continues.
(6) Shortages are allowed and fully backlogged except for the final cycle.
(7) The cost of items changes as a Continuous-Time Markov-Process, in each cycle except for the last cycle, over the time horizon.

3. Model formulation

According to the assumptions, we formulated mathematically the proposed inventory model. According to the model of Mirzazadeh et al. (2009), the graphical presentation of the proposed inventory is shown in Fig. 1. It's assumed the length of planning horizon H = n.T; where n is an integer decision variable representing the number of replenishments to be made during H; and T is time between two replenishments. The time horizon, H, is divided into n equal cycle each of length T. Each inventory cycle except the last cycle can be divided into four parts. The production starts at time zero and the inventory level is increasing. This fact continues till the production stops at time α. Then, the level of inventory is decreasing by consumption and deterioration rates. At the moment of kT, the inventory level leads to zero and shortages occur. During the time interval [kT, β], we do not have any deterioration, and therefore the shortages level linearly increases by the demand rate. At time β, the production starts again and the shortages level linearly decreases until the moment of T. In this moment, the second cycle starts and this behavior continues till the end of (n-1) cycle. In the last cycle, shortages are not allowed and each inventory cycle can be divided into two parts. The production stops at time (n-1)T+ α and then the inventory level decreases until the end of time horizon.

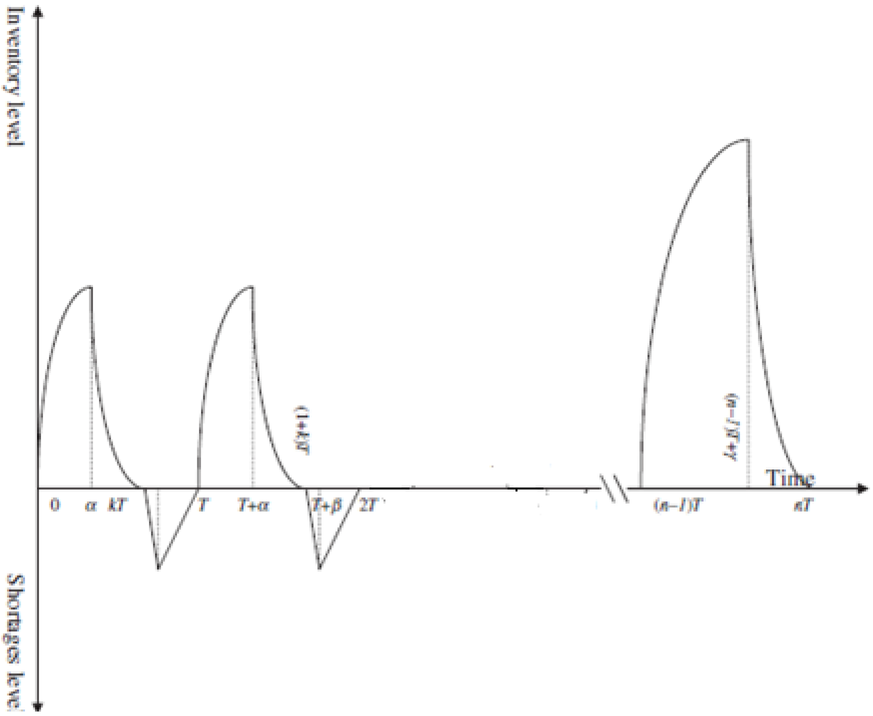


Figure1. Graphical representation of the inventory system

4. The mathematical modelling and analysis

So according to this description the changes of inventory respect to time can be shown using the following differential Equations:

$$\frac{dI_1(t_1)}{dt_1}+\theta I_1(t_1) = p - D,\ 0 \le t_1 \le \alpha \quad (1)$$

During [0, kT), the inventory level can be described as follows:

$$\frac{dI_2(t_2)}{dt_2}+ \theta I_2(t_2) = -D, \quad 0 \le t_2 \le kT - \alpha \quad (2)$$

During [kT, β), we have no deterioration. Therefore, the shortages level is governed by

$$\frac{dI_3(t_3)}{dt_3} = -D,\ \ 0 \le t_3 \le \beta - kT - \alpha \quad (3)$$

Finally, during [β, T) the shortages level is represented by

$$\frac{dI_4(t_4)}{dt_4} = P - D, \qquad 0 \le t_4 \le T - \beta \quad (4)$$

In the last cycle, shortages are not allowed and the inventory level is governed by the following differential equations:

($I_i(t_i)$) denotes the inventory level at any time t_i in (i-4)th part of last cycle that is i = 5, 6).

$$\frac{dI_5(t_5)}{dt_5} + \theta I_5(t_5) = P - D, \quad 0 \le t_5 \le \gamma \tag{5}$$

$$\frac{dI_6(t_6)}{dt_6} + \theta I_6(t_6) = -D, \quad 0 \le t_6 \le T - \gamma \tag{6}$$

The solutions of the above differential equations after applying the following boundary conditions:

