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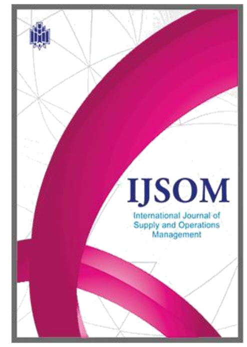

A New Multi-Objective Location Routing Problem with Hybrid Fuzzy-Stochastic Approach by Considering Capacity Restrictions: Model, Solution and Application

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Abstract

This study proposes a multi-objective location-routing problem considering the capacity of vehicles to decline the system's costs. The model considers probabilistic times of traveling, service, and waiting by vehicles while guaranteeing the least probability which the cumulative values of these parameters are less than a pre-determined value when minimization of this value is considered an objective function. To cope with uncertainty, fuzzy numbers for important parameters of customer demand, vehicle capacity, variable and fixed transportation costs, and depot opening costs are used. Moreover, the nonlinear constraints are linearized to reduce computational time. We also use a fuzzy ranking method to transform the presented model into an equivalent auxiliary crisp model. As the model is NP-hard, we introduce a novel Multi-Objective Imperialist Competitive Algorithm (MOICA) to address the issue. The efficacy of the presented MOICA is evaluated by comparing its performance against two well-established multi-objective metaheuristics, Pareto Archived Evolution Strategy (PAES), and Non-Dominated Sorting Genetic Algorithm-II (NSGA-II). Leveraging Response Surface Methodology (RSM), the mutation and crossover operators employed by each algorithm were meticulously tuned. Subsequently, the performance of all three algorithms was examined using four benchmark comparison metrics across a range of established benchmark examples. The results demonstrably substantiate the superiority of the proposed MOICA in achieving optimal solutions.

Keywords: Imperialist Competitive Algorithm (ICA); Facility location; Vehicle routing; Fuzzy mathematical programming; Linearization; Location-routing problem (LRP).

1. Introduction

Distribution network design hinges on two critical decisions: route planning and facility location. Traditionally, these problems are addressed sequentially due to their inherent complexity. However, this approach frequently brings about suboptimal solutions, as evidenced by Salhi & Rand (1989). The LRP addresses this limitation by integrating facility location and vehicle routing decisions (Vincent et al., 2010, Tordecilla et al., 2023, Shi et al., 2023). This integrated approach offers practical and cost-effective solutions for various real-world applications, including the distribution of perishable food products (Govindan et al., 2014), waste collection (Caballero et al., 2007, Han et al., 2024), mission planning in space exploration (Ahn et al., 2012), hub location and routing (Çetiner et al., 2010), parcel delivery (Wasner & Zäpfel, 2004), and blood bank location (Or & Pierskalla, 1979, Kaya & Ozkok, 2020).

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In recent years, numerous studies have developed combined Location-Routing Problems (LRPs) with varying characteristics. A recent survey by Drexl and Schneider (2015) indicates that most of the literature assumes deterministic data due to the additional challenges posed by incorporating uncertainty (Drexl and Schneider, 2015). However, it is frequently unrealistic to consider that all parameters are precisely known in advance, as real-world applications are rife with uncertainties. Consequently, the volume of research on stochastic LRPs is significantly smaller compared to deterministic problems. Fuzzy logic has been employed in LRP issues to address vague or uncertain parameters. Zarandi et al. (2011) were the first to formulate LRP using fuzzy parameters. However, previous studies primarily considered customer demand or travel time as the only fuzzy parameters. For a more realistic approach, it is essential to account for the fact that the most critical parameters are not precisely known in advance and must be treated as fuzzy data.

Traditional Location-Routing Problems (LRPs) typically focus on minimizing the total cost. However, real-world scenarios often involve multiple, conflicting objectives. Research on LRPs with multiple objectives remains limited (Nagy & Salhi, 2007). Therefore, investigating LRPs that incorporate both monetary and non-monetary objectives simultaneously is a worthwhile pursuit.

This research proposes a multi-objective LRP with fuzzy parameters and a homogeneous fleet of vehicles with capacity constraints. The model employs probabilistic travel, service, and waiting times due to the inherent difficulty or impossibility of precisely determining these parameters in real-world situations with factors like variable traffic conditions and unforeseen events. Additionally, real-world applications necessitate timely customer deliveries within predefined time windows. To balance these competing objectives of on-time delivery and efficient location-routing, the model minimizes the probability of exceeding a predetermined threshold for the total travel, service, and waiting times. This objective function prioritizes meeting customer time constraints. The second objective minimizes the overall system cost, encompassing variable transportation costs, fixed transportation costs, and depot opening costs.

The proposed model's capability expands to handle a wider range of uncertainties by incorporating fuzzy numbers for crucial parameters. These parameters include customer demand, vehicle capacity, fixed and variable transportation costs, and depot opening costs. This approach aligns better with real-world scenarios where precise knowledge of these parameters beforehand is often unrealistic. This paper addresses the transformation of a fuzzy Logic Regression Problem (LRP) model into a corresponding crisp multi-objective model. To achieve this, a ranking methodology is employed that leverages the comparative analysis of expected intervals associated with the fuzzy numbers.

However, the inherent NP-hardness of the issue is exacerbated by the introduction of non-linear constraints, significantly increasing solution time (Alizadeh et al., 2015). To address this challenge, we propose a simplification of the model by transforming the non-linear constraint into a linear one using a uniform distribution function.

Because the proposed model is NP-hard, obtaining optimal solutions within reasonable computational times, especially for large-scale instances, necessitates the use of heuristic algorithms. This research leverages MOICA to answer the model (Atashpaz-Gargari & Lucas, 2007, Golmohammadi and Abedsoltan, 2023). Prior research suggests that the Imperialist Competitive Algorithm (ICA) exhibits superior performance compared to population-based approaches (Mozafari et al., 2012; Shiripour et al., 2012; Rahimi et al., 2013; Nia et al., 2015). Inspired by these findings, we implement ICA to address the proposed model. The performance of the presented MOICA is evaluated against two well-established evolutionary meta-heuristics: PAES and NSGA-II. This evaluation employs four benchmark comparison metrics across a range of established benchmark instances. To facilitate an efficient search of the solution space, RSM is utilized to define and customize mutation and operators tailored to each algorithm. In summary, this paper presents several novel contributions to the field of multi-objective LRPs:

- Incorporating fuzzy parameters for customer demand, vehicle capacity, costs, and depot opening costs, reflecting real-world uncertainties.
- Modeling probabilistic travel, service, and waiting times to account for inherent variability.
- Proposing a crisp equivalent by the use of a ranking method based on expected fuzzy number intervals.
- Guaranteeing a minimum probability of on-time delivery by minimizing the total travel, service, and waiting times.
- Simplifying the model by transforming a non-linear constraint to a linear one.

- Employing MOICA, a recently developed multi-objective evolutionary algorithm, to answer the problem.
- Evaluating MOICA's performance against established algorithms (NSGA-II and PAES) using four comparison metrics on benchmark instances.

The proposed model incorporates a unique combination of features, including, Fuzzy parameters reflecting real-world uncertainties in customer demand, vehicle capacity, costs, and depot opening costs, probabilistic travel, service, and waiting times to account for inherent variability, classic LRP constraints related to distribution centers, vehicle capacity, sub-tour elimination, and graph-based considerations. To the best of our knowledge, this combination has not been previously addressed in the literature, resulting in a model that closely resembles real-world logistics problems. The complexity introduced by these assumptions aligns the model with the inherent complexities of real-world scenarios. The research is structured as follows,

The section 2 presents a review of relevant past works. Section 3 defines and formulates the addressed problem. Section 4 details the simplification techniques applied to the proposed model. Section 5 explains the transformation of the fuzzy model into a crisp equivalent. Section 6 discusses the proposed MOICA algorithm, comparative metaheuristics, parameter settings, comparison metrics, test problem generation, and computational results, including a comparison of the three algorithms. Section seven presents suggestions for future research and concludes the paper.

2. Literature Review

2.1 Seminal Works and Variants of LRP

LRPs have garnered less research attention compared to the extensive literature on individual VRPs or location variants (Zarandi et al., 2011). While the concept of integrating facility location and vehicle routing originated nearly five decades ago, with early studies recognizing the interdependence of these decisions (Prodhon & Prins, 2014), optimization techniques and computational power were insufficient for a comprehensive model (Watson-Gandy & Dohrn, 1973; Boventer, 1961; Christofides & Eilon, 1969; Maranzana, 1964; Webb, 1968). Although these pioneering works identified the close link between location and routing decisions, they did not capture the full complexity of LRPs.

The pivotal work by Salhi and Rand (1989) quantified the benefits of integrating VRP during facility location. Their findings demonstrated that separate decision-making can lead to suboptimal solutions, even for long-term location decisions (Salhi & Nagy, 1999). This crucial insight spurred a surge in LRP research, as evidenced by the comprehensive survey by Nagy and Salhi (2007). Several review papers have contributed to the LRP literature, including those by Prodhon and Prins (2014), Drexl and Schneider (2015), Nagy and Salhi (2007), Min et al. (1998), and Balakrishnan et al. (1987). These reviews provide classifications of LRP variants and discuss recent advancements in the field.

Standard LRPs are typically formulated as deterministic, static, discrete, single-objective, and single-echelon problems, excluding inventory decisions (Drexl & Schneider, 2015, Prodhon & Prins, 2014). Recent literature investigations by Drexl & Schneider (2015), and Prodhon & Prins (2014) highlight this standard form while acknowledging extensions that incorporate multiple objectives, uncertainties, and additional distribution echelons.

