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Inventory Model for deteriorating Items with Four level System and Shortages

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Abstract
This paper presents an inventory model for deteriorating items in which shortages are allowed. It is assumed that the production rate is proportional to the demand rate. The production rate is considered to be greater than demand rate. The inventory model is developed by considering four different circumstances. The optimal of the problem is obtained with the help of Mathematica 7 software. Numerical example is given to illustrate the model. Sensitivity analysis of the model has been developed to examine the effect of changes in the values of the different parameters for optimal inventory policy.

Keywords: Inventory, constant demand, deterioration, shortages, production rate

1. Introduction

In the past decades, inventory models for deteriorating items have been widely studied by a large number of researchers. Most of the items deteriorate over time. Deterioration is applicable to many inventories in practice, such as volatile liquids, medicines, agriculture products, radioactive materials, blood, etc. The deterioration of goods is a realistic phenomenon in many inventory systems. The controlling of deteriorating items is a measure problem in any inventory system. Thus, to develop an optimal inventory policy, the loss due to deterioration cannot be ignored. Ghare and Schrader (1963) developed inventory model for deteriorating items with constant rate of deterioration. Covert and Philip (1973) extended Ghare and Schrader (1963) model for variable rate of deterioration by assuming 2-parameter Weibull distribution function. The effect of deterioration on an inventory system has been examined by several authors such as Phillip (1974), Chakrabarty, Giri and Choudhury (1998), Misra (1975), Mehar et al (2012), Teng et al (2005), Tripathy and Mishra (2011) ,etc. Goyal (2011) established a single item inventory model under permissible delay in payments. Chung (1998) developed an alternative approach to determine the economic order quantity under the condition of permissible delay in payments. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payment. Chang et al (2001) extended Aggarwal and Jaggi (1995) model with linear trend demand.

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In inventory management, demand is the major factor. In the study of EOQ models, four types of demand are assumed, i.e. (i) constant demand (ii) time dependent demand (iii) probabilistic demand and (iv) stock- dependent demand. In classical economic order quantity model, the demand rate is assumed to be constant. In reality, the demand rate is not always constant, but it varies over time. The stock- dependent demand is more relevant, at present, in the study of inventory models. Mandal and Phaujdar (1989) proposed an inventory model for stock-dependent consumption rate. Tripathy and Mishra (2010) developed an inventory model with time- dependent Weibull demand rate where shortages are allowed. Silver and Meal (1969), were the first to develop the EOQ model for the case of varying demand. Silver and Meal (1973) established an appropriate solution procedure for general case of a deterministic, time dependent demand pattern. Aggarwal et al (2009) developed an inventory model by considering demand rate as exponentially increasing function of time. Lin and Julian (2012) investigated an inventory model with stock at the beginning and shortages allowed and then partially backlogged. There are so many productions in the real world that demand is time-varying or time- dependent. Gupta and Vrat (1986) studied an inventory model for stock-dependent demand rate. Ray and Chaudhuri (1997) developed a finite time- horizon deterministic economic order quantity inventory model with shortages where the demand rate at any instant depends on the stock- level at that instant. In the past few decades many researchers have developed inventory models by considering time- dependent demand rate. Dutta and pal (1991) investigated a finite time- horizon inventory model following the approach to Misra (1979) with linearly time – dependent demand rate allowing shortages and considering the effect of inflation and time value of money. Among the important papers published so far considering the time- dependent and stock- level dependent demand rate, the works of Baker and Urban (1988), Datta and Pal (1990), Urban (2012), Tripathi (2011), Tripathi et al (2011) and Tripathi and Kumar (2011), Teng et al (2012), etcetera, are worth mentioning. These authors developed inventory models with time- dependent demand rate, with and without considering the inflationary effects.