$I_1(0) = 0$, $I_2(kT-\alpha) = 0$, $I_3(0) = 0$, $I_4(T-\beta) = 0$, $I_5(0) = 0$ and $I_6(T-\gamma) = 0$, are:

$$I_1(t_1) = \frac{p - D}{\theta}\left(1 - e^{-\theta t_1}\right), 0 \le t_1 \le \alpha \tag{7}$$

$$I_2(t_2) = \frac{-D}{\theta}\left(1 - e^{\theta(KT-\alpha-t_2)}\right), \quad 0 \le t_2 \le kT - \alpha \tag{8}$$

$$I_3(t_3) = -\mathrm{D}t_3, \qquad 0 \le t_3 \le \beta - kT \tag{9}$$

$$I_4(t_4) = (\mathrm{P} - \mathrm{D})(t_4 - T + \beta), \qquad 0 \le t_4 \le \mathrm{T} - \beta \tag{10}$$

$$I_5(t_5) = \frac{p - D}{\theta}\left(1 - e^{-\theta t_5}\right), 0 \le t_5 \le \gamma \tag{11}$$

$$I_6(t_6) = \frac{-D}{\theta}\left(1 - e^{\theta(T-\gamma-t_6)}\right), 0 \le t_6 \le T - \gamma \tag{12}$$

Using the mentioned equations, we can calculate the values of α, β and γ with respect to k and T.

Solving $I_1(\alpha) = I_2(0)$ for α we have,

$$\alpha = \frac{1}{\theta}\ln\left(\frac{P-D(1-e^{\theta kT})}{P}\right) \tag{13}$$

β can be calculated by solving $I_3(\beta\text{-}kT) = I_4(0)$

$$\beta = \frac{[P-D(1-K)]T}{P} \tag{14}$$

Finally, solving $I_5(\gamma) = I_6(0)$ for γ we have

$$\gamma = \frac{1}{\theta}\ln\left(\frac{P-D(1-e^{\theta T})}{P}\right) \tag{15}$$

5. Markov process

A Markov process is a stochastic process with the following properties:
(a.) The number of possible outcomes or states is finite.
(b.) The outcome at any stage depends only on the outcome of the previous stage.
(c.) The probabilities are constant over time.

If x_0 is a vector which represents the initial state of a system, then there is a matrix M such that the state of the system after one iteration is given by the vector Mx_0. Thus we get a chain of state vectors: $x_0, Mx_0, M^2x_0, \ldots$ where the state of the system after n iterations is given by $M^n x_0$. Such a chain is called a Markov chain and the matrix M is called a transition matrix.

In this study, it is assumed that inflation rate changes as a markov chain and then it will be to predict and estimate the stationary distribution of inflation rate, as shown in appendix A.
The objective of the problem is minimization of the total expected present value of costs over the time horizon. Considering ECP as the expected present value (EPV) of costs of purchasing, ECH as the EPV of costs of holding, ECS as the EPV of costs of shortages and ECR as the EPV of costs of replenishment, respectively. The total expected present value of costs over time horizon (ETVC) is:

$$\text{ETVC}(n, k) = \text{ECR} + \text{ECP} + \text{ECH} + \text{ECS} \tag{16}$$

The detailed analysis is given as follows:

The EPV of ordering cost (ECR)

Consider CR as the ordering cost, therefore,

$$ECR = \sum_{i=-2}^{6} f(i) \; s(1 + (\sum_{j=1}^{n-1} e^{(-(r-i/100)\left(jT + \frac{[p-D(1-k)]T}{P}\right))})) \quad (17)$$

The EPV of purchasing cost (ECP)

Let ECP_1 be the EPV of the purchase cost. The EPV of the purchase cost in the last cycle is shown with ECP_2. The first purchase is ordered at time zero and equals to: cPα. Then, next purchase will occur at time β. therefore,

$$ECP_1 = \{ \sum_{i=-2}^{6} f(i) (\sum_{j=1}^{n-1} c.p. (\frac{\ln\left(\frac{p-D\left(1-e^{\theta kT}\right)}{P}\right) e^{(-(j-1)(r-i/100)T)}}{\theta} +$$

$$(c + c[\int_{\frac{1}{\theta}\ln\left(\frac{P-D\left(1-e^{\theta KT}\right)}{P}\right)}^{\infty} \lambda . \exp(-\lambda . t_1)\, dt_1 . Q_{01}) . \left(\int_{kT-\frac{1}{\theta}\ln\left(\frac{P-D\left(1-e^{\theta KT}\right)}{P}\right)}^{\infty} \lambda . \exp(-\lambda . t_2) dt_2 . Q_{12} \right) .$$

$$(\int_{\frac{(p-D(1-k))T}{P} - kT}^{\infty} \lambda . \exp(-\lambda . t_3) dt_3\, Q_{23}]) . (. \int_{T - \frac{(p-D(1-k))T}{P}}^{\infty} \lambda \, \exp(-\lambda . t_4) dt_4 \, . Q_{c_3 c_4})$$

$$. \; e^{(-\left(\frac{[p-D(1-k)]T}{P} + (j-1)T)\left(r - \frac{i}{100}\right)\right)))} (T - \frac{(p-D(1-k))T}{P})$$