A growing trend exists towards addressing more intricate and integrated problems in LRP research. Drexl & Schneider (2015) propose a classification scheme for LRP variants and extensions published since the last comprehensive investigation by Nagy & Salhi (2007). This classification encompasses issues with fuzzy and stochastic data, multi-echelon distribution networks, multiple objectives multi-period planning horizons, complex route structures, and inventory decisions. Drexl & Schneider (2015) further summarize the core ideas of each classified work and identify promising research directions.

Since in the present paper, we propose a multi-objective LRP with stochastic and fuzzy data, and some metaheuristic approaches are suggested to solve the proposed model, so here we categorize the existing literature paying more attention to recent developments in these main approaches.

2.2 Deterministic vs. Stochastic vs. Fuzzy Data.

Deterministic models, a common approach in LRPs, assume all problem data are known with perfect accuracy. However, real-world scenarios are inherently uncertain, making this assumption often unrealistic (Drexl & Schneider, 2015). Stochastic data addresses this by representing uncertain parameters (e.g., customer demands, and travel times) as probability distributions. Fuzzy data, on the other hand, utilizes fuzzy numbers to model imprecise data. While incorporating uncertainty adds complexity, a significant portion of the literature continues to rely on deterministic models (Drexl & Schneider, 2015).

Research on stochastic LRPs remains limited. According to Nagy & Salhi (2007), existing stochastic studies primarily focus on customer demand as the sole variable and restrict themselves to single-depot, single-vehicle scenarios (Traveling Salesman Location Problem). Similarly, Drexl & Schneider (2015) identified only four and five papers published between 2006 and 2014 that addressed stochastic and fuzzy data, respectively. This limited focus on stochastic and fuzzy data highlights the potential for further research in these areas compared to the abundance of work on deterministic formulations.

Several studies have addressed LRP complexities by incorporating uncertainty. Hassan-Pour et al. (2009) propose a two-stage solution for a multi-objective LRP with stochastic facility and transport link availability. They solve the facility location issue mathematically and then employ a hybridized simulated annealing (SA) algorithm for the multi-objective VRP. Pioneering work by Zarandi et al. (2011) introduced fuzzy variables to LRP modeling. They consider an LRP with capacitated vehicles and facilities, where travel times are demonstrated as triangular fuzzy numbers. The model leverages credibility theory (Baoding, 2004) and utilizes an SA algorithm for the solution. Zare Mehrjerdi & Nadizadeh (2013) focus on an LRP with uncertain customer demands modeled as fuzzy data. Their model, based on fuzzy credibility theory, is solved using a greedy clustering method incorporating stochastic simulation.

2.3 Multi-Objective LRPs

Traditional LRPs typically focus on minimizing total cost, encompassing depot opening costs and variable and fixed transportation costs (Nagy & Salhi, 2007). However, real-world scenarios often involve conflicting objectives. While research on multi-objective LRPs remains limited (Nagy & Salhi, 2007), recent studies have begun to address this gap.

Building upon prior research, Tavakkoli-Moghaddam et al. (2010) present a model for a bi-objective LRP that accommodates the presence of optional customers. This model simultaneously optimizes for both non-monetary and monetary objectives. The monetary objective focuses on minimizing total cost expenditures, encompassing facility opening costs, fixed and variable depot costs, and variable transportation costs. In contrast, the non-monetary objective prioritizes maximizing the total consumer demand served. This study employs two metaheuristics (Elite TS (Gu et al., (2007)), and Multi-Objective Scatter Search (MOSS)) to solve the issue and demonstrate that MOSS outperforms Elite TS in terms of robustness based on computational results.

Wang et al. (2014) propose a non-linear LRP model for relief distribution, considering reliability with split delivery, total cost, and travel time simultaneously. They employ Non-Dominated Sorting Differential Evolution Algorithm (NSDE), and Non-Dominated Sorting Genetic Algorithm (NSGA) for solution (Wang et al., 2014). Martínez-Salazar et al. (2014) presented a two-echelon VRP model that incorporates direct transports on the first level and routing options on the second. Minimizing total distribution cost and balancing route duration are the addressed objectives. They propose two metaheuristics: Scatter Tabu Search for Multi-Objective Optimization (SSPMO) and NSGA-II. Their findings suggest SSPMO's efficiency for smaller instances, while NSGA-II performs better with increasing problem size (Martínez-Salazar et al., 2014). Niu et al. (2024) investigated a multi-objective LRP problem in the waste management system. The goal was to achieve equilibrium among residential satisfaction, carbon emission, and total cost.

2.4 Solution Approaches and Metaheuristics for LRP Problems

LRP inherently combines two NP-hard sub-problems: vehicle routing and facility location (Megiddo & Supowit, 1984; Salhi & Nagy, 1999). This characteristic renders exact solution methods impractical for large-scale LRP instances encountered in real-world applications (Nia et al., 2015, Diabat, 2014). Metaheuristic search algorithms offer an alternative for tackling such complex optimization issues. These algorithms are designed to obtain high-quality (though not necessarily optimal) solutions within reasonable computational times. Several successful applications of metaheuristics have been documented in LRP research. Prominent examples include Scatter Search (Tavakkoli-Moghaddam et al., 2010), Ant Colony Optimization (Sim & Sun, 2003; Ting & Chen, 2013), Multi-objective dragonfly algorithm (Golmohammadi et al., 2024), Tabu Search (Burks Jr, 2006; Goścień et al., 2015), Simulated

Annealing (Vincent et al., 2010; Yu & Lin, 2015), Genetic Algorithms (Karakatič & Podgorelec, 2015; Khalili-Damghani et al., 2015), Particle Swarm Optimization (Norouzi et al., 2015, Marinakis et al., 2013, Golmohammadi et al., 2024), Neural Networks (Schwardt & Fischer, 2008; Schwardt and Dethloff, 2005), and Evolutionary Algorithms (Prodhon, 2011; Koç et al., 2015). Among these, population-based approaches like Genetic Algorithms are often preferred due to their generally superior performance (Nia et al., 2015).

ICA, an evolutionary algorithm inspired by socio-political processes (Atashpaz-Gargari & Lucas, 2007), presents promising features for LRP applications. Notably, ICA's search process effectively reduces the likelihood of getting trapped in local optima for issues with large search spaces (Jula et al., 2015). This is achieved through well-designed operators that frequently relocate solutions within the search space. Additionally, ICA's relative youth compared to other established algorithms presents ample opportunities for further development (Jula et al., 2015).

Empirical evidence suggests ICA's effectiveness in solving LRP and related problems. Studies have demonstrated ICA's superior performance compared to other similar algorithms in terms of solution quality and computational efficiency (Nia et al., 2015). Shiripour et al. (2012) compared ICA and GA for a multi-facility location problem, finding ICA preferable in both solution accuracy and computation time. Similar results favoring ICA were obtained when applied to non-convex dynamic economic power dispatch (Mohammadi-Ivatloo et al., 2012), project scheduling (Rahimi et al., 2013), and composite material design (Mozafari et al., 2012). ICA's versatility is further demonstrated by its successful application in various optimization problems, including vehicle routing with time windows (Wang et al., 2011), multi-period location problems (Amiri-Aref et al., 2013), hub location problems (Mohammadi et al., 2014), and generalized traveling salesman problems (Ardalan et al., 2015).

3. Assumption and Problem Definition

In the present study, solving the LRP with capacitated vehicles, fuzzy parameters, and also probabilistic times of traveling, service, and waiting by vehicles are considered. It is assumed that $G=(V,E)$ is an undirected simple graph in which V is a set of nodes including a subset I of P potential depot locations and a subset $J=V/I$ of n consumers respectively. The arc set E that composed of the pairs $e=(i,j)$ in which a normal distribution of travel times of mean $E(t)$ and Variance $V(t)$ between nodes and also a fuzzy travel cost is related to each element of E. A fuzzy opening cost $\tilde{F}$ is considered for each depot $i \epsilon I$. A fuzzy demand $\widetilde{d_j}$ is considered for each customer $j \epsilon J$ which must be met by a single vehicle. The transportation unit cost is denoted as a fuzzy variable $\widetilde{C_{ij}}$. A set K of homogeneous vehicles with fuzzy capacity $\tilde{Q}$ exists. A dependent fuzzy cost $\widetilde{FV_k}$ is incurred when each vehicle is used by a depot i, and performs a single route. Each route should begin from and terminate at the same depot, and its total load should not exceed vehicle capacity and also no route is considered between depots. Furthermore, a normal distribution of mean $E(t)$ and Variance $V(t)$ for waiting times w_{jk} and service times s_{jk} by vehicles are considered, in which there is a least probability of β that the sum of the total traveling times, service times, and waiting times by vehicles be lower than a static value of B when minimization of this value is considered as an objective function.

The target of the present research is to simultaneously optimize the location and number of depots, obtain the optimum allocation of consumers to distribution centers, the optimal number of transportation vehicles, and the allocation of customers to them in a fuzzy environment. The addressed problem encompasses two key decisions: facility location and vehicle routing. The target is to simultaneously obtain optimal facility locations, transportation routes for vehicles, and the most efficient sequence for serving customers while considering vehicle capacities. The proposed model incorporates both non-monetary and monetary objective functions. The monetary objective minimizes the total system cost, which includes the sum of variable and fixed transportation costs, and fixed facility location costs. The non-monetary objective minimizes a predetermined value B, representing the trade-off between customer satisfaction (served within a predetermined time) and facility location and vehicle routing efficiency.

