

A stochastic programming approach for a multi-site supply chain planning in textile and apparel industry under demand uncertainty

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Abstract

In this study, a new stochastic model is proposed to deal with a multi-product, multi-period, multi-stage, multi-site production and transportation supply chain planning problem under demand uncertainty. A two-stage stochastic linear programming approach is used to maximize the expected profit. Decisions such as the production amount, the inventory level of finished and semi-finished product, the amount of backorder and the quantity of products to be transported between upstream and downstream plants in each period are considered. The robustness of production supply chain plan is then evaluated using statistical and risk measures. A case study from a real textile and apparel industry is shown in order to compare the performances of the proposed stochastic programming model and the deterministic model.

Keywords: multi-site; supply chain planning; stochastic programming; textile; robustness.

1. Introduction

Modern process industries operate no more as traditional single-plant but as multi-site supply chain structure where different production facilities are serving a global market. In the last decades, supply chain management has received a remarkable interest in order to cope with highly

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continuous competition. Supply chain planning is an important process within the supply chain management involving decisions undertaken by a company from the procurement of raw materials to the shipping of end products to the customer. The supply chain planning problem can be classified following the time horizon into three categories: strategic, tactical, and operational (Chopra and Meindl 2010). The strategic level concerns the design and the structure of the supply chain over a long time horizon between five and ten years. The operational level is related to short term decisions lasting from few days to few weeks such as scheduling, lot sizing and sequencing. The tactical planning model is between these two extremes and includes procurement, production, and distribution decisions.

This study is particularly motivated by a tactical supply chain planning problem faced by multi-site supply network from textile and apparel industry. Textile manufacturing process consists of knitting and dyeing, cutting, embroidery, cloth making, and packaging stages. Each production stage may include more than one plant, forming a multi-site, multi-stage manufacturing environment.

The fluctuation of products demand is among the most important sources of uncertainty in the textile and apparel industry. In fact, the customer demand could be determined only at the end of the planning horizon. The under-estimation of overall demand leads either to loss sales or unsatisfied customers. However, the over-estimation of the products demand results in high production and inventory costs.

In this paper, we deal with a multi-product, multi-period, multi-stage, multi-site supply chain planning problem under customer demand uncertainty. A two-stage stochastic programming model is developed in order to incorporate the effects of the uncertainty in the considered problem. Decision variables such as amounts of production and quantity to be transported between different manufacturing facilities are considered as first-stage variables and are assumed to be made before the realization of the uncertainty. Otherwise, decision variables related to the inventory level, backorder amount, and transportation amount of end products to be shipped to the customer are considered as second-stage variables made after the realization of uncertain demand. Subsequently, the robustness of production planning solution is evaluated. Statistical metric and financial risk metric, such as value at risk (*VaR*) and conditional *VaR* (*CVaR*), are calculated in order to evaluate the robustness of planning solutions generated by the stochastic programming model compared to the deterministic model. Besides, a real example from a textile and apparel manufacturer case in Tunisia is illustrated to compare the proposed stochastic model with the traditional deterministic supply chain planning model.

The main scientific contribution of this work is to develop a new stochastic model for a multi-product, multi-period, multi-stage, multi-site supply chain production and transportation planning problem under customer demand uncertainty. Besides, the proposed model and the evaluation approach are applied to a real case study from textile and apparel industry.

The rest of the paper is organized as follows. In the next section, we present the literature review of related topics. Section 3 describes the textile and apparel supply network under consideration. In section 4, a two-stage stochastic formulation is proposed in order to incorporate demand

uncertainty in the supply chain planning problem. Section 5 describes the stochastic programming algorithm. By conducting a real case study, Section 6 verifies the effectiveness and the robustness of the proposed stochastic model compared to the deterministic model. Finally, conclusions, limitations of the developed model, and future research directions are drawn in Section 7.

2. Literature review

To cope with highly competitive and global markets, the structure of manufacturing companies has changed from traditional single-site to multi-site structure. Multi-site production planning problems have received a lot of attention in the literature.

Most of the papers dealing with multi-site production planning problem focus on deterministic approaches. Toni and Meneghetti (2000) addressed the production planning problem of a textile-apparel industry supply chain. The authors investigate the influence of production planning period length as well as color assortment in the system's time performance. A real case study from an Italian network of firms was treated using a simulation model. Lin and Chen (2007) developed a monolithic model of a multi-stage multi-site multi-item production planning problem. The proposed model combined simultaneously two different time scales, i.e., monthly and daily time buckets. A practical example from the thin film transistor-liquid crystal display (TFT-LCD) industry is illustrated to explain the planning model. Leung et al (2003) studied a multi-site aggregate production planning problem of a multinational lingerie company located in Hong Kong using a goal programming approach. Three major objective functions were considered, which are minimization of the cost of workers hiring and laying-off, the maximization of profit and the minimization of the over-or under-utilization of import quotas of different products. Shah and Ierapetritou (2012) treated the integrated planning and scheduling problem for multi-site, multi-product batch plants using the augmented Lagrangian decomposition method. Given the fixed demand forecast, the model aims to minimize production, storage, shipping, and backorder costs. Felfel et al (2014) proposed a multi-objective, multi-stage, multi-product, and multi-period model for production and transportation planning in a multi-site manufacturing network. The developed model aims simultaneously to minimize the total cost and to maximize products' quality level. It should be noted that most of the papers dealing with multi-site production planning problem focus on deterministic solution. However, real production planning problems are characterized by several sources of uncertainty. Hence, the assumption that all model parameters are known with certainty will lead to non-optimal and even unrealistic results.