(18)

In the last cycle, one order will occur at time (n-1)T and the order quantity is γP. the EPV of the purchase cost in the last cycle will be one of the following phrases:

$$ECP_2 = \{ cp(\sum_{i=-2}^{6} f(i) \left(\frac{1}{\theta} . \ln\left(\frac{P-D(1-e^{\theta T})}{P}\right) e^{-(n-1)(r-i/100)T}\right) \} \quad (19)$$

The total expected purchase cost over the time horizon would be

$$ECP = ECP_1 + ECP_2 \quad (20)$$

The EPV of holding cost (ECH)

Let ECH_1 be the EPV of the holding cost. The EPV of the holding cost during the last cycle, can be defined with ECH_2.

ECH_1 = {

$$\sum_{i=-2}^{6} f(i)\,\Big(\sum_{j=1}^{n-1}\Big(c_1 e^{(-(j-1)(r-i/100)T)} \int_0^{\frac{1}{\theta}\ln(\frac{P-D(1-e^{\theta KT})}{P})} \frac{\left(p-D\left(1-e^{-(\theta t_1)}\right)\right) e^{(-(r-i/100)t_1)}}{\theta}$$

dt_1+

$$\int_0^{kT-\frac{1}{\theta}\ln\left(\frac{P-D\left(1-e^{\theta KT}\right)}{P}\right)} \frac{-D\left(1-e^{\left(\theta\left(kT-\frac{\ln\left(\frac{p-D\left(1-e^{(\theta kT)}\right)}{p}\right)}{\theta}-t_2\right)\right)}\right)e^{(i/100-r)t_2}}{\theta}$$

$$dt.\, e^{\left(-\frac{\ln\left(\frac{p-D\left(1-e^{(\theta kT)}\right)}{p}\right)(r-i/100)}{\theta}\right)}\Big)\Big)\ \} \qquad (21)$$

$$ECH_2 = \{c_1\Big(\int_0^{\frac{1}{\theta}\ln\left(\frac{P-D\left(1-e^{\theta kT}\right)}{P}\right)} \frac{\left(p-D\left(1-e^{-(\theta t_5)}\right)\right)e^{(-(r-i/100)t_5)}}{\theta}\,dt_5.$$

$$e^{(-(r-i/100)(n-1)T)} \qquad +$$

$$\int_0^{T-\frac{1}{\theta}\ln\left(\frac{P-D\left(1-e^{\theta T}\right)}{P}\right)} \frac{-D\left(1-e^{\left(\theta\left(T-\frac{\ln\left(\frac{p-D\left(1-e^{(\theta T)}\right)}{p}\right)}{\theta}-t_6\right)\right)}\right)e^{(-(r-i/100)t_6)}}{\theta}\,dt_6\Big).$$

$$e^{\left(-(r-i/100)(n-1)T+\frac{1}{\theta}\ln\left(\frac{\left(P-D\left(1-e^{\theta T}\right)\right)}{P}\right)\right)}\Big)$$

$+$

$$\int_0^{T-\frac{1}{\theta}\ln\left(\frac{P-D\left(1-e^{\theta T}\right)}{P}\right)} \frac{-D\left(1-e^{\left(\theta\left(T-\frac{\ln\left(\frac{p-D\left(1-e^{(\theta T)}\right)}{p}\right)}{\theta}-t6\right)\right)}\right)e^{(-(r-i/100)t6)}}{\theta} dt_6).e^{\left(-(r-i/100)(n-1)T+\frac{1}{\theta}\ln\left(\frac{(P-D\left(1-e^{\theta T}\right)}{P}\right)\right)})$$

$$\}$$

(22)

So, the total EPV of the holding costs over the time horizon is

$$ECH = ECH_1 + ECH_2 \quad (23)$$

The EPV of shortage costs (ECS)

ECS shows the EPV of the shortage costs. Shortages are not allowed in the last cycle.

$$\textbf{ECS} = \{\sum_{i=-2}^{6} f(i)(\sum_{j=1}^{n-1} c_2(\int_0^{\frac{(P-D(1-k))T}{P}-kT} -dt_3.e^{(-(r-i/100).t_3)}\, dt_3).e^{(-kT\left(r-\frac{i}{100}\right))}$$

$$+(\int_0^{T-\frac{(P-D(1-k))T}{P}} (D-p).(-t_4-T-\frac{(P-D(1-k))T}{P}).e^{(-\left(r-\frac{i}{100}\right)t4)}\, dt_4) .$$

$$e^{\left(-\frac{(p-D(1-k))T\left(r-\frac{i}{100}\right)}{p}\right)}).e^{(-(j-1)\left(r-\frac{i}{100}\right)T)})\} \quad (24)$$

6. Solution algorithm

The steps of the optimum solution procedure are described below. The problem is to determine the optimal values of n, k^*, so as to minimize the total expected inventory system costs.