In some cases, customers have to wait for inventory system in case of shortages. This is due to specific characteristic or the outstanding quality of the product. For example, cheese can only be found in a specific shop. The question of the inventory shortages was not considered by above researchers. Deb and Chaudhuri (1987) were the first to incorporate shortages into the inventory lot sizing problem with a linearly increasing time- varying demand. Ghiami et al. (2013) investigated a two- echelon supply chain model for deteriorating inventory in which the retailer's warehouse has a limited capacity. Bhunia et al. (2014) presented an inventory model for single deteriorating item with two separate warehouses having different preserving facilities and shortages.Yang et al (2010) established an EOQ model for deteriorating items with stock- dependent demand and partial backlogging. Ouyang and Chang (2013) extended the effects of the reworking imperfect quality item and trade credit on the EPQ model with imperfect production process and complete backlogging. Soni (2013) developed an EOQ model considering (i) the demand rate as multivariate function of price and level of inventory (ii) delay in payment is permissible. Tripathi and Pandey (2013) established an inventory model for deteriorating items with Weibull distribution time dependent demand rate under permissible delay in payments. Sarkar (2012) developed an EOQ model for finite replenishment rate where demand and deterioration rate are both time dependent. Tripathi (2011) formulated an inventory model for non- deteriorating item and time dependent demand rate under inflation when supplier offers a permissible delay to the purchaser, if the order quantity is greater than or equal to a predetermined quantity.

In this paper, an attempt has been made to develop a deterministic inventory model by considering four different situations in which production rate is greater than demand rate. Shortages are allowed and inventory over time is taken into account.
The rest of the paper is organized as follows: In section 2, we describe the notation and assumptions and mathematical models. Next, in section 3, numerical example is given followed by sensitivity analysis in section 4 to demonstrate the applicability of the proposed model. In section 5, we draw conclusions and future research direction.

2. Mathematical Model and Analysis

In this section, the present study develops total average cost under four different circumstances. The following notation and assumptions are used throughout this paper:

2.1.Notations:

C_1 : Carrying cost per unit time

C_2 : Shortage cost per unit time

C_3 : Set up cost per production run

$I(t)$: The inventory level at any time‘t’

D : Constant demand rate

p : The production rate i.e. $p = \lambda D$, where $\lambda > 1$ a constant

T_1 : Start time production

T_2 : Time to finish backlog

T_3 : The stop time of production

T_4 : The time to complete the cycle

A : Shortage level at time $t = T_1$

B : Stock level at time $t = T_3$

T_1^* : Optimal start time of production

T_2^* : Optimal time to finish backlog

T_3^* : Optimal stop time of production

T_4^* : Optimal time to complete the cycle

SC : Total shortage cost

HC : Holding cost

SC*: Optimal total shortage cost

AIC : The total average cost for the production cycle

AIC*: The optimal total average cost for the production cycle

θ : Constant deterioration rate

2.2.Assumptions

Next, the following assumptions are made to establish the inventory model:

1. The demand rate ‘D’ of the system is constant
2. The production rate is $p = \lambda D$, where $\lambda > 1$, and is constant
3. Shortages are allowed and are completely backlogged
4. Lead time is zero
5. The on hand inventory deteriorate with time
6. C_1, C_2, C_3 are all assumed to be known and fixed during production cycle

2.3.The Model formulation and solution

In this section (,) we will develop mathematical model to find optimal solution.
At initial stage (i.e. at time t = 0), the inventory level is zero. The shortage starts at t = 0 which accumulates unto the level A at time $t=T_1$. At time $t=T_1$ the production starts and the backlog is finished at time $t=T_2$. At time $t=T_3$ the stock level reaches a level B . The production is stopped at this point. Thus the inventory level decreases gradually due to meet demand and becomes zero at time $t=T_4$. After time $t=T_4$ the cycle repeats itself (Figure for four level inventory system is given in the appendix) .The differential equation describing the inventory level I(t) in the interval $(0,T_4)$ are given by

$$\frac{dI(t)}{dt}=-D \quad , \ 0\le t\le T_1 \tag{1}$$

$$\frac{dI(t)}{dt}=p-D , \ T_1\le t\le T_2 \tag{2}$$

$$\frac{dI(t)}{dt}+\theta I(t)=p-D , \ T_2\le t\le T_3 \tag{3}$$

$$\frac{dI(t)}{dt}+\theta I(t)=-D \quad ,T_3\le t\le T_4 \tag{4}$$

Solution of Equations (1) to (4) with $p=\lambda D$ and with the conditions I(0) = 0, $I(T_1)=-A, I(T_2)=0, I(T_3)=B, I(T_4)=0$, is given by

$$I(t)=-Dt \quad , \ 0\le t\le T_1 \tag{5}$$

$$I(t)=(\lambda-1)D(t-T_2) \quad , \ T_1\le t\le T_2 \tag{6}$$

$$I(t)=\frac{(\lambda-1)D}{\theta}\left\{1-e^{\theta(T_2-t)}\right\}, \ T_2\le t\le T_3 \tag{7}$$

$$I(t)=\frac{D}{\theta}\left\{e^{\theta(T_4-t)}-1\right\} \quad , \ T_3\le t\le T_4 \tag{8}$$

