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Robust Optimization Approach for Design for a Dynamic Cell Formation Considering Labor Utilization: Bi-objective Mathematical Model

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Abstract

In this paper, robust optimization of a bi-objective mathematical model in a dynamic cell formation problem considering that labor utilization with uncertain data is carried out. The robust approach is used to reduce the effects of fluctuations of the uncertain parameters with regard to all the possible future scenarios. In this research, cost parameters of the cell formation and demand fluctuations are subjects to uncertainty and a mixed-integer programming (MIP) model is developed to formulate the related robust dynamic cell formation problem. Then the problem is transformed into a bi-objective linear one. The first objective function seeks to minimize relevant costs of the problem including machine procurement and relocation costs, machine variable cost, inter-cell movement and intra-cell movement costs, overtime cost and labor shifting cost between cells, machine maintenance cost, inventory, holding part cost. The second objective function seeks to minimize total man-hour deviations between cells or indeed labor utilization of the modeled.

Keywords: Dynamic Cellular Manufacturing; Robust Optimization; man-hour deviations; bi-objective mathematical model.

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1. Introduction

Cellular manufacturing systems that have combined the flexibility of job shop systems and high production rate of flow lines are introduced as a system superior to other production systems. One of the difficult steps designing a cellular manufacturing system is the cell formation problem including grouping the parts into part families and grouping the machines into manufacturing cells. This is done in a way that the parts having similar manufacturing process are produced in the same cell. It is usually assumed in modeling this problem that operations of a particular machine are specified. Cellular manufacturing system is used in production environments in order to increase flexibility and efficiency. Cellular manufacturing is an innovative production strategy based on group production. The designing method of cellular manufacturing is usually considered for one period that the product composition (the number of pieces of each type to be produced) and the demand for each part (The number of pieces of each type demanded) are considered constant in each period of planning. Since in the real world these two amounts are not constant in many cases and it is impossible to predict demand dynamic cell manufacturing is utilized. Dynamic cell manufacturing system is actually reconfiguration of manufacturing cells including part families and grouping machinery in each period. In order to reach optimal production cell and machinery configuration can be changed in each period. Due to the advancement of technology, the increase of demand and followed by an increase in production as well as more complex manufacturing systems, a model with high reliability and flexibility is needed for the manufacturing planning. Although models with deterministic approach are efficient in many cases, however it’s obvious that in a real production and planning environment many of the effective elements in manufacturing are indefinite and uncertain. However the solution to these problems is different from deterministic problems. The approach expanded to deal with the uncertainty of the data in recent years is robust optimization where the opportune optimization will be done in the worst cases which may lead to a maximum minimizing objective function. In this approach researchers seek to near to optimal solutions which are highly probable to be feasible. In other words, being a bit regardless of the objective function guarantees the feasible results obtained. However, in the case of uncertainty in the coefficients of the objective function, being slightly regardless of optimal value of objective function researchers seek to find an answer that actual answers are better than it with a high probability. A robust optimization bi-objective function is utilized in this paper in order to minimize costs and maximize labor utilization in a dynamic cell formation problem in a state of data uncertainties. In most of the cell production system models input data are assumed to be certain and deterministic. However, in real world conditions most of the parameters are uncertain and ambiguous such as part demand and processing time.

2. Literature review

The beginning of using technology group dates back to about 1920, the time when Taylor used a classification system to group the parts in need of special operations. Group technology was first introduced as a single machine concept by Mitrofanoff in 1959 in Russia. That’s in a way that some of the similar parts are grouped and added to machine works subsequently in order to take advantage of continuous production of parts or reduce machine setup time as once for all parts. Cellular Manufacturing (CM), which is an innovative manufacturing strategy derived from group technology concept, is an approach that can be used to improve both flexibility and efficiency in today modern competitive manufacturing environments, such as flexible manufacturing system and just-in-time production. The basic idea behind GT/CM is to decompose a manufacturing system into subsystems by identifying and exploiting the similarities amongst part and machines (Venugopal, V., and T. T. Narendran, 1992). CM is an efficient way to cut down the costs, improve the quality of the products and strengthen the manufacturing flexibility (Zhang, Zhifeng, and Renbin Xiao, 2009). The aim of CM is to reduce setup and flow times and therefore reduce inventory and market response times. Setup times are reduced by using part-family tooling and sequencing, whereas flow times are reduced by minimizing setup and move times, wait times for moves and by using small transfer batches (Wemmerlöv, Urban, and Nancy L. Hyer, 1989). Design of a cellular manufacturing system is composed of four phases of (1) cell formation (i.e., grouping parts with similar processing requirements into part families and corresponding machines into machine cells), (2) group layout (i.e., laying out machines within each cell, called intra- cell layout, and laying out cells within a shop floor, called inter-cell layout), (3) group scheduling (i.e., scheduling part families), and (4) resource allocation (i.e., assigning tools, human and material resources) (Wemmerlöv et al., 1986).