Step 0. Set n=1,

Step 1. Substitute all the given values (s, r, p, etc) into (16) and take the partial derivation of ETVC (n, k^*) with respect to k and Equate the partial derivation to zero, as follow:

$$\frac{dETVC(n,k)}{dk} = 0$$

Step 2. Substitute k^* into Equation(16).

.

Step 3. Set n=n+1, If n=2, then go to step 1.

Step 4. If ETVC (n-1, k^*) < ETVC (n-2, k^*), then go to step 1.

Else, The ((n-2)*, k*) and ETVC ((n-2)*, k*) values constitute the optimal solution, and then stop.

The (n*, k*) and ETVC (n*, k*) values satisfy the following conditions:

ΔETVC (n*-1, k*)≥0

Where ΔETVC (n*, k*) = ETVC (n* + 1, k*) – ETVC (n*, k*)

To ensure convexity of the objective function, the derived values of (n*, k*) must satisfy the following sufficient conditions;

$$\frac{d^2ETVC(n,k)}{dk^2} \geq 0$$

7. Numerical example

According to the results, the following numerical example is providing to illustration. the time horizon, H, is 10 years. The company interest rate is 10% and the deterioration rate of the on-hand inventory per unit time is 0.02. The constant annual production rate is 4000 units. Also, let r=$0.2/$/year; θ = 0.01; P = 4000 units/year. The ordering, production, holding and shortage costs at the beginning of the time horizon are: S = $50/order; c = $5/unit; c1 = $0.1/unit/year; c2 = $0.2/unit/year and the demand rate is D = 1000.

The problem is to determine the optimal ordering policy for minimizing the EPV of the total inventory system costs (ETVC (n, k)). Considering the above information and using the numerical methods, the problem i solved and the results are illustrated in Table 1. It can be seen that the number of replenishment = 41 and time interval between replenishments is

$T^* = \frac{10}{2} = 0.244$ year.

The shortages occur after elapsing 39% of the cycle time. (k*= 0.39). The minimum value of the ETVC (n, k,) with these values is 3 250 000.

Table 1. Optimal solution for numerical example

n	k	ETVC (n,k)	n	k	ETVC (n,k)
1	0.39	0.587 * 10^7	34	0.37	0.339 * 10^7
4	0.39*	0.495 * 10^7*	41	0.39	0.325 * 10^7
9	0.38	0.473 * 10^7	15	0.39	0.452 * 10^7
14	0.39	0.428* 10^7	20	0.37	0.497 * 10^8
18	0.39	0.380 * 10^7	50	0.39	05.72 * 10^8
26	0.39	0.352 * 10^7	100	0.38	0.672* 10^8

8. Sensitivity analysis

In order to assess the relative impact of the different parameters on the solution quality, systematic sensitivity analysis was performed. On the basis of sensitivity analysis of the parameters. The value of each given parameters, demand (D), the time horizon (H), the deterioration rate (θ),the discount rate (r), the ordering cost (s), the production rate (p) and unit cost of the item (c) was changed, This fact is done by changes (-50%, -20%,0%, +20%, +50%) of different parameters value in Table 2. On the basis of sensitivity analysis of the parameters, the following features are observed.

1. The number of replenishments (n) is sensitive to changes of value of demand, Changing of values of the parameters the deterioration rate (θ),the discount rate (r), the production rate (p) and unit cost of the item (c) have little effect on the number of replenishments (n) and it is slightly sensitive to changes of this parameters. In addition the number of replenishments (n) is highly sensitive to changes of the values of the ordering cost (s) and the time horizon (H).

2. If production rate (p) increases, then the number of production increases which implies the increasing value of total expected cost (ETVC). Changing of values of the parameters demand (D), the time horizon (H) and the discount rate (r) changes of value of total expected cost (ETVC). But value of total expected cost doesn't change much with changes of the parameters the deterioration rate (θ), unit cost of the item (c) and the ordering cost (s).

3. The optimal value of k is slightly sensitive to changes of the parameters demand (D), the production rate (p), the ordering cost (S), the time horizon (H) and the deterioration rate (θ).is sensitive to changes value of the discount rate (r), but not much, and it is sensitive to changes of value of parameter of unit cost of the item (c).

Table 2. Effects of changes in model parameters on n, k and optimal expected system cost

		-50%	**-20%**	**0%**	**20%**	**50%**
D	**n**	17	26	41	45	54
	k	0.40185	0.40239	0.40161	0.40376	0.40464
	ETVC	0.134. 10^7	0.314. 10^7	0.423. 10^8	0.586. 10^8	0.731. 10^8
P	**n**	41	41	41	41	42
	k	0.42431	0.41466	0.40161	0.41351	0.40713
	ETVC	0.152. 10^7	0.240. 10^8	0.423. 10^8	412. 10^8	0.593. 10^8
S	**n**	58	46	41	38	34
	k	0.40161	0.41162	0.40161	0.41261	0.40652
	ETVC	0.235. 10^8	0.274.. 10^8	0.423. 10^8	0.219. 10^8	0.184. 10^8
C	**n**	39	40	41	42	43
	k	0.59191	0.43328	0.41161	0.39944	0.37557
	ETVC	0.412. 10^8	0.1419. 10^8	0.423. 10^8	0429. 10^8	0.438. 10^8
H	**n**	20	33	41	50	63
	k	0.40161	0.41161	0.40161	0.40171	0.40178
	ETVC	0.186. 10^8	0.287. 10^8	0423. 10^8	0.654. 10^8	0796. 10^8
r	**n**	40	41	41	41	42
	k	0.42341	0.42341	0.40161	0.39141	0.39234
	ETVC	0.543. 10^8	0.456. 10^8	0.423. 10^8	0.376. 10^7	0.245. 10^7
θ	**n**	41	41	41	41	42
	k	0.42253	0.41563	0.40161	0.39213	0.38342
	ETVC	0.505. 10^8	0.477. 10^8	0.423. 10^8	0.392. 10^8	0.342 . 10^8