3.1 Notations

3.1.1 Sets

K	Transportation vehicles $k \in \{1, \ldots, k\}$
I	Potential depot locations $i \in \{1, \ldots, m\}$
J	Customers $j \in \{1, \ldots, n\}$

3.1.2 Parameters

$\tilde{Q}$	The fixed fuzzy loading capacity of transportation vehicles;
w_{jk}	Waiting time of vehicle k in the place of the costumer j;
s_{jk}	Service time of vehicle k in the place of the costumer j;
$\tilde{d}_j$	The fuzzy demand of the costumer j;
$\widetilde{FV}_k$	The fixed fuzzy cost of transportation by vehicle k;
D_{ij}	Distance among point i and point j $(i, j \in I \cup J)$;
$\tilde{C}_{ij}$	The fuzzy unit cost of transportation from point i to point $(i, j \in I \cup J)$;
$\tilde{F}_i$	The fixed fuzzy cost of opening a depot at site i;
t_{ij}	Traveling time from point i to point $(i, j \in I \cup J)$;
K	Number of available vehicles;
P	Number of depot locations;
N	Number of customers;

3.1.3 Decision Variables

U_{lk}	Auxiliary variables used in sub-tour elimination constraints;
z_{ij}	If customer j is served by depot I, it is equal to 1; 0 otherwise;
y_i	If a depot is located at site I, it is equal to 1; 0 otherwise;
x_{ijk}	If point j is immediately met after point i by vehicle k $(i, j \in I \cup J, k \in K)$, it is equal to 1; 0 otherwise

3.1.4 The Mathematical Model

The formulation of the proposed multi-objective stochastic and fuzzy LRP can be stated as follows:

$$Min \sum_{i\in I} \tilde{F}_i y_i + \sum_{k\in K} \widetilde{FV}_k \sum_{i\in I}\sum_{j\in J} x_{ijk} + \sum_{i\in J}\sum_{j\in J}\sum_{k\in K} \tilde{C}_{ij} D_{ij} x_{ijk} \tag{1}$$

$$Min \quad B \tag{2}$$

$Subject\ To$:

$$\sum_{i\in I} y_i = P \tag{3}$$

$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1 \qquad \forall j \in J \tag{4}$$

$$\sum_{j\in I\cup J} x_{ijk} - \sum_{j\in I\cup J} x_{jik} = 0 \qquad \forall k \in K, i \in I \cup J \tag{5}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1 \qquad \forall k \in K \tag{6}$$

$$\sum_{j\in J}\tilde{d}_j\sum_{i\in I\cup J}x_{ijk}\le\tilde{Q} \qquad \forall k\in K \qquad (7)$$

$$-z_{ij}+\sum_{u\in I\cup J}\left(x_{iuk}+x_{ujk}\right)\le 1 \qquad \forall i\in I, j\in J, k\in K \qquad (8)$$

$$P\left\{\sum_{i\in I\cup J}\sum_{j\in I\cup J}t_{ij}x_{ijk}+\sum_{j\in J}(s_{jk}+w_{jk})\sum_{j\in I\cup J}x_{ijk}\le B\right\}\ge\beta \qquad \forall k\in K \qquad (9)$$

$$z_{ij}\le y_i \qquad \forall i\in I, j\in J \qquad (10)$$

$$U_{gk}-U_{jk}+Nx_{gjk}\le N-1 \qquad \forall g,j\in J, k\in K \qquad (11)$$

$$x_{ijk}, y_i, z_{ij}\in\{0,1\} \qquad \forall i,j\in I\cup J, k\in K \qquad (12)$$

$$U_{lk}\ge 0 \qquad \forall l\in I, k\in K \qquad (13)$$

The model employs two objective functions and a set of constraints to optimize vehicle routing and facility location decisions. Objective Function 1 (Eq. 1): Minimizes total system cost, encompassing depot opening costs, and variable and fixed transportation costs. Objective Function 2 (Eq. 2): Minimizes total travel time, service time, and waiting time for vehicles. This is achieved by minimizing a predetermined value B, introduced in constraint (9).

Constraint Set 3 ensures the number of open depots meets a predetermined value. Constraint Set 4 (Eq. 4) guarantees each consumer is assigned to a single open depot and served by one vehicle. Constraint Set 5 represents vehicle flow, mandating each route starts and ends at the same depot (a vehicle cannot leave a customer unserved). Constraint Set 6 limits the existence of a direct route between any two customers to at most one. Constraint Set 7 enforces vehicle capacity limitations. Constraint Set 8 permits customer demand allocation only on routes that include both a depot and customers. Constraint Set 9 ensures a minimum probability (β) that the total travel, service, and waiting times for vehicles fall below a specified value (B). Constraint Set 10 links depot service to the establishment; a depot can serve consumers only if it is open. Constraint Set 11 eliminates sub-tours, ensuring efficient routing. Constraint sets 12-13 define the decision variables' nature.

4. Presented Model Simplification

LRPs are NP-hard issues, and the inclusion of non-linear constraints further exacerbates their computational complexity (Megiddo & Supowit, 1984). Consequently, obtaining optimal solutions for models with non-linear constraints can be highly time-consuming. To address this challenge, we propose the linearization of non-linear constraint (9). This linearization enhances the efficiency of our presented model by enabling the application of efficient solution algorithms. The non-linear constraint (9) is linearized using a uniform distribution function, as follows:

First, we need to define an integer variable y as follows,

$$y=\sum_{i=1}^{N}\sum_{j=1}^{N}t_{ij}+\sum_{i=0}^{N}(s_{jk}+w_{jk})-B \qquad (14)$$

Considering the normal distribution of mean E (t) and Variance V(t) for traveling times, service times, and waiting times by vehicles, the variance and mean of variable y can be stated as follows:

$$E(y)=\sum_{i=1}^{N}\sum_{j=1}^{N}E(t_{ij})+\sum_{i=0}^{N}(E(s_{jk})+E(w_{jk}))-B \qquad (15)$$

$$V(y) = \sum_{i=1}^{N}\sum_{j=1}^{N} V(t_{ij}) + \sum_{i=0}^{N}(V(s_{jk}) + V(w_{jk})) \tag{16}$$

So, it is evident that the following term has a standard normal distribution:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} t_{ij} + \sum_{i=0}^{N}(s_{jk} + w_{jk}) - B - E(y) \Big/ \sqrt{V(y)} \sim N(0,1) \tag{17}$$

Then, constrain (9) is equal to the following term:

$$\frac{\sum_{i=1}^{N}\sum_{j=1}^{N} t_{ij} + \sum_{i=0}^{N}(s_{jk} + w_{jk}) - B - E(y)}{\sqrt{V(y)}} \leq -\frac{E(y)}{\sqrt{V(y)}} \tag{18}$$

Also, in normal distribution if $\theta \sim N(0,1)$, we have:

$$P\left\{\theta \leq -E(y)/\sqrt{V(y)}\right\} \geq \beta \tag{19}$$

So, the service level constraint can be established only when we have the following term:

$$\emptyset^{-1}(\beta) \leq -E(y)/\sqrt{V(y)} \tag{20}$$

Finally, considering constraint (20), constrain (9) is linearized as follows:

$$\emptyset^{-1}(\beta)\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} V(t_{ij})x_{ijk} + \sum_{i=0}^{N}(V(s_{jk}) + V(w_{jk}))\sum_{j=1}^{N} x_{ijk}} + \sum_{i=1}^{N}\sum_{j=1}^{N} E(t_{ij})x_{ijk} + \sum_{i=0}^{N}(E(s_{jk}) + E(w_{jk}))\sum_{j=1}^{N} x_{ijk} \leq B \tag{21}$$

5. The Auxiliary Crisp Multi-Objective Model

Fuzzy numbers lack a strict linear order, typically exhibiting a partial order (Lai & Hwang, 1993). To address this, various ranking methods have been presented in the literature to transform fuzzy numbers into a totally ordered set (JIMÉNEZ, 1996; Inuiguchi & Ramık, 2000; Baykasoglu & Gocken, 2010; Hatami-Marbini et al., 2011). These approaches play a crucial role in possibility models by facilitating the ranking of objective function values and constraint feasibility. The selection of a ranking method is guided by several desirable attributes, including rationality, distinguishability, adherence to fuzzy representation, and robustness (Jiménez et al., 2007). This study adopts the ranking method based on expected interval comparison, as introduced by JIMÉNEZ (1996). This approach not only satisfies the essential properties of existing methods but also offers computational efficiency for linear problems due to its linearity-preserving nature (Jiménez et al., 2007).