Using the condition $I(T_1)=-A$ in equation (5), we obtain

$$A=DT_1 \tag{9}$$

Similarly using the condition
$I(T_1)=-A$ in equation (6), we obtain

$$A=(\lambda-1)D(T_2-T_1) \tag{10}$$

Equating equations (9) and (10), we obtain

$$T_1=\frac{(\lambda-1)T_2}{\lambda} \tag{11}$$

Again using the condition $I(T_3) = B$ in equations (7) and (8), we obtain

$$B=\frac{(\lambda-1)D}{\theta}\left\{1-e^{\theta(T_2-T_3)}\right\} \tag{12}$$

$$B=\frac{D}{\theta}\left\{e^{\theta(T_4-T_3)}-1\right\} \tag{13}$$

Equating equations (12) and (13), we obtain

$$e^{\theta(T_4-T_3)}+(\lambda-1)e^{\theta(T_2-T_3)}-\lambda=0 \quad \text{0r} \quad T_3=\frac{1}{\theta}\log\left(\frac{(\lambda-1)e^{\theta T_2}+e^{\theta T_4}}{\lambda}\right) \tag{14}$$

The total shortage cost in the system is given by

$$SC = C_2\left[\int_0^{T_1} -I(t)dt + \int_{T_1}^{T_2} -I(t)dt\right] = \frac{DC_2}{2}\left\{T_1^2 + (\lambda-1)(T_2-T_1)^2\right\} \tag{15}$$

The total holding cost in the system will be

$$HC = C_1\left[\int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_3} I(t)dt\right]$$
$$= \frac{DC_1}{\theta}\left[(\lambda-1)\left\{(T_3-T_2) + \frac{e^{\theta(T_2-T_3)}-1}{\theta}\right\} + \left\{\frac{e^{\theta(T_4-T_3)}-1}{\theta} - (T_4-T_3)\right\}\right] \tag{16}$$

The total average inventory cost in the system is given by

$$AIC = \frac{1}{T_4}\left(SC + HC + C_3\right) \tag{17}$$

Since, it is difficult to handle equation (17) to find the exact optimal solutions; therefore, we make use of the second order approximation for exponential terms, which follows as:

$$e^{\theta(T_2-T_3)} \approx 1 + \theta(T_2-T_3) + \frac{\theta^2(T_2-T_3)^2}{2}, etc \tag{18}$$

Using equation (18), (17) reduces to

$$AIC = \frac{1}{T_4}\left[\frac{DC_2}{2}\left(\frac{\lambda-1}{\lambda}\right)T_2^{\,2} + \frac{C_1D}{2}\left\{(\lambda-1)(T_3-T_2)^2 + (T_4-T_3)^2\right\} + C_3\right] \tag{19}$$

Also truncated Taylor's series is used for finding closed form solution of exponential terms i.e. $e^{\theta T_4} \approx 1 + \theta T_4 + \dfrac{\theta^2 T^2}{2}$. From equation (10), we obtain

$$e^{\theta(T_4-T_3)} + (\lambda-1)e^{\theta(T_2-T_3)} - \lambda = 0, \quad \text{or} \quad T_3 = \frac{(\lambda-1)T_2\left(1+\frac{\theta T_2}{2}\right) + T_4\left(1+\frac{\theta T_4}{2}\right)}{\lambda} \tag{20}$$

Taking first and second order partial derivatives of equation (19) with respect to T_2 and T_4 we obtain

$$\frac{\partial(AIC)}{\partial T_2} = \frac{(\lambda-1)D}{\lambda T_4}\left\{C_2T_2 + C_1(1-\lambda\theta T_2 + \theta T_2) - C_1(T_4-T_3)(1+\theta T_2)\right\} \tag{21}$$

$$\frac{\partial(AIC)}{\partial T_4} = -\frac{N}{T_4^{\,2}} + \frac{C_1D}{\lambda T_4}\left\{(\lambda-1)(T_3-T_2)(1+\theta T_4) + (T_4-T_3)(\lambda-1-\theta T_4)\right\} \tag{22}$$