9. Conclusion

We developed an inventory control model to manage the deteriorating items when shortage is permitted. The present value of total cost during the planning horizon in this inventory system is developed first, the proposed model is illustrated through numerical example and the sensitivity analysis is performed. In many inventory models under inflationary conditions, it has been assumed that the inflation rate is constant over the planning horizon, but the effect of inflation and time-value of money cannot be ignored. In this study, an inventory model developed to manage the deteriorating items under markovian inflationary conditions when Shortages are taken into account. It's assumed assumed the demand and deteriorating rates are constant and planning horizon is finite. Inflation rate changes as a chain markov, in addition it considers the changes of cost of items in each cycle except for the last cycle as a Continuous-Time Markov- Process, over the time horizon. The objective of the problem is to determine the optimal ordering policy for minimizing the total costs of the inventory system. The proposed model is illustrated through the numerical example and the Sensitivity analysis is performed for evaluation and validation.

Refrences

Aggarwal, S.P. and Jaggi, C.K. (1995). Ordering policies of deteriorating items under permissible delay in payments, *Journal of the Operational Research Society*, Vol.46, 658-662.

Alizadeh et al, (2011). A modified (S _ 1,S) inventory system for deteriorating items with Poisson demand and non-zero lead time. Elsevier.

Balkhi, Z.T. (2004b), ‘‘On the Optimality of Inventory Models with Deteriorating Items for Demand and Onhand Inventory Dependent Production Rate,’’ *IMA Journal Management Mathematics*, 15, 67–86.

Buzacott, J. A. (1975). Economic order quantities with inflation. *Operational research quarterly*, 553-558.

S.R. Chakravarthy, J.K. Daniel, A Markovian inventory system with random shelf time and back orders, Comput. Ind. Eng. 47 (2004) 315–337.

Chang, C. T. (2004). An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. *International journal of production economics*, 88(3), 307-316.

Chen, J. M. (1998). An inventory model for deteriorating items with time-proportional demand and shortages under inflation and time discounting. *International Journal of Production Economics*, 55(1), 21-30.

Chung, K. L. 1960. " Marko_ chain with stationary transition probabilities." Berlin: Springer.

Covert, R. P. Philip, G. C. (1973). An EOQ model for items with Weibull distribution deterioration. AIIETrans. 5, 323–326.

Dave, U. & Patel, L.K. (T, Si) policy inventory model for deteriorating items with time proportional demand, J. Oper. Res. Soc. 32 (1981) 137–142.

Diaz et al, (2016). Analyzing a lost-sale stochastic inventory model with Markov-modulated demands: A simulation-based optimization study. Elsevier.

Doob, J. L. (1953). *Stochastic processes* (Vol. 101). Wiley: New York.

Hariga, M., Ben-daya, M. Optimal time-varying lot sizing models under inflationary conditions, Eur. J. Oper. Res. 89 (1996) 313–325.

Ghare, P.M., Schrader,S.F. (1963) .A model for exponentially decaying inventory.J. Ind. Eng.14,238–243.

Ghoreishi et al. (2014). Optimal pricing and ordering policy for non-instantaneous deteriorating items under inflation and customer returns. *Optimization*, 63(12), 1785-1804.

Larsen, Ch., & Turkensteen, M. (2014). A vendor managed inventory model using continuous approximations for route length estimates and Markov chain modeling for cost estimates. Elsevier.

Lee, Ch., & Chang, Ch. (2012). An Inventory Model for Deteriorating Items in a Supply Chain with System Dynamics Analysis. Elsevier.

Liao, H. C., & Chen, Y. K. (2003). Optimal payment time for retailer's inventory system. *International Journal of Systems Science*, 34(4), 245-253.

Loa, S.T., Wee, H. M., Huang, W.C., 2007. An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. International Journal of Production Economics 106, 248–260.

Mirzazadeh et al. (2009). An inventory model under uncertain inflationary conditions, finite production rate and inflation-dependent demand rate for deteriorating items with shortages. *International Journal of Systems Science*, 40(1), 21-31.

Mirzazadeh, A. (2010). Effects of uncertain inflationary conditions on an inventory Model for deteriorating items with shortages. *Journal of Applied Sciences* (Faisalabad), 10(22), 2805-2813.

Pall et al, (2015). A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness. Elsevier.