Dynamic Cellular Manufacturing System (DCMS) has been discussed by Rheault et al. (1995).DCMS considers a multi-period planning horizon and involves differences in product mix and demand requirements. As a result, the formed cells in one period may not be optimal for the successive periods and reconfiguration is inescapable. Literature body of dynamic cell formation (DCF) problem has evolved since the first research paper denoted to this problem by Burbidge in 1960s (Burbridge, J.L., 1971). In last decades, many researches have been done on cell formation problems and many methods such as meta-heuristic algorithms (like genetic algorithms, simulated annealing, and taboo search) and mathematical programming have been proposed. Gupta et al. (1995) developed a GA for Minimizing total inter-cell and intra-cell moves in cellular manufacturing system. Khaksar et al. (Khaksar- Haghani, 2013) presented a new integer linear programming model for designing multi-floor layout of CMS. An aspect of the model was concurrently making the cell formation and group layout decision to achieve an optimal design solution in a multi-floor factory. Kumar Saxena et al. (2011) proposed a mixed-integer nonlinear programming model to design the dynamic cellular manufacturing systems (DCMSs) under dynamic environment. Kia et al. (2012) presented a new mixed-integer nonlinear programming model for an intra-cell layout design for a DCMS. One of the new aspects of the proposed model was the concurrent making of the cell formation and intra-cell layout decisions in a dynamic environment. The other aspect of the model was utilization of a multi-row machine layout for machines of unequal-area. Defersha and Chen (Defersha, et al. 2008) proposed a comprehensive mathematical model for dynamic cell formation problem considering cell reconfiguration, alternative routings, and sequence of operations, duplicate machines, machine capacity, workload balancing and production cost. But the matter of concern is that effective parameters in the manufacturing system are considered constant in previous studies while that is not true in reality. In the real world, there are different types of uncertainties influencing manufacturing procedure. These uncertainties are divided into two main categories: a) environmental uncertainty and b) system uncertainty. Environmental uncertainty includes uncertainties belonging to manufacturing procedure such as demand and supplier uncertainty. System uncertainty is related to uncertainties inside the manufacturing procedure such as uncertainty at delivery time, uncertainty in quality, sudden failure in manufacturing system and product changes. In fact if the models are stable, the risk of incorrect use will be much less. Stability means that the output of the model should not be much sensitive about the exact values of the parameters and inputs. Robust optimization is applied to model problems related to optimization where data uncertainties are relevant and get an answer which is good enough about all or most of the uncertain parameters. Robust optimization can be considered as a complementary option for sensitivity analysis and stochastic planning. In classical paradigms of mathematical programming input data of the model (the parameters) are certain (deterministic) and equivalent to the nominal values. This attitude does not take into account the effect of uncertainty on the quality and feasibility of the model. In fact, different amounts of data available from their nominal values may lead to some constraints to be violated; the optimum solution is not optimal in the long run, or even is not feasible any more. These solutions are called robust. Based on previous studies, robust optimization is one of the approaches that work very efficiently in the presence of uncertainty (Feizollahi et al., 2012). Robust optimization was introduced by Sweester in 1973. The model proposed by Singh is very conservative and is the most pessimistic approach (Singh, C., 1982). Many attempts have been done to provide sustainable and manageable models in past two decades in order to solve different optimization problems with uncertain data. Mulvey et al. proposed a model which decision maker can undertake incompatibility risk or the level of service function and propose a set of solutions that are less sensitive to data realization in a set of scenarios. Two types of robustness are proposed in this approach: solution robust (remains close to optimal for all scenarios) and model robust (remains almost feasible for all data scenarios). Optimized solution gained by robust optimization model is called robust. If input data changes and remains close to optimal, it will be called solution robust. A solution is robust when remains feasible despite small changes in input data which is called model robust (Mulvey et al., 1995). Ben-Tal et al. proposed models as well as robust linear programming called a conic quadratic programming (Ben-Tal et al., 2009). These models are less conservative and provide better solutions. Meanwhile, Bertsimas et al. made an evolution in robust optimization proposing a model with adjustable levels of conservative work as well as robust linear programming. Their work can be applied on optimization problems with discrete variables (Bertsimas et al., 2004). One of the basic assumptions in optimization models in mentioned investigations is that the resources and associated costs are considered certain, while many of these parameter values are uncertain in real world and instead of an exact value a range of values are allocated to them. Due to the uncertainty of the data, it is possible that the optimum solution by the models is not feasible. Sensitivity analysis and stochastic planning approaches are used in classic method for taking into account the uncertainty of the data. In the first approach, the impact of uncertainty on the model data is ignored primarily and later for endorsing the results obtained the sensitivity analysis is used. But the sensitivity analysis is only a tool for analysis of appropriateness of solutions and can’t be used to produce robust solutions. In addition, simultaneously the sensitivity analysis on parameters is not practical in models having a lot of uncertain data. In the middle of 1950s, Dantzig presented stochastic planning as an approach to model the uncertainty of the data. This approach takes scenarios with different probabilities into consideration to assign values to parameters and solution feasibility is expressed by the chance constraints. There are three main problems with this approach:

1) Understanding the data distribution function and create values from scenarios getting values by this distributions. 2) Chance constraints eliminate the convex feature of the main problem and add too much to the complexity of problem. 3) The dimensions of obtained optimization model increases too much with the rise of scenarios that causes major computing challenges (Dantzig, George B., 1955). Sustainable optimization approach is presented in recent years to deal with the uncertainty of the data. This approach seeks to related optimal solutions having a high probability. In other words, being a bit regardless of the objective function, feasibility of the objective function is guaranteed. In recent years, special attention is associated to the development of sustainable models (Pan et al., 2010). Integrated logistics optimization and production costs related to the supply chain is proposed based on the scenario where the demand parameter is considered uncertain. Robust optimization model formulation with expected costs, cost variability due to demand uncertainty and expected penalties are proposed in this paper. Vafaeinezhad et al. formulated robust optimization of a mathematical model of a dynamic cell formation problem integrating CF, production planning and worker assignment that implemented with uncertain scenario-based data (Vafaeinezhad et al., 2016). In their research, miscellaneous cost parameters of the cell formation and demand fluctuations are subject to uncertainty and a mixed-integer nonlinear programming model is developed to formulate the related robust dynamic cell formation problem Zanjani et al. focused on a multi-period, multi-product sawmill production planning problem considering uncertain quality of raw materials (Zanjani et al., 2010). They proposed two robust optimization models with different variability measures and the tradeoff between plant stability as well as consumption of raw materials and expected backorder/inventory cost is proposed. Robust optimization model for the general cell formation problem considering machinery layout is presented. Intracellular transport costs of materials, cost of sending materials from parts to cells, cost of materials left from parts to cells, inventory costs and the part demand parameter under scenario are considered in (Paydar et al., 2014). The objective of this model including the total cost of the intracellular and intercellular transport, the investment cost of the machinery and inventory cost is minimized. A robust optimization model for manufacturing planning problem is considered in (Rahmani et al. 2013). In this model, the total cost parameters including setup costs, production cost, labor cost, inventory cost, labor transportation cost and nonlinear demand parameter under scenario is considered. In this model the relationship between response robustness and model robustness is analyzed. Aalaei and Davoudpour (Aalaei and Davoudpour, 2015) designed a bi-objective optimization model for a dynamic virtual cellular manufacturing system and supply chain design. Their presented model considers important manufacturing features thoroughly such as multi-plants and facility locations, multi-markets allocations, multi-period production planning under uncertain demand of products and capacities of resources. Also their model is converted into an equivalent auxiliary crisp model. Then, to solve the model, a revised multi-choice goal programming approach is applied to find a favorite solution. Sakhaeii and et al (2016) presented a robust optimization approach a new integrated mixed-integer linear programming (MILP) model to solve a dynamic cellular manufacturing system (DCMS) with unreliable machines and a production planning problem simultaneously. Their model is incorporated with dynamic cell formation, inter-cell layout, machine reliability, operator assignment, alternative process routings and production planning concepts. To cope with the parts processing time uncertainty, a robust optimization approach immunized against even worst-case is adopted. Deep and Singh (Deep and

Singh, 2015) designed a comprehensive mathematical model for designing robust machine cells for dynamic part production. The proposed model incorporates machine cell configuration design problem bridged with the machines allocation problem, the dynamic production problem and the part routing problem. For some applications of robust optimization can refer to (Saffarian et al., 2051). In this study researcher proposed a bi-objective model for relief chain logistic in uncertainty condition including uncertainty in traveling time and also amount of demand in damaged areas. And other, (Shishebori and Ghaderi, 2015) considers the combined facility location/network design problem with regard to transportation link disruptions and develops a mixed integer linear programming formulation to model it.