Many approaches have been proposed in the literature to cope with uncertainty. According to Sahinidis (2004), these approaches can be classified into four major categories: fuzzy programming approach, robust optimization approach, stochastic programming approach, and stochastic dynamic programming approach. Stochastic programming approach (Birge and Louveaux, 1997; Dantzig, 1955) is one of the most widely spread techniques in the literature used in supply chain planning problem under uncertainty. In this approach, the decision variables of the optimization problem are divided into two sets. The decision variables of the first stage called "here and now" decisions have to be made before the realization of uncertainty. Subsequently, the second-stage decision variables are chosen after the presence of uncertain parameters in order to

correct the infeasibilities caused by uncertainty realization (“wait and see” decisions). Therefore, the value of the objective function is the sum of first-stage decision variables and the second-stage expected recourse variables.

Several works in the literature have been interested in stochastic programming model for supply chain planning problem.

Gupta and Maranas (2000) proposed a multi-site midterm supply-chain planning problem under demand uncertainty using two-stage stochastic programming approach. The supply chain decisions are devised into two categories: manufacturing decisions and logistics decisions. The manufacturing decisions are taken “here and now” before the realization of uncertainty while the logistics decisions are postponed in a “wait and see” mode. A single period is considered in the developed model.

Leung et al (2006) developed a two-stage stochastic programming model in order to optimize a multi-site aggregate medium-term production planning problem under an uncertain environment. The first-stage decisions include the amount of manufactured product in regular-time and overtime, volume of subcontracted products and number of required workers, hired workers and laid-off workers. Decisions such as inventory level of products, and the amount of under-fulfilment products are considered as second-stage decisions. The effectiveness of the proposed model was highlighted through a real-world case study from a multinational lingerie company situated in Hong Kong. Karabuk (2008) considered a yarn production planning problem under demand uncertainty in a textile manufacturing supply chain. The author developed a stochastic programming model where the rover configuration, frame configuration, and production quantity are the first-stage decisions. Inventory level is considered as recourse decision. A two-step preprocessing algorithm is developed to solve the optimization problem and to reduce computational complexities of the large-scale resulting model. Nevertheless, these works didn’t consider transportation in the mathematical optimization model.

Nagar and Jain (2008) studied a multi-period supply chain planning problem for new product launches under demand uncertainty. A two-stage stochastic programming approach is developed in order to incorporate uncertainty. Production quantity, raw material procurement, and capacity utilization are presented as “here and now” decisions. Outsourcing, inventory and shipping of end product to customer are proposed as “wait and see” decisions until the realization of uncertain demand. Subsequently, this model is extended using a multi-stage stochastic programming formulation. Mirzapour Al-e-hashem et al (2011a) proposed a mid-term multi-product, multi-period, multi-site production-distribution planning problem under cost and demand uncertainties. A two-stage stochastic programming model was developed to incorporate the uncertain parameters. Mirzapour Al-e-Hashem *et al* (2011b) developed a multi-site, multi-product, multi-period aggregate production planning problem. To solve this problem, a new robust multi-objective mixed integer nonlinear programming model was proposed. The common critic of these works is the consideration of a single production stage in the planning problem.

Awudu and Zhang (2013) developed a two-stage stochastic programming model for a production planning problem in a biofuel supply chain under uncertainty in order to maximize the expected

profit. Amount of products to be produced, and amount of raw materials to be purchased and consumed are considered as the first-stage decisions. Decisions such as backlog, lost sales, and sold products quantity are considered as second-stage decisions. A case study from a biofuel supply chain is illustrated to demonstrate the effectiveness of the proposed model. A single period and a single production stage are taken into account in this work.

In the context of supply chain planning, robustness can be defined as a measure of resilience of the objective function, usually cost or profit, to change under random events and uncertain parameters. Therefore, the evaluation of robustness represents an important issue in order to assess the performance of the supply chain planning in the face of parameter uncertainty. Vin and Ierapetritou (2001) developed a strategy to quantify scheduling robustness in the face of uncertainty under uncertainty. To do so, several robustness metrics were used, such as the corrected standard deviation, the deterministic standard deviation and the extent of violation. Lin et al (2011) proposed a stochastic programming model for strategic capacity planning in thin film transistor-liquid crystal display industry. The robustness of the capacity plan is evaluated using financial risk measures, such as the value at risk and the conditional value at risk. Although the evaluation of robustness was performed in scheduling and strategic planning, this concept was not extended to tactical multi-site supply chain production and transportation planning problem.

3. Problem statement

In this paper, we considered a supply network from the textile and apparel industry wherein the finished product is processed by means of different production stages. The textile and apparel manufacturing process consists of five main stages: knitting and dyeing, cutting, embroidery, cloth making, and packaging. Each production stage may include more than one plant establishing a multi-site supply network manufacturing environment as illustrated in Figure 1. The considered supply network is composed of an internal plant (Textile-International “TE-INTER”) and four subcontractors: a dyer and a knitter, an embroiderer, and three cloth makers. The TE-INTER company is formed of three manufacturing departments which are cutting, packaging, and cloth making. Other activities such as knitting, dyeing, and embroidery are subcontracted because of the lack of technical competence and resources. Cloth making operation can be also subcontracted in order to extend the production capacity and to fulfill all the customer demand.