Where N is the Numerator of equation (19)

$$\frac{\partial^2(AIC)}{\partial T_2^{\,2}} = \left[C_2\lambda + C_1\left\{(1-\lambda\theta T_2 + \theta T_2)^2 + \lambda(\lambda-1)\theta(T_3-T_2)\right.\right.$$
$$\left.\left.+(\lambda-1)(1+\theta T_2)^2 - \lambda\theta(T_4-T_3)\right\}\right] \tag{23}$$

$$\frac{\partial^2(AIC)}{\partial T_4^{\,2}} = \frac{2N}{T_4^{\,3}} - \frac{2C_1D}{\lambda T_4^{\,2}}\left\{(\lambda-1)(T_3-T_2)(1+\theta T_4) + (T_4-T_3)(\lambda-1-\theta T_4)\right\}$$
$$+\frac{C_1D}{\lambda^2T_4}\left\{\lambda(\lambda-1)\theta(T_3-T_2) + (1+\theta T_4)^2 - \lambda\theta(T_4-T_3) + (\lambda-1-\theta T_4)^2\right\} \tag{24}$$
$$+2\lambda C_3 - 2\lambda C_1DT_4\left\{(T_4-T_2)(\lambda-1-\theta T_4) + \lambda\theta T_4(T_3-T_2)\right\} = 0$$

$$\frac{\partial^2(AIC)}{\partial T_2 \partial T_4} = -\frac{(\lambda-1)D}{\lambda T_4^{\,2}}\left[C_2T_2 + C_1\left\{(T_2-T_3)(1-\lambda\theta T_2+\theta T_2)-(T_4-T_3)(1+\theta T_2)\right\}\right]$$
$$-\frac{C_1(\lambda-1)D}{\lambda^2 T_4}\left\{(1-\lambda\theta T_2+\theta T_2)(1+\theta T_4)+(\lambda-1-\theta T_4)(1+\theta T_2)\right\} \quad (25)$$

Optimal solution is obtained by solving $\frac{\partial(AIC)}{\partial T_2}=0$ and $\frac{\partial(AIC)}{\partial T_4}=0$, simultaneously, we obtain

$$(C_1+C_2)T_2 - C_1T_4 - C_1\theta T_2\left\{(\lambda-1)T_2-\lambda T_3+T_4\right\}=0 \quad (26)$$

$$(\lambda-1)(\lambda C_1+C_2)T_2^{\,2}+\lambda C_1 D\left\{\lambda T_3^{\,2}+T_4^{\,2}-2T_3(\lambda T_2-T_2+T_4)\right\}$$
$$+2\lambda C_3 - 2\lambda C_1 D T_4\left\{(T_4-T_2)(\lambda-1-\theta T_4)+\lambda\theta T_4(T_3-T_2)\right\}=0 \quad (27)$$

$$\left(\frac{\partial^2(AIC)}{\partial T_2^{\,2}}\right)\left(\frac{\partial^2(AIC)}{\partial T_4^{\,2}}\right)-\left(\frac{\partial^2(AIC)}{\partial T_2\partial T_4}\right)^2>0, \text{ and } \frac{\partial^2(AIC)}{\partial T_2^{\,2}}>0,\quad \frac{\partial^2(AIC)}{\partial T_4^{\,2}}>0.$$

The numerical optimal (minimum) values of $T_2 = T_2^*$ and $T_4 = T_4^*$ are obtained by solving equations (26) and (27) simultaneously.

3. Numerical Example

In this section, the present study provides the following numerical example to illustrate the theoretical results as reported in section 2.3.

The numerical example is given below to illustrate the proposed model.

Let us take the parameter values of the inventory system as C_1 = 20, C_2 = 30, C_3 = 100, λ = 1.05 and D = 10^4 in appropriate units. Solving equations (26) and (27), we obtain the optimal values of T_2 and T_4 respectively. After getting optimal values of T_2 and T_4 , we obtain optimal values of T_1 and T_3 from equations (11) and (20) respectively. We obtain $T_1 = T_1^*$ = 0.00334648 year, $T_2 = T_2^*$ = 0.702716 year, $T_3 = T_3^*$ = 0.173666 year, $T_4 = T_4^*$ = 0.175723 year AIC = AIC^* = $ 1076.39.

4. Sensitivity Analysis

In this section, we introduce sensitivity analysis with the variation of different parameters. The total average cost for the production cycle is the real solution in which the model parameters are assumed to be static values. It is reasonable to study the effect of making changes in the model parameter over a given optimal solution. It is important for finding the effects on different system parameter measures such as carrying cost, shortage cost, setup cost etc. For this purpose, it is required to observe whether the current situations remain changed or unchanged with respect to various system parameters. If current situations remain unchanged, this situation becomes infeasible etc.