Here, the framework of the implementation of the ranking method is presented (JIMÉNEZ, 1996):

Suppose that we have the following linear programming issue with fuzzy parameters:

$$\begin{aligned} &min\ \tilde{C}x \\ &s.t.: x \in \left\{x \in R^n \middle| \tilde{A}x \geq \tilde{B}, x \geq 0\right\} \end{aligned} \tag{22}$$

The membership function of a fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$will be defined thus:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; \forall x \in (a_4, \infty) \\ f_A(x) & ; \forall x \in [a_1, a_2] \\ 1 & ; \forall x \in [a_2, a_3] \\ g_A(x) & ; \forall x \in [a_3, a_4] \\ 0 & ; \forall x \in (a_4, \infty) \end{cases} \quad (23)$$

In order to demonstrate the existence of the inverse functions $f_A^{-1}(x)$ and $g_A^{-1}(x)$, considered that $f_A(x)$ be continuous and increasing but $g_A(x)$ be decreasing and continuous. Then, the expected interval of a fuzzy number is calculable (JIMÉNEZ, 1996).

$$EI(\tilde{A}) = [E_1^{\tilde{A}}, E_2^{\tilde{A}}] = \left[\int_{a_1}^{a_2} x df_A(x), - \int_{a_3}^{a_4} x dg_A(x)\right] \quad (24)$$

Integrating by parts and also changing the variable $\alpha = f_A(x)$, $\alpha = g_A(x)$, the expected interval of a fuzzy number $\tilde{A}$, noted $EI(\tilde{A})$, can be stated by:

$$EI(\tilde{A}) = [E_1^{\tilde{A}}, E_2^{\tilde{A}}] = \left[\int_0^1 f_A^{-1}(\alpha) d\alpha, - \int_0^1 g_A^{-1}(\alpha) d\alpha\right] \quad (25)$$

If $f_A(x)$and $g_A(x)$are linear, the expected interval for a triangular or trapezoidal fuzzy number $\widetilde{A}$, noted $EI(\tilde{A})$, is easily calculated as eq.(26) and the expected value of a fuzzy number, noted $EV(\tilde{A})$, is the half point of its expected interval, as follows:

$$EI(\tilde{A}) = \left[\frac{1}{2}(a_1 + a_2), \frac{1}{2}(a_3 + a_4)\right] \quad (26)$$

$$EV(\tilde{A}) = \frac{E_1^{\tilde{A}} + E_2^{\tilde{A}}}{2} \quad (27)$$

Which for a trapezoidal fuzzy number can be written as follows:

$$EV(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4}{4} \quad (28)$$

On the other hand, for any pair of fuzzy numbers $\widetilde{\mathrm{A}}$ and $\widetilde{\mathrm{B}}$, based on the ranking method of Jimenez, the degree to which $\widetilde{A}$ is bigger than $\widetilde{B}$ is defined as:

$$\mu_M(\tilde{A}, \tilde{B}) = \begin{cases} 0 & if E_2^a - E_1^b < 0 \\ \dfrac{E_2^A - E_1^B}{E_2^A - E_2^B - [E_1^A - E_2^B]} & if \ \ 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & if E_1^a - E_2^b > 0 \end{cases} \quad (29)$$

Where $[E_1^A, E_2^A]$ *and* $[E_1^B, E_2^B]$ are expected intervals of $\tilde{\mathrm{A}}$ and $\tilde{\mathrm{B}}$. When $\mu_M(\tilde{A}, \tilde{B}) = 0.5$, it implies that $\widetilde{A}$ and $\tilde{B}$ are indifferent and when $\mu_M(\tilde{A}, \tilde{B}) \geq \alpha$, $\widetilde{A}$ is greater than, or equal to $\widetilde{B}$ at least in a degree of preferences α, that will be denoted by $\tilde{A} \geq_\alpha \tilde{B}$.

Also, for any pair of fuzzy numbers $\tilde{A}$ and $\tilde{B}$, when $\tilde{A}$ is indifferent to $\widetilde{B}$ in degree of α, the following relationships can be defined simultaneously (Parra, Terol et al. 2005, Jiménez, Arenas et al. 2007):

$$\tilde{A} \leq_{\alpha/2} \tilde{B} \quad \tilde{A} \geq_{\alpha/2} \tilde{B} \tag{30}$$

Eq. (30) can be rewritten as follow:

$$\alpha/2 \leq \mu_M(\tilde{A}, \tilde{B}) \leq 1 - \alpha/2 \tag{31}$$

Moreover, if $min\{\mu_M(\tilde{A}x, \tilde{B})\} = \alpha$, the decision vector $x \in R^n$ will be feasible in a degree of α, that is stated as $\widetilde{A}x \geq_\alpha \tilde{B}$ (Jiménez, Arenas et al. 2007). So, according to the above explanation, eq. (29) can be rewritten as the follows:

$$[(1-\alpha).E_2^A + \alpha.E_1^A].x \geq \alpha.E_2^B + (1-\alpha).E_1^B \quad or$$
$$\frac{E_2^{Ax} - E_1^B}{E_2^{Ax} - E_1^{Ax} + E_2^B - E_1^B} \geq \alpha \tag{32}$$

Jimenez et al. (2007) proved that the feasible solution x^0 is α-acceptable optimal solution of the model eq.(22), if it is verified that:

$$\tilde{C}x \geq \tilde{C}x^0$$
$$\forall x \in \{x \in R^n | \tilde{A}x \geq_\alpha \tilde{B}, x \geq 0\} \tag{33}$$

And at least in the degree of ½, we can say x^0 is a better solution in opposition to the other feasible vectors. So eq. (33) can be written as follows:

$$\frac{E_1^{\tilde{C}x} + E_2^{\tilde{C}x}}{2} \geq \frac{E_1^{\tilde{C}x^0} + E_2^{\tilde{C}x^0}}{2} \tag{34}$$

In the end, according to above explanation, with substitution of fuzzy parameters with the expected interval and expected values, model (22) can be transformed into an equivalent crisp α-parametric linear model as follows:

$$min\, EV(\tilde{C}).x$$
$$s.t.: x \in \{x \in R^n | \tilde{A}x \geq_\alpha \tilde{B}, x \geq 0\} \tag{35}$$

Where $EV(\tilde{C})$ represents the expected value of the fuzzy vector $\tilde{C}$.

Therefore, considering the above explanations and definitions, after the use of proper linearization and substitution of fuzzy parameters with the expected interval and expected values, the presented model is converted to the equivalent auxiliary crisp multi-objective model as follows:

$$Min \sum_{i\in I} EV(\tilde{F}_i)y_i + \sum_{k\in K} EV(\widetilde{FV}_k) \sum_{i\in I}\sum_{j\in J} x_{ijk} + \sum_{i\in J}\sum_{j\in J}\sum_{k\in K} EV(\tilde{C}_{ij})D_{ij}x_{ijk} \tag{36}$$

$$MinB \tag{37}$$

$Subject\ To$:

$$\sum_{i\in I} y_i = P \tag{38}$$

$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1 \qquad \forall j \in J \tag{39}$$

$$\sum_{j\in I\cup J} x_{ijk} - \sum_{j\in I\cup J} x_{jik} = 0 \qquad \forall k \in K, i \in I \cup J \qquad (40)$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1 \qquad \forall k \in K \qquad (41)$$

$$\sum_{j\in J}(\alpha.E_2^{d_j} + (1-\alpha).E_1^{d_j}) \sum_{i\in I\cup J} x_{ijk} \le \left[(1-\alpha).E_2^{Q} + \alpha.E_1^{Q}\right] \qquad \forall k \in K \qquad (42)$$

$$-z_{ij} + \sum_{u\in I\cup J}\left(x_{iuk} + x_{ujk}\right) \le 1 \qquad \forall i \in I, j \in J, k \in K \qquad (43)$$

$$\emptyset^{-1}(\beta)\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} V(t_{ij})x_{ijk} + \sum_{i=0}^{N}(V(s_{jk}) + V(w_{jk}))\sum_{j=1}^{N} x_{ijk}} + \sum_{i=1}^{N}\sum_{j=1}^{N} E(t_{ij})x_{ijk} + \sum_{i=0}^{N}(E(s_{jk}) + E(w_{jk}))\sum_{j=1}^{N} x_{ijk} \le B \qquad \forall k \in K \qquad (44)$$

$$z_{ij} \le y_i \qquad \forall i \in I, j \in J \qquad (45)$$

$$U_{gk} - U_{jk} + N x_{gjk} \le N - 1 \qquad \forall g, j \in J, k \in K \qquad (46)$$

$$x_{ijk}, y_i, z_{ij} \in \{0,1\} \qquad \forall i, j \in I \cup J, k \in K \qquad (47)$$

$$U_{lk} \ge 0 \qquad \forall l \in I, k \in K \qquad (48)$$

6. Solution Approaches and Proposed Algorithms

The NP-hard nature of LRPs necessitates the use of metaheuristic search algorithms for obtaining near-optimal solutions within practical timeframes (Salhi & Nagy, 1999). This study proposes a novel MOICA to address the proposed multi-objective LRP model. To benchmark MOICA's effectiveness, we compare its performance with two established multi-objective evolutionary algorithms: PAES and NSGA-II. Additionally, to enhance the search efficiency within the solution space, we customize mutation and crossover strategies for each algorithm using RSM. This customization aims to improve the algorithms' ability to explore and exploit promising regions of the search space. The subsequent sections detail the solution representation scheme (Section 6.1) and the proposed MOICA (Section 6.2). A comprehensive comparison study will be conducted using several benchmark instances and performance metrics to validate MOICA's efficacy. Section 6.3 describes comparative metaheuristic algorithms, Section 6.4 explains parameter setting, Section 6.5 introduces comparison metrics, Section 6.6 describes the initialization of numerical test problems and finally, Section 6.7 presents a comparison of metaheuristic algorithms.

6.1 Solution Method and Implementation

A candidate solution in our LRP model must represent three crucial aspects:

1. Customer Allocation: This determines which consumers are served by each vehicle.
2. Depot Selection: It identifies which depots will be opened to serve customers.
3. Vehicle Routing: This defines the sequence that consumers are visited by each vehicle, starting and ending at the same depot.

We propose a string-based solution representation scheme using a permutation of n consumers, d potential depots, and m vehicles. This string comprises three distinct parts:

- Part 1 (n elements): This encodes the customer sequence for each route.
- Part 2 (m elements): This indicates the consumer indices served by each vehicle's route.
- Part 3 (m elements): This specifies the starting depots for each vehicle's route.

Decoding the Solution Representation:

- Part 1: The customer sequence in each route is determined by reading elements from left to right.
- Part 2: Consecutive elements define a customer subset served by a single vehicle. The values are sorted from smallest to largest, ensuring each consumer is served by one vehicle.
- Part 3: This part identifies the starting depot for each vehicle. Duplicate values represent opened depots. A new route begins at each opened depot, serving customers assigned to that depot until vehicle capacity is reached.