Sharmila, D., & Uthaya kumar, R. (2015). Inventory model for deteriorating items involving fuzzy with shortages and exponential demand. *International Journal of Supply and Operations Management, Vol.2, No.3, 888-904.*

Sivakumar, A perishable inventory system with retrial demands and a finite population, J. Comput. Appl. Math. 224 (2009) 29–38

Tripathi, P. P., (2014). Inventory Model for deteriorating Items with Four level System and Shortages. *International Journal of Supply and Operations Management*, Volume 1, Issue 2, pp. 216-227.

Yang et al. (2003). Control of singularly perturbed Markov chains: A numerical study. The ANZIAM Journal, 45(01), 49-74.

Yin et al. (2001). Discrete-time dynamic systems arising from singularly perturbed Markov chains. Nonlinear analysis: theory, methods & applications, 47(7), 4763-4774.

Yin, G. G., & Zhang, Q. (2012). Continuous-time Markov chains and applications: a two-time-scale approach (Vol. 37). Springer Science & Business Media.

Yin, G., & Zhang, Q. (Eds.). (1997). Mathematics of Stochastic Manufacturing Systems: AMS-SIAM Summer Seminar in Applied Mathematics, June 17-22, 1996, Williamsburg, Virginia (Vol. 33). American Mathematical Soc..

Yin et al. (1995). Approximating the optimal threshold levels under robustness cost criteria for stochastic manufacturing systems. In Proceeding IFAC Conference of Youth Automation YAC (Vol. 95, pp. 450-454).

Appendix A

We want to determine the stationary distribution of inflation rate in 1991 to 2011in Iran.
Let: X_n: inflation rate in the state n.

Assumptions:

1. Each state has long been a month.

2. Using of the wholesale price of goods, inflation rare is calculated in each situation compared with the previous situation.

3. Inflation rate in the state n than to the state n-1 is calculated as follows. The inflation rate of (n-1)-th month to n-th month is:

$X_n = [(C_n - C_{n-1}) / C_{n-1}] .100$

C_n is price index in month n and is reported by the Iranian Central Bank. The base year (the year is that the index is set equal to 100) is 2004.

Annual growth of index in the year t than to the year t-1: [Average of the year (t) - average of the year (t-1)] / average of the year (t − 1)

Table A.1. The total price index of consumer goods and services in urban areas (2004=100)

Month / Year	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Average of year	Inflation rate
1991	6.7	6.6	6.6	6.6	6.5	6.6	6.6	6.7	6.9	7.0	7.1	7.3	6.8	90
1992	7.4	7.6	7.6	7.7	7.9	8.1	8.2	8.2	8.4	8.6	8.9	9.4	8.2	20.7
1993	9.8	9.6	9.8	9.8	9.9	10.1	10.1	10.1	10.3	10.5	10.9	11.2	10.2	24.4
1994	14.7	11.5	11.7	11.7	11.9	12.1	12.4	12.5	13.0	13.3	13.7	14.4	12.5	22.9
1995	22.2	15.0	15.3	15.5	15.7	16.3	17.1	17.1	17.4	18.1	19.5	20.8	16.9	35.2
1996	30.3	23.7	23.9	23.7	23.9	24.5	25.0	25.6	26.4	27.2	27.8	28.4	25.2	49.4
1997	34.5	30.0	29.8	29.9	30.0	30.4	30.8	31.2	31.9	32.3	32.8	33.3	31.0	23.2
1998	40.9	35.1	35.3	35.3	35.7	35.9	36.0	36.3	36.9	37.8	38.9	39.4	36.4	17.3
1999	49.5	41.0	41.0	41.5	41.7	42.1	42.6	43.2	44.4	45.0	45.8	46.9	43.0	18.1
2000	56.0	49.9	50.3	49.9	50.2	50.6	51.2	51.5	52.8	53.9	54.6	55.3	51.6	20.1

Table A.1. Continued

Month **Year**	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Average of year	Inflation rate
2001	62.7	56.1	56.9	56.4	57.0	57.5	57.8	58.4	59.6	60.0	60.5	61.6	58.2	12.6
2002	70.1	62.7	62.8	63.3	63.7	64.0	64.4	64.7	65.9	66.9	67.7	68.8	64.8	11.4
2003	82.6	71.7	72.8	73.0	73.2	74.4	74.2	75.3	76.7	78.3	79.8	80.0	75.0	15.8
2004	94.5	83.4	84.6	85.2	85.4	85.3	85.9	86.9	88.9	90.0	90.6	92.1	86.7	15.6
2005	110.0	95.7	96.9	98.1	98.7	98.9	100.1	102.9	102.0	103.3	104.5	106.4	100.0	15.2
2006	115.8	109.1	108.9	108.2	108.0	108.4	109.2	110.3	111.4	112.3	113.3	115.1	110.4	10.4
2007	135.3	116.9	119.0	118.9	119.4	121.7	123.4	124.4	127.4	129.9	131.6	133.0	123.5	11.9
2008	168.0	136.3	138.1	139.2	140.0	143.5	145.7	148.2	152.4	154.8	158.2	162.9	146.2	18.4
2009	194.0	170.8	174.6	175.5	178.7	185.7	188.7	190.2	192.6	191.0	191.1	191.9	183.3	25.4
2010	214.0	196.4	199.9	200.0	202.1	202.9	203.1	204.2	206.8	208.1	208.1	211.9	203.0	10.8
2011	256.1													