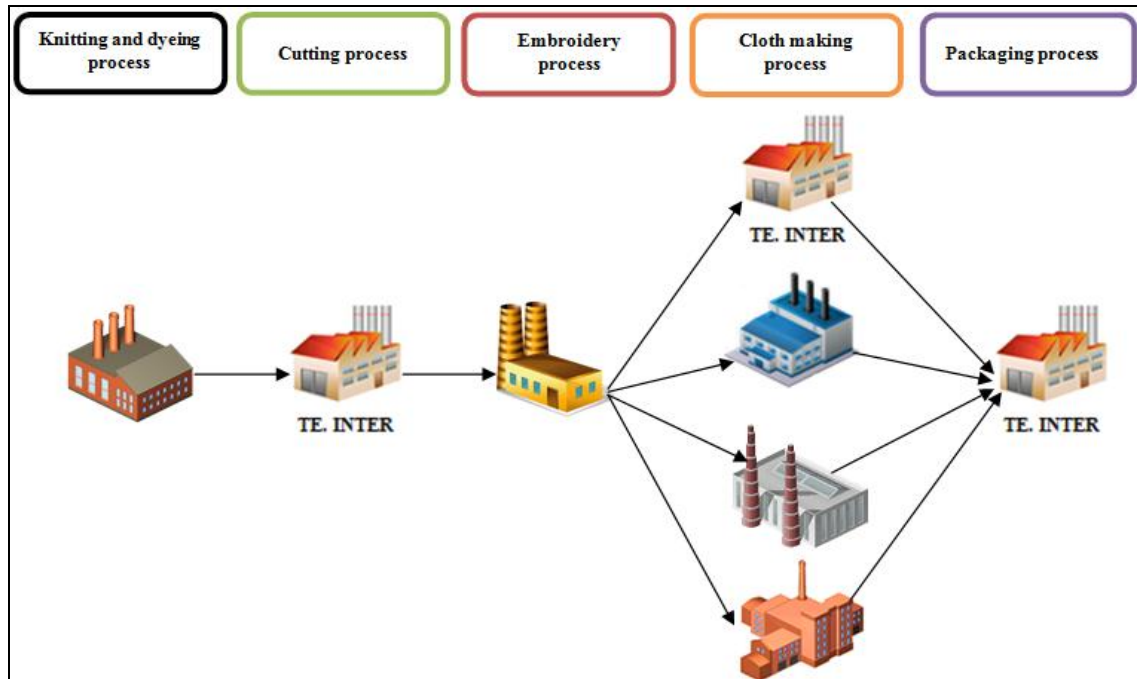


Figure 1. The multi-site supply network environment of textile and apparel industry

In textile and apparel industry, products are usually characterized by volatile demand and short life cycle. The objective of the production supply network planning is to maximize the expected profit, computed by subtracting the expected total costs from the total revenues. The expected total costs comprise production costs, storage costs, shortage cost, and transportation costs. Decisions to be decided include the production amount at each plant, the amount of inventory of finished or semi-finished products, and the flows of materials between different plants taking into account product demand uncertainty.

The multi-site supply chain planning model is built on the following assumptions:

- It is assumed that there is no initial amount of inventory and backorder.
- Since the demand is uncertain, shortage of products may occur in each period, which is assumed to be backordered.
- The uncertain demand is defined under different scenarios and it is assumed to follow a discrete distribution associated with known probability.
- A distribution lead time is taken into account in shipping the finished products to the customers and semi-finished products between different plants of the network.
- There is no waste of products during the transportation of finished and semi-finished products.

To formulate the mathematical model, we introduce the following indices parameters and decision variables:

Indices

L_i	Set of direct successor plant of i .
ST_j	Set of stages ($j= 1,2, \dots, N$).
i, i'	Production plant index ($i, i' = 1, 2, \dots, I$) where i belongs to stage n and i' belongs to stage $n+1$.
k	Product index ($k = 1,2, \dots, K$).
t	Period index ($t = 1,2, \dots, T$).
s	Scenario index ($s = 1,2, \dots, S$).

Decision variables

P_{ikt}	Production amounts of product k at plant i in period t in regular-time.
S_{ikt}^s	Amounts of end of period inventory of product k for scenario s at plant i in period t .
JS_{ikt}^s	Amounts of end of period inventory of semi-finished product k for scenario s at plant i in period t .
BD_{kt}^s	Backorder amounts of finished product k for scenario s in period t .
$TR_{i \rightarrow i', kt}$	Amounts of product k transported from plant i to i' in period t .
$TR_{i \rightarrow CUS, kt}^s$	Amounts of product k transported from the last plant i to customer for scenario s in period t .
$Q_{i,k}$	Amounts of product k received by plant i for scenario s in period t .

Parameters

cp_{ik}	Unit cost of production for product k in regular-time at plant i .
$ct_{i \rightarrow i', k}$	Unit cost of transportation between plant i and i' of production for product k .
$ct_{i \rightarrow CUS, k}$	Unit cost of transportation between the last plant i and the customer.
cs_{ik}	Unit cost of inventory of finished or semi-finished product k at plant i .
cb_k	Unit cost of backorder of product k .
pr_k	Unit sales price of finished product k .