6.1.1 Effectiveness of the Representation:

This solution representation offers several advantages:

- Comprehensiveness: It encodes all essential decision variables (customer allocation, depot selection, and vehicle routing) within a single structure.
- Efficiency: The decoding process is straightforward and computationally efficient.
- Constraint Satisfaction: The design ensures that vehicle capacity constraints are not violated.

Figure 1 provides this solution representation for a sample problem.

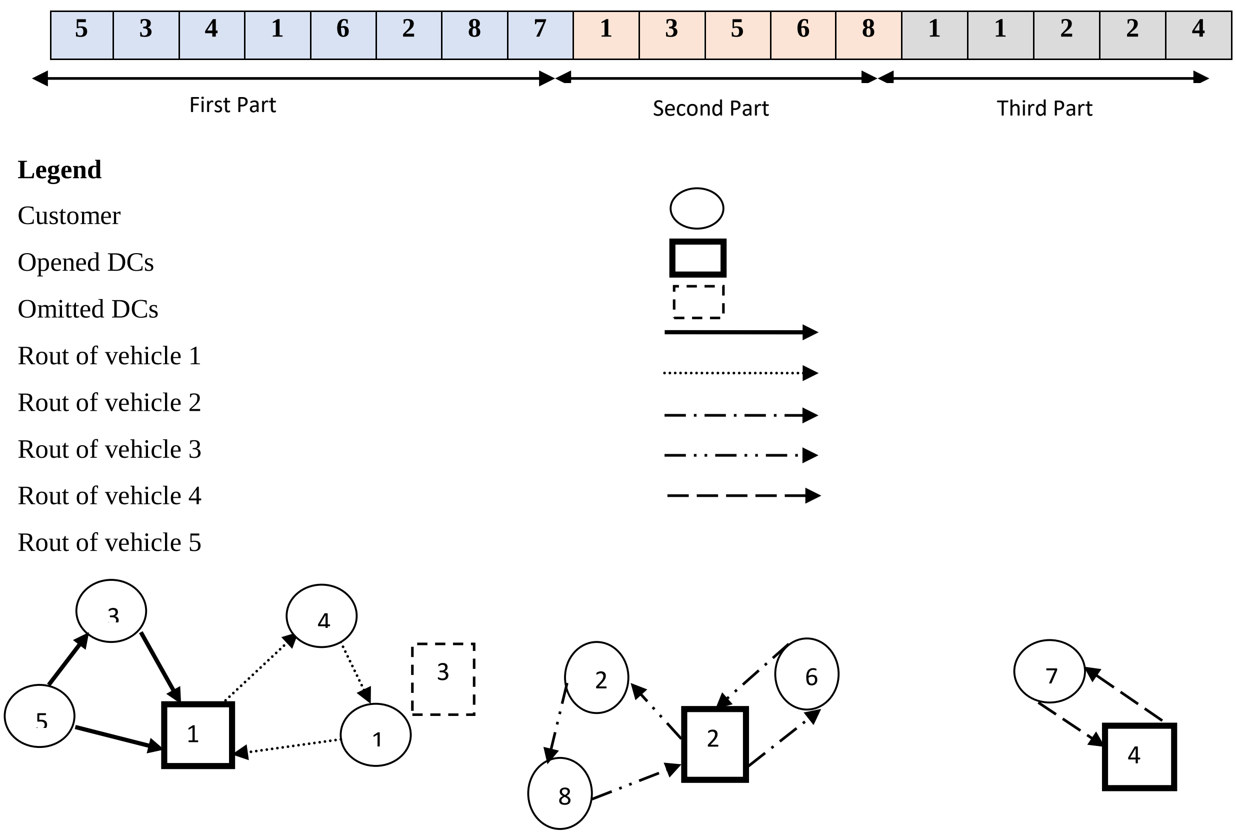

Figure 1. Sample solution

6.2 Multi-Objective Imperialist Competitive Algorithm

The ICA is a recent evolutionary optimization algorithm inspired by socio-political processes (Atashpaz-Gargari & Lucas, 2007). It has demonstrated effectiveness in achieving fast convergence rates and finding high-quality solutions (global optima) for various optimization problems (Nazari-Shirkouhi et al., 2010). ICA utilizes an initial population, with each individual referred to as a "country." The following subsections will provide a detailed explanation of ICA's key steps.

6.2.1 Generating Initial Empires

Within the domain of optimization, the objective lies in identifying an optimal solution defined by a set of decision variables structured as an array. This array is referred to as a "chromosome" within the framework of GA, while the terminology "country" is employed within the context of ICA. In the case of N-dimensional optimization problems, a "country" is constructed as an $1 \times N$ array, as detailed below:

$$Country = [p_1, p_2, p_3, \dots, p_N] \tag{49}$$

The constituent elements of a country are represented by floating-point numerical values, with each element corresponding to a specific parameter, such as cultural attributes, linguistic characteristics, or economic policies. The determination of a country's cost is predicated upon the application of predefined cost functions. Subsequently,

$$Cost = f[Country] = f[p_1, p_2, p_3, \dots, p_N] \tag{50}$$

To initiate the optimization algorithm, an initial population of size N_{pop} is generated. Subsequently, a subset of $Nimp$ countries deemed the most influential, are designated as imperialists. The remaining population, comprising $Ncol(Ncol = Npop - Nimp)$ individuals, is partitioned into colonies, each affiliated with a specific empire. The cost value associated with each objective function is computed, enabling the calculation of the cost value for each imperialist through the following equation:

$$Cost_{i,n} = \frac{\left|f_{i,n}^{p} - f_{i,n}^{p,best}\right|}{f_{i,total}^{p,max} - f_{i,total}^{p,min}} \tag{51}$$

In the aforementioned equation, $Cost_{i,n}$ denotes the normalized value of the i^{th} objective function for the n^{th} imperialist, while $f_{i,n}^{p}$ represents the corresponding raw value. The optimal value of the i^{th} objective function at each iteration is denoted by $f_i^{p,best}$, with $f_{i,total}^{p,max}$ and $f_{i,total}^{p,min}$ representing the maximum and minimum values, respectively. The normalized value of the cost of each imperialist is determined by summing the normalized values of all objective functions, as expressed by the following formula:

$$Total\ Cost_n = \sum_{i=1}^{r} Cost_{i,n} \tag{52}$$

Where r denotes the number of objective functions. Subsequent to the computation of normalized objective function values, the power of each imperialist is determined based on the following equation. Based on the imperialists' respective powers, the colonies are subsequently partitioned among them. Through this process,

$$p_n = \left|\frac{Total\ Cost_n}{\sum_{i=1}^{N_{imp}} Total\ Cost_i}\right| \tag{53}$$

Ultimately, the initial population of colonies within an empire is computed using the following equation,

$$NC_n = round\{p_n . N_{col}\} \tag{54}$$

In the equation, NC_n represents the initial number of colonies assigned to the n^{th} imperialist, while N_{col} signifies the total number of colonies. Subsequently, a random selection of NC_n colonies is undertaken and assigned to the respective imperialists. These colonies, in conjunction with their corresponding imperialists, constitute the n^{th} empire. Consequently, empires with greater power are characterized by a larger number of colonies, whereas those with lesser power exhibit a smaller colony count.

6.2.2 Total Power of an Empire

The aggregate power of an empire is computed as the sum of the imperialist's power and a percentage of the mean power of its constituent colonies, as formalized in Equation (55). Subsequently, the imperialistic competition commences, resulting in the elimination of weaker empires from the competitive landscape.

$$TP\ Emp_n = (Total\ Cost(imperialist_n) + \xi mean\{Total\ Cost(colonies\ of\ empire_n)\}) \tag{55}$$

Where $TP\ Emp_n$ is the total power of then^{th} empire and ξ(zeta) is a positive small number, that is advised to be less than 1 in order to augment the role of the colonies in determining the total power of an empire.

In Equation (55), $TP\ Emp_n$ represents the total power of the n^{th} empire. The parameter ξ (zeta), a positive constant less than unity, is introduced to amplify the influence of colonies on the determination of an empire's overall power.

6.2.3 Assimilation

The absorption policy inherent to the ICA algorithm mandates the translocation of colonies towards their respective imperialist counterparts. This movement is influenced by a constellation of socio-political factors, visually depicted in Figure 2. The directional vector, originating from the colony and terminating at the imperialist, defines the trajectory of this movement. As illustrated in Figure 2, the scalar quantity 'd' represents the inter-colonial distance between the imperialist and its colony. The actual displacement (x) of the colony towards its imperialist is modeled as a random variable uniformly distributed within the interval [0, d].

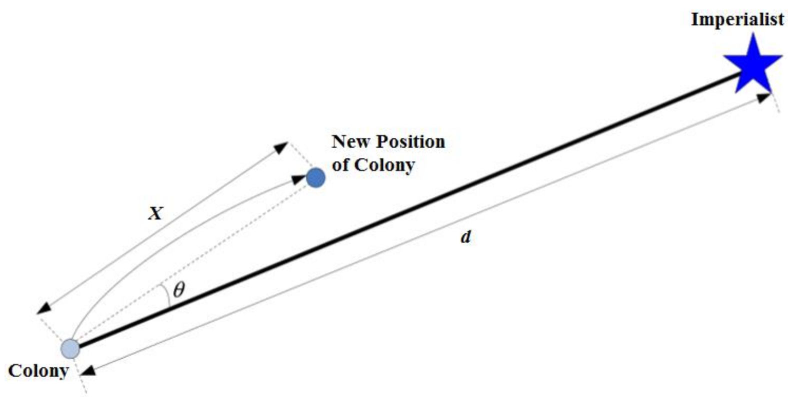

Figure 2. Moving colonies toward the imperialists with a random angle θ

6.2.4 Crossover Policy

During this phase, information exchange among colonies is facilitated through the application of crossover operators. The proportion of the population participating in the crossover operation is denoted by *p*-Crossover.