In this paper, a bi-objective optimization model is proposed in order to minimize the costs and maximize the labor utilization in dynamic cell formation problem while data uncertainty. In most of cell manufacturing system models input data are considered certain and deterministic. In order to model the dynamic cell formation problem, a bi-objective function is considered including minimization of cell formation costs and also minimizing the total man-hour deviations among cells in different periods. Therefore, the main objective of this paper is presenting a bi-objective model in order to reduce cell formation costs and the total man-hour deviations simultaneously. In this regard, the first objective of this paper is minimizing the cell dynamic formation problem costs including machinery and transportation cost, the variable machinery cost, the intracellular and intercellular transportation cost, the overtime cost, the cost of labor transfer between cells and the maintenance cost of inventory. The second objective is minimizing the total man-hour deviations among cells and actually maximizing labor utilization in dynamic cell formation problem.

3. Model description

A new model is proposed in this part in order to robust optimization considering the objectives of minimizing the costs primarily and then minimizing the total man-hour deviations among cells.

3-1 Assumptions

- *Each type of part has several operations which should be processed based on the number of operations.*
- *Operation time and the amount of manual labor required for all operations are clear according to one part on different machineries.*
- *The demand for different parts in each period is uncertain and under scenario.*
- *Time capacity of all machineries in normal time in the planning horizon is clear and consistent.*
- *Time capacity of machinery in overtime period in the planning horizon is clear and consistent.*
- *The purchase price of any machinery in the entire time period is known.*
- *Fixed cost of any machinery is uncertain and under the scenario. Even when the machine is idle is assumed for each machine.*
- *The variable cost of each type of machinery is uncertain and under the scenario. The variable cost covers the operations associated to workload assigned to the machine.*

- *The inventory and backorder between periods is allowed considering costs under scenario.*
- *Overtime operations on all types of machinery are determined and constant and the maximum time to work on any machinery is determined and limited.*
- *The maximum size of cell is known and constant in each period.*
- *All machineries operate multitask. Therefore the switch cost for one or more operations should be considered.*
- *Total number of human resources is fixed for all periods. Layoffs and hiring are not allowed.*
- *Intracellular transportation cost for each machine between periods is uncertain and under scenario.*
- *Batch sizes to move parts between and inside cells are fixed, but the size of intercellular and intracellular batches are different. Researchers assumed that carrying batches between cells are of similar cost and intercellular movement of batches cost the same.*
- *Labor intracellular transportation cost for each period is uncertain and under scenario.*
- *The cost of intercellular and intracellular in each batch is uncertain and under scenario.*
- *The purchase cost of machinery is uncertain and under scenario.*
- *The variable cost of processing on machinery in the whole time of each period is uncertain and under scenario.*
- *The final income proceeded from sale of machinery is uncertain and under scenario.*
- *The available time to manpower is constant.*

2-3 *Mathematical model*

Sets & Notations

C=1,2…C	cell production set	C
m=1,2…M	set of different types of machines	M
p=1,2…P	set of different types of parts	P
h=1,2…H	set of time periods	H
j=1,2…OP	set of operations related to part p	J
s=1,2…S	set of scenarios	S

Parameters

M	the number of different types of machines
L	the total number of human resources
C	the maximum number of cells that could be formed
D_{phs}	the demand for part *P* in period *h* under scenario *S*
ϑ_{phs}	1 if part *p* is produced in period *h* under scenario *S*, otherwise 0

B_p^{inter}	the batch size for part *P* intercellular movement
B_p^{intra}	the batch size for part *P* intracellular movement
γ_s^{inter}	the cost of intercellular movement in each batch under *scenario s*
γ_p^{intra}	the cost of intracellular movement in each batch under *scenario s*
φ_{ms}	the cost of purchasing a machine type *m* under *scenario s*
h_{phs}	the inventory cost for part *P* maintenance at the end of *h* period under *scenario s*
W_{ms}	Final income by selling machine type *m* under scenario *s*
α_{ms}	Overhead cost of machine type m in each period under *s* scenario
ρ_{hs}	Constant cost of intracellular movement of labor in period *h* under *scenario s*
β_{ms}	variable cost of machine *m* for each unit of time in a certain time under *scenario s*
δ_{ms}	the transportation cost of machine *m* under *scenario s*
T_{mh}	time capacity of the machine *m* in period *h* in normal time
T'_{mh}	time capacity of the machine *m* in period *h* in overtime
θ_{mhs}	variable processing time on machine *m* in each part of the whole time in period *h* under scenario *s*
UB	the maximum cell size
P_s	the probability of each scenario
WT	available time for each labor
t_{jpm}	the processing time needed to do operation *j* from part *p* on machine *m*
t'_{jpm}	the amount of manual work required to do operation *j* from part *p* on machine *m*
a_{jpm}	1 if operation *j* from part *p* is done on machine *m*, otherwise 0
λ_1	the weight assigned to total costs response variance (risk aversion range of the first objective function)

λ_2	the weight assigned to total man-hour variations response variance (risk aversion range of the second objective function)
ω	the penalty of deviation from uncertain parameters of the problem

Decision variables

N_{mch}	the number of machine *m* assigned to cell *c* in period *h*
K^{+}_{mch}	the number of machine *m* added to cell *c* in period *h*
K^{-}_{mch}	the number of machine *m* omitted from cell *c* in period *h*
I^{+}_{mh}	the number of machine *m* purchased in period *h*
I^{-}_{mh}	the number of machine *m* sold in period *h*
X_{jpmchs}	1 if operation *j* from part *p* on machine *m* in cell *c* in period *h* under scenario *s* is one, otherwise 0
L_{ch}	the number of labor assigned to cell *c* in period *h*
WM_{chs}	rate of man-hour workload in cell *c* in period *h* under scenario *s*
AM_{chs}	the average rate of man-hour workload in cell *c* in period *h* under scenario *s*
T'_{mch}	additional time needed for machine *m* in cell *c* in period *h*
δ_{phs}	unsatisfied demand of part *p* in period *h* under scenario *s*
Q_{phs}	the number of parts *p* manufactured in period *h* under scenario *s*
I_{phs}	the inventory rate of part *p* at the end of period *h* under scenario *s*