Sensitivity analysis is performed by changing parameters C_1, C_2 , C_3, λ and θ, and taking one parameter at a time, keeping the remaining parameters at their original values (as in numerical example. The sensitivity analysis of each of the decision variables C_1, C_2, C_3, λ and θ, to changes in each of the parameters $T_1 = T_1^*$, $T_2 = T_2^*$, $T_3 = T_3^*$, $T_4 = T_4^*$,SC = SC* and AIC = $AIC^{*.}$

Table1. Sensitivity analysis on carrying cost C_1 on optimal values of T_1, T_2, T_3, T_4 and AIC.

C_1	T_1^*	T_2^*	T_3^*	T_4^*	HC*	SC*	AIC*
21	0.00339550	0.0713062	0.171244	0.173215	52.8426	36.32	1092.06
22	0.00344219	0.0722859	0.169011	0.170900	51.8491	37.32	1106.92
23	0.00348646	0.0732156	0.166944	0.168756	50.8914	38.29	1121.03
24	0.00352852	0.0740989	0.165026	0.166764	49.9689	39.22	1134.46
25	0.00356855	0.0749396	0.163240	0.164908	49.0788	40.11	1147.26
26	0.00360670	0.0757407	0.161575	0.163175	48.2217	40.98	1159.48
27	0.00364310	0.0765050	0.160015	0.161551	47.3925	41.81	1171.15
28	0.00367786	0.0772351	0.158552	0.160027	46.5917	42.61	1182.30
29	0.00371111	0.0779333	0.157177	0.158594	45.8180	43.38	1192.99
30	0.00374294	0.0786017	0.155882	0.157243	45.0697	44.13	1203.223

Table2. Sensitivity analysis on shortage cost C_2 on optimal values of T_1, T_2, T_3, T_4 and AIC.

C_2	T_1^*	T_2^*	T_3^*	T_4^*	SC*	AIC*
31	0.00326069	0.0684744	0.172523	0.174652	34.61	1083.25
32	0.00317946	0.0667686	0.171443	0.173639	33.97	1089.80
33	0.00310227	0.0651476	0.170419	0.17268	33.35	1096.07
34	0.0030288	0.0636049	0.16945	0.171771	32.75	1102.08
35	0.00295881	0.0621351	0.168529	0.170908	32.17	1107.83
40	0.00265306	0.0557142	0.164543	0.167174	29.56	1133.39
45	0.00240535	0.0505123	0.161358	0.164192	27.33	1154.59
50	0.00220043	0.0462091	0.158756	0.161756	25.42	1172.47
55	0.00202801	0.0425883	0.156588	0.159728	23.75	1187.75
60	0.00188087	0.0394982	0.154754	0.158012	22.29	1200.97
70	0.001642886	0.0345006	0.1518198	0.155269	19.84	1222.70
80	0.001458643	0.0306315	0.1495757	0.153172	17.87	1239.81
90	0.001311724	0.0275462	0.1478037	0.151517	16.26	1253.64
100	0.00119179	0.0250277	0.1463684	0.150177	14.91	1265.06

Table3. Sensitivity analysis on setup cost C_3 on optimal values of T_1, T_2, T_3, T_4 and AIC.

C_3	T_1^*	T_2^*	T_3^*	T_4^*	SC*	AIC*
110	0.00350451	0.0735948	0.183029	0.184037	38.69	1129.56
120	0.00365508	0.0767567	0.190001	0.191949	42.08	1180.44
130	0.00379890	0.0797769	0.197626	0.199506	45.46	1229.31
140	0.00393672	0.0826712	0.204944	0.206749	48.82	1276.41
150	0.00406917	0.0854526	0.211987	0.21371	52.16	1321.93
160	0.00419677	0.0881321	0.218782	0.220417	55.48	1366.02
170	0.00431995	0.0907190	0.225350	0.226892	58.78	1408.82
180	0.00443910	0.093221	0.231710	0.233155	62.07	1450.44
190	0.00455453	0.0956452	0.237880	0.239223	65.34	1490.98
200	0.00466654	0.0979974	0.243874	0.245112	68.60	1530.52

Table4. Sensitivity analysis on λ on optimal values of T_1, T_2, T_3, T_4 and AIC.