$capp_{it}$ Production capacity at plant i in normal working hours in period t .

$caps_{it}$ Storage capacity at plant i in period t .

$captr_{i \rightarrow i',t}$ Transportation capacity at plant i in period t .

D_{kt}^s Demand of finished product k for scenario s in period t .

b_k Time needed for the production of a product entity k [min].

DL Delivery time of the transported quantity.

π^s The occurrence probability of scenario s where $\sum_{s=1}^S \pi^s = 1$

4. Proposed two-stage stochastic programming model

Due to the uncertainty of finished products demand, the deterministic model is inappropriate to optimize the expected net profit. Therefore, a two-stage stochastic programming model is proposed in order to incorporate uncertainty in the decision-making. It should be noted that the stages of the stochastic programming model correspond to different steps of decision-making and it is not related to time periods. Due to the considerable lead times required in the production process, the production amounts in each plant and the product amounts to be transported between upstream and downstream plants are taken “here and now” before the realization of the uncertainty. Other decision variables such as inventory, backorder size and flow of finished products to be shipped to the customer can be achieved in a “wait and see” mode.

Consequently, the two-stage stochastic programming model can be formulated as follows. The objective function (1) aims to maximize the expected profit obtained by subtracting the total expected cost from the expected revenue. The occurrence probability of each scenario is considered in order to calculate the expected revenue and the expected cost. The total cost includes production cost, inventory cost, backorder cost, transportation cost of semi-products between upstream and downstream plants, and transportation cost of finished products to customer.

$$\begin{aligned}
 \text{Max } E[\text{Profit}] = & \sum_{s=1}^S \pi^s \sum_{t=1}^T \sum_{k=1}^K \sum_{i=1}^I pr_k TR_{i \rightarrow \text{CUS},kt}^s - cs_{ik} (S_{ikt}^s + JS_{ikt}^s) \\
 & - ct_{i \rightarrow \text{CUS},k} TR_{i \rightarrow \text{CUS},kt}^s - cb_k BD_{k,t}^s - \sum_{t=1}^T \sum_{k=1}^K \sum_{i=1}^I cp_{ik} P_{ikt} + ct_{i \rightarrow i',k} TR_{i \rightarrow i',kt}
 \end{aligned} \tag{1}$$

Constraint (2) is the balance for the inventory level of products in each production stage excluding the last stage.

$$S_{ik,t}^s = S_{ik,t-1}^s + P_{ikt} - \sum_{i' \in L_i} TR_{i \rightarrow i',kt}, \quad \forall i \in ST_{j < N}, \forall k, t, s \quad (2)$$

Constraint (3) provides the balance for end of period inventory in the last production stage.

$$\sum_{i=1}^I S_{ik,t}^s = \sum_{i=1}^I S_{ik,t-1}^s + P_{ikt} - TR_{i \rightarrow CUS,kt}^s, \quad \forall i \in ST_{j=N}, k, t, s \quad (3)$$

Equation (4) represents the inventory balance for the semi-finished products.

$$JS_{ik,t}^s = JS_{ik,t-1}^s + Q_{ikt} - P_{ikt}, \quad \forall i, k, t, s \quad (4)$$

Constraint (5) represents the balance equation for shortage in end product demand.

$$BD_{kt}^s = BD_{k,t-1}^s + D_{kt}^s - TR_{i \rightarrow CUS,kt}^s, \quad \forall k, t, s \quad (5)$$

Constraint (6) provides the balance for transportation between different production plants.

$$Q_{i'k,t+DL} = \sum_{i' \in L_i} TR_{i \rightarrow i',kt}, \quad \forall i, k, t, s \quad (6)$$

Constraints (7) ensure that the production capacity is respected.

$$\sum_{k=1}^K b_k P_{ikt} \leq capp_{it}, \quad \forall i, t \quad (7)$$

Equation (8) is the storage capacity constraint.

$$\sum_{k=1}^K S_{ikt}^s + JS_{ikt}^s \leq caps_{it}, \quad \forall i, t, s \quad (8)$$

Constraint (9) guarantees that the transportation capacity is respected.

$$\sum_{k=1}^K TR_{i \rightarrow i', kt} \leq captr_{it}, \quad \forall i, t, s \quad (9)$$

Constraint (10) is the non-negativity restriction on the decision variables.

$$P_{ikt}, S_{ikt}^s, JS_{ikt}^s, TR_{i \rightarrow i', kt}, TR_{i \rightarrow CUS, kt}^s, Q_{i,k}, BD_{kt}^s \geq 0, \quad \forall i, k, t, s \quad (10)$$

5. Stochastic programming algorithm

The major steps of the two-stage stochastic programming algorithm to solve the proposed model are given below:

Step 1: make the first-stage decisions including the production amount in each plant and the product amount to be transported between upstream and downstream plants.

Step 2: Compute the first stage cost, $Cost1$ as follows:

$$Cost1 = \sum_{t=1}^T \sum_{k=1}^K \sum_{i=1}^I cp_{ik} P_{ikt} + ct_{i \rightarrow i', k} TR_{i \rightarrow i', kt} \quad (11)$$

Step 3: at the beginning of stage 2, the realization of all uncertain demand occurs.