Table A.2. Monthly rate of change of the price index (monthly inflation rate) from 1991 until 2011

Month / Year	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
1991	1	2	1	1	2	2	1	0	2	2	2	5
1992	4	-2	2	0	1	2	0	0	1	1	3	2
1993	2	0	1	0	1	1	2	0	4	2	3	5
1994	2	2	2	1	1	3	4	0	1	4	7	6
1995	6	6	0.84	-0.84	0.84	2	2	2	3	3	2	4
1996	2	0	-0.66	0.33	0.33	1	1	1	2	1	1	1
1997	3	1	1	0	1	0.56	0.27	0.83	1	2	0.52	1
1998	3	0.24	0	1	0.48	0.95	1	1	2	1	1	2
1999	5	0	0	1	0	1	-0.7	0.58	0.58	2	1	1
2000	1	0.17	1	1	-0.87	0.87	1	0.52	2	0.67	0.83	1
2001	2	0	0.15	0.79	0.63	0.47	0.62	0.15	1	1	1	1
2002	0.43	0.85	1	0.27	0.27	1	-0.26	1	1	1	1	1
2003	21	0.96	1	0.7	0.23	-0.11	0.7	1	2	1	1	1
2004	2	1	1	1	0.61	0.2	1	0.79	1	1	1	1
2005	3	-0.18	-0.18	-0.91	-0.81	0.37	0.73	1	0.99	0.8	0.89	1
2006	0.6	0.94	1	-0.08	0.42	1	1	0.18	2	1	1	1
2007	١	0.37	1	0.72	0.57	2.5	1	1	2	1	2	2
2008	٣	1	2	0.51	1	3	1	0.79	1	-0.51	-0.52	0.41
2009	1	1	1	0.05	0.5	0.49	0.09	0.04	1	0.49	0.48	1
2010	٠/٩٩	0.42	0.69	0.46	1	0.76	1	1	2	0.99	2	3
2011	0.39											

Figures in Table3, indicate the state space. Since elements of the state space must be integer, we convert the figures as follows: Suppose k is an integer.

If X_n € (k-0.5 , k+0.5) then X_n = k, So, we have: -2,-1, 0, 1, 2, 3, 4, 5, 6}

Table A.3.

Month / Year	Apr	May	June	July	Aug		Sep	Oct	Nov	Dec	Jan	Feb	Mar
1991	1	2	0	1	2		2	1	0	2	2	2	5
1992	4	-2	2	0	1		2	0	0	1	1	3	2
1993	2	0	1	0	1		1	2	0	4	2	3	5
1994	2	2	2	1	1		3	4	0	1	4	7	6
1995	6	6	1	-1	1		2	2	2	3	3	2	4
1996	2	0	-1	0	0		1	1	1	2	1	1	1
1997	3	1	1	0	1		1	0	1	1	2	1	1
1998	3	0	0	1	0		1	1	1	2	1	1	2
1999	5	1	1	-1	1		1	1	1	1	2	1	1
2000	1	0	1	-1	1		1	1	1	2	1	1	1
2001	2	0	0	1	1		0	1	0	1	1	1	1
2002	0	1	1	0	0		1	0	1	0	2	1	1
2003	2	1	1	1	0		0	1	1	2	1	1	1
2004	2	1	1	1	1		0	1	1	1	1	1	1
2005	3	-1	0	-1	0		0	1	1	1	1	1	1
2006	1	1	1	0	0		1	1	1	2	1	1	1
2007	1	1	1	1	1		3	1	1	2	1	2	2
2008	3	1	2	1	1		3	1	1	1	-1	-1	0
2009	1	1	1	0	0		0	0	0	1	0	0	1
2010	1	0	1	0	1		1	1	1	1	2	2	3
2011	0												

Table A.4. Frequency Matrix

	-2	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
-2	0	0	0	0	1	0	0	0	0
-1	0	1	4	3	0	0	0	0	0
0	0	2	12	18	1	0	1	0	0
1	0	4	16	68	14	2	1	0	0
2	0	0	6	12	93	12	0	0	0
3	0	1	2	0	2	1	1	1	0
4	1	0	1	0	2	0	0	0	0
5	0	0	0	1	1	0	1	0	0
6	0	0	0	1	1	0	0	0	1

f_{ij} represents the number of observations from the state of i to the state of j.