Mathematical model

$$Min\ Z_1$$

$$\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C} N_{mch}\,\alpha_{ms} \tag{1}$$

$$+\sum_{h=1}^{H}\sum_{m=1}^{M} I^{+}_{mh}\,\varphi_{ms} - \sum_{h=1}^{H}\sum_{m=1}^{M} I^{-}_{mh}\,W_{ms} \tag{2}$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O}\sum_{m=1}^{M}\beta_{ms}\,Q_{phs}\,t_{jpm}\,X_{jpmchs} \tag{3}$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\gamma_s^{inter}\left[\frac{Q_{phs}}{B_p^{inter}}\right]\left|\sum_{m=1}^{M}X_{(j+1)pmchs}-\sum_{m=1}^{M}X_{jpmchs}\right| \tag{4}$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\gamma_s^{intra}\left[\frac{Q_{phs}}{B_p^{intra}}\right]\left(\left|\sum_{m=1}^{M}X_{(j+1)pmchs}-\sum_{m=1}^{M}X_{jpmchs}\right|-\left|\sum_{m=1}^{M}X_{(j+1)pmchs}-\sum_{m=1}^{M}X_{jpmchs}\right|\right) \tag{5}$$

$$+\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}T'_{mch}\,\theta_{mch} \tag{6}$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\rho_{hs}\left(\left|L_{c(h+1)}-L_{ch}\right|\right) \tag{7}$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\delta_{ms}\left(K_{mch}^{+}+K_{mch}^{-}\right) \tag{8}$$

$$+\sum_{h=1}^{H}\sum_{p=1}^{P}h_{phs}\,I_{phs} \tag{9}$$

$$Min\ Z_2=\sum_{c=1}^{C}\sum_{h=1}^{H}\left|WM_{chs}-AM_{chs}\right| \tag{10}$$

$s.t$

$$\sum_{c=1}^{C}\sum_{m=1}^{M}X_{jpmchs}\,a_{jpm}=\vartheta_{phs}\qquad\forall\, j,p,h \tag{11}$$

$$X_{jpmchs}\leq a_{jpm}\qquad\forall\, j,p,m,c,h \tag{12}$$

$$\sum_{p=1}^{P}\sum_{j=1}^{O}X_{jpmchs}\,Q_{phs}\,t_{jpm}\leq T_{mh}\,N_{mch}+T'_{mch}\qquad\forall\, m,c,h \tag{13}$$

$$\sum_{c=1}^{C}N_{mch}-\sum_{c=1}^{C}N_{mc(h-1)}=I_{mh}^{+}-I_{mh}^{-}\qquad\forall\, m,h \tag{14}$$

$$N_{mc(h-1)}+K_{mch}^{+}-K_{mch}^{-}=N_{mch}\qquad\forall\, m,c,h \tag{15}$$

$$\sum_{c=1}^{C}T'_{mch}=T'_{mh}\qquad\forall\, m,h \tag{16}$$

$$\sum_{c=1}^{C} L_{ch} = \mathrm{L} \qquad \forall\, h \qquad (17)$$

$$\sum_{m=1}^{M} N_{mch} = UB \qquad \forall\, \mathrm{c}, h \qquad (18)$$

$$WM_{chs} = \sum_{j=1}^{O}\sum_{p=1}^{P}\sum_{m=1}^{M} X_{jpmchs}\, Q_{phs}\, t'_{ipm} \qquad \forall\, \mathrm{c}, h \qquad (19)$$

$$AM_{chs} = \left(\frac{\sum_{j=1}^{O}\sum_{p=1}^{P}\sum_{m=1}^{M} X_{jpmchs}\, Q_{phs}\, t'_{ipm}}{C}\right) \qquad \forall\, \mathrm{c}, h \qquad (20)$$

$$\sum_{j=1}^{O}\sum_{p=1}^{P}\sum_{m=1}^{M} X_{jpmchs}\, Q_{phs}\, t'_{ipm} \leq WT\, L_{ch} \qquad \forall\, \mathrm{c, h} \qquad (21)$$

$$D_{phs} = Q_{phs} - I_{phs} + I_{p(h-1)s} \qquad \forall\, \mathrm{p}, h \qquad (22)$$

$$Q_{phs} \leq M\, \vartheta_{phs} \qquad \forall\, \mathrm{p}, h, s \qquad (23)$$

$$X_{jpmchs} \qquad \forall\, j, \mathrm{p}, \mathrm{m}, \mathrm{c}, h, s \qquad (24)$$

$$L_{ch}, N_{mch}, K^{+}_{mch}, K^{-}_{mch}, I^{+}_{mch}, I^{-}_{mch} \qquad \forall\, \mathrm{m}, \mathrm{c}, h \qquad (25)$$

The first objective includes nine cost parameters. The first expression is the constant cost of the machine; second one is the difference between the machine purchase cost and revenue, the third one is the variable cost of the machine, the forth one is the cost of intercellular movement, the fifth one is the cost of intracellular movement, the sixth one is the overtime cost of the machine, the seventh one is the cost of labor transportation between cells, the eighth one is the cost of movement of the machine, and the ninth one is the cost of part inventory. In the second objective function the tenth expression is minimizing total man-hour deviations between cells in different periods and actually labor utilization. The first constraint proposed in inequality (11) guaranties that each operation is assigned to one machine and one cell, if assuming that the constraint is equal to 1, and then all products are manufactured in every period. The constraint (12) does not let x to be equal to 1 while the parameter corresponding to “a” is equal to 0. The constraint (13) guarantees that the capacity of the machine does not exceed and prevent the repeat of machines to meet the demand. The constraint (14) shows the number of machines purchased or sold in each period. The constraint (15) shows the balance of the machine. The number of different machine in current period in a specific cell is equal to the number of different machines in the previous period in addition to the number of machines added minus the number of machines removed from the cell. The constraint (16) guarantees that the sum of time assigned to each cell and each kind of machine can’t exceed capacity in overtime period. The constraint (17) guarantees the sum of labor assigned to each cell is equal to the total number of labor in each period. The constraint (18) specifies the upper limit for the size of the cell. The constraint (19) shows the man-hour workload in cell *c* in period *h* under scenario *s*. The constraint (20) shows the average of man-hour workload in cell c in period h under scenario s. The constraint (21) guarantees that the work time of an operator does not exceed the available time that an operator is allowed to work. The constraint (22) shows the inventory balance between periods for each kind of parts which means that the level of inventory of each part at the end of period is equal to the level of inventory in the previous period in addition to the amount of production minus the demand for the part. The constraint (23) is the complementary of constraint (11). This constraint guarantees some of the demand of the part can be produced in each period if its corresponding operation in constraint (11) is satisfied. The constraints (24) to (26) specify type of decision variables in the model.

4. Linearization of the proposed model

The proposed model of the problem is mixed-integer nonlinear programming because the absolute exists in the fourth, fifth and seventh expression of the first objective function and the tenth expression in the second objective function and also multiplying the decision variable to the second, fourth and fifth expressions of the first objective function and the constraints number 13, 19, 20, 21. Linearization is performed in two steps. In the first step the absolute expression is linearized as follows. Non-negative variables of $Z^1{}_{jpchs}$ and $Z^2{}_{jpchs}$ are defined and the fourth expression of the objective function is rewritten as follows.

$$\frac{1}{2}\gamma_s^{inter}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\left[\frac{Q_{phs}}{B_p^{inter}}\right]\left(Z_{jpchs}^1+Z_{jpchs}^2\right) \tag{27}$$

Then the following expressions are added to the constraints of the original model in order to complete the linearization:

$$Z_{jpchs}^1-Z_{jpchs}^2=\sum_{m=1}^{M}X_{(j+1)pmchs}-\sum_{m=1}^{M}X_{jpmchs} \tag{28}$$