Step 4: at the end of stage 2, having seen the realization of the uncertainty, and the first-stage decisions, make the second-stage decisions including inventory and backorder size as well as the amount of product to be shipped to the customer.

Step 5: compute the second-stage scenario cost, $Cost2^s$ as well the scenario revenue, $Revenue^s$. The second-stage scenario cost is equal to:

$$Cost2^s = \sum_{t=1}^T \sum_{k=1}^K \sum_{i=1}^I cs_{ik} (S_{ikt}^s + JS_{ikt}^s) + ct_{i \rightarrow CUS, k} TR_{i \rightarrow CUS, kt}^s + cb_k BD_{k,t}^s \quad (12)$$

The revenue of each scenario is given by:

$$Revenue^s = \sum_{t=1}^T \sum_{k=1}^K \sum_{i=1}^I pr_k TR_{i \rightarrow CUS,kt}^s \quad (13)$$

Step 6: Calculate the expected total profit $E[Profit]$ as follows:

$$E[Profit] = \sum_{s=1}^S \pi^s (Revenue^s - Cost2^s) - Cost1 \quad (14)$$

The proposed two-stage programming model is solved using the stochastic programming solver for multistage stochastic programs with recourse of Lingo 14.0 software.

6. Computational experiments

The main purpose of this section is to evaluate the effectiveness and the robustness of stochastic model in comparison with the deterministic model using real case industrial data from textile and apparel industry. In Section 6.1, the related input data are described. Then, the deterministic and stochastic models are solved and the quality of the obtained solutions is compared using stochastic programming parameters in Section 6.2. It is worthwhile mentioning that the deterministic model is widely used in the literature (Kall and Wallace, 1994; Birge and Louveaus, 1997; Awudu and Zhang, 2013) to evaluate the performance of the stochastic programming model. To solve the deterministic model, the random parameters are assumed to be known with certainty and thus only one scenario with mean random values is considered.

Giving the simulation results, we evaluate the robustness of the proposed model through many statistical and risk metrics as detailed in Section 6.3. Section 6.4 gives other case studies under randomly generated customer demand to validate the obtained results. The experiments are conducted using LINGO 14.0 package program and MS-Excel 2010 with an INTEL(R) Core (TM) and 2 GB RAM.

6.1. Industrial case description

In this section, real data is provided from a medium and small enterprise located in Tunisia in textile and apparel industry. The planning horizon of the planning problem covers two months and the length of a period is one week. On the basis of past sales records and future long-term and short-term contracts, the future economy can be assumed to be one of four scenarios: poor, fair, good, or boom. The market demand of the finished product P1 and P2 under each scenario is reported in Table1. Different plant indices are listed in Table 2. Table 3 described the production capacities of different plants. It should be noted that the production capacity varies from one period to another because of the absenteeism. Table 4 provides information about production and inventory unit cost. The transportation unit cost and capacity are shown in Table 5. The processing times of different manufacturing process are reported in Table 6.

Table 1. Finished product demand.

Scenario	Period										
	T1 → T5		T6			T7			T8		
	P1	P2	P1	P2	π^s	P1	P2	π^s	P1	P2	π^s
1	0	0	2350	2230	0.25	2300	2310	0.25	2 430	2210	0.2
2			2650	2520	0.35	2730	2770	0.4	2720	2530	0.3
3			1860	1550	0.2	1930	2040	0.15	1950	1860	0.25
4			1240	1020	0.2	1180	1060	0.2	1010	930	0.25

P1: product 1, P2: product 2.

Table 2. Plant indices and designation.

Plants	Designation
A1	Knitting and dyeing process– Subcontractor1
A2	Cutting- TE-INTER
A3	Embroidery Subcontractor2
A4	Cloth making- TE-INTER
A5	Cloth making –Subcontractor3
A6	Cloth making –Subcontractor4
A7	Cloth making –Subcontractor5
A8	Packaging- TE-INTER

Table 3. Production capacity per week [min].

Plants	Period							
	T1	T2	T3	T4	T5	T6	T7	T8
A1	57600	54720	57600	60480	54720	54720	51840	54720
A2	28800	31680	34560	25920	31680	23040	28800	23040
A3	43200	40320	46080	37440	40320	40320	43200	40320
A4	86400	77760	74880	83520	77760	83520	89280	83520
A5	31680	34560	25920	28800	34560	37440	31680	37440
A6	54720	48960	46080	60480	48960	63360	51840	63360
A7	17280	20160	20160	14400	20160	17280	23040	17280
A8	17280	20160	14400	17280	20160	23040	20160	23040

Table 4. Unit production cost and inventory unit cost.

Unit cost	Product	Plants							
		A1	A2	A3	A4	A5	A6	A7	A8
<i>cp</i>	P1	1.72	0.72	0.9	1.75	1.9	1.65	1.5	0.38
	P2	2.5	0.57	1.42	2.6	2.3	2.83	2.1	0.29
<i>cs</i>	P1, P2	0.3	0.1	0.15	0.12	0.1	0.11	0.1	0.2

Table 5. Unit cost and capacity of transportation per week.