6.2.5 Revolution

A random selection process is employed to identify a subset of colonies, which are subsequently replaced by an equivalent number of newly generated individuals. This mechanism, analogous to the mutation in Genetic Algorithms, is termed 'revolution'. The revolution operation induces random modifications to a colony's socio-political attributes. The implementation of the revolution operation enhances the algorithm's diversity, augments its capacity to evade local optima, and mitigates the risk of premature convergence.

6.2.6 Exchanging Positions of a Colony and the Imperialist

Moving colonies toward their imperialist, a colony might reach a better position with lower cost than the imperialist. In this case, when a colony becomes more powerful than its imperialist, its position will be reversed and the total cost of each imperialism will be up-to-date.

6.2.7 Uniting similar empires

During the search for the global minimum, convergence of imperialist positions can occur. When the distance between two imperialists falls below a predefined threshold, they are merged into a single empire. This process involves combining all their colonies to form the colony set of the newly formed empire.

6.2.8 Imperialistic competition

During the imperialistic competition phase, empires engage in a contest to acquire a subset of the weakest colonies belonging to the least powerful empire. The probability of each empire successfully obtaining these colonies is determined by its respective possession probability, as calculated according to the following Equation,

$$NTP\ Emp_n = max\{TP\ Emp_i\} - TP\ Emp_n \qquad (56)$$

Where $NTP\ Emp_n$ represents the total power of the n^{th} empire and $TP\ Emp_n$ its corresponding normalized total power. Given the normalized total power of all empires, the normalized power or possession probability of each empire can be expressed by the following equation:

$$P_{p_n} = \left| \frac{NTP\ Emp_n}{\sum_{i=1}^{N_{imp}} TNTP\ Emp_i} \right| \qquad (57)$$

Subsequently, the aforementioned colony is assigned to an empire through the application of the roulette wheel selection method. To facilitate the distribution of the weakest colonies among the empires, the vector P is defined as follows,

$$P = [p_{p_1}, p_{p_2}, p_{p_3}, \dots, p_{p_{N_{imp}}}] \qquad (58)$$

Then, the vector *R* with the same size as *P* is created in which its elements are random numbers generated by the uniform distribution function between 0 and 1.

$$R = [r_1, r_2, r_3, \dots, r_{N_{imp}}] \qquad (59)$$

Also, vector *D* is expressed by subtracting vector *R* from *P*, as follows,

$$D = P - R = \left[D_1, D_2, D_3, \dots, D_{N_{imp}}\right] = \left[p_{p_1} - r_1, p_{p_2} - r_2, p_{p_3} - r_3, \dots, p_{p_{N_{imp}}} - r_{N_{imp}}\right] \qquad (60)$$

By examining vector *D*, the element possessing the maximum value is identified. The corresponding index of this element determines the empire that will acquire the colony (or colonies). The process of empire selection bears resemblance to the roulette wheel selection method employed in Genetic Algorithms for parent selection. However, this mechanism exhibits a significantly enhanced computational efficiency compared to the traditional roulette wheel, thereby accelerating the overall execution speed.

6.2.9 Eliminating the Powerless Empires

The core competition mechanism in ICA drives the elimination of weaker empires. This occurs when weak empires lose all their colonies, effectively removing them from the competition. This process fosters a gradual augment in the power of strong empires and a corresponding decline in the power of weak ones. Over time, this competition ideally leads to the collapse of all empires except the most powerful one, resulting in the convergence of all colonies into a single empire. Consequently, the solutions (countries) within this final empire converge towards the cost function global minimum, signifying the termination of the algorithm.

6.2.10 Stopping Criteria

The termination criteria for ICA typically encompass several factors including maximum Iterations, predefined CPU time, and single remaining empire: If only one empire persists after the competition process, the algorithm terminates, indicating potential convergence. In this study, we adopt the single remaining empire criterion for termination. This signifies the elimination of all weaker empires, suggesting convergence towards a global minimum within the final remaining empire. Additionally, the algorithm will be terminated if the best solution remains stagnant for a consecutive period, but this criterion is not employed here.

6.3 Comparative Meta-Heuristics

This study aims to evaluate the effectiveness of the presented MOICA by comparing its performance with two established multi-objective evolutionary algorithms: PAES and NSGA-II. The evaluation will be conducted using four key performance metrics, which will be detailed in Section 6.5.

6.3.1 Non-Dominated Sorting Genetic Algorithm II (NSGAII)

The NSGA-II, introduced by Deb et al. (2000) and further elaborated by Deb et al. (2002), is a prominent algorithm for multi-objective optimization issues. It builds upon the foundation of GA to identify the Pareto-optimal front, a set of solutions where no improvement in one objective can be achieved without sacrificing another (Deb, 2001). NSGA-II serves as an improvement over NSGA (Srinivas & Deb, 1994) by incorporating elitism through non-dominated sorting and maintaining population diversity using crowding distance sorting. Similar to GAs, NSGA-II utilizes mutation and crossover operators to generate new solutions. After population initialization, the following key steps are employed.

6.3.2 Non-Dominated Sorting

This is a core principle in multi-objective optimization algorithms. It aims to classify candidate solutions into distinct fronts. Within each front, no solution dominates any other solution according to all objective functions simultaneously. In simpler terms, for any two solutions in the same front, there is no way to improve one objective function without worsening another. Then, after sorting these small groups the best of them will be selected. For an n objective functions model, solution x_1 is superior to the solution x_2 if:

1. For all of the objective functions, solution x_1 is not worse than solution x_2.
2. For at least one of the individual objective functions, x_1 is exactly better than x_2

Employing this technique, all solutions that are non-dominated by any other are classified into the first front. Subsequently, solutions dominated exclusively by members of the first front are grouped into the second front.

6.3.3 Crowding Distance

Crowding distance sorting, a key component of NSGA-II, refines the solution selection process within each non-dominated front. It aims to maintain a diverse population by favoring solutions located on the periphery of the fronts, promoting exploration of the entire Pareto-optimal frontier. The crowding distance of a solution is a metric that estimates its neighbor density within the front. Solutions with a higher crowding distance reside in less crowded regions and are thus more likely to be chosen for the next generation, fostering diversity. The calculation of crowding distance in NSGA-II is presented in Equation (61):

$$CD_i = \sum_{r=1}^{n} \frac{f_{r,i+1}^{P} - f_{r,i-1}^{P}}{f_{r,Total}^{P.max} - f_{r,Total}^{P.min}} \tag{61}$$

Wherein n denotes the number of objective functions, $f_{r,i+1}^{p}$ represents the value of the r^{th} objective function for the $(i+1)^{th}$ solution, and $f_{r,i-1}^{p,}$ represents the value of the rth objective function for the $(i-1)^{th}$ solution when the population is ordered based on the crowding distance of the r^{th} objective function. Additionally, $f_{r,total}^{p,max}$ and $total$ signify the maximum and minimum values, respectively, of the r^{th} objective function.

6.3.4 Selection and Recombination

Individual selection is conducted via tournament selection, incorporating a crowding comparison operator. This process is executed subsequent to the ranking and assignment of individuals based on non-domination and crowding distance criteria. The first step of the tournament selection mechanism is selecting two solutions for the population size. This methodology amalgamates the current and offspring populations to form a combined pool from which individuals for the subsequent generation are selected. The new population is sequentially populated by each non-dominated front until the prescribed population size is attained.

6.3.5 Pareto Archived Evolution Strategy (PAES)

The PAES is a metaheuristic algorithm designed to address multi-objective optimization problems (Knowles and Corne, 1999; Corne et al., 2000). Leveraging a simple $(1+1)$ local search evolution strategy, PAES effectively identifies diverse solutions within the Pareto optimal set. To maintain population diversity along the Pareto front and assess the quality of newly generated candidate solutions, the algorithm employs an archive to store non-dominated solutions. Initially, the algorithm commences with a single randomly generated chromosome, which undergoes evaluation using the specified multi-objective cost function. Subsequently, at each iteration, a new candidate solution is produced through a random mutation operation. The newly generated solution is then compared to the current solution, and an update to the archive is performed accordingly (Knowles and Corne, 1999). This iterative process continues until the termination criterion is met.

6.4 Parameter Setting

The success of optimization algorithms is highly contingent upon appropriate parameter settings. Inappropriate parameter selection can hinder the algorithm's ability to find high-quality solutions. To address this challenge, we employ RSM to identify parameter configurations that lead to optimal solutions for Location-Routing Problems (LRPs) of varying sizes (small and large). Compared to factorial design (Montgomery, 1997), RSM offers the advantage of continuous parameter optimization. The initial step involves determining statistically significant factors influencing elapsed time and makespan for each algorithm. Each factor will be evaluated at two levels, coded as "1" for the high value and "-1" for the low value. The specific coded variables will be presented as follows:

$$X_i = \frac{r_i - \left(\frac{h+l}{2}\right)}{\left(\frac{h-l}{2}\right)} \tag{62}$$

Where x_i and r_i denote coded and natural variables, respectively, with h and l representing high and low factor levels. To facilitate the comparison of diverse parameter combinations within the context of the multi-objective LRP model, a quality index is introduced. This metric aggregates Pareto solutions obtained from various parameter settings, subjecting them to a collective non-domination analysis. The percentage of Pareto solutions attributable to a specific parameter combination serves as its quality index. For computational efficiency, the number of function evaluations (NFC) is set to 30,000 for small-scale problems and 100,000 for larger instances.

6.4.1 MOICA

Factors and their values for large and small size issues in the MOICA algorithm are demonstrated in Table 1.