The fifth expression of the first objective function is replaced as follows because of the existence of absolute values of $Y^1{}_{jpmchs}$ and $Y^2{}_{jpmchs}$ and the fifth expression of the first objective function is rewritten as follows:

$$\frac{1}{2}\gamma_s^{inter}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\left[\frac{Q_{phs}}{B_p^{inter}}\right]\left(\sum_{m=1}^{M}\left(Y_{jpmchs}^1+Y_{jpmchs}^2\right)-\left(Z_{jpchs}^1+Z_{jpchs}^2\right)\right) \tag{29}$$

Following items are added to the constraints of the original model in order to complete the linearization:

$$Y_{jpmchs}^1-Y_{jpmchs}^2=X_{(j+1)pmchs}-X_{jpmchs} \tag{30}$$

In the seventh expression of the first objective function two expressions as $W^1{}_{ch}$ and $W^2{}_{ch}$ are replaced to the absolute expression and the objective function is rewritten as follows:

$$\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\rho_{hs}\,(W_{ch}^{1}+W_{ch}^{2}) \tag{31}$$

The following items are added to the constraints of the original model in order to complete the linearization:

$$W_{ch}^{1}-W_{ch}^{2}=L_{c(h+1)}-L_{ch} \tag{32}$$

In the tenth expression of the second objective function two expressions as $G^1{}_{chs}$ and $G^2{}_{chs}$ are replaced to the absolute expression and the objective function is rewritten as follows:

$$\sum_{c=1}^{C}\sum_{h=1}^{H}(G_{chs}^{1}+G_{chs}^{2}) \tag{33}$$

Following items are added to the constraints of the original model in order to complete the linearization:

$$G_{chs}^{1}-G_{chs}^{2}=WM_{chs}-AM_{chs} \tag{34}$$

As it is clear the objective function (27) is still nonlinear because of multiplying of two variables of Q_{ph} $(Z^1{}_{jpchs}+Z^2{}_{jpchs})$. Therefore the expression $\varphi^1{}_{jpchs}$ is added to make it linear and the objective function is rewritten as follows:

$$\frac{1}{2}\gamma_s^{inter}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\left[\frac{\varphi_{jpchs}^{1}}{B_p^{inter}}\right] \tag{35}$$

Following items are added to the constraints of the original model in order to complete the linearization:

$$\varphi_{jpchs}^{1}\geq Q_{phs}-M\left(1-Z_{jpchs}^{1}-Z_{jpchs}^{2}\right) \tag{36}$$

$$\varphi_{jpchs}^{1}\leq Q_{phs}+M\left(1-Z_{jpchs}^{1}-Z_{jpchs}^{2}\right) \tag{37}$$

Also, the objective function (5) is nonlinear because of multiplying of two variables. Therefore the expression $\varphi^2{}_{jpchs}$ is added to make it linear and the objective function is rewritten as follows:

$$\frac{1}{2}\gamma_s^{inter}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\left[\frac{\varphi_{jpchs}^{2}}{B_p^{inter}}\right] \tag{38}$$

Following items are added to the constraints of the original model in order to complete the linearization:

$$\varphi_{jpchs}^{2}\geq Q_{phs}-M\left\{1-\sum_{m=1}^{M}\left(Y_{jpmchs}^{1}+Y_{jpmchs}^{2}\right)+\left(Z_{jpchs}^{1}+Z_{jpchs}^{2}\right)\right\} \tag{39}$$

$$\varphi_{jpchs}^{2}\leq Q_{phs}+M\left\{1-\sum_{m=1}^{M}\left(Y_{jpmchs}^{1}+Y_{jpmchs}^{2}\right)+\left(Z_{jpchs}^{1}+Z_{jpchs}^{2}\right)\right\} \tag{40}$$

In the third expression of the first objective function, the constraints (21), (20), (19), (13) are nonlinear because of multiplying of two variables as $(X_{jpmchs}\times Q_{phs})$. Therefore the non- negative expression φ_{jpmchs} is added to the objective function to make it linear. So the following items are added to the constraints:

$$\varphi_{jpmchs} \geq Q_{phs} - M((1 - X_{jpmchs}) \quad (40)$$

$$\varphi_{jpmchs} \leq Q_{phs} + M((1 - X_{jpmchs}) \quad (41)$$

Finally linear model of the dynamic cell formation problem is written as follows:

$$MinZ_1 = \sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O}\sum_{m=1}^{M} \beta_{ms}\, t_{jpm}\, \varphi_{jpmchs}$$

$$+Eq(1) + Eq(2) + Eq(6) + Eq(8) + Eq(9) + Eq(31) + Eq(35) + Eq(38)$$

$$MinZ_2 = Eq(33)$$

$$s.t$$

$$Eq(11), Eq(12), Eq(14), Eq(18), Eq(22), Eq(28), Eq(30), Eq(32)$$

$$, Eq(34), Eq(36), Eq(37), Eq(39), Eq(42)$$

$$\sum_{p=1}^{P}\sum_{j=1}^{O} \varphi_{jpmchs} t_{jpm} \leq T_{mh} N_{mch} + T'_{mch} \qquad \forall\ m, c, h \quad (42)$$

$$WM_{chs} = \sum_{j=1}^{O}\sum_{p=1}^{P}\sum_{m=1}^{M} \varphi_{jpmchs}\, t'_{jpm} \qquad \forall\ c, h, s \quad (43)$$

$$AM_{chs} = \left(\frac{\sum_{j=1}^{O}\sum_{p=1}^{P}\sum_{m=1}^{M} \varphi_{jpmchs}\, t'_{jpm}}{C}\right) \qquad \forall\ c, h \quad (44)$$

$$\sum_{j=1}^{O}\sum_{p=1}^{P}\sum_{m=1}^{M} \varphi_{jpmchs}\, t'_{jpm} \leq WT\, L_{ch} \qquad \forall\ c, h \quad (45)$$

5. The framework of the robust optimization model

The robust optimization acquires a set of solutions which are stable to fluctuations in parameters (input data) in the future. The robust optimization approach is proposed by Mulvey which decision maker can undertake incompatibility risk or the level of service function and propose a set of solutions that are less sensitive to data realization in a set of scenarios. Two types of robustness are proposed in this approach: solution robust (remains close to optimal for all scenarios) and model robust (remains almost feasible for all data scenarios). Optimized solution gained by robust optimization model is called robust. If input data changes and remains close to optimal, it will be called solution robust. A solution is robust when remains feasible despite small changes in input data which is called model robust. Robust optimization includes two specific constraints: 1) Structural constraint 2) control constraint. The structural constraint is a concept of linear programming and input data are constant and determined and far from any disturbance, while the control constraints are formulated as assistant constraints that have been affected by uncertain data. The robust optimization framework is explained briefly.