Plant i → Plant j	Capacity (captr)	Unit Cost (ct) (P1,P2)
A1 → A2	9100	0.6
A2 → A3	8700	0.45
A3 → A4	7500	0.37
A3 → A5	7500	0.52
A3 → A6	7500	0.65
A3 → A7	7500	0.34
A4 → A8	-----	0
A5 → A8	2500	0.49
A6 → A8	5000	0.35
A7 → A8	1500	0.27
A8 → Customer	10000	0.5

Table 6. Processing time [min].

Product	Plants							
	A1	A2	A3	A4	A5	A6	A7	A8
P1	8	4	4.5	11	10.5	12	13	3
P2	10	2.5	6.5	16.5	15.5	14	16	2.5

6.2. Computational results

In order to evaluate the impact of uncertainty parameters on the planning decisions, two stochastic well-known measures were used: the expected value of perfect information (*EVPI*) and the value of stochastic solution (*VSS*) (Birge and Louveaus 1997). The *EVPI* parameter helps to determine the expected profit loss under uncertainty. It can be calculated as:

$$EVPI = WS - TSP \tag{15}$$

Where *TSP* is the objective value of two-stage stochastic programming model and *WS* represents the objective value of the “wait and see” model. The *WS* model involves a family of linear programming models. Each model is associated with an individual scenario. The solution of the *WS* model is obtained by weighting each individual scenario with its corresponding probability. Such a model would allow to always make the best decision regardless of the uncertain parameters which is not possible in practice.

The *VSS* parameter calculates the possible profit from solving the two-stage stochastic programming model over the deterministic model. If the *VSS* is positive, it implies that the solutions of stochastic programming model are better than those of the deterministic model. It is defined as:

$$VSS = TSP - EEV \tag{16}$$

Where *EEV* represents the expected solution of deterministic model.

The *EVPI* is then computed: $EVPI = WS - TSP = 127716.8 - 109267.2 = 18449.6$.

According to the results mentioned in Table 7, the *EVPI/WS* ratio is equal to 14.45%, which shows the big influence of product demand uncertainty on the obtained solution. Therefore, it is worthwhile to have better forecast about the demand scenarios. Then, the *VSS* is calculated: $VSS = TSP - EEV = 109267.2 - 101688.6 = 7578.6$. Therefore, the two-stage stochastic model can lead to 7.45% more gain than the deterministic model as shown in Table 7.

Table 7. Stochastic programming parameters

<i>WS</i>	<i>TSP</i>	<i>EEV</i>	<i>EVPI = WS - TSP</i>	<i>VSS = TSP - EEV</i>	<i>EVPI/WS (%)</i>	<i>VSS/EEV (%)</i>
127716.8	109267.2	101688.6	18449.6	7578.6	14.45%	7.45%

6.3. Robustness evaluation

6.3.1. Robustness evaluation metrics

In order to evaluate the robustness of the production planning, different statistical and risk metrics were used. These metrics are basically:

1. Mean value (μ). It is defined as follows :

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i \quad (17)$$

Where $x_{i=1..N}$ are the values of the sample items.

2. Standard deviation of profit distribution (SD): It measures the dispersion or variation of a set of data from its mean. A high standard deviation indicates a larger dispersion or variability. A low standard deviation implies that the data points are close to the average value. It can be formulated as:

$$SD = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2} \quad (18)$$

3. Value at risk (VaR): it is a percentile-based metric widely used in the literature for risk measurement purposes. It is defined as the minimal return or the maximal loss of a production planning over a specific time horizon at a specified confidence level (α). The VaR can be defined as the minimal portfolio return or the minimal profit at a pre-specified confidence level α as follows (Topaloglu and al 2002):

$$VaR(x, \alpha) = \min\{u : F(x, u) \geq 1 - \alpha\} = \min\{u : P\{R(x, \tilde{r}) \leq u\} \geq 1 - \alpha\}. \quad (19)$$

Where \tilde{r} : the return vector, $\tilde{r} = (\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n)^T$

R : Uncertain return of the portfolio at the end of the holding period.

F : Distribution function and $F(x, u) = P\{R(x, \tilde{r}) \leq u\}$

4. Conditional value at risk (CVaR): It is also called the mean shortfall, the mean excess loss, and tail VaR. It is a more consistent and coherent measure of risk than the VaR since it gives information about the average loss which exceeds the VaR. Topaloglou et al (2002) have introduced a general definition of the CVaR for continuous and discrete distributions as follows:

$$VaR(x, \alpha) = \left(1 - \frac{\sum_{\{s \in \Omega | R(x, r_s) \leq z\}} p_s}{1 - \alpha}\right)z + \frac{1}{1 - \alpha} \sum_{\{s \in \Omega | R(x, r_s) > z\}} p_s R(x, r_s) \tag{20}$$

Where $z = VaR(x, \alpha)$;

p_s : Associated probability to the return value $\tilde{r}_s = (\tilde{r}_{1s}, \tilde{r}_{2s} \dots \tilde{r}_{ns})^T$ under a scenario s .

Both VaR and CVaR are calculated for different risk level (α): 0.85, 0.9, and 0.95.

6.3.2. Measurement of robustness

The case study is solved with a sample size of 64 scenarios. The profit distribution results for the two-stage stochastic and deterministic model are illustrated in Figure 2. According to these distributions, we make a risk assessment analysis by comparing different distributions and we evaluate the robustness of the production planning of the stochastic and deterministic model.