6.4.2 NSGA-II

The initial population is configured at 200 individuals for small-scale problems and 300 for larger instances. Crossover and mutation rates are fixed at 0.8 and 0.2, respectively. The termination criterion is governed by the NFC, set to 30,000 for large-scale problems and 100,000 for smaller ones.

Table 1. Tuned value factors of the proposed MOICA.

Factors	Optimal real value	
	S	L
N-imp	5	8
n-Pop	193	300
β	1.8	2.15
ξ	0.195	0.125
P_R	0.12	0.32
P_C	0.6	0.6
P_A	0.54	0.64

6.4.3 PAES

- The archive is maintained at a capacity of 150 solutions.
- The termination condition is based on NFC, set at 30,000 for small-scale and 100,000 for large-scale problem instances.

6.5 Comparison Metric

To evaluate the efficacy of the presented MOICA algorithm, four established comparison metrics tailored to multi-objective optimization are employed.

6.5.1 Quality Metric (QM)

This evaluation metric, known as Pareto Front Coverage (PFC), assesses the ability of an algorithm to explore the Pareto-optimal front (Moradi et al., 2011). It considers all non-dominated solutions identified by the algorithms collectively. The PFC value for each algorithm is then calculated as the percentage of the overall Pareto front covered by its solutions. A higher PFC value indicates a more comprehensive exploration of the Pareto-optimal region, signifying a better-performing algorithm in this regard.

6.5.2 Mean ideal distance (MID)

The MID metric quantifies the distance between the ideal point and the Pareto front, computed according to Equation [Equation number]. Unlike the QM, a lower MID value indicates a superior solution according to this metric.

$$MID = \frac{\sum_{i=1}^{n} \sqrt{\left(\frac{f_{1i} - f_1^{best}}{f_{1,total}^{max} - f_{1,total}^{min}}\right)^2 + \left(\frac{f_{2i} - f_2^{best}}{f_{2,total}^{max} - f_{2,total}^{min}}\right)^2}}{n} \quad (63)$$

Where n represents the cardinality of the non-dominated solution set, $f_{1,total}^{max}$ denotes the maximum value of the i^{th} fitness function among all non-dominated solutions generated by the algorithms, $f_{1,total}^{min}$ signifies the minimum value of the ith fitness function within the same set, and f_1^{best} represents the optimal value of the ith fitness function.

6.5.3 Diversification metric (DM):

The DM measures the spread of a Pareto solution set, and is defined by:

$$DM = \sqrt{\left(\frac{max\, f_{1i} - min\, f_{1i}}{f_{1.total}^{max} - f_{1.total}^{min}}\right)^2 + \left(\frac{max\, f_{2i} - min\, f_{2i}}{f_{2.total}^{max} - f_{2.total}^{min}}\right)^2} \quad (64)$$

A better performance results from the algorithm with a higher value due to this metric.

6.5.4 Spacing Metric (SM):

SM provides a quantitative assessment of the uniformity of distribution within the non-dominated solution set, as formulated by Nekooghadirli, Tavakkoli-Moghaddam, et al. (2014).

$$SM = \frac{\sum_{i=1}^{n-1}\left|\bar{d} - d_i\right|}{(n-1)\bar{d}} \tag{65}$$

Where d_i represents the Euclidean distance between consecutive solutions within the obtained non-dominated set, and $\bar{d}$ denotes the average of these distances. The SM leverages these values to quantify the uniformity of distribution among the solution set points. Due to the discontinuous nature of the test problems, the associated trade-off surface exhibits irregularities, complicating the interpretation of this metric. Our methodological approach for analyzing the SM is analogous to the ANOVA-based analysis of the number of non-dominated solutions, with the focus shifted to the spacing metric itself. A lower SM value is indicative of superior algorithm performance.

6.5.5 Generating Numerical Test Problems

Metaheuristic algorithms are well-known for their sensitivity to initial solutions, which significantly impacts their ability to converge to optimal solutions (Talbi, 2009). To comprehensively evaluate the effectiveness of the presented model and algorithms, computational experiments using a set of benchmark LRP problems of varying sizes are conducted. This allows for a comparative analysis of the algorithms' performance on problems with different complexities. Fuzzy parameter determination employs triangular fuzzy numbers using the methodology established by Lai and Hwang (1992). In this approach, the most likely value serves as the crisp equivalent, while the optimistic and pessimistic values are calculated using the following equations, as presented by Niakan and Rahimi (2015).

$$\begin{aligned} n^0 &= (1+d_1)n^m \\ n^P &= (1+d_2)n^m \end{aligned} \qquad (d_1, d_2) \sim Uniform\ (0.2,0.8) \tag{66}$$

Where, in this paper d_1 and d_2 result from the uniformly distributed interval of (0.2, 0.8). Following parameter adjustments outlined in Table 2, a variable number of potential depot locations were determined for each test problem based on the corresponding customer count, as detailed in Table 3. This process resulted in the generation of 67 distinct problem classes for the purpose of algorithm evaluation and computational experimentation. A concise problem identifier was established using a convention combining the number of customers, a "#" symbol, and the number of candidate depots. For instance, an issue involving 50 customers and four potential depots is designated as 50#4.

Table 2. Corresponding parameter distribution

Parameters	Corresponding distribution
D_{ij}	$U \sim (10,20)$
C_{ij}	$Fuzzy(50)$
t_{ij}	$N \sim (40,6)$
F_i	$Fuzzy\ (2000)$
FV_k	$Fuzzy\ (200)$
d_j	$Fuzzy\ (100)$
w_{jk}	$N \sim (8,2)$
s_{jk}	$N \sim (12,4)$
Q	$Fuzzy(400)$

Table 3. Numbers of potential depots for each test problem.

No. of costumers	10	15	20	25	30	40	50	70	100
No. of Potential Depots	3 to 4	3 to 5	3 to 6	3 to 6	3 to 8	3 to 10	3 to 12	3 to 16	3 to 18

6.6 Comparison of Meta-Heuristic Algorithms

This section presents a comprehensive comparison of the presented MOICA with established algorithms, PAES and NSGA-II. To achieve this, all 67 benchmark problem classes were tackled by each algorithm four times. The best solution gained from each run was considered the final output for performance evaluation. The algorithms' performance was then compared using four key metrics: QM, DM, MID), and SM. The detailed results for both small-sized and large-sized issues are proposed in Tables 4 to 9. An analysis of the comparison metrics reveals that MOICA consistently achieves superior performance. Specifically, MOICA exhibits higher QM and DM values, indicating better solution quality and population diversity across all test problems compared to NSGA-II and PAES. Additionally, MOICA demonstrates a lower average SM value compared to the other algorithms in most cases, signifying a well-distributed population within the Pareto-optimal front. While MOICA performs well on both problem sizes, a clear trend emerges: MOICA's advantage is particularly pronounced for large-sized issues (Table 9) compared to small-sized issues (Table 4). This suggests that MOICA's capabilities are well-suited for handling complex LRP instances.

Table 3. Comparison results between MOICA, PAES and NSGA-II in terms of QM and SM for small-size problems

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	**PAES**	**MOICA**	**NSGA-II**	**PAES**	**MOICA**
10#3	0.235	0	0.765	0.827	0.625	0.741
10#4	0.105	0	0.895	0.661	0.495	0.778
15#3	0.250	0	0.750	0.791	0.788	0.920
15#4	0	0	1	0.693	0.785	0.868
15#5	0	0	1	0.571	1.092	0.634
20#3	0	0	1	1.184	0.999	0.881
20#4	0.235	0	0.765	0.778	1.257	0.973
20#5	0.434	0	0.565	0.560	1.036	0.874
20#6	0.347	0.217	0.434	0.733	1.031	0.924
25#3	0.100	0	0.900	1.207	1.120	1.361
25#4	0.272	0	0.727	1.001	0.878	1.287
25#5	0.292	0	0.708	0.977	1.360	1.220
25#6	0.190	0	0.809	0.940	0.977	1.481
30#3	0.167	0.083	0.750	0.651	1.084	1.116
30#4	0.059	0.294	0.647	1.059	1.322	0.810
30#5	0	0	1	0.942	0.965	0.978
30#6	0	0	1	0.670	0.916	0.980
30#7	0.118	0	0.882	0.986	1.478	1.041
30#8	0	0	1	0.586	0.911	0.642

Table 4. Comparison results between MOICA, PAES and NSGA-II in terms of DM and MID for small size problems.

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	NSGA-II	PAES	MOICA	NSGA-II	PAES	MOICA
10#3	1.397	0.996	1.574	0.633	0.632	0.518
10#4	1.102	1.067	1.414	0.704	0.597	0.581
15#3	0.652	0.208	1.414	0.873	0.608	0.242
15#4	0.404	1.125	0.664	0.712	0.872	0.348
15#5	0.203	1.232	0.444	0.339	0.708	0.230
20#3	1.323	1.270	1.087	0.776	0.696	0.523
20#4	0.714	1.268	0.896	0.440	0.674	0.399
20#5	0.958	1.029	1.169	0.518	0.845	0.538
20#6	1.063	1.075	1.161	0.575	0.751	0.621
25#3	1.295	0.436	1.381	0.601	0.437	0.247
25#4	0.960	0.943	1.297	0.663	0.762	0.718
25#5	1.105	0.775	1.314	0.536	0.577	0.511
25#6	0.559	0.911	1.414	0.482	0.576	0.287
30#3	0.566	1.012	1.279	0.697	0.731	0.632
30#4	1.160	0.954	1.178	0.762	0.547	0.485
30#5	1.103	0.860	1.478	0.781	0.846	0.500
30#6	0.484	1.021	1.010	0.297	0.481	0.379
30#7	0.733	1.267	0.947	0.479	0.646	0.276
30#8	0.699	0.696	1.184	0.579	0.860	0.554

Table 6. Comparison results between MOICA, PAES and NSGA-II for problem size 40 with various potential depots.