Transfer Matrix:

This matrix calculated using the frequency matrix and considering the following equation:

$P_{ij} = f_{ij} / \sum f_{ij}$ j= -2 … 6

Table A.5. Transition Matrix

	-2	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
-2	0	0	0	0	1	0	0	0	0
-1	0	0.125	0.5	0.375	0	0	0	0	0
0	0	0.05	0.35	0.52	0.029	0	0.029	0	0
1	0	0.03	0.15	0.64	0.13	0.019	0.009	0	0
2	0	0	0.04	0.09	0.75	0.0	0	0	0
3	0	0.125	0.5	0	0.5	0.125	0.125	0.125	0
4	0.25	0	0.25	0	0.5	0	0	0	0
5	0	0	0	0.33	0.33	0	0.33	0	0
6	0	0	0	0.33	0.33	0	0	0	0.33

[(f(-2), f(-1), f(0), f(1), f(2), f(3), f(4), f(5), f(6)] . A = [(f(-2), f(-1), f(0), f(1), f(2), f(3), f(4), f(5), f(6)]

So we have:

f(-2) = 0.01, f(-1) = 0.06, f(0) = 0.3, f(1) = 0.15, f(2) = 0.3, f(3) = 0.07, f(4) = 0.04,

f(5) = 0.03, f(6) = 0.04

Appendix B

Now, It's assumed that the cost of items in first (n-1) cycle changes as a Continuous - Time Markov-Process. If we consider cost of items in each cycle at time (j-1) T, (j=1...n) is c_0, then we want to predict the values of cost of items at time (j-1) T $+\beta$, (j=1...n). Now consider state space as follow:

$S = \{c_0, c_1, c_2, c_3, c_4\}$

To determine a Continuous – Time - Process, it must be specified Q_{xy}, $f_x(t)$.

T_x**: Stop Time at the state x.**

$f_x(t)$**: Probability Density Function of** T_x**.**

Q_{xy}: **Probability of** Change from the state x to the state y. $x \neq y$.

$Q_{xx} = 0$, $\sum_y Q_{xy} =$ ١

The Continuous -Time Process will have the Markovian property, if $f_x(t)$ **has an exponential distribution. Therefore, have:**

$f_x(t) = \lambda e^{-\lambda t}$ $t \geq 0$

If It's assumed the states c_0, c_1, c_2, c_3, c_4 have the poisson distribution in each cycle, so the interval space between each two the subsequent states follows an exponential distribution as follow:

$$P_{xy}(t) = \begin{cases} \frac{e^{-\lambda t}(\lambda t)^{y-x}}{(y-x)!} & y \geq x \\ 0 & y < x \end{cases}$$

$P_{xy}(t)$: **The probability that the** Process starts from the state x and will be in the state y at $t \geq 0$.

$$P_{xy}(t) = \frac{e^{-\lambda t}(\lambda t)^{y-x}}{(y-x)!}$$

$$P'_{xy}(t) = -\lambda e^{-\lambda t} \frac{(\lambda t)^{y-x}}{(y-x)!} + \frac{(y-x)\lambda e^{-\lambda t}(\lambda t)^{y-x-1}}{(y-x)!}$$

$$P'_{xy}(t)_{t=0} = q_{xy}$$

$$p'_{xy}(0) = \begin{cases} \lambda & y = x+1 \\ -\lambda & y = x \\ 0 & otherwise \end{cases}$$

$$P_{xx}(t) = e^{-\lambda t}$$

$$P'_{xx}(t) = -\lambda e^{-\lambda t}$$

$$P'_{xx}(0) = -\lambda$$

$q_{xx} = -\lambda$, $x \geq 0$

$$q_{xy} = \begin{cases} \lambda & y = x+1 \\ -\lambda & y = x \\ 0 & otherwise \end{cases}$$

$q_x = -q_{xx}$

$Q_{xy} = -\frac{q_{xy}}{q_{xx}}$

Therefore, have:
$Q_{xy} = 1$ if $y = x+1$

The time horizon, H, is divided into n equal cycle. It is assumed that the Process is in the state c_0 at time (j-1) T, (j=1...n), in the state c_1 at time (j-1) T +α, (j=1...n), in the state c_2 at time (j+k-1) T, (j=1...n), in the state c_3 at time (j-1) T +β, (j=1...n) and in the state of c_4 at time Tj, (j=1...n)

X: The Probability of that the process stays at c_0 for at least α units of time and then enters to c_1,
Y: The Probability of that the process stays at c_1 for at least (kT-α) units of time and then enters to c_2.
Z: The Probability of that the process stays at c_2 for at least (β –kT) units of time and then enters to c_3.
W: The Probability of that the process stays at c_3 for at least (T – β) units of time and then enters to C_4.

Therefore, Changes of the process of the state c_0 to the state c_4 is shown as follows:

$$\int_{\alpha}^{\infty} \lambda\, e^{-\lambda t_1}\, dt_1 \,.Q_{c_0c_1} \,.\, \int_{kT-\alpha}^{\infty} \lambda\, e^{-\lambda t_2}\, dt_2 \,.Q_{c_1c_2} \,.\, \int_{\beta-kT}^{\infty} \lambda\, e^{-\lambda t_3}\, dt_3 \,.Q_{c_2c_3} \,.\, \int_{T-\beta}^{\infty} \lambda\, e^{-\lambda t_4}\, dt_4 \,.Q_{c_3c_4}$$