First, $x \epsilon R^{n}{}_1$ the design variable vector and $y \epsilon R^{n}{}_1$ the control variable vector are considered. The robust optimization model is as follows:

$$Min\ c^T x + d^T y$$

$$Ax = b \quad (1)$$

$$Bx + Cy = e \quad (2)$$

$$x, y \geq 0 \quad (3)$$

The constraint (1) is a structural constraint and the constant variables are determined and certain. The constraint (2) is the control constraint and their variables are uncertain and under scenario. The constraint (3) guarantees the non-negativity of the variables.

The robust optimization problem formulation includes a set of scenarios such as τ={1,2,…S} .

Under each scenario $\tau \in S$, the coefficients of control constraints are equal to constant P_s probability of { ds,Bs,Cs,es} that Ps stands for probability of happening of each scenario and ΣsPs=1. The optimum solution of this model is robust when solution remains close to optimal for each specific scenario of $(\tau \in S)$. This is called model robustness.

There are conditions that solutions obtained by the model are not both feasible and optimum for all scenarios $(\tau \in S)$. The relation between robustness of answer and robustness of model is determined using the concept of multi-criteria decision-making. The robust optimization model is formulated to measure this equation. First, the control variable *Ys* for each scenario $\tau \in S$ and also the fault vector δs that measures admissible non feasibility in control constraints under scenario *s* are introduced.

Because of uncertain parameters of the model it can be unjustified for some of the scenarios. Therefore δs show non feasibility of the model under scenario *s*. If the model is feasible, δs will be equal to 0; otherwise δs is a positive value according to constraint (57). In fact the robustness of the model measures unsatisfied demand for part production. The robust optimization model is formulated as follows based on mathematical programming problem (1)-(4):

$$Min\ \sigma(x, y_1, \ldots, y_s) + \omega\ \rho(\delta_1, \delta_2, \ldots, \delta_s)$$

$$AX = b \qquad 4$$

$$B_s x + C_s y_s + \delta_s = e_s \qquad 5$$

$$x \geq 0\ , y \geq 0$$

It should be noted that because robust optimization model considers multiple scenarios, the first expression from the primary objective function is a unique selection for the previous objectives of (47) and $ℶ_s = c^T x + d^T y$ is a random variable having the random value of $ℶ_s = c^T x + ds^T ys$ and probability of P_s under scenario $\tau \in S$.

In random linear programming formulation the average value of $\sigma(0) = \sum_s ℶ_s\, p_s$ is used and shows the first expression of solution robustness. The second expression in objective function as $\rho(\delta 1, \delta 2, \ldots, \delta s)$ is the feasible penalty function that penalties control constraints violations under some of the scenarios. The violation of control constraints means that non feasible solution is gained under some of the scenarios. Using the weight of ω the relation between solution robustness measured by the first expression σ(0) and model robustness measured by penalty function of ρ(0), it can be modeled under multi-criteria decision making. As an instance, ω(0) stands to minimize σ(0) and the solution is non-feasible. While if ω is extended enough, ρ(0) will dominate and lead to higher costs. Surveying the selection of the appropriate type of ρ (0) and σ (0) can be referred to [3]. The expression of σ $(x, _1, \ldots, y_s)$ is proposed as follows by [3]:

$$\sigma(0) = \sum_{s}^{S} ℶ_s\, p_s + \lambda \sum_{s}^{S} p_s \left(ℶ_s - \sum_{s'}^{S} ℶ_{s'}\, p_{s'} \right) \qquad 6$$

In order to show robustness the variance of equation (5) shows that the decision has a high risk. In the other words a small variable in uncertain parameters can cause big changes in the value of measurement function. Λ is the assigned weigh to solution variance. As it can be seen, there is a quadratic expression in equation (6). In order to reduce computer operations an absolute expression is used instead of the quadratic expression as follows:

$$\sigma(0) = \sum_{s}^{S} ℶ_s\, p_s + \lambda \sum_{s}^{S} p_s \left| ℶ_s - \sum_{s'}^{S} ℶ_{s'}\, p_{s'} \right| \qquad 7$$

5.1 *the suggested robust optimization model*

In this paper, the overhead cost of the machine, the purchase cost of the machine, the revenue gained by machine sale, the variable cost of the machine, the cost of intercellular movement in each batch, the cost of intracellular movement in each batch, the cost of overtime of the machine, the cost of labor transportation between cells, the cost of machine movement, the cost of part inventory and the part demand parameter are considered uncertain and under scenario. The explained robustness model above for this dynamic cell formation problem is as follows:

$$TC_s = \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\alpha_{ms} + \sum_{h=1}^{H}\sum_{m=1}^{M} I_{mh}^{+}\,\varphi_{ms} - \sum_{h=1}^{H}\sum_{m=1}^{M} I_{mh}^{-}\,W_{ms}$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O}\sum_{m=1}^{M}\beta_{ms}\varphi_{jpmchs}t_{jpm}$$

$$+\frac{1}{2}\gamma_s^{inter}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\left[\frac{\varphi_{jpchs}^{1}}{B_p^{inter}}\right]$$

$$+\frac{1}{2}\gamma_s^{inter}\sum_{h=1}^{H}\sum_{p=1}^{P}\sum_{j=1}^{O-1}\sum_{c=1}^{C}\left[\frac{\varphi_{jpchs}^{2}}{B_p^{inter}}\right]$$

$$+\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}T'_{mch}\,\theta_{mch} + \frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\rho_{hs}\,(W_{ch}^{1}+W_{ch}^{2})$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\delta_{ms}\,(K_{mch}^{+}+K_{mch}^{-}) + \sum_{h=1}^{H}\sum_{p=1}^{P}h_{phs}\,I_{phs}$$

$$BW_s = \sum_{c=1}^{C}\sum_{h=1}^{H}(G_{chs}^{1}+G_{chs}^{2})$$

Therefore the robust optimization model of dynamic cell formation problem is as follows:

$$Min\ Z_1 = \sum_{s} P_s\,TC_s + \lambda_1 \sum_{s} P_s \left|TC_s - \sum_{s'} P_{s'}\,TC_{s'}\right| + \omega \sum_{s}\sum_{i}\sum_{h} P_s\,\delta_{ihs}$$

$$Min\ Z_2 = \sum_{s} P_s\,BW_s + \lambda_1 \sum_{s} P_s \left|BW_s - \sum_{s'} P_{s'}\,BW_{s'}\right| + \omega \sum_{s}\sum_{i}\sum_{h} P_s\,\delta_{ihs}$$

$S.t$

$$D_{phs} = \delta_{phs} + Q_{phs} - I_{phs} + I_{p(h-1)s}$$

Other Constraints

The first and second expressions of the first objective function are the average and variance of total costs respectively. Actually, these two expressions measure the solution robustness. The third expression of the first objective function measures the robustness of the model considering non-feasibility of control constraint under scenario *s*. The first and second expressions of the second objective function are the average and variance of total man-hour deviations respectively.

Actually, these two expressions measure the solution robustness. The third expression of the second objective function measures the robustness of the model considering non-feasibility of control constraint under scenario s.