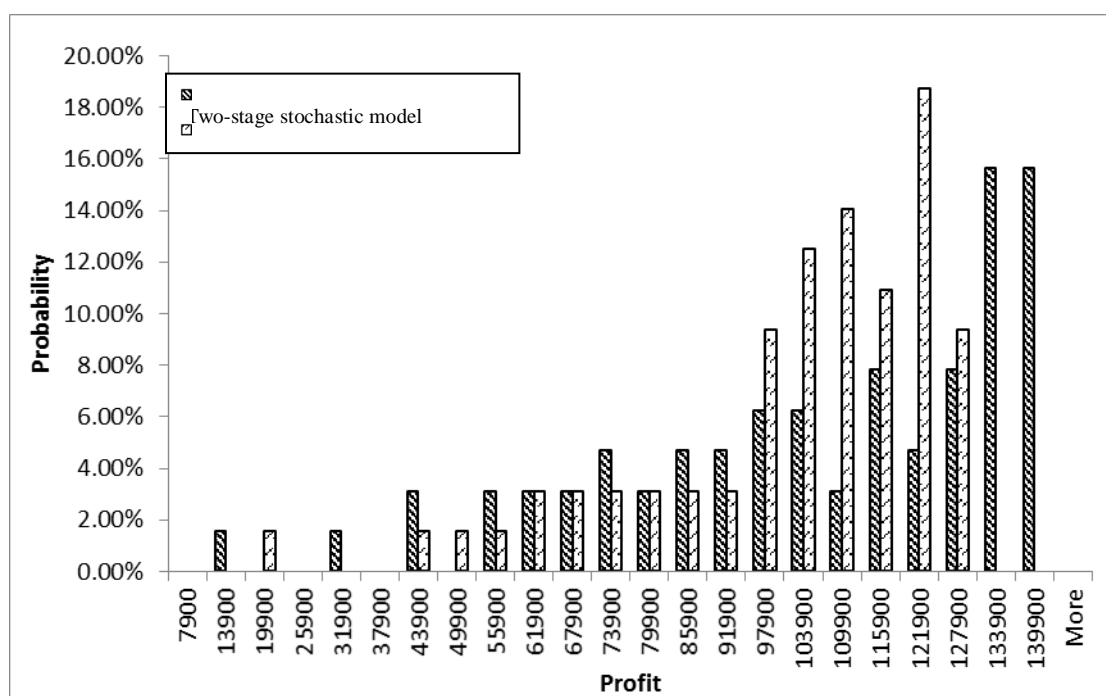


Figure 2. Profit distributions for stochastic and deterministic model

Based on these distributions, we calculate the mean, the standard deviation, and the *VaR* and *CvaR* values for different risk levels ($\alpha=0.85$, $\alpha=0.95$, $\alpha=0.9$) as shown in Table 8. The standard deviations calculated from these results are 31623.55 and 23371.93, respectively, for the two-stage stochastic and deterministic model. This result implies that a larger spread of values is obtained when we take uncertainty into account. From the Table 8, we also observe that the mean is greater for stochastic programming model. However, the *VaR* and *CVaR* for different coefficients ($\alpha=0.9$, $\alpha=0.95$, $\alpha=0.99$) are greater for deterministic model. The computational results show that the deterministic model provides a more robust solution than the stochastic model. In order to measure the improvement performance gap of the stochastic model over the deterministic model, a metric is defined as follows:

$$Improvement\ gap\ (\%) = 100 \frac{Expected\ stochastic\ value - Expected\ deterministic\ value}{Expected\ deterministic\ value} \quad (21)$$

We can see from Table 9 an improvement of 3% in mean value. Besides, we see a negative improvement in *VaR* by -12.21%, -13.55% and -18.41% for ($\alpha=0.85$), ($\alpha=0.9$) and ($\alpha=0.95$), respectively. Moreover, we can note a negative improvement in *CVaR* by -13.79%, -16.53% and -22.66% for ($\alpha=0.85$), ($\alpha=0.9$) and ($\alpha=0.95$), respectively, which means that the robustness of the obtained solutions is reduced after applying the stochastic programming model.

Table 8. Statistical and risk metrics of the stochastic and deterministic model

Model	SD	Mean	VAR (85%)	CVAR (85%)	VAR (90%)	CVAR (90%)	VAR (95%)	CVAR (95%)
SPM	31623.55	102315.52	68179.36	53691.66	57026.46	45186.98	42019.46	32356.37
DTM	23371.93	99335.22	77661.97	62281.20	65963.17	54134.27	51502.07	41838.98

Table 9. Improvement gap (%) of the stochastic model over the deterministic model

Improvement gap (%)						
Mean	<i>VaR</i> (85%)	<i>CVaR</i> (85%)	<i>VaR</i> (90%)	<i>CVaR</i> (90%)	<i>VaR</i> (95%)	<i>CVaR</i> (95%)
3.00%	-12.21%	-13.79%	-13.55%	-16.53%	-18.41%	-22.66%

6.4. Validation by other case studies

In order to validate the obtained results in Sections 6.2 and 6.3, we simulate five sets of industrial data for randomly generated customer demand. Then, we calculate the stochastic programming parameters and the statistical and risk metrics as shown in Table 10 and Table 11, respectively. As it is seen in Table 10, the VSS in all scenarios are positive with an average of 8.21%. Therefore, the use of the stochastic model can gain an average of 8.21% over the deterministic model in different scenarios. Table 10 also reports that the EPVI/WS average ratio is 16.99 %, which reflects the big impact of uncertain parameters on the solution model.