λ	T_1^*	T_2^*	T_3^*	T_4^*	HC*	SC*	AIC^*
1.10	0.0044776	0.0492535	0.1178320	0.123149	49.8572	33.08	1485.50
1.15	0.0051640	0.0395909	0.0921107	0.0989853	46.1010	30.67	1785.80
1.20	0.0056292	0.0337752	0.0766115	0.0844429	42.8321	28.52	2029.20
1.25	0.0059594	0.0297970	0.0676427	0.0744959	40.5041	26.64	2243.61
1.30	0.0061993	0.0268635	0.0582254	0.0671612	37.4919	24.98	2419.14
1.35	0.0063753	0.0245904	0.0522102	0.061478	35.2890	23.52	2583.11
1.40	0.0065045	0.0227659	0.0474053	0.0569164	33.3301	22.21	2732.82
1.45	0.0065986	0.0212622	0.0434674	0.0531569	31.5168	21.04	2871.16
1.50	0.0066657	0.0199971	0.0401749	0.0499939	29.9985	19.99	3000.22
1.55	0.0067118	0.0189151	0.0373776	0.0472886	28.5703	19.04	3121.54
1.60	0.0067413	0.0179769	0.0349690	0.0449429	27.2718	18.18	3236.33

Table5. Sensitivity analysis on deterioration rate θ on optimal values of T_1, T_2, T_3, T_4 and AIC.

θ	T_1^*	T_2^*	T_3^*	T_4^*	SC*	AIC^*
0.01	0.002354324	0.0494408	0.116931	0.123602	17.46	1483.25
0.02	0.002355426	0.0494394	0.116998	0.123599	17.46	1483.26
0.03	0.002354143	0.049437	0.117063	0.123593	17.46	1483.29
0.04	0.002353990	0.0494337	0.117126	0.123585	17.45	1483.33
0.05	0.002353780	0.0494294	0.117187	0.123575	17.45	1483.38
0.08	0.002352900	0.0494109	0.117356	0.12353	17.44	1483.60
0.09	0.002352520	0.0494029	0.117407	0.12351	17.43	1483.70
0.10	0.002352090	0.0493939	0.117457	0.123489	17.43	1483.81
0.15	0.002349290	0.0493351	0.117670	0.123344	17.39	1484.52
0.18	0.002347090	0.0492888	0.117774	0.123234	17.35	1485.07
0.19	0.002351062	0.0492716	0.117805	0.123193	17.34	1485.28

The results of sensitivity analysis are summarized in Tables 1 to 5.
The following inferences can be made from the results obtained:
(i). When carrying cost per unit time C_1, shortage cost per unit time C_2, setup cost per unit time C_3, and production rate parameter λ increase or decrease, the total average cost AIC will also increase or decrease. That is the change in C_1, C_2, C_3 and λ will lead the positive change in AIC.
(ii).When deterioration rate θ increases or decreases, the total average cost (AIC) is approximately constant. That is change in θ will lead AIC unchanged.
(iii) When carrying cost per unit time C_1 increases optimal total shortage cost SC* increases and slight decrease with the increase of shortage cost per unit time C_2.
(iv) When setup cost per unit time C_3 and λ increase result decrease and increase in optimal total shortage cost SC* respectively.
(v) When deterioration rate 'θ' increases or decreases Optimal shortage cost SC* remains constant.
(vi)The higher value of C_1 and λ causes lower value of holding cost
Table 5 shows that total average cost (AIC) and total shortage cost SC* are not sensitive to changes in the parameter deterioration rate θ.

5. Conclusion and Future Research

In this paper, we develop an EOQ inventory model for deteriorating item by considering four different circumstances. Shortages are taken into account. Second order approximation is used in exponential terms to provide closed form numerical optimal solution for finding the optimal time for all four stages and optimal total average cost for the production cycle. We derive the first and second order conditions for finding the optimal solution. Numerical example shows the applicability of the proposed model. Finally, we make the sensitivity analysis of parameters on the optimal solutions. The sensitivity of the solution to changes in the value of different parameters is quite sensitive and provides a useful reference for managerial decision making and administration except for deterioration rate.

The proposed model can be extended in several ways. For instance, we may extend the constant deterioration rate to a time dependent deterioration rate. We could consider the demand as a function of selling price, stock- dependent, time product quantity, etc. Finally, we could generalize the model for quantity discount and others.

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Appendix

Figure for four level inventory system.