The first objective function is nonlinear because of absolute expression and the problem changes into linear programming with two new variables q_{1s}, p_{1s}. Following constraint is added to the original model:

$$p_{1s} - q_{1s} = TC_s - \sum_{s'} P_{s'} TC_{s'}$$

Also, the second objective function is nonlinear because of absolute expression and the problem changes into linear programming with two new variables q_2 , p_{2s} . Following constraint is added to the original model:

$$p_{2s} - q_{2s} = BW_s - \sum_{s'} P_{s'} BW_{s'}$$

Therefore the robust optimization of dynamic cell formation problem is written as follows:

$$Min\ Z_1 = \sum_s P_s TC_s + \lambda_1 \sum_s P_s \left| TC_s - \sum_{s'} P_{s'} TC_{s'} \right| + \omega \sum_s \sum_i \sum_h P_s \delta_{ihs}$$

$$Min\ Z_2 = \sum_s P_s BW_s + \lambda_1 \sum_s P_s \left| BW_s - \sum_{s'} P_{s'} BW_{s'} \right| + \omega \sum_s \sum_i \sum_h P_s \delta_{ihs}$$

$S.t$

$$D_{phs} = \delta_{phs} + Q_{phs} - I_{phs} + I_{p(h-1)s}$$

Other constraints

6. The suggested procedure to solve the model

The robust optimization proposed in previous section is a multi-objective programming problem. First, the problem should be changed into an equal problem having an objective function. In this section, the problem can be replaced with an objective function in order to solve multi objective models using $LP-metric$ method. Because these two objective functions are not scaled similarly, they should be normalized using the following relation that Z^*_i is the optimum value for each objective function. Two objective functions with following equations are replaced and leads the problem to have one objective. In this paper, researchers assumed that two objective functions are named as Z_1 ، Z_2. Based on $LP-metric$ method the robust optimization model of dynamic cell formation for each of these two objective functions are solved separately. The formulation of objective function related to $LP-metric$ model is as follows:

$$Min\ Z_3 = \left[\alpha \frac{z_1 - z_1^*}{z_1^*}\right] + (1+\alpha)\frac{z_1 - z_1^*}{z_1^*}$$

Where 0≤ α ≤1. Weight coefficients belong to parameters of the objective function in above equation. Using above equation, the problem changes to have one objective that can be solved easily.

7. Numerical Example

In order to solve dynamic cell production problem using robust optimization approach a numerical example is utilized. Following similar data in the literature (Vafaeinezhad et al. 2016). First, inputs and information of the problem enter the model and the problem is solved using software and then output results will be analyzed. This problem that is proposed in uncertain environment includes 5 parts, 3 periods, and 3 cells. In each part the number of operations is assumed 3 which must be performed consecutively and in terms of processing time. Also, it is assumed that financial scenarios of the future would be appropriate for four probable scenarios as excellent, good, fair and poor respectively with 45%, 25%, 20% and 15% probabilities respectively. In this example manufacturing costs are selected from uniform random sets and some of the other parameters are according to the following table.

Table 1. Time capacity of machine type *m* in period *h* in normal time and overtime

machine	*Normal time*	*overtime*
M1	500	200
M2	500	200
M3	500	200
M4	500	200
M5	500	200

Table 2. The processing time needed to perform operation *j* from part *p* on machine type *m*

	P_1			P_2			P_3			P_4			P_5		
t_{jpm}	J_1	J_2	J_3	J_1	J_2	J_3	J_1	J_2	J_3	J_1	J_2	J_3	J_1	J_2	J_3
M_1	0	0	0	176	0	39	0	0	0	0	83	0	0	0	0
M_2	0	0	0	0	0	0	99	0	33	0	0	74	0	0	0
M_3	73	93	0	0	0	0	0	0	0	45	0	0	0	0	0
M_4	0	0	46	0	81	0	0	0	0	0	0	0	0	26	0
M_5	0	0	0	0	0	0	0	48	0	0	0	0	12	0	75

Table 3. The manual work time needed to perform operation *j* from part *p* on machine type *m*

	P$_1$			P$_2$			P$_3$			P$_4$			P$_5$		
t_{jpm}	J$_1$	J$_2$	J$_3$	J$_1$	J$_2$	J$_3$	J$_1$	J$_2$	J$_3$	J$_1$	J$_1$	J$_3$	J$_1$	J$_2$	J$_3$
M$_1$	0	0	0	176	0	39	0	0	0	0	0	0	0	0	0
M$_2$	0	0	0	0	0	0	99	0	33	0	0	74	0	0	0
M$_3$	73	93	0	0	0	0	0	0	0	45	45	0	0	0	0
M$_4$	0	0	46	0	81	0	0	0	0	0	0	0	0	26	0
M$_5$	0	0	0	0	0	0	0	48	0	0	0	0	12	0	75

Because of the importance of two objective functions the total costs and total man-hour deviations between cells three models are proposed simultaneously to perform sensitivity analysis:

1- The Z_1 model: includes total costs of manufacturing system.
2- The Z_2 model: includes total man-hour deviations between cells in different periods and under different scenarios considering related constraints.
3- The Z_3 model: the LP_metric a combination of Z_1 and Z_2 model considering related constraints.

Therefore a series of multi-objective solutions are purchased for the model considering different values of α.

After solving the model using the software, the following results are gained:

Table 4. Unsatisfied demand of the parts in three periods

δ_{phs}	senario	P_1	P_2	P_3	P_4	P_5
h_1	Boom	0	17.391	0	0	0
	Good	0	0	0	0	0
	Fair	0	0	0	0	0
	poor	0	0	0	0	0
h_2	Boom	75.301	117.391	0	0	0
	Good	0	0	0	0	0
	Fair	0	0	0	0	0
	poor	0	0	0	0	0
h_3	Boom	0	0	0	0	0
	Good	0	0	0	0	0
	Fair	0	0	0	0	0
	poor	0	0	0	0	0

Table 5. Analysis of the components of the objective function in different scenarios in three periods

	Total Cost	Workload imbalanced	Machine constant cost	Machine variable cost	Purchasing machine cost	Inter cell movement	Intra cell movement	Inventory cost	Overhead cost
Boom	275040.319	687.879	30000	87218	100000	12571.792	1862.174	0	28813.413
Good	189211.564	244.109	27000	28888.091	88000	4422.727	1689.962	388.506	25826.087
Fair	179160.169	370.321	24300	25414.862	81000	3313.027	1226.513	634.94	22807.261
poor	134714.601	111	22000	9528	74000	1266.667	1250	1569.697	19829.935

Figure 1 shows the cell configuration in the first period for the main model of dynamic cell formation problem. In this figure three cells are formed for each period. The parts, the machine, the operation assignment of the part and machine allocation are also shown in following figure. For sample 2, machine type 3 is allocated to cell one in period one. The cell is shown in a rectangular shape and the numbers inside show the operations of the parts. Part 1 in period one performs operation 1 on machine 3 in cell 1 and with an intercellular movement performs operation 2 in cell 3 on machine 3, then with an intercellular movement performs operation 3 in cell 1 on machine 4. Part 2 performs operation 1 in cell 1 on machine 1 and with an intercellular movement performs operation 2 in cell 2 on machine 4 then with an intercellular movement performs operation 3 in cell 3 on machine 1. Part 4 performs operation 1 in cell 3 on machine 3 and with an intercellular movement performs operation 2 in cell 2 on machine 1 then with an intercellular movement performs operation 3 on machine 2. Therefore in order to process parts in period 1 six intracellular movements are done.