Table 10. Stochastic programming parameters under other industrial case studies.

Case	WS	TSP	EEV	EVPI=WS-TSP	VSS=TSP-EEV	EVPI/WS (%)	VSS/EEV (%)
1	114125.8	94567.47	87115.42	19558.33	7452.05	17.14%	8.55%
2	111782.2	92836.5	85592.06	18945.7	7244.44	16.95%	8.46%
3	117844.5	100436.2	94720.9	17408.3	5715.3	14.77%	6.03%
4	118476	97928.61	90186.84	20547.39	7741.77	17.34%	8.58%
5	108886.2	88460.2	80821.45	20426	7638.75	18.76%	9.45%
Average						16.99%	8.21%

From Table 11, we observe that the SD of the stochastic distribution is higher than those of the deterministic distribution. It is clear that a larger dispersion and variability is obtained when we consider uncertainty. We can also see from Table 11 that the mean values are greater for the stochastic model. Besides, the VaR values are greater for the deterministic model in all scenarios at different coefficients α expect for ($\alpha=0.85$). Moreover, CVaR values are higher for deterministic model in all scenarios. The performance improvement of the stochastic model over the deterministic model is summarized in Table 12. The average performance improvement in mean value is 12.14%. However, the average performance improvement in VaR is 3.03%, -2.90% and -12.49% for ($\alpha=0.85$), ($\alpha=0.9$), and ($\alpha=0.95$), respectively. In addition, the average performance improvement in CVaR is -15.37%, -17.60% and -21.04% for ($\alpha=0.85$), ($\alpha=0.9$) and ($\alpha=0.95$), respectively. This finding suggests that the stochastic programming model improves profitability. However, it reduces the robustness of supply chain planning results in comparison with the deterministic model under demand uncertainties.

Table 11. Statistical and risk metrics of the stochastic and deterministic model under other industrial case studies.

Case	Mode I	SD	Mean	VAR (85%)	CVAR (85%)	VAR (90%)	CVAR (90%)	VAR (95%)	CVAR (95%)
1	SPM	23407.33	97051.29	72644.10	47229.49	62797.20	41862.03	46947.60	33922.50
	DTM	17740.87	88384.74	70125.75	55110.32	66031.02	49676.22	55164.85	42139.75
2	SPM	22271.98	96518.20	73067.70	46286.98	66306.10	39318.89	54116.20	29463.07
	DTM	16738.18	86888.04	71306.34	53558.06	64760.34	48896.60	59279.34	38357.51
3	SPM	20158.92	103115.20	84228.70	59491.84	69924.80	57760.26	62631.80	50594.38
	DTM	15840.12	93215.74	76408.80	67316.43	70747.80	63257.36	67106.76	57079.91
4	SPM	25292.45	102639.02	72848.10	46808.37	66464.80	39823.24	49599.90	32448.35
	DTM	18402.98	89782.71	69436.64	56574.10	64655.21	51366.09	56944.64	42960.88
5	SPM	26016.38	91962.82	60471.20	38849.96	48820.50	34223.94	39533.70	25892.22
	DTM	17236.30	80066.20	64374.31	48623.87	56516.21	43605.30	48958.41	35316.93

Table 12. Improvement gap (%) of the stochastic model over the deterministic model under other industrial case studies.

Improvement gap (%)							
Case	Mean	VaR (85%)	CVaR (85%)	VaR (90%)	CVaR (90%)	VaR (95%)	CVaR (95%)
1	9.81%	3.59%	-14.30%	-4.90%	-15.73%	-14.90%	-19.50%
2	11.08%	2.47%	-13.58%	2.39%	-19.59%	-8.71%	-23.19%
3	10.62%	10.23%	-11.62%	-1.16%	-8.69%	-6.67%	-11.36%
4	14.32%	4.91%	-17.26%	2.80%	-22.47%	-12.90%	-24.47%
5	14.86%	-6.06%	-20.10%	-13.62%	-21.51%	-19.25%	-26.69%
Average	12.14%	3.03%	-15.37%	-2.90%	-17.60%	-12.49%	-21.04%

7. Conclusion

In this paper, we propose a multi-product, multi-period, multi-stage, multi-site production and transportation supply chain planning problem faced by textile and apparel industry under demand uncertainty. In order to incorporate the effects of the uncertainty in the supply chain planning problem, a two-stage stochastic model is developed. A real case study from a textile and apparel supply network is illustrated to verify the effectiveness and the robustness of the developed model. According to the computational results, the proposed stochastic programming model provides a higher expected profit and profit mean value than the deterministic model under demand uncertainty. However, the proposed stochastic model leads to less robust solutions in comparison with the deterministic model. Improving the robustness of the planning solutions in the face of uncertainty represents an interesting future work. This perspective can be addressed by means of risk management models that incorporate risk measures into the stochastic programming model.

Acknowledgements

This research and innovation is carried out in the context of a thesis MOBIDOC funded by the European Union under the program PASRI. The authors are grateful to LINDO Systems Inc for providing us with a free educational research license of the extended version of LINGO 14.0 software package.

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