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	PAES	MOICA	NSGA-II	PAES	MOICA
40#3	0.071	0	0.928	1.302	1.417	1.390
40#4	0.3634	0.272	0.364	1.019	1.167	1.267
40#5	0.357	0	0.642	1.628	1.295	1.649
40#6	0.318	0.182	0.500	1.059	1.272	1.321
40#7	0.370	0	0.630	0.984	1.374	1.372
40#8	0.0416	0	0.958	1.037	1.296	0.935
40#9	0.240	0	0.760	1.2678	1.049	1.410
40#10	0.111	0.111	0.778	0.979	1.422	0.928
Problem No.	**Diversity Metric (DM)**			**Mean Ideal Distance (MID)**		
	NSGA-II	**PAES**	**MOICA**	**NSGA-II**	**PAES**	**MOICA**
40#3	1.244	1.105	1.267	0.707	0.776	0.430

40#4	1.019	1.225	1.175	0.609	0.750	0.452
40#5	0.986	1.264	1.176	0.554	0.813	0.377
40#6	1.032	1.105	1.179	0.664	0.677	0.408
40#7	0.702	1.192	1.064	0.731	0.765	0.563
40#8	0.458	1.170	0.827	0.430	0.767	0.322
40#9	1.203	0.802	1.132	0.506	0.601	0.431
40#10	1.036	1.349	1.350	0.692	0.823	0.360

Table 5. Comparison results between MOICA, PAES and NSGA-II for problem size 50 with various potential depots.

Problem No.	**Quality Metric (QM)**			**Spacing Metric (SM)**		
	NSGA-II	**PAES**	**MOICA**	**NSGA-II**	**PAES**	**MOICA**
50#3	0.434	0.086	0.478	1.339	1.461	1.479
50#4	0.105	0	0.895	1.342	1.181	1.292
50#5	0.238	0	0.762	1.465	1.548	1.466
50#6	0	0	1	1.076	1.346	0.764
50#7	0	0	1	1.042	1.249	1.446
50#8	0	0	1	1.469	1.212	0.893
50#9	0	0	1	0.753	1.202	1.432
50#10	0	0.107	0.892	1.127	1.043	0.795
50#11	0.160	0	0.840	0.952	0.998	0.863
50#12	0	0	1	1.001	1.020	1.033
Problem No.	**Diversity Metric (DM)**			**Mean Ideal Distance (MID)**		
	NSGA-II	**PAES**	**MOICA**	**NSGA-II**	**PAES**	**MOICA**
50#3	0.722	1.289	1.340	0.641	0.731	0.658
50#4	0.633	0.826	1.145	0.526	0.679	0.492
50#5	1.342	1.084	1.175	0.603	0.601	0.443
50#6	0.799	1.266	0.920	0.554	0.715	0.482
50#7	0.443	0.985	1.388	0.490	0.517	0.383
50#8	1.137	1.174	0.752	0.457	0.608	0.281
50#9	0.875	0.961	1.155	0.504	0.734	0.369
50#10	1.077	0.500	1.232	0.702	0.526	0.373
50#11	0.904	1.222	1.414	0.585	0.668	0.640
50#12	0.918	0.887	1.112	0.680	0.551	0.230

Table 6. Comparison results between MOICA, PAES and NSGA-II for problem size 70 with various potential depots.

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	PAES	MOICA	NSGA-II	PAES	MOICA
70#3	0	0	1	0.672	0.501	0.905
70#4	0	0	1	0.517	1.299	0.811
70#5	0	0	1	0.586	0.593	0.878
70#6	0	0	1	0.737	0.402	0.752
70#7	0.200	0	0.800	0.826	0.514	0.953
70#8	0	0	1	0.495	1.032	0.427
70#9	0	0.076	0.924	1.230	0.559	0.893
70#10	0	0	1	0.994	0.789	0.806
70#11	0	0	1	0.726	1.119	0.850
70#12	0	0	1	0.632	0.904	0.608
70#13	0	0	1	1.019	1.069	1.071
70#14	0.352	0	0.647	0.721	1.024	0.721
70#15	0.273	0	0.727	0.491	1.151	0.993
70#16	0	0	1	1.039	0.550	0.673
Problem No.	**Diversity Metric (DM)**			**Mean Ideal Distance (MID)**		
	NSGA-II	**PAES**	**MOICA**	**NSGA-II**	**PAES**	**MOICA**
70#3	0.470	0.122	1.00	0.492	0.992	0.125
70#4	0.110	1.095	0.417	0.519	0.743	0.250
70#5	0.362	0.249	1.043	0.959	0.806	0.365
70#6	0.815	0.443	1.367	0.672	0.519	0.174
70#7	1.066	0.484	0.808	0.692	0.832	0.336
70#8	0.709	1.189	0.749	0.509	0.846	0.364
70#9	1.203	0.445	1.041	0.643	0.563	0.257
70#10	1.081	0.605	0.600	0.696	0.758	0.222
70#11	0.321	1.184	0.891	0.275	0.668	0.250
70#12	0.561	1.109	0.723	0.643	0.832	0.127
70#13	0.604	1.246	0.820	0.518	0.737	0.220
70#14	0.918	1.136	0.751	0.491	0.753	0.459
70#15	0.173	1.009	1.051	0.456	0.724	0.256
70#16	0.724	0.552	0.912	0.847	1.021	0.210

Table 7. Comparison results between MOICA, PAES and NSGA-II for problem size 100 with various potential depots.

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	PAES	MOICA	NSGA-II	PAES	MOICA
100#3	0	0	1	0.468	1.231	.653
100#4	0.5	0	0.5	1.037	0.509	1.364
100#5	0	0	1	0.013	0.199	1.177
100#6	0	0.250	0.750	0.499	0.298	0.568
100#7	0.333	0.0833	0.583	0.659	0.823	1.044
100#8	0	0	1	1.704	0.285	0.547
100#9	0	0	1	0.892	1.487	0.454
100#10	0	0	1	1.035	0.841	0.963
100#11	0	0	1	0.7051	1.000	0.580
100#12	0	0	1	1.069	0.062	0.711
100#13	0	0	1	0.633	0.968	0.392
100#14	0	0	1	1.240	0.357	1.028
100#15	0	0	1	0.911	0.861	1.355
100#16	0	0	1	0.628	1.076	1.114
100#17	0	0	1	1.105	0.901	0.765
100#18	0	0	1	0.977	1.106	1.036

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	NSGA-II	PAES	MOICA	NSGA-II	PAES	MOICA
100#3	0.231	0.649	0.656	0.832	1.349	0.243
100#4	1.039	0.512	1.080	0.449	0.749	0.440
100#5	1.042	0.422	1.164	0.798	0.852	0.099
100#6	0.869	1.290	0.834	0.727	0.761	0.397
100#7	0.442	1.042	1.016	0.310	0.364	0.230
100#8	1.174	0.255	0.367	0.673	0.606	0.147
100#9	0.187	1.100	0.562	0.394	0.872	0.027
100#10	1.131	0.855	0.955	0.653	0.590	0.275
100#11	0.289	1.146	0.680	0.467	0.830	0.275
100#12	1.047	0.232	1.191	0.590	0.207	0.340
100#13	0.585	0.958	0.657	0.571	0.880	0.175
100#14	1.058	0.724	0.895	0.624	0.523	0.218
100#15	1.211	0.559	1.121	0.692	0.734	0.482
100#16	0.871	0.760	1.156	0.963	0.794	0.392
100#17	0.660	1.064	0.891	0.758	0.916	0.179
100#18	0.447	1.187	0.943	0.815	0.765	0.280

7. Conclusions and Suggestions for Future Research

This study introduced a MOICA for addressing multi-objective Location-Routing Problems (LRPs) with homogeneous and capacitated vehicle fleets. The proposed model offers enhanced capabilities by incorporating fuzzy parameters to represent uncertainties in vehicle capacity, customer demand, depot opening costs, and transportation costs. Additionally, the model considers probabilistic travel, service, and waiting times while guaranteeing a minimum probability threshold for their combined value when minimizing this sum is an objective. The other objective minimizes the overall system cost.

To facilitate the solution, a fuzzy number ranking method based on expected interval comparisons was employed to transform the fuzzy model into an equivalent crisp model. Subsequently, the model was simplified by linearizing a non-linear constraint, reducing computational time and improving efficiency. Response surface methodology was then utilized to tailor crossover and mutation strategies for each algorithm employed in the study. A comprehensive comparison evaluated MOICA's performance against established evolutionary algorithms, NSGA-II and PAES. This evaluation, based on four key performance metrics across various benchmark instances, demonstrated that MOICA consistently outperforms the other algorithms, particularly on large-sized problems.

Future research directions encompass incorporating additional real-world complexities into the LRP model. These include facility availability constraints (time windows), heterogeneous vehicle fleets, and models with pickup and delivery demands. Furthermore, hybridizing MOICA with novel local search procedures presents a promising area for further investigation. The superior performance of MOICA, compared to established algorithms, highlights its potential as a valuable tool for solving complex multi-objective LRPs. Moreover, the consideration of shared logistics resources in the model as a way to use the maximum capacity of vehicles as well as reduce greenhouse gas emissions is an interesting topic for future research. Considering the collaborative contracts that ease the use of shared resources is another suggestion for future research. Additionally, the investigation of blockchain-based and IoT-based solutions for optimizing the LRP problem and tracking is another subject that can be addressed.

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