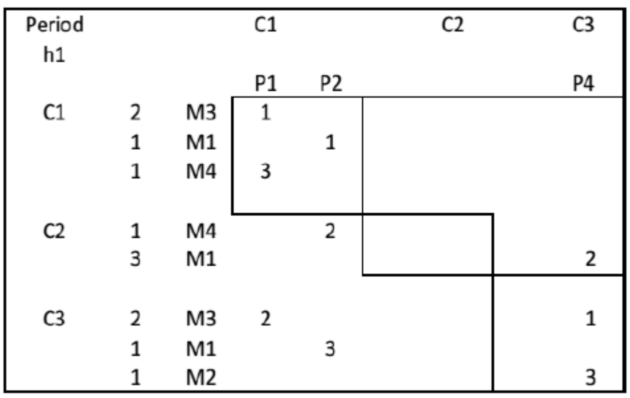

Figure 1. Cell configuration in the original model in period one under an excellent scenario

Figure 2 shows the cell configuration in the first period for robust optimization of dynamic cell formation problem model. In period 1, one type 3 machine and one type 4 machine in cell 1, one type 1 machine and one type 3 machine in cell 2 and one type 1 machine and two type 1 machine and two type 1 machine and one type 4 machine and one type 2 machine are assigned to cell 3. Actually this is less than the number of machines in the original model. Part 4 performs operation 1 in cell 1 on machine 3 and with an intercellular movement performs operation 2 in cell 3 on machine 1 then performs operation 3 in cell 3 on machine 2. Part 2 performs operation 1 in cell 2 on machine 1 and with an intercellular movement performs operation 2 in cell 1 on machine 4 then with an intracellular movements performs operation 3 in cell 2 on machine 3. Part 1 performs operation 1 in cell 3 on machine 3 and with an intercellular movement performs operation 2 in cell

2 on machine 3 then with another intracellular movement performs operation 3 in cell 3 on machine 4. Actually cell configuration in robust model in period 1 has less intracellular movements and machines compared to the original model. Therefore the cost of movement of the machines and constant cost of the machine and also the cost of intracellular and intercellular movements are reduced.

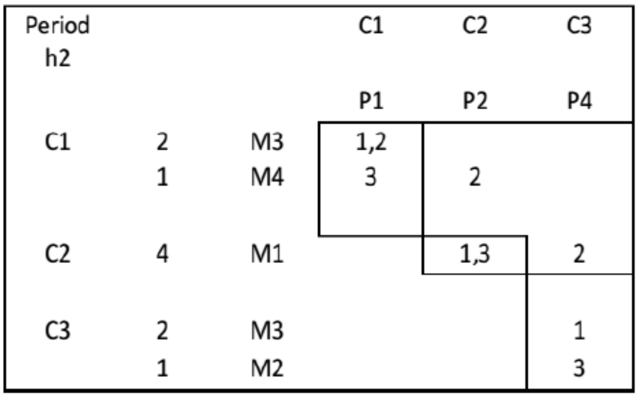

Figure 2. Cell configuration in the robust optimized model in period one under an excellent scenario

In order to show the robustness of dynamic cell formation problem by MIP model, expected value of uncertain parameters in the original uncertain linear integer programming model and the certain values of parameters, are called the average of the original model and compared to robust optimization model. As mentioned in previous sections, robust optimization obtains solution robustness against uncertain parameters fluctuations in the future. Some of the parameters are uncertain at the beginning of the planning horizon and only at the time of implementation, the actual values of uncertain parameters will be specified. In this problem considered 10 random samples of uncertain parameters and the sum of the first objective function (total cost of cell formation) and the second objective function (sum of the man-hour deviations in the cell) is shown as a total objective function. The total objective function of each sample for dynamic cell formation problem is compared to robust and original models. Table 5 shows the value of total objective function for scenarios with the probabilities of 45%, 25%, 20% and 15%.

Table 6. The total value of the objective function obtained from the robust model and the original model considering scenarios mentioned in the problem

instances	١	٢	٣	٤	٥	٦	٧	٨	٩	١٠
Robust Model	257361.8	286941.8	294127.3	285993.3	271391.6	301591.6	298255.4	308646.9	279743.5	309443
Main Model	316642.7	394204.2	389787.9	421225.5	410668.2	432788.6	371037.8	439419.8	406668.8	434360.9

8. Conclusion

Problems in dynamic cellular production systems vary greatly and many tools have been used to solve them. Literature and history of the dynamic cell formation problem shows that so far, solving the dynamic cell formation problem is not considered regarding labor utilization uncertainly. One of the main assumptions in optimization model in mentioned researches is considering the resources and their corresponding costs certain. While many of these parameter values in real conditions is uncertain and instead of an accurate value, they allocate a range of values to themselves. So, it is possible for the optimized solution of the models to be non-feasible because of uncertainty of data. On this paper robust optimization of a bi-objective mathematical model in a dynamic cell formation problem considering labor utilization with uncertain data is carried out. Some of the uncertain costs of cell formation model are as follows: The cost of purchasing machinery, costs of movements, variable costs of the machine, the cost of the intracellular and intercellular movements, the cost of overtime work, the cost of labor transportation between cells and the maintenance inventory cost of parts. Robust optimization is a model proposed to deal with data uncertainty in recent years. This approach seeks for close to optimal solutions having a high probability. In other words, with a bit ignorance of the objective function, it guarantees the feasibility of the gained solution. This model was solved considering robust optimization approach. The robust optimization approach reduces the effect of uncertain parameters fluctuations under specific scenarios. In this paper, the cell formation cost parameter and part demand fluctuations are considered uncertain in each period. This problem was formulated as the bi-objective mixed integer non-linear programming primarily and then changed into a linear model. This robust optimization bi-objective model is solved by LP method as a mono-objective model. The computational experiments obtained from a set of used data in (Vafaeinezhad et al. 2016) from a set of virtual data for a factory show that the proposed robust model is more practical for handling uncertain parameters in the production environments. The tradeoff between optimality and infeasibility is used for obtaining robust solution based on the opinion of decision-makers. The results show the robustness and effectiveness of the model in real-word practical production planning problem. Also, the results obtained by the robust MIP model (Requires less execution time) indicate the advantages of robust optimization in generating more robust production plans over the considering expected value of uncertain parameters in deterministic programming